1. FUNDAMENTALS OF ENGINEERING MECHANICS

ENGINEERING MECHANICS: The subject of Engineering Mechanics is that branch of Applied Science, which deals with the **laws and principles of Mechanics**, along with their applications to engineering problems. The subject of Engineering Mechanics may be divided into the following two main groups:

1. Statics, and 2. Dynamics

STATICS: It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies at rest.

DYNAMICS: It is that branch of Engineering Mechanics, which deals with the forces and their effects, while acting upon the bodies in motion. The subject of Dynamics may be further sub-divided into the following two branches:

1. Kinetics, and 2. Kinematics

KINETICS: It is the branch of Dynamics, which deals with the bodies in motion due to the application of forces.

KINEMATICS: It is that branch of Dynamics, which deals with the bodies in motion, without any reference to the forces which are responsible for the motion.

RIGID BODY: A rigid body (also known as a rigid object) is a solid body in which **deformation is zero or so small it can be neglected.** The distance between any two given points on a rigid body remains constant in time regardless of external forces exerted on it. A rigid body is usually considered as a continuous distribution of mass.

Nobody is perfectly rigid, however rigid body is defined as a body in which particles do not change their relative positions under the action of any force or torque. Rigid body is ideal body. When the body does not deform under the action of A force or A torque, body is said rigid.

Particle: A particle is ideally dimensionless. But it has a very small mass.

Deformable body: When a body deforms due to A force or A torque it is said deformable body. Material generates stresses against deformation.

FORCE: It is defined as an agent which **produces or tends to produce, destroys or tends to destroy motion**. *e.g.*, a horse applies force to pull a cart and to set it in motion. Force is also required to work on a bicycle pump. In this case, the force is supplied by the muscular power of our arms and shoulders.

CHARACTERISTIC OF A FORCE: In order to determine the effects of a force, acting on a body, we must know the following characteristics of a force:

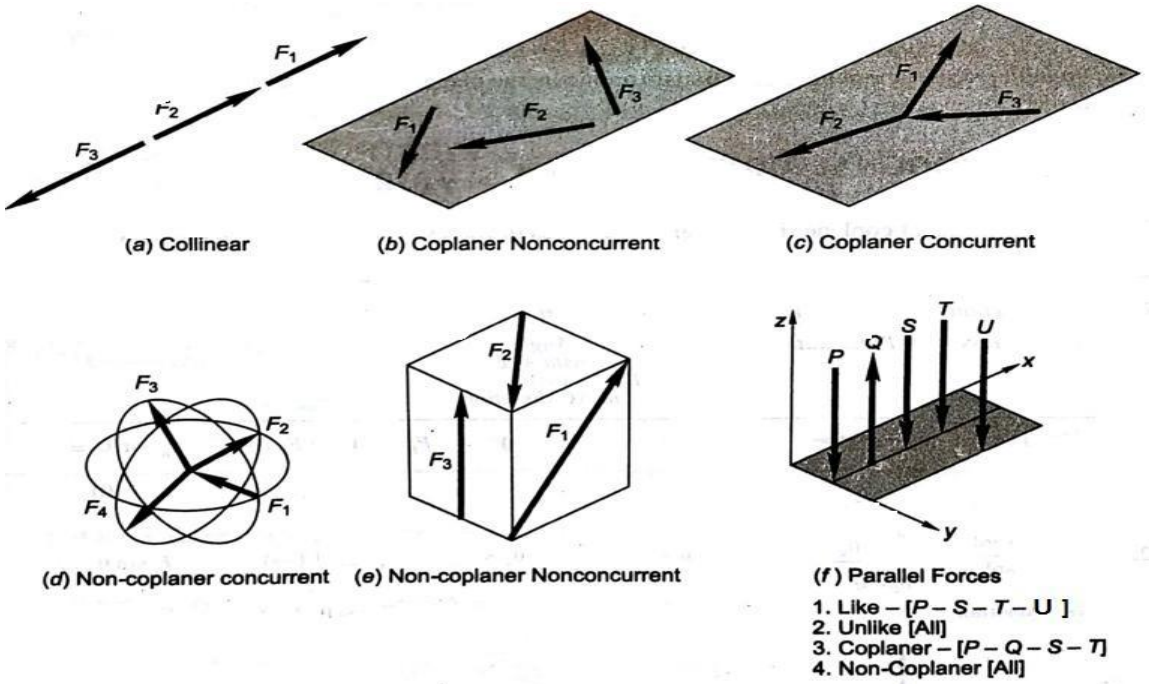
1. **Magnitude** of the force (*i.e.*, 100 N, 50 N, 20 kN, 5 kN, etc.)
2. The **direction of the line**, along which the force acts (*i.e.*, along *OX*, *OY*, at 30° North of East etc.). It is also known as line of action of the force.
3. **Nature of the force** (*i.e.*, whether the force is push or pull). This is denoted by placing an arrowhead on the line of action of the force.
    - Tensile force
    - Compressive force
    - Pull force
    - Push force
4. **The point at which** (or through which) the force acts on the body
5. It is a vector quantity

EFFECTS OF A FORCE: A force may produce the following effects in a body, on which it acts:

1. It may change the motion of a body. i.e. if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelerate it.
2. It may retard the motion of a body.
3. It may retard the forces, already acting on a body, thus bringing it to rest or inequilibrium.
4. It may give rise to the internal stresses in the body, on which it acts.

SYSTEM OF FORCES: When two or more forces act on a body, they are called to form a system of forces. Following systems of forces are important from the subject point of view;

1. **Coplanar forces**: The forces, whose lines of action lie on the same plane, are known as



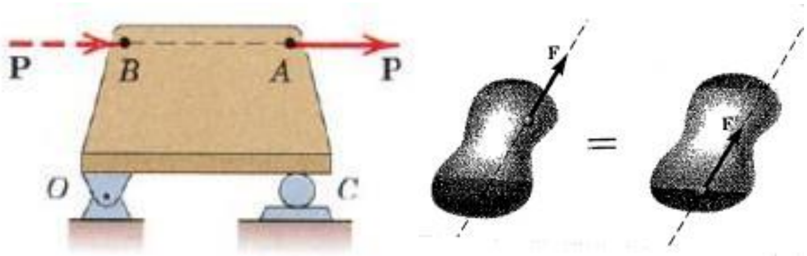
coplanar forces.

2. **Collinear forces**: The forces, whose lines of action lie on the same line, are known ascollinear forces
3. **Concurrent forces:** The forces, which meet at one point, are known as concurrent forces. The concurrent forces may or may not be collinear.
4. **Coplanar concurrent forces**: The forces, which meet at one point and their lines of action also lie on the same plane, are known as coplanar concurrent forces.
5. **Coplanar non-concurrent forces:** The forces, which do not meet at one point, but their lines of action lie on the same plane, are known as coplanar non-concurrent forces.
6. **Non-coplanar concurrent forces**: The forces, which meet at one point, but their lines of action do not lie on the same plane, are known as non-coplanar concurrent forces.
7. **Non-coplanar non-concurrent forces**: The forces, which do not meet at one point and their lines of action do not lie on the same plane, are called non-coplanar non-concurrent forces.

Principle of transmissibility:

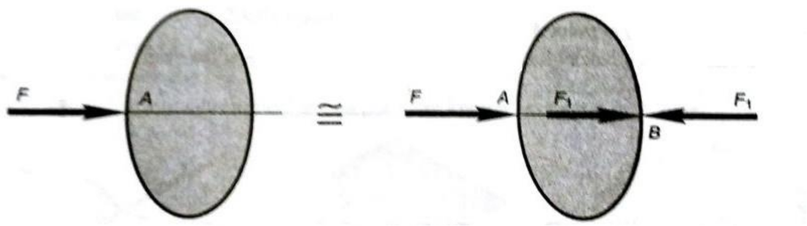
The principle of transmissibility states that a force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a sliding vector, and we need specify only the magnitude, direction, and line of action.

It states, "If a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."



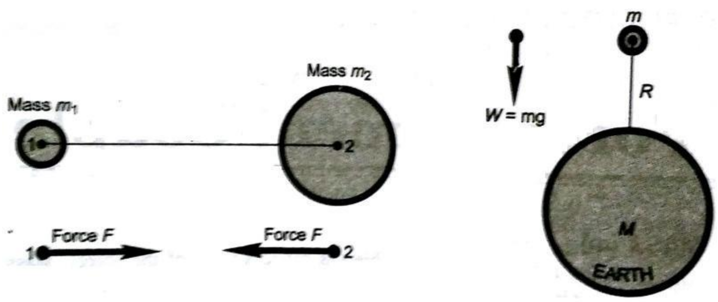
Principle of superposition:

If two equal, opposite and collinear forces are added to or removed from the system of forces, there will be nochange in the position of the body. This is known as principle of superposition of forces



Law of Gravitation:

Magnitude of gravitational force of attraction between two particles is proportional to the product of their masses and inversely proportional to the square of the distance between their centers



ACTION AND REACTION FORCE: Forces always act in pairs and always act in opposite directions. When you push on an object, the object pushes back with an equal force. Think of a pile of books on a table. The weight of the books exerts a downward force on the table. This is the action force. The table exerts an equal upward force on the books. This is the reaction force.

RESOLUTION OF A FORCE: The process of splitting up the given force into a number of components, without changing its effect on the body is called resolution of a force. A force is, generally, resolved along two mutually perpendicular directions. In fact, the resolution of a force is the reverse action of the addition of the component vectors.

COMPOSITION OF FORCES: The process of finding out the resultant force, of a number of given forces, is called composition of forces or compounding of forces.

RESULTANT FORCE: If a number of forces, P, Q, R ... etc. are acting simultaneously on a particle, then it is possible to find out a single force which could replace them i.e., which would produce the same effect as produced by all the given forces. This single force is called resultant force and the given forces R ...etc. are called component forces

PRINCIPLE OF RESOLUTION: It states, “The algebraic sum of the resolved parts of a no. of forces, in a given direction, is equal to the resolved part of their resultant in the same direction.”

Note: In general, the forces are resolved in the vertical and horizontal directions.

Resultant of Coplanar concurrent forces:

R**esultant Force:**If number of Forces acting simultaneously on a particle, it is possible to find out a single force which could re-place them or produce the same effect as of all the given forces is called resultant force.

Methods of Finding Resultant:

1) Parallelogram Law of Forces (For 2 Forces)
2) Triangle Law (For 2 Forces)
3) Lami‟s theorem (For 3 forces)
4) Method of resolution (For more than 2 Forces)

METHODS FOR THE RESULTANT FORCE:

Though there are many methods for finding out the resultant force of a number of givenforces, yet the following are important from the subject point of view:

1. Analytical method. 2. Method of resolution.

PARALLELOGRAM LAW OF FORCES:

It states, “If two forces, acting simultaneously on a particle, be represented in magnitude and direction by the two adjacent sides of a parallelogram ; their resultant may be represented in magnitude and direction by the diagonal of the parallelogram, which passes through their point of intersection.”

Mathematically, resultant force,

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

and

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

where F_1 and F_2 = Forces whose resultant is required to be found out,

θ = Angle between the forces F_1 and F_2, and

α = Angle which the resultant force makes with one of the forces (say F_1).

LAWS FOR THE RESULTANT FORCE:

The resultant force, of a given system of forces, may also be found out by the following laws

1. Triangle law of forces. **2.** Polygon law of forces.

TRIANGLE LAW OF FORCES:

It states, “If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order ; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order.”

POLYGON LAW OF FORCES:

It is an extension of Triangle Law of Forces for more than two forces, which states, “If a number of forces acting simultaneously on a particle, be represented in magnitude and direction, by the sides of a polygon taken in order ; then the resultant of all these forces may be represented, in magnitude and direction, by the closing side of the polygon, taken in opposite order.”

METHOD OF RESOLUTION:

- Resolve all the forces horizontally and find the algebraic sum of all the horizontal components .
- Resolve all the forces vertically and find the algebraic sum of all the verticalcomponents
- The resultant *R* of the given forces will be given by the equation;

$$R = \sqrt{(\sum H)^2 + (\sum V)^2}$$

- The resultant force will be inclined at an angle , with the horizontal, such that

$$\tan\theta = \frac{\sum V}{\sum H}$$

GRAPHICAL (VECTOR) METHOD FOR THE RESULTANT FORCE:

It is another name for finding out the magnitude and direction of the resultant force by the polygon law of forces. It is done as discussed below:

- **Construction of space diagram (position diagram)**: It means the construction of a diagram showing the various forces (or loads) along with their magnitude and linesof action.
  - **Use of Bow's notations:** All the forces in the space diagram are named by using the Bow's notations. It is a convenient method in which every force (or load) is named by two capital letters, placed on its either side in the space diagram.
- **Construction of vector diagram (force diagram):** It means the construction of a diagram starting from a convenient point and then go on adding all the forcesvectorially one by one (keeping in view the directions of the forces) to some suitable scale. Now the closing side of the polygon, taken in opposite order, will give the magnitude of the resultant force (to the scale) and its direction.

MOMENT OF A FORCE: It is the turning effect produced by a force, on the body, on which it acts. The moment of a force is equal to the product of the force and the perpendicular distance of the point, about which the moment is required and the line of action of the force.

Mathematically, moment,

M = P × l

where P = Force acting on the body, and

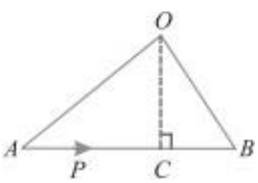
l = Perpendicular distance between the point, about which the moment is required and the line of action of the force.

GRAPHICAL REPRESENTATION OF A MOMENT: Consider a force *P* represented, in magnitude and direction, by the line *AB*. Let *O* be a point, about which the moment of this force is required to be found out, as shown in Fig. From *O*, draw *OC* perpendicular to *AB*. Join *OA* and *OB*.

Now moment of the force *P* about *O*

= P × OC = AB × OC

But *AB* × *OC* is equal to twice the area of triangle *ABO*. Thus the moment of a force, about any point, is equal to twice the area of the triangle, whose base is the line to some scale representing the force and whose vertex is the point about which the moment is taken.



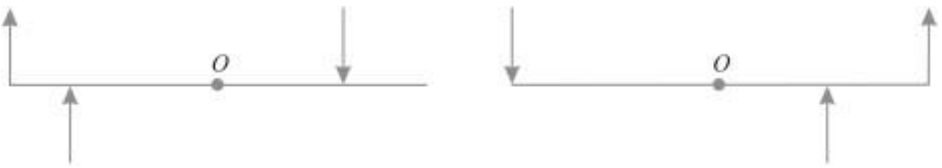
UNITS OF MOMENT:

Since the moment of a force is the product of force and distance, therefore the units of the moment will depend upon the units of force and distance. Thus, if the force is in Newton and the distance is in meters, then the units of moment will be Newton-meter (briefly written as N-m). Similarly, the units of moment may be kN-m (*i.e.* kN × m), N-mm (*i.e.* N × mm) etc.

TYPES OF MOMENTS:

Broadly speaking, the moments are of the following two types:
1. Clockwise moments. 2. Anticlockwise moments.

CLOCKWISE MOMENT:



(*a*) Clockwise moments (*b*) Anticlockwise moments

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the same direction in which hands of a clock move as shown in Fig.

ANTICLOCKWISE MOMENT:

It is the moment of a force, whose effect is to turn or rotate the body, about the point in the opposite direction in which the hands of a clock move as shown in Fig.(*b*). **Note.** The general convention is to take clockwise moment as positive and anticlockwise moment as negative.

VARIGNON'S PRINCIPLE OR LAW OF MOMENTS:

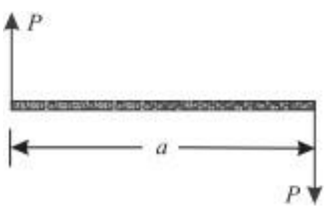
It states, "If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point."

COUPLE: A pair of two equal and unlike parallel forces (*i.e.*forces equal in magnitude, with lines of action parallel to each other and acting in opposite directions) is known as a couple.

As a matter of fact, a couple is unable to produce any translatory motion (*i.e.*, motion in a straight line). But it produces a motion of rotation in the body, on which it acts. The simplest example of a couple is the forces applied to the key of a lock, while locking or unlocking it.

ARM OF A COUPLE:

The perpendicular distance (*a*), between the lines of action of the two equal and opposite parallel forces, is known *as arm of the couple* as shown in Fig.



MOMENT OF A COUPLE: The moment of a couple is the product of the force (*i.e.*, one of the forces of the two equal and opposite parallel forces) and the arm of the couple.
Mathematically:
Moment of a couple = $P \times a$

where P = Magnitude of the force, and
a = Arm of the couple.

CLASSIFICATION OF COUPLES:

The couples may be, broadly, classified into the following two categories, depending upon their direction, in which the couple tends to rotate the body, on which it acts:

1. Clockwise couple, and 2. Anticlockwise couple

(*a*) Clockwise couple

(*b*) Anticlockwise couple

CLOCKWISE COUPLE:

A couple, whose tendency is to rotate the body, on which it acts, in a clockwise direction, is known as a clockwise couple as shown in Fig. (*a*). Such a couple is also called positive couple.

ANTICLOCKWISE COUPLE:

A couple, whose tendency is to rotate the body, on which it acts, in an anticlockwise direction, is known as an anticlockwise couple as shown in Fig. (*b*). Such a couple is also called a negative couple.

CHARACTERISTICS OF A COUPLE:

A couple (whether clockwise or anticlockwise) has the following characteristics:

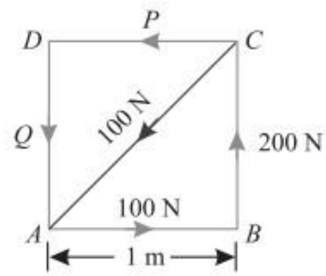
1. The algebraic sum of the forces, constituting the couple, is zero.
2. The algebraic sum of the moments of the forces, constituting the couple, about any point is the same, and equal to the moment of the couple itself.
3. A couple cannot be balanced by a single force. But it can be balanced only by a couple of opposite sense.
4. Any no. of co-planer couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

EXAMPLE: A square ABCD has forces acting along its sides as shown in Fig. 4.13. Find the values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.

Solution. Given : Length of square = 1 m

Values of P and Q

We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions must be zero. Resolving the forces horizontally,



$$100 - 100 \cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100 \cos 45° \text{ N}$$

$$= 100 - (100 \times 0.707) = 29.3 \text{ N} \quad \textbf{Ans.}$$

Now resolving the forces vertically,

$$200 - 100 \sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - (100 \times 0.707) = 129.3 \text{ N} \quad \textbf{Ans.}$$

Magnitude of the couple

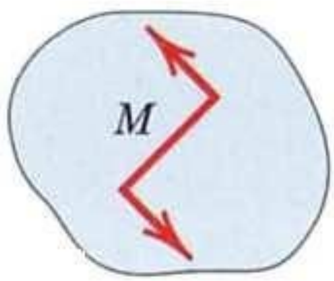
We know that moment of the couple is equal to the algebraic sum of the moments about any point. Therefore moment of the couple (taking moments about A)

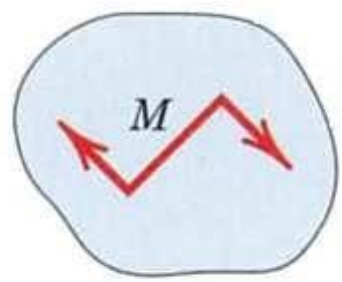
$$= (-200 \times 1) + (-P \times 1) = -200 - (29.3 \times 1) \text{ N-m}$$

$$= -229.3 \text{ N-m} \quad \textbf{Ans.}$$

Since the value of moment is negative, therefore the couple is anticlockwise.

The sense of the moment M is established by the right-hand rule.



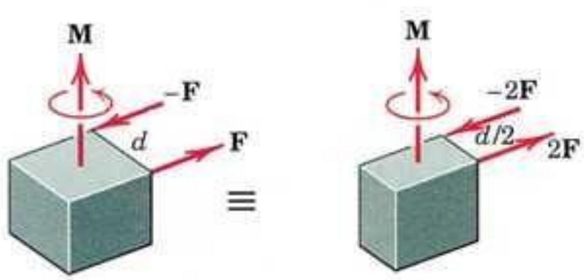


Counter clockwise couple (-) Clock wise couple(+)

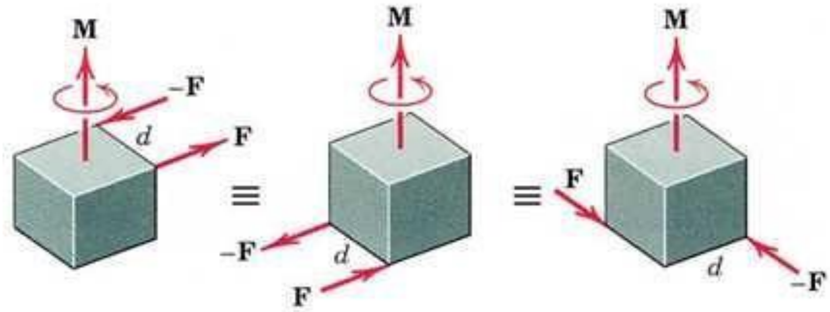
The magnitude of the couple is independent of the distance.

Equivalent Couples

Changing the values of F and does not change a given couple as long as the product Fd remains the same.



A couple is not affected if the forces act in a different but parallel plane.



Force-CoupleSystems

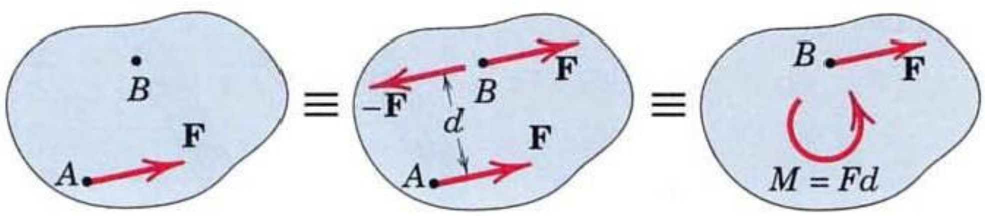
The effect of a force acting on a body is:

a) The tendency to push or pull the body in the direction of the force, and

b) To rotate the body about any fixed axis which does not intersect

The line of action of the force (force does not go through the mass center of the body).

We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.



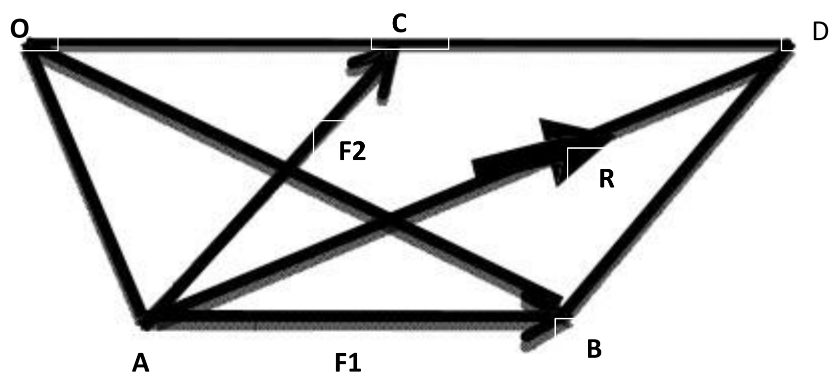
Also we can combine a given couple and a force which lies in the plane of the couple to produce a single, equivalent force.

Varignon's principle of moments:

If a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point.

Proof:

For example, consider only two forces F1 and F2 represented in magnitude and direction by AB and AC as shown in figure below.



Let be the point, about which the moments are taken. Construct the parallelogram ABCD and complete the construction as shown in fig.

By the parallelogram law of forces, the diagonal AD represents, in magnitude and direction, the resultant of two forces F1 and F2, let R be the resultant force.

By geometrical representation of moments

- The moment of force about O= 2 **x** Area of triangle AOB

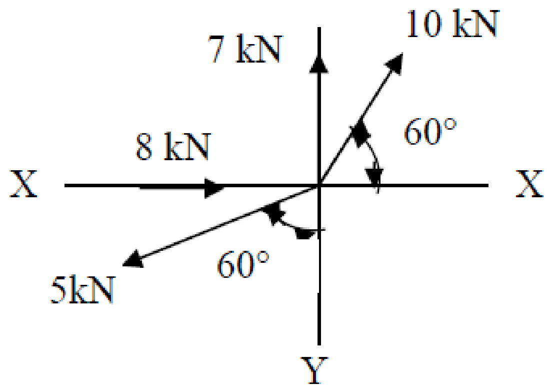
- The moment of force about O= 2 **x** Area of triangle AOC
- The moment of force about O = 2 **x** Area of triangle AOD But,
- Area of triangle AOD = Area of triangle AOC + Area of triangle ACD
- Area of triangle ACD = Area of triangle ADB = Area of triangle AOB
- Area of triangle AOD = Area of triangle AOC + Area of triangle AOB

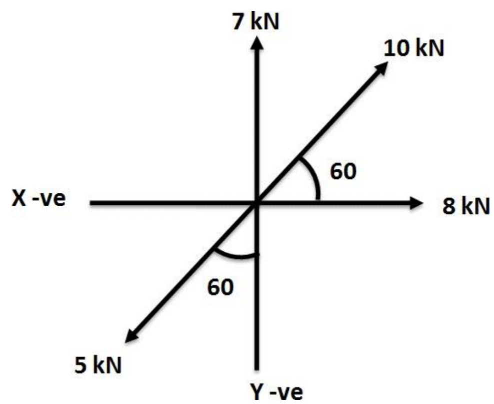
Multiplying throughout by 2, we obtain

2 **x** Area of triangle AOD= 2 **x** Area of triangle AOC + 2 **x** Area of triangle AOB

i.e. Moment of force R about O = Moment of force F1about O + Moment of force F2 about O

Example 1: - Find resultant of a force system shown in Figure





Answer:

1) **Given Data**

P1 = 8 kN
P2 = 10 kN
P3 = 7 kN
P4 = 5 kN

$\theta_1 = 0$

$\theta_2 = 60$

$\theta_3 = 90$

$\theta_4 = 270 - 60 = 210$

2) **Summation of horizontal force**

$$\sum H = P_1 \cos\theta_1 + P_2 \cos\theta_2 + P_3 \cos\theta_3 + P_4 \cos\theta_4 = 8.67kN(\rightarrow)$$

3) **Summation of vertical force**

$$\sum V = P_1 \sin\theta_1 + P_2 \sin\theta_2 + P_3 \sin\theta_3 + P_4 \sin\theta_4 = 13.16kN(\uparrow)$$

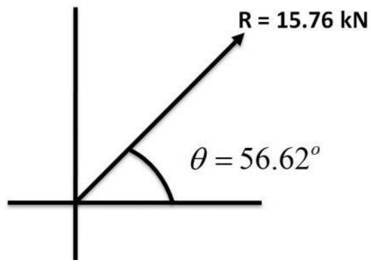
4) **Resultant force**

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = 15.76kN$$

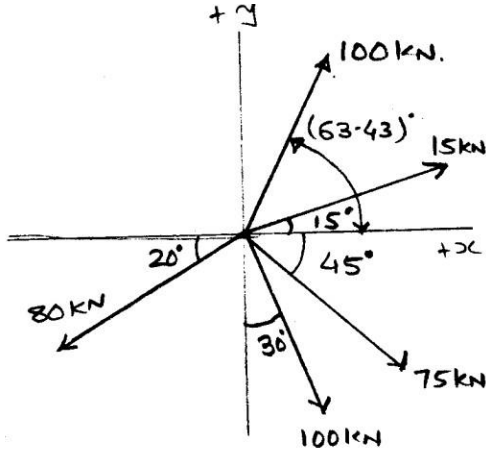
5) **Angle of resultant**

$$\tan\theta = \left|\frac{\sum V}{\sum H}\right| = 1.518$$

$$\theta = 56.62$$



Example 2 Find magnitude and direction of resultant for a concurrent force system shown in Figure



Answer

1) **Summation of horizontal force**

$\rightarrow$ **(+Ve)** $\leftarrow$ **(-Ve)**

$$\sum H = +15 \text{ Cos } 15^o + 100\text{Cos } (63.43)^o - 80\text{Cos}20^o + 100\text{Sin}30^o + 75\text{Cos}45^o = +87.08 \text{ kN} (\rightarrow)$$

2) **Summation of vertical force**

$\uparrow$ **(+Ve)** $\downarrow$ **(-Ve)**

$$\sum V = +15 \text{ Sin } 15^o + 100\text{Sin } (63.43)^o - 80\text{Sin}20^o + 100\text{Cos}30^o + 75\text{Sin}45^o = -73.68 \text{ kN} (\downarrow)$$

3) **Resultant force**

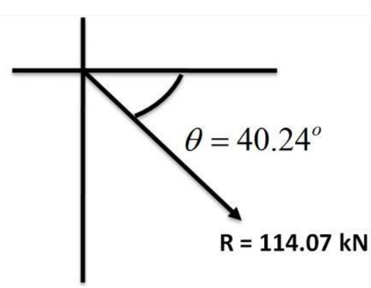
$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = 114.07kN$$

4) **Angle of resultant**

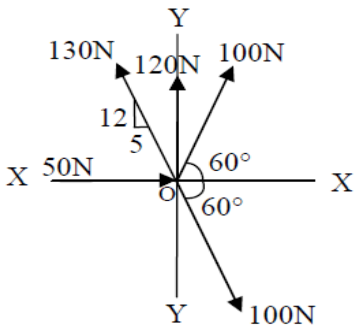
$$\tan\theta = \left|\frac{\sum V}{\sum H}\right| = 0.846$$

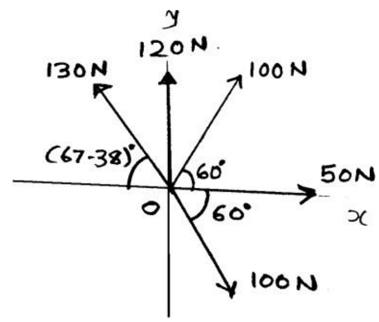
$$\theta = 40.24$$

5) **Angle of resultant with respect to positive x – axis**



Example 3 Determine magnitude and direction of resultant force of the force system shown in fig.





Answer

$$\tan\beta = \frac{12}{5} = 2.4 \qquad \therefore \beta = 67.38^0$$

1) **Summation of horizontal force**

$\rightarrow$ **(+Ve)** $\qquad\qquad\qquad$ $\leftarrow$ **(-Ve)**

$$\sum H = +50 + 100\text{ Cos }60^{\circ} - 130\text{Cos }(67.38)^{\circ} + 100\text{Cos}30^{\circ} + 100\text{Cos}60^{\circ} = +100\text{ N } (\rightarrow)$$

2) **Summation of vertical force**

$\uparrow$ **(+Ve)** $\qquad\qquad\qquad$ $\downarrow$ **(-Ve)**

$$\sum V = +100\text{Sin}60^{\circ} + 120 + 130\text{Sin }(67.38)^{\circ} - 100\text{Sin}60^{\circ} - 100\text{Sin}60^{\circ} = +240\text{ N } (\uparrow)$$

3) **Resultant force**

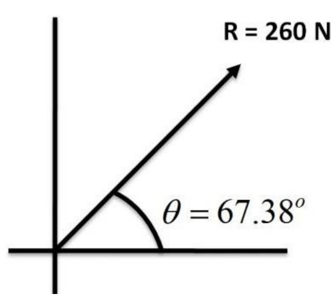
$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = 260N$$

4) **Angle of resultant**

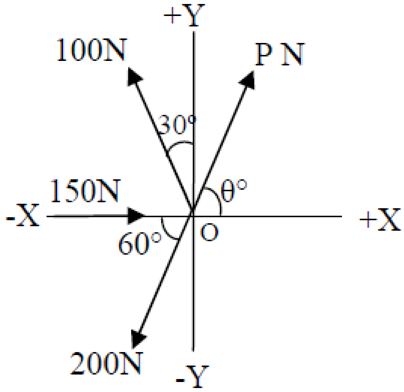
$$\tan\theta = \left|\frac{\sum V}{\sum H}\right| = 2.4$$

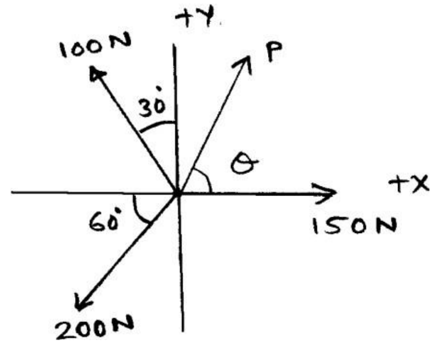
$$\theta = 67.38^{o}$$

5) **Angle of resultant with respect to positive x – axis**



Example: 4 A system of four forces shown in Fig. has resultant 50 kN along + X - axis. Determine magnitude and inclination of unknown force P.





Answer

As the R= 50N & directed along + X – axis.

$$\sum H = +50N \text{ and } \sum V = 0N$$

Now, $\sum H = +150 + \text{P Cos}\,\theta - 100\text{ Sin }30° - 200\text{ Cos }60° = 50\text{ N}$

$$\therefore \text{P Cos}\theta = 50 \_\_\_\_\_\_\_\_\_\_ (1)$$

Now, $\sum \text{V} = +\text{P Sin}\,\theta - 100\text{ Cos }30° - 200\text{ Sin }60° = 0$

$$\therefore \text{P Sin}\theta = 86.60 \_\_\_\_\_\_\_\_\_\_ (2)$$

From Equation (1) & (2).

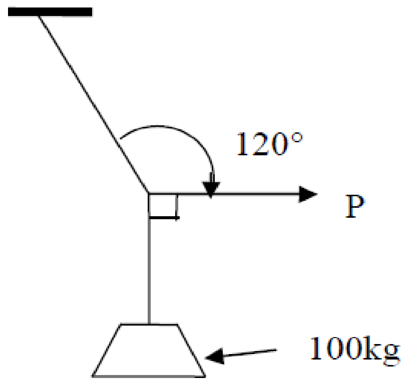
$$\tan\theta = \frac{86.60}{50}$$

$$\tan\theta = 1.732$$

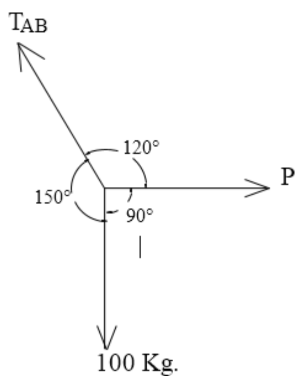
$$\therefore \theta = 60°$$

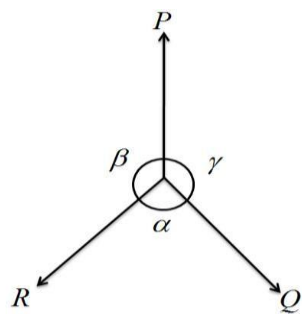
$$\therefore \text{P} = 100\text{ N}$$

Example: 5 Find the magnitude of the force P, required to keep the 100 kg mass in the position by strings as shown in the Figure



Answer:





Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami‟s theorem.

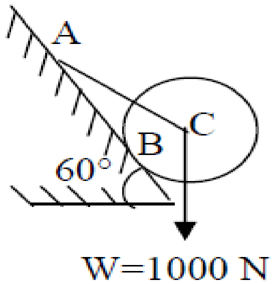
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\therefore \frac{P}{\text{Sin } 150} = \frac{TAB}{\text{Sin } 90} = \frac{100}{\text{Sin } 120}$$

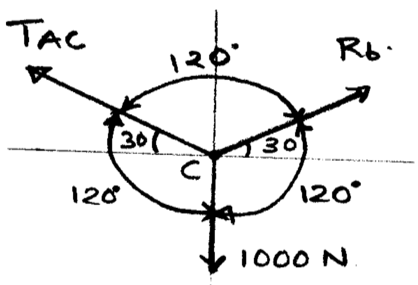
$$P = 566.38 \text{ N}$$

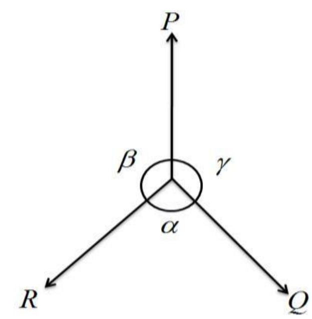
$$T_{AB} = 1132.76 \text{ N}$$

Example: 6 A cylindrical roller 600mm diameter and weighing 1000 N is resting on a smooth inclined surface, tied firmly by a rope AC of length 600mm as shown in fig. Find tension in rope and reaction at B



Answer:





Free Body Diagram will be as show in fig. and there are three coplanar concurrent forces which are in equilibrium so we can apply the lami"s theorem.

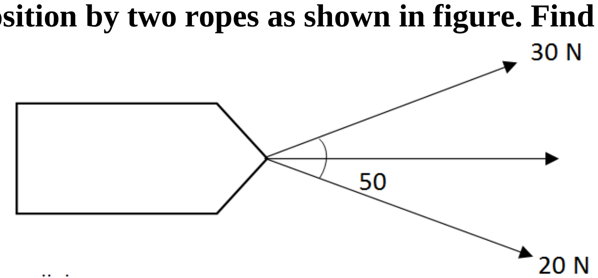
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

$$\therefore \frac{\text{Tac}}{\text{Sin } 120} = \frac{\text{Rb}}{\text{Sin } 120} = \frac{1000}{\text{Sin } 120}$$

$$\text{Tac} = 1000 \text{ N}$$

$$R_B = 1000 \text{ N}$$

Example: 7 A boat kept in position by two ropes as shown in figure. Find the drag force on the boat.



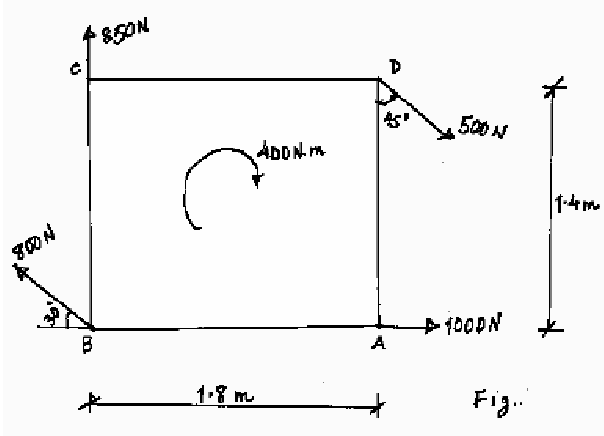
Answer:

According to law of parallelogram

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{20^2 + 30^2 + 2\times 20\times 30\cos 50} = 45.51N$$

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{30\sin 50}{20 + 30\cos 50} \qquad \therefore \alpha = 30.32^0$$

Example: 8 For a coplanar, non-concurrent force system shown in Fig. determine magnitude, direction and position with reference to point A of resultant force.



Answer

To find out magnitude & direction of R

Summation of horizontal force

$$\Sigma H = +500 \text{ Sin } 45º - 800\text{Cos } 30º + 1000 = +660.73 \text{ N } (\rightarrow)$$

Summation of vertical force

$$\sum V = -500 \text{ Cos}45º + 850 + 800\text{Sin}30º = +896.45 \text{ N} \quad (\uparrow)$$

Resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = \sqrt{(660.73)^2 + (896.45)^2} = 1113.64\ N$$

Angle of resultant

$$\tan\theta = \frac{896.45}{660.73}$$

$$\therefore\ \theta = 53.61°$$

Here, we have to also locate the „R" @ pt. A Let the „R" is located at a dist^n x from A in the horizontal direction.

Now this dist^n „X" can be achived by using varignon"s principle.

First, Take the moment @ A of all the forces.

$$\mathbf{M}_{ALL} = + (500 \text{ Sin } 45° \text{ X } 1.4) + (850 \text{ X } 1.8) + (800 \text{ Sin } 30° \text{ X } 1.8) + 400$$

$$= + 3144.97 \text{ N-m } [\,↴\,] \_\_\_\_\_ (1)$$

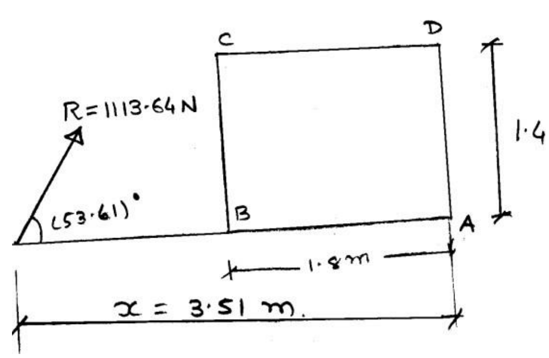
Now moment of „R" @ point „ A „

$\mathbf{M}_R = + (\text{R Sin } \theta . X) = + (\sum Fy.\ ) = 896.45.x$ ______ (2)

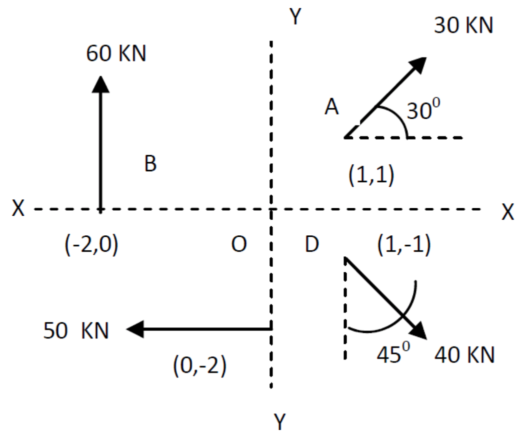
(1) = (2)

896.45 X = 3144.97

X = 3.51 m



Example: 9 Find magnitude, direction and location of resultant of force system with respect to point 'O' shown in fig.



Answer

Summation of horizontal forces

$$\Sigma H = +30\, Cos\, 30^o - 50 + 40 Sin\, 45^o = +4.265\, KN \quad (\rightarrow)$$

Summation of vertical forces

$$\Sigma V = +30\, Sin 30^o + 60 - 40\, Cos\, 45^o = +46.72\, KN \quad (\uparrow)$$

Resultant force

$$R = \sqrt{\left(\sum H\right)^2 + \left(\sum V\right)^2} = \sqrt{(4.265)^2 + (46.72)^2} = 46.91\, KN$$

Angle of resultant

$$\tan\theta = \frac{46.72}{4.265}$$

$$\therefore\ \theta = 84.78$$

Now,as we requred to find out the position of „R" with respect to the point „O". Take the moment of all the forces @ point „O „ we have,

$$M_0 = +(30\ Cos30° \ X\ 1) - (30\ Sin30° \ X\ 1) + (60\ X\ 2) + (50\ X\ 2) - (40\ Cos45° \ X\ 1) + (40\ Sin45° \ X\ 1)$$

$$\mathbf{M_0 = +230.98\ KN\text{- unit} \quad (\curvearrowright)\ \_\_\_\_\_\_\_\_\ (1)}$$

Now, moment of „R" @ Pt. „O"

(considering „R" lies at a distance x from the point „O" in the horizontal direction)

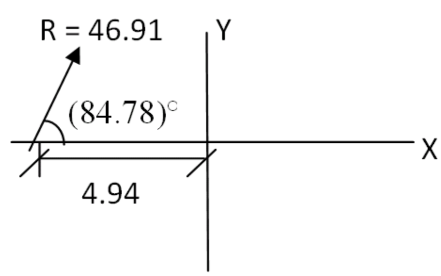
$$M_R = +(R\ Sin\theta\ X) = (\sum Fy.x)$$

$$\mathbf{M_R = +46.72.X\ \_\_\_\_\_\_\_\_\_\_\ (2)}$$

According to varignon's principle

$$\therefore 46.72\ X = 230.98$$

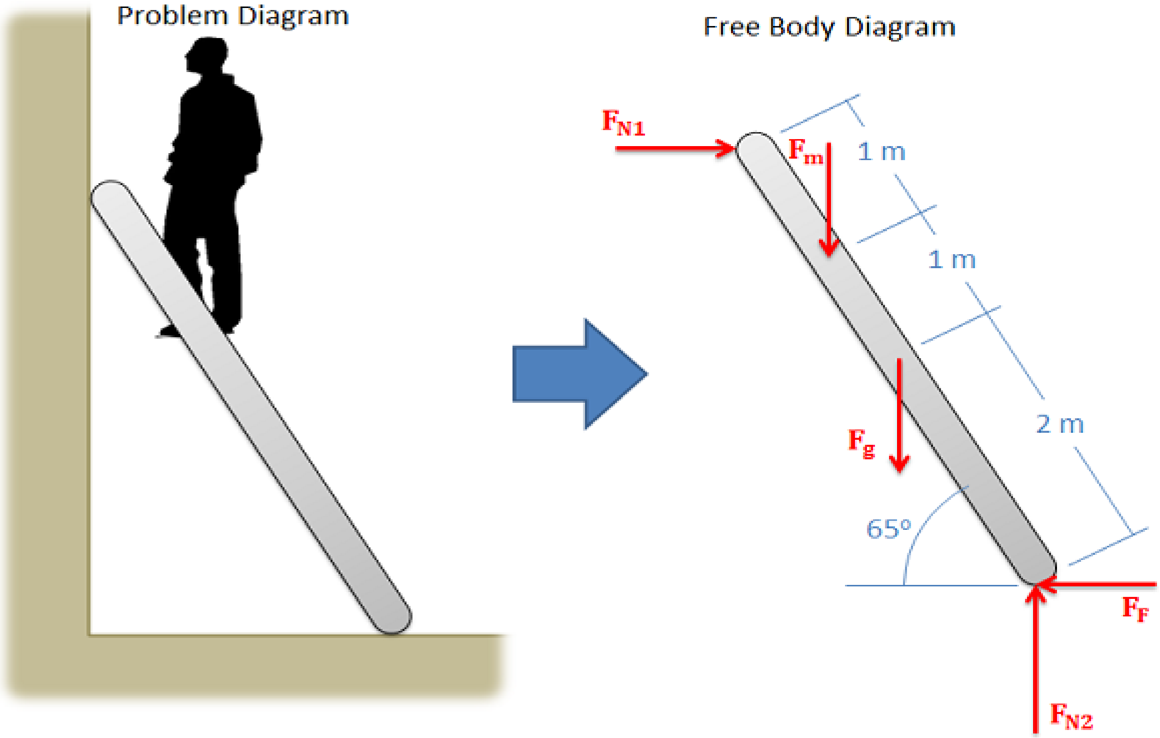
$$\therefore X = 4.94 \text{ unit}$$



Free Body Diagrams

A free body diagram is a tool used to solve engineering mechanics problems. As the name suggests, the purpose of the diagram is to "free" the body from all other objects and surfaces around it so that it can be studied in isolation. We will also draw in any forces or moments acting on the body, including those forces and moments exerted by the surrounding bodies and surfaces that we removed.

The diagram below shows a ladder supporting a person and the free body diagram of that ladder. As you can see, the ladder is separated from all other objects and all forces acting on the ladder are drawn in with key dimensions and angles shown.



A ladder with a man standing on it is shown on the left. Assuming friction only at the base, a free body diagram of the ladder is shown on the right.

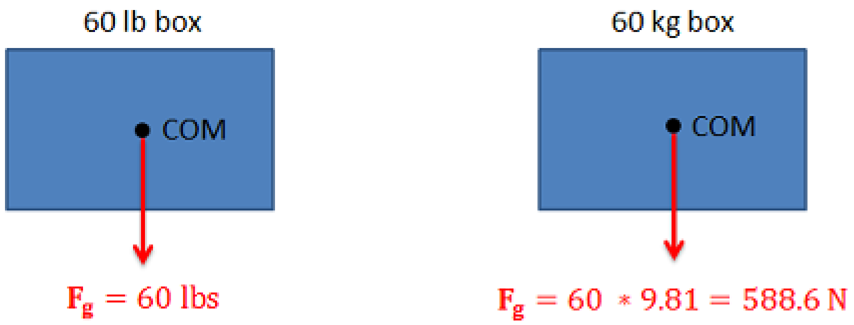
Constructing the Free Body Diagram

The first step in solving most mechanics problems will be to construct a free-body diagram. This simplified diagram will allow us to more easily write out the equilibrium equations for statics or strengths of materials problems. To construct the diagram we will use the following process.

1. First draw the body being analyzed, separated from all other surrounding bodies and surfaces. Pay close attention to the boundary, identifying what is part of the body, and what is part of the surroundings.

2. Second, draw in all **external** forces and moments acting directly on the body. Do not include any forces or moments that do not directly act on the body being analyzed. Do not include any forces that are **internal** to the body being analyzed. Some common types of forces seen in mechanics problems are:

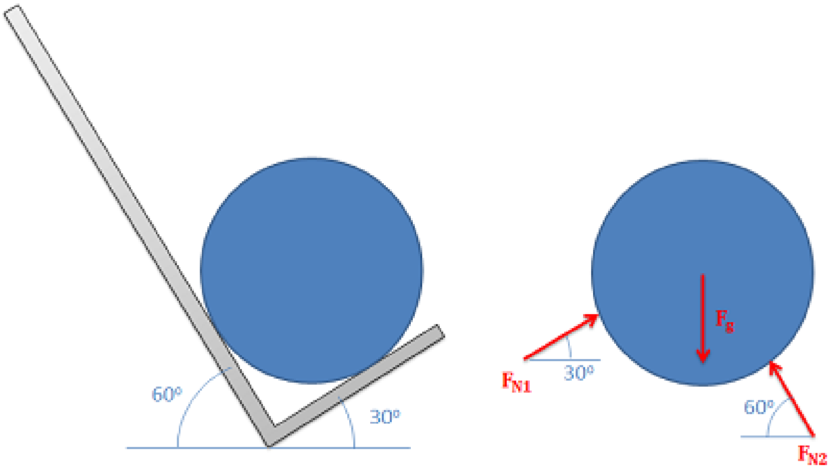
- **Gravitational Forces:** Unless otherwise noted, the mass of an object will result in a gravitational weight force applied to that body. This force will always point down towards the center of the earth and act on the center of mass of the body.



Gravitational forces always act downward on the center of mass.

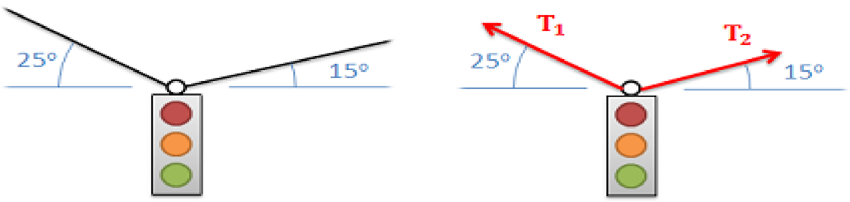
Normal Forces (or Reaction Forces): Every object in direct contact with the body will exert a normal force on that body. Note that only objects in direct contact can exert normal forces on the body.

An object in contact with another object or surface will experience a normal force that is perpendicular (hence normal) to the surfaces in contact.



Normal forces always act perpendicular to the surfaces in contact. The barrel in the hand truck shown on the left has a normal force at each contact point.

Tension in Cables: Cables, wires or ropes attached to the body will exert a tension force on the body in the direction of the cable. These forces will always pull on the body, as ropes, cables and other flexible tethers cannot be used for pushing.

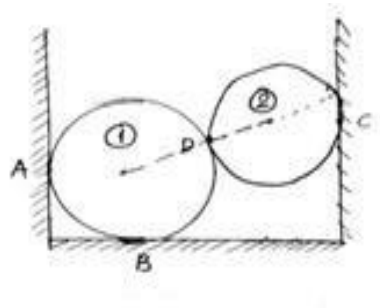


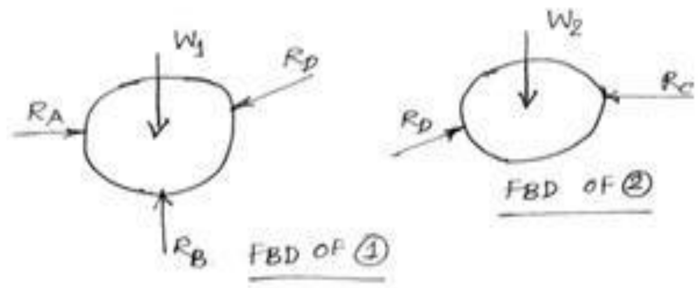
The tension force in cables always acts along the direction of the cable and will always be a pulling force.

The above forces are the most common, but other forces such as pressure from fluids, spring forces and magnetic forces may exist and may act on the body.

Once the forces are identified and added to the free body diagram, the last step is to label any key dimensions and angles on the diagram.

An example for Free Body Diagram is shownbelow.





EQUILIBRIUM:

If the resultant of a number of forces, acting on a particle is zero, the particle will be in equilibrium. Such a set of forces, whose resultant is zero, are called equilibrium forces. The force, which brings the set of forces in equilibrium, is called an equilibrant.

PRINCIPLES OF EQUILIBRIUM:

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. Two force principle:

As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2. **Three force principle**:

As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with thethird force.

3. Four force principle:

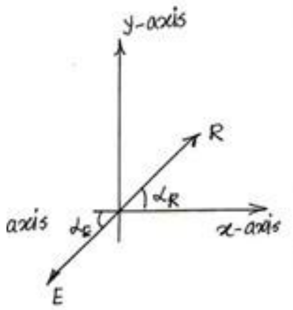
As per this principle, if a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with theresultant of the other two forces.

CONDITIONS OF EQUILIBRIUM: Equilibrium is the status of the body when it is subjected to a system of forces. If the body is completely at rest, it necessarily means that there is neither a resultant force nor a couple acting on it. A little consideration will show that in this case the following conditions are already satisfied:

$$\sum \mathbf{H} = \mathbf{0} \ \sum \mathbf{V} = \mathbf{0} \text{ and } \sum \mathbf{M} = \mathbf{0}$$

The above mentioned three equations are known as the conditions of equilibrium. For a system of coplanar concurrent forces for the resultant to be zero hence $\sum H = 0 \ \sum V = 0$

Equilibrant: Equilibrant is a single force which when added to a system of forces brings the status of equilibrium. Hence this FORCE is of the same magnitude as the resultant but opposite in sense. This is depicted in figure.

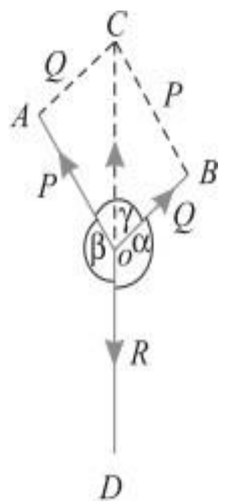


METHODS FOR THE EQUILIBRIUM OF COPLANAR FORCES:

Though there are many methods of studying the equilibrium of forces, yet the following are important from the subject point of view :
1. Analytical method. 2. Graphical method.

LAMI'S THEOREM:
It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."
Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



Proof

Consider three coplanar forces P, Q, and R acting at a point O. Let the opposite angles to three forces be α, β and γ as shown in Fig. 5.2.

Now let us complete the parallelogram $OACB$ with OA and OB as adjacent sides as shown in the figure. We know that the resultant of two forces P and Q will be given by the diagonal OC both in magnitude and direction of the parallelogram $OACB$.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.

From the geometry of the figure, we find

$$BC = P \text{ and } AC = Q$$

$$\therefore \quad \angle AOC = (180° - \beta)$$

and $$\angle ACO = \angle BOC = (180° - \alpha)$$

$$\therefore \quad \angle CAO = 180° - (\angle AOC + \angle ACO)$$
$$= 180° - [(180° - \beta) + (180° - \alpha)]$$
$$= 180° - 180° + \beta - 180° + \alpha$$
$$= \alpha + \beta - 180°$$

But $\alpha + \beta + \gamma = 360°$

Subtracting 180° from both sides of the above equation,

$$(\alpha + \beta - 180°) + \gamma = 360° - 180° = 180°$$

or $\angle CAO = 180° - \gamma$

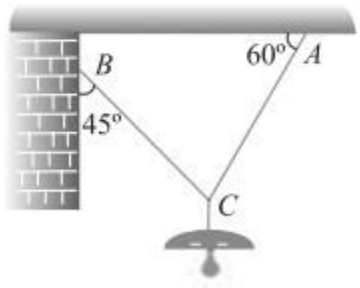
We know that in triangle AOC,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\frac{OA}{\sin (180° - \alpha)} = \frac{AC}{\sin (180° - \beta)} = \frac{OC}{\sin (180° - \gamma)}$$

or $$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

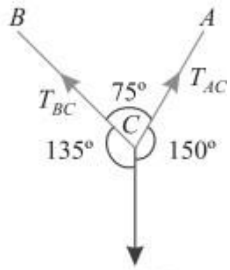
EXAMPLE: An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig. Using Lami's theorem, or otherwise, determine the force in the strings AC.



Solution. Given : Weight at $C = 15$ N

Let T_{AC} = Force in the string AC, and

T_{BC} = Force in the string BC.



The system of forces is shown in Fig. 5.4. From the geometry of the figure, we find that angle between T_{AC} and 15 N is 150° and angle between T_{BC} and 15 N is 135°.

$$\therefore \quad \angle ACB = 180° - (45° + 60°) = 75°$$

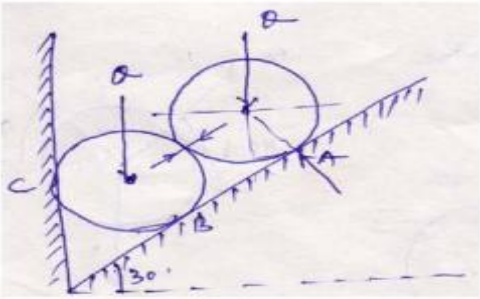
Applying Lami's equation at C,

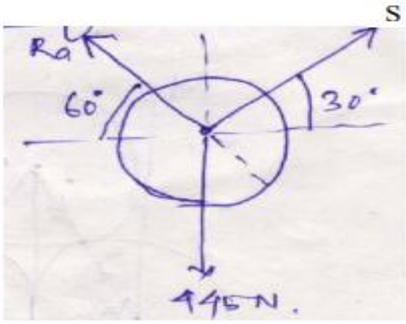
$$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 135°} = \frac{T_{BC}}{\sin 150°}$$

or $$\frac{15}{\sin 75°} = \frac{T_{AC}}{\sin 45°} = \frac{T_{BC}}{\sin 30°}$$

$$\therefore \quad T_{AC} = \frac{15 \sin 45°}{\sin 75°} = \frac{15 \times 0.707}{0.9659} = 10.98 \text{ N} \quad \textbf{Ans.}$$

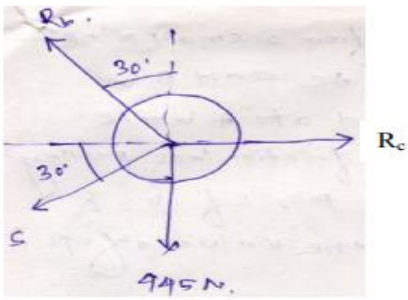
Problem: Two identical rollers each of weight Q = 445 N are supported by an inclined plane and a vertical wall as shown in the figure. Assuming smooth surfaces, find the reactions induced at the points of support A, B and C.





$$\frac{R_a}{\sin 120} = \frac{S}{\sin 150} = \frac{445}{\sin 90}$$

$$\Rightarrow R_a = 385.38N$$
$$\Rightarrow S = 222.5N$$



Resolving vertically

$$\sum Y = 0$$
$$R_b \cos 60 = 445 + S \sin 30$$
$$\Rightarrow R_b \frac{\sqrt{3}}{2} = 445 + \frac{222.5}{2}$$
$$\Rightarrow R_b = 642.302N$$

Resolving horizontally

$$\sum X = 0$$
$$R_c = R_b \sin 30 + S \cos 30$$
$$\Rightarrow 642.302 \sin 30 + 222.5 \cos 30$$
$$\Rightarrow R_c = 513.84N$$

NUMERICALS: EXAMPLE: Find the magnitude of the two forces, such that if they act at right angles, their resultant is 10 N . But if they Act at 60°, their resultant is 13 N.

Solution. Given : Two forces = F_1 and F_2.

First of all, consider the two forces acting at right angles. We know that when the angle between the two given forces is 90°, then the resultant force (R)

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2}$$

or $$10 = F_1^2 + F_2^2 \qquad \text{...(Squaring both sides)}$$

Similarly, when the angle between the two forces is 60°, then the resultant force (R)

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60°}$$

$$\therefore \qquad 13 = F_1^2 + F_2^2 + 2F_1 F_2 \times 0.5 \qquad \text{...(Squaring both sides)}$$

or $$F_1 F_2 = 13 - 10 = 3 \qquad \text{...(Substituting } F_1^2 + F_2^2 = 10\text{)}$$

We know that $(F_1 + F_{2]})^2 = F_1^2 + F_2^2 + 2F_1F_2 = 10 + 6 = 16$

$$\therefore \qquad F_1 + F_2 = \sqrt{16} = 4 \qquad \text{...}(i)$$

Similarly $$(F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1F_2 = 10 - 6 = 4$$

$$\therefore \qquad F_1 - F_2 = \sqrt{4} = 2 \qquad \text{...}(ii)$$

Solving equations (i) and (ii),

$$F_1 = 3 \text{ N} \quad \text{and} \quad F_2 = 1 \text{ N} \quad \textbf{Ans.}$$

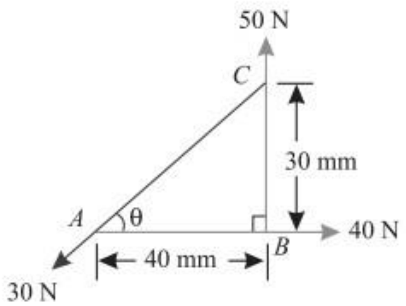
EXAMPLE: A triangle ABC has its side AB = 40 mm along positive x-axis and side BC = 30 mm along positive y-axis. Three forces of 40 N, 50 N and 30 N act along the sides AB, BC and CA respectively. Determine magnitude of the resultant of such a system of forces.

Solution. The system of given forces is shown in Fig. 2.3.

From the geometry of the figure, we find that the triangle ABC is a right angled triangle, in which the *side $AC = 50$ mm. Therefore

$$\sin\theta = \frac{30}{50} = 0.6$$

and $$\cos\theta = \frac{40}{50} = 0.8$$



Resolving all the forces horizontally ($i.e.$, along AB),

$$\Sigma H = 40 - 30 \cos\theta$$
$$= 40 - (30 \times 0.8) = 16 \text{ N}$$

and now resolving all the forces vertically ($i.e.$, along BC)

$$\Sigma V = 50 - 30 \sin\theta$$
$$= 50 - (30 \times 0.6) = 32 \text{ N}$$

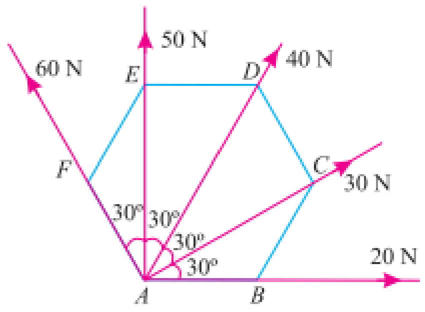
We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(16)^2 + (32)^2} = 35.8 \text{ N} \quad \textbf{Ans.}$$

ROBLEMS ON COPLLANAR CONCURRENT AND NON CONCURRENT FORCE SYSTEMS

Problem No 1: The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.

The system of given forces is shown in Fig.



Magnitude of the resultant force

Resolving all the forces horizontally (*i.e.*, along *AB*),

$$\Sigma H = 20 \cos 0° + 30 \cos 30° + 40 \cos 60° + 50 \cos 90° + 60 \cos 120° \text{ N}$$

$$= (20 \times 1) + (30 \times 0.866) + (40 \times 0.5) + (50 \times 0) + 60\,(-0.5) \text{ N}$$

$$= 36.0 \text{ N} \qquad ...(i)$$

and now resolving the all forces vertically (*i.e.*, at right angles to *AB*),

$$\Sigma V = 20 \sin 0° + 30 \sin 30° + 40 \sin 60° + 50 \sin 90° + 60 \sin 120° \text{ N}$$

$$= (20 \times 0) + (30 \times 0.5) + (40 \times 0.866) + (50 \times 1) + (60 \times 0.866) \text{ N}$$

$$= 151.6 \text{ N} \qquad ...(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(36.0)^2 + (151.6)^2} = 155.8 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

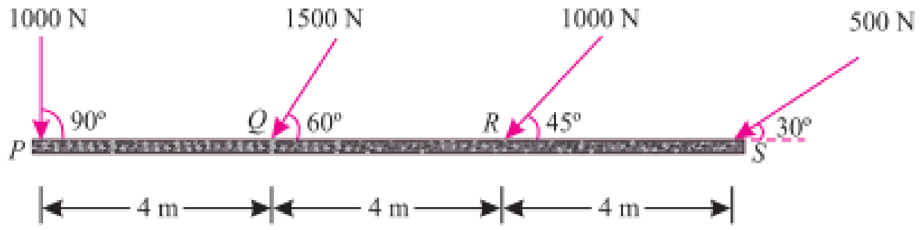
Let θ = Angle, which the resultant force makes with the horizontal (*i.e.*, *AB*).

We know that

$$\tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{151.6}{36.0} = 4.211 \quad \text{or} \quad \theta = 76.6° \quad \textbf{Ans.}$$

Note. Since both the values of ΣH and ΣV are positive, therefore actual angle of resultant force lies between 0° and 90°.

Problem No 2: A horizontal line PQRS is 12 m long, where PQ = QR = RS = 4 m. Forces of 1000 N, 1500 N, 1000 N and 500 N act at P, Q, R and S respectively with downward direction. The lines of action of these forces make angles of 90°, 60°, 45° and 30° respectively with PS. Find the magnitude, direction and position of the resultant force



Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = 1000 \cos 90° + 1500 \cos 60° + 1000 \cos 45° + 500 \cos 30° \text{ N}$$
$$= (1000 \times 0) + (1500 \times 0.5) + (1000 \times 0.707) + (500 \times 0.866) \text{ N}$$
$$= 1890 \text{ N} \quad \ldots(i)$$

and now resolving all the forces vertically,

$$\Sigma V = 1000 \sin 90° + 1500 \sin 60° + 1000 \sin 45° + 500 \sin 30° \text{ N}$$
$$= (1000 \times 1.0) + (1500 \times 0.866) + (1000 \times 0.707) + (500 \times 0.5) \text{ N}$$
$$= 3256 \text{ N} \quad \ldots(ii)$$

We know that magnitude of the resultant force,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(1890)^2 + (3256)^2} = 3765 \text{ N} \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant force makes with PS.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{3256}{1890} = 1.722 \quad \text{or} \quad \theta = 59.8° \quad \textbf{Ans.}$$

Note. Since both the values of ΣH and ΣV are +ve. therefore resultant lies between 0° and 90°.

Position of the resultant force

Position of the resultant force

Let x = Distance between P and the line of action of the resultant force.

Now taking moments* of the vertical components of the forces and the resultant force about P, and equating the same,

$$3256\,x = (1000 \times 0) + (1500 \times 0.866)\,4 + (1000 \times 0.707)8 + (500 \times 0.5)12$$
$$= 13\,852$$

$$\therefore \quad x = \frac{13\,852}{3256} = 4.25 \text{ m} \quad \textbf{Ans.}$$

Problem No 3: ABCD is a rectangle, such that AB = CD = a and BC = DA = b. Forces equal to P act along AD and CB and forces equal to Q act along AB and CD respectively. Prove that the perpendicular distance between the resultants of P and Q at A and that of P and Q at C

$$= \frac{(P \times a) - (Q \times b)}{\sqrt{(P^2 + Q^2)}}$$

Solution. Given : The system of forces is shown in Fig. 4.14.

Let x = Perpendicular distance between the two resultants.

We know that the resultant of the forces P and Q at A,

$$R_1 = \sqrt{P^2 + Q^2} \qquad ...(i)$$

and resultant of the forces P and Q at C,

$$R_2 = \sqrt{P^2 + Q^2} \qquad ...(ii)$$

$\therefore$ Resultant $R = R_1 = R_2$ ...[from equations (i) and (ii)]

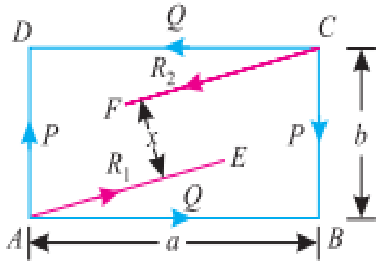


Fig. 4.14.

We know that moment of the force (P) about A,

$$M_1 = P \times a \qquad ...(+\text{ Due to clockwise})$$

and moment of the force (Q) about A,

$$M_2 = -Q \times b \qquad ...(-\text{ Due to anticlockwise})$$

$\therefore$ Net moment of the two couples

$$= (P \times a) - (Q \times b) \qquad ...(iii)$$

and moment of the couple formed by the resultants

$$= R \times x = \sqrt{P^2 + Q^2} \times x \qquad ...(iv)$$

Equating the moments (iii) and (iv),

$$\sqrt{P^2 + Q^2} \times x = (P \times a) - (Q \times b)$$

$$\therefore \qquad x = \frac{(P \times a) - (Q \times b)}{\sqrt{P^2 + Q^2}} \quad \textbf{Ans.}$$

Problem No 4: A square ABCD has forces acting along its sides as shown in Fig. 4.13. Find the values of P and Q, if the system reduces to a couple. Also find magnitude of the couple, if the side of the square is 1 m.

Solution. Given : Length of square = 1 m

Values of P and Q

We know that if the system reduces to a couple, the resultant force in horizontal and vertical directions must be zero. Resolving the forces horizontally,

$$100 - 100\cos 45° - P = 0$$

$$\therefore \quad P = 100 - 100\cos 45° \text{ N}$$

$$= 100 - (100 \times 0.707) = 29.3 \text{ N} \quad \textbf{Ans.}$$

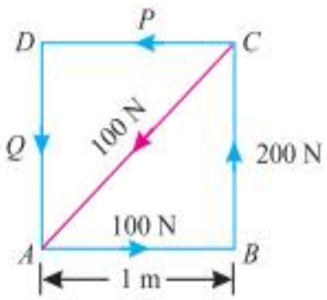


Fig. 4.13.

Now resolving the forces vertically,

$$200 - 100\sin 45° - Q = 0$$

$$\therefore \quad Q = 200 - (100 \times 0.707) = 129.3 \text{ N} \quad \textbf{Ans.}$$

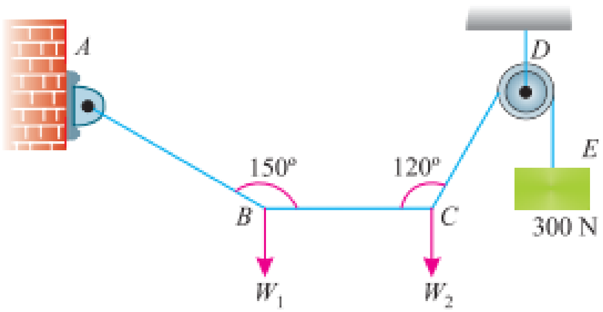
Magnitude of the couple

We know that moment of the couple is equal to the algebraic sum of the moments about any point. Therefore moment of the couple (taking moments about *A*)

$$= (-200 \times 1) + (-P \times 1) = -200 - (29.3 \times 1) \text{ N-m}$$

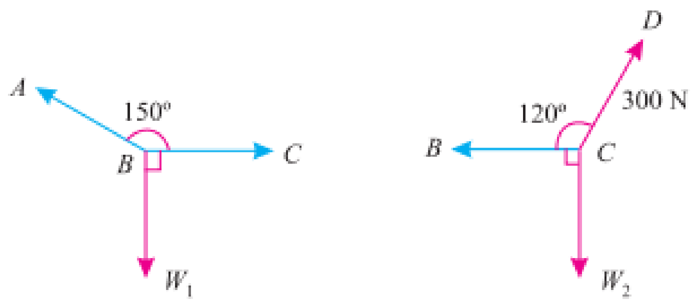
$$= -229.3 \text{ N-m} \quad \textbf{Ans.}$$

Problem No 5: A light string ABCDE whose extremity A is fixed, has weights W1 and W2 attached to it at B and C. It passes round a small smooth peg at D carrying a weight of 300 N at the free end E as shown in Fig



If in the equilibrium position, BC is horizontal and AB and CD make 150° and 120° with BC, find (i) Tensions in the portion AB, BC and CD of the string and (ii) Magnitudes of W1 and W2.

Solution. Given : Weight at E = 300 N



(*a*) Joint *B* (*b*) Joint *C*

(*i*) *Tensions is the portion AB, BC and CD of the string*

Let T_{AB} = Tension in the portion *AB*, and

T_{BC} = Tension in the portion *BC*,

We know that tension in the portion *CD* of the string.

$$T_{CD} = T_{DE} = 300 \text{ N} \quad \textbf{Ans.}$$

Applying Lami's equation at C,

$$\frac{T_{BC}}{\sin 150°} = \frac{W_2}{\sin 120°} = \frac{300}{\sin 90°}$$

$$\frac{T_{BC}}{\sin 30°} = \frac{W_2}{\sin 60°} = \frac{300}{1} \qquad ...[\because \sin(180° - \theta) = \sin\theta]$$

$\therefore$ $$T_{BC} = 300 \sin 30° = 300 \times 0.5 = 150 \text{ N} \quad \textbf{Ans.}$$

and $$W_2 = 300 \sin 60° = 300 \times 0.866 = 259.8 \text{ N}$$

Again applying Lami's equation at *B*,

$$\frac{T_{AB}}{\sin 90°} = \frac{W_1}{\sin 150°} = \frac{T_{BC}}{\sin 120°}$$

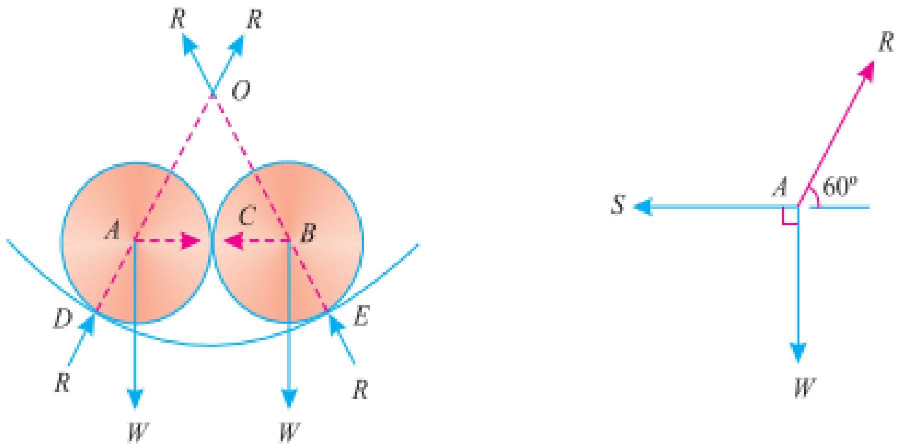
$$\frac{T_{AB}}{1} = \frac{W_1}{\sin 30°} = \frac{150}{\sin 60°} \qquad ...[\because \sin(180° - \theta) = \sin\theta]$$

$\therefore$ $$T_{AB} = \frac{150}{\sin 60°} = \frac{150}{0.866} = 173.2 \text{ N} \quad \textbf{Ans.}$$

and $$W_1 = \frac{150 \sin 30°}{\sin 60°} = \frac{150 \times 0.5}{0.866} = 86.6 \text{ N}$$

Problem No 6: Two equal heavy spheres of 50 mm radius are in equilibrium within a smooth cup of 150 mm radius. Show that the reaction between the cup of one sphere is double than that between the two spheres. Solution.

Given : Radius of spheres = 50 mm and radius of the cup = 150 mm.



The two spheres with centres A and B, lying in equilibrium, in the cup with O as centre are shown in Fig. 5.11 (a). Let the two spheres touch each other at C and touch the cup at D and E respectively.

Let R = Reactions between the spheres and cup, and

S = Reaction between the two spheres at C.

From the geometry of the figure, we find that OD = 150 mm and AD = 50 mm. Therefore OA = 100 mm. Similarly OB = 100 mm. We also find that AB = 100 mm. Therefore OAB is an equilateral triangle. The system of forces at A is shown in Fig. 5.11 (b).

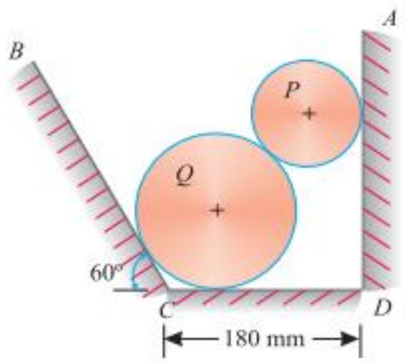
Applying Lami's equation at A,

$$\frac{R}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{S}{\sin 150^\circ}$$

$$\frac{R}{1} = \frac{W}{\sin 60^\circ} = \frac{S}{\sin 30^\circ}$$

$$\therefore \quad R = \frac{S}{\sin 30^\circ} = \frac{S}{0.5} = 2S$$

Problem No 7: Two cylinders P and Q rest in a channel as shown in Fig



The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N

If the bottom width of the box is 180 mm, with one side vertical and the other inclined at 60°, determine the pressures at all the four points of contact

Solution. Given : Diameter of cylinder P = 100 mm ; Weight of cylinder P = 200 N ; Diameter of cylinder Q = 180 mm ; Weight of cylinder Q = 500 N and width of channel = 180 mm

From the geometry of the figure, we find that

$$ED = \text{Radius of cylinder } P = \frac{100}{2} = 50 \text{ mm}$$

Similarly $$BF = \text{Radius of cylinder } Q = \frac{180}{2} = 90 \text{ mm}$$

and $$\angle BCF = 60°$$

$\therefore$ $$CF = BF \cot 60° = 90 \times 0.577 = 52 \text{ mm}$$

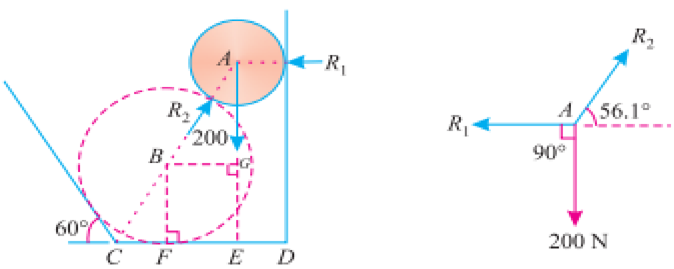
$\therefore$ $$FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

and $$AB = 50 + 90 = 140 \text{ mm}$$

$\therefore$ $$\cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

or $$\angle ABG = 56.1°$$

The system of forces at A is shown in Fig. 5.14 (b).



(a) Free body diagram (b) Force diagram

Applying Lami's equation at A,

$$\frac{R_1}{\sin(90° + 56.1°)} = \frac{R_2}{\sin 90°} = \frac{200}{\sin(180° - 56.1°)}$$

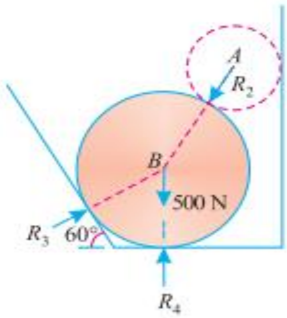
$$\frac{R_1}{\cos 56.1°} = \frac{R_2}{1} = \frac{200}{\sin 56.1°}$$

$\therefore$ $$R_1 = \frac{200 \cos 56.1°}{\sin 56.1°} = \frac{200 \times 0.5571}{0.830} = 134.2 \text{ N} \quad \textbf{Ans.}$$

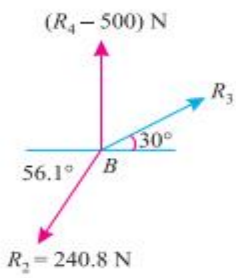
and $$R_2 = \frac{200}{\sin 56.1°} = \frac{200}{0.8300} = 240.8 \text{ N} \quad \textbf{Ans.}$$

Now consider the equilibriXum of the cylinder Q. It is in equilibrium under the action of the following four forces, which must pass through the centre of the cylinder as shown in Fig. 5.15 (*a*).

1. Weight of the cylinder Q (500 N) acting downwards.
2. Reaction R_2 equal to 240.8 N of the cylinder P on cylinder Q.
3. Reaction R_3 of the cylinder Q on the inclined surface.
4. Reaction R_4 of the cylinder Q on the base of the channel.



(*a*) Free body diagram



(*b*) Force diagram

A little consideration will show, that the weight of the cylinder Q is acting downwards and the reaction R_4 is acting upwards. Moreover, their lines of action also coincide with each other.

$\therefore$ Net downward force = $(R_4 - 500)$ N

The system of forces is shown in Fig. 5.15 (*b*).

Applying Lami's equation at B,

$$\frac{R_3}{\sin(90° + 56.1°)} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin(180° + 30° - 56.1°)}$$

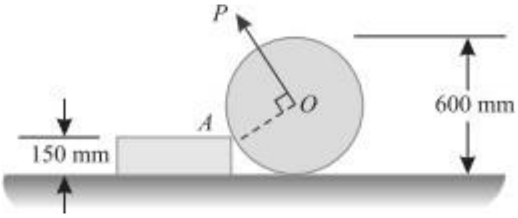
$$\frac{R_3}{\cos 56.1°} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin 26.1°}$$

$\therefore$ $$R_3 = \frac{240.8 \times \cos 56.1°}{\sin 60°} = \frac{240.8 \times 0.5577}{0.866} = 155 \text{ N} \quad \textbf{Ans.}$$

and $$R_4 - 500 = \frac{240.8 \times \sin 26.1°}{\sin 60°} = \frac{240.8 \times 399}{0.866} = 122.3 \text{ N}$$

$\therefore$ $$R_4 = 122.3 + 500 = 622.3 \text{ N} \quad \textbf{Ans.}$$

EXAMPLE: A uniform wheel of 600 mm diameter, weighing 5 kN rests against a rigid rectangular block of 150 mm height as shown in Fig.



Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner A of the block. Also find the reaction on the block. Take all the surfaces to be smooth.

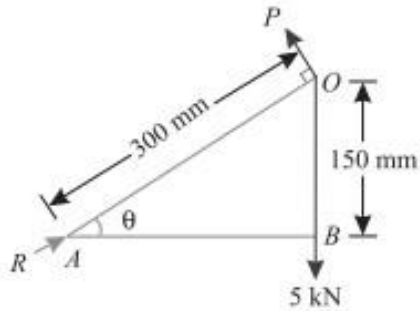
Solution. Given : Diameter of wheel = 600 mm; Weight of wheel = 5 kN and height of the block = 150 mm.

Least pull required just to turn the wheel over the corner.

Let P = Least pull required just to turn the wheel in kN.

A little consideration will show that for the least pull, it must be applied normal to AO. The system of forces is shown in Fig. 3.9. From the geometry of the figure, we find that

$$\sin\theta = \frac{150}{300} = 0.5 \quad \text{or} \quad \theta = 30°$$



and

$$AB = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm}$$

Now taking moments about A and equating the same,

$$P \times 300 = 5 \times 260 = 1300$$

$$\therefore \quad P = \frac{1300}{300} = 4.33 \text{ kN} \quad \textbf{Ans.}$$

Reaction on the block

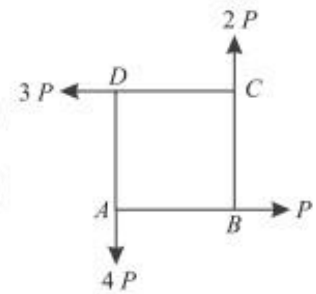
Let R = Reaction on the block in kN.

Resolving the forces horizontally and equating the same,

$$R\cos 30° = P\sin 30°$$

$$\therefore \quad R = \frac{P\sin 30°}{\cos 30°} = \frac{4.33 \times 0.5}{0.866} = 2.5 \text{ kN} \quad \textbf{Ans.}$$

EXAMPLE: Four forces equal to P, 2P, 3P and 4P are respectively acting along the four sides of square ABCD taken in order. Find the magnitude, direction and position of the resultant force.



Solution. The system of given forces is shown in Fig. 3.12.

Magnitude of the resultant force

Resolving all the forces horizontally,

$$\Sigma H = P - 3P = -2P \quad ...(i)$$

and now resolving all forces vertically,

$$\Sigma V = 2P - 4P = -2P \quad ...(ii)$$

We know that magnitude of the resultant forces,

$$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(-2P)^2 + (-2P)^2}$$

$$= 2\sqrt{2}P \quad \textbf{Ans.}$$

Direction of the resultant force

Let θ = Angle, which the resultant makes with the horizontal.

$$\therefore \quad \tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-2P}{-2P} = 1 \quad \text{or} \quad \theta = 45°$$

Since ΣH as well as ΣV are –ve, therefore resultant lies between 180°and 270°. Thus actual angle of the resultant force = 180° + 45° = 225° **Ans.**

Position of the resultant force

Let x = Perpendicular distance between A and the line of action of the resultant force.

Now taking moments of the resultant force about A and equating the same,

$$2\sqrt{2}P \times x = (2P \times a) + (3P \times a) = 5P \times a$$

$$\therefore \quad x = \frac{5a}{2\sqrt{2}} \quad \textbf{Ans.}$$

Note. The moment of the forces P and $4P$ about the point A will be zero, as they pass through it.

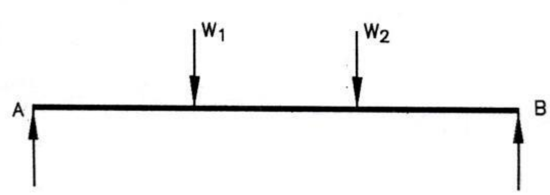
SUPPORT REACTIONS

Types of load

1) Point load
2) Uniformly distributed load
3) Uniformly varying load

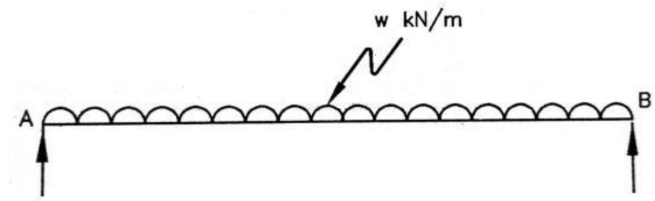
Point load

- Load concentrated on a very small length compare to the length of the beam, is known as point load or concentrated load. Point load may have any direction.
- For example truck transferring entire load of truck at 4 point of contact to the bridge is point load.



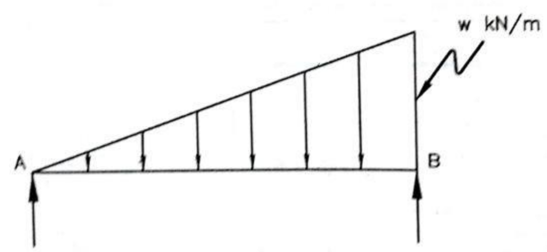
Uniformly distributed load

- Load spread uniformly over the length of the beam is known as uniformly distributed load.
- Water tank resting on the beam length
- Pipe full of water in which weight of the load per unit length is constant.



Uniformly varying load

- Load in which value of the load spread over the length if uniformly increasing or decreasing from one end to the other is known as uniformly varying load. It is also called triangular load.

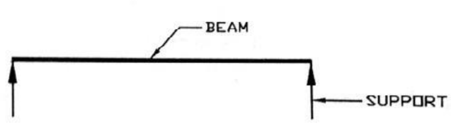


Type of beam

1) Simply supported beam
2) Cantilever beam
3) Fixed beam
4) Continuous beam
5) Propped cantilever beam

Simply supported beam

- It is the beam which is rest on the support. Here no connection between beam and support.

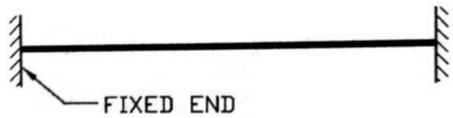


Cantilever beam

- If beam has one end fixed and other end free then it is known as cantilever beam

Fixed beam

- If both end of beam is fixed with support then it is called as fixed beam



Continuous beam

- If beam has more than two span, it is called as continuous beam

Propped cantilever beam

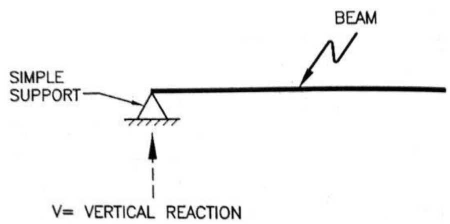
- If one end of beam is fixed and other is supported with prop then it is known as propped cantilever beam.

Type of support

1) Simple support
2) Roller support
3) Hinged support
4) Fixed support

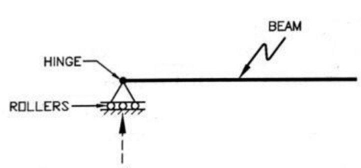
Simple support

- In this type of support beam is simply supported on the support. There is no connection between beam and support.Only vertical reaction will be produced.



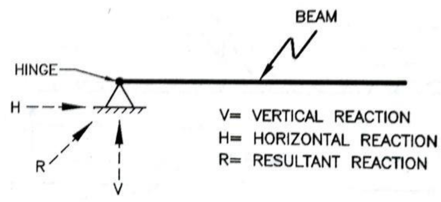
Roller support

- Here rollers are placed below beam and beam can slide over the rollers. Reaction will be perpendicular to the surface on which rollers are supported.
- This type of support is normally provided at the end of a bridge.



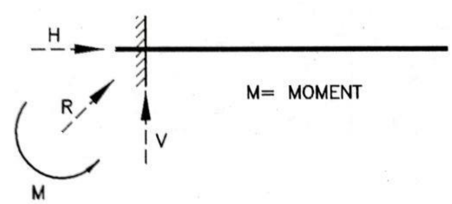
Hinged support

- Beam and support are connected by a hinge.Beam can rotate about the hingeReaction may be vertical, horizontal or inclined.

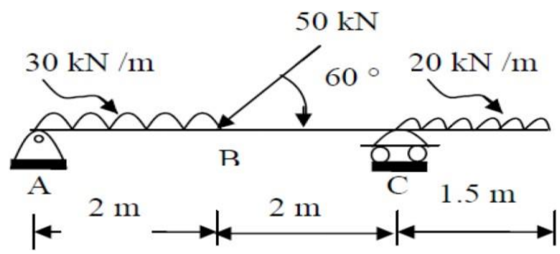


Fixed support

- Beam is completely fixed at end in the wall or support. Beam cannot rotate at end.Reactions may be vertical, horizontal, inclined and moment.



Example 1 Find out the support reactions for the beam.



Answer:

1)Now, Applying $\sum M = 0$(↻ +ve ↺ -ve)

Now, Taking moment @ pt. A, we have,

$$+ (30 \times 2 \times 1) + (50 \sin 60^\circ \times 2) - (R_c \times 4) = (20 \times 1.5 \times 4.75) = 0$$

$$\mathbf{R_c = 61.45\ kN}$$

2) Now $\sum$ Fy $= 0$

$$+ R_{AV} - (30 \times 2) - (50 \sin 60^\circ) + R_C - (20 \times 1.5) = 0$$

$$R_{AV} = 71.85\ kN.$$

3) Now, $\sum$ Fx $= 0$

$$+ R_{AV} - (50 \cos 60^\circ) = 0$$

$$R_{AV} = 25.0\ KN$$

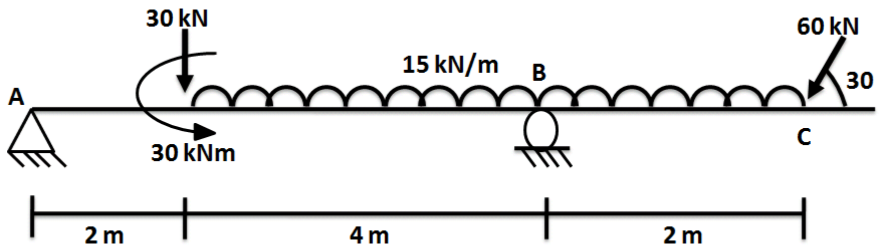
$$R_A = \sqrt{R_{AV}^2 + R_{AH}^2}$$

$$R_A = 76.08\ kN$$

$$\tan\theta = \frac{R_{AV}}{R_{AH}}$$

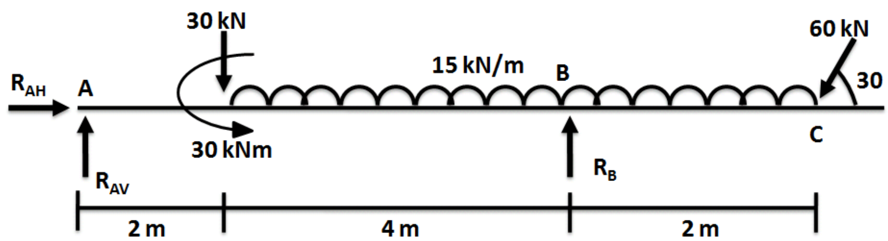
$$\boldsymbol{\theta = (70.81)^\circ}$$

Example- 2 Determine the reactions at support A and B for the beam loaded as shown in figure



Answer:

The F.B.D. of the beam is shown below



1)Applying $\sum$ M = 0 ↻ + ve ↺ -ve

Take the moment @ pt. A, we have,

$$+ (30 \text{ X } 2) - (30) - (R_B \text{ X } 6) + (15 \text{ X } 6 \text{ X } 5) + (60 \text{ Sin } 30^\circ) = 0$$

$$R_{AV} - 30 - (15 \text{ X } 6) + R_B - (60 \text{ Sin} 30^\circ) = 0$$

$$R_B = 120 \text{ kN}$$

2) $\sum$ Fy = 0

$$\therefore R_{AV} = 30 \text{ kN}$$

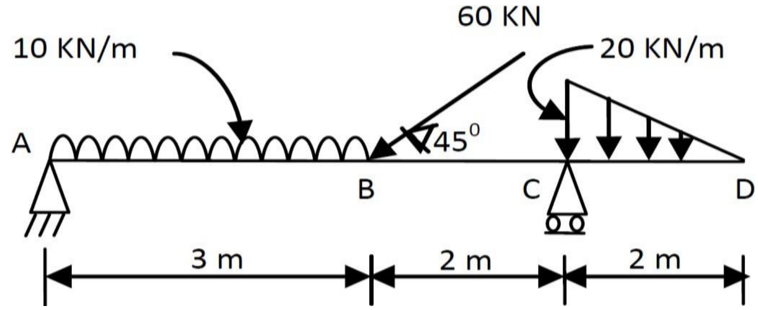
3) ΣFy = 0

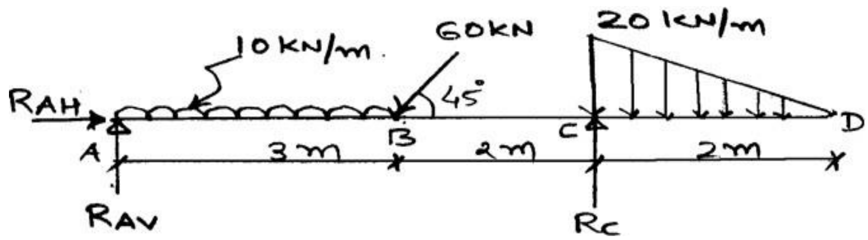
$$R_{AH} - 60 \text{ Cos } 30^\circ = 0$$

$$\therefore R_{AH} = +51.96 \text{ kN}$$

$$\text{Now, } R_A = \sqrt{R_{AV}^2 + R_{AH}^2} = 60 \text{ kN}$$

Example: 3 **Calculate reactions at support due to applied load on the beam as shown in Figure**





Answer:

Showing the reactions at support.

1) Applying $\sum M = 0$

Take the moment @ pt. A, we have,

$+ (10 \times 3 \times 1.5) + (60 \sin 45^\circ \times 3) - (R_C \times 5) + (1/2 \times 20 \times 2 \times 5.66) = 0$

$\therefore R_C = 57.096$ KN $(\uparrow)$

2) $\sum V = 0 \uparrow + Ve \quad \downarrow - Ve$

$+ R_{AV} - (10 \times 3) - (60 \sin 45^\circ) + R_C - (1/2 \times 20 \times 2) = 0$

Putting value of RC, we have.

$R_{AV} = 35.33$ KN

3) $\sum$ **H = 0**

$R_{AH} - 60 \cos 45^\circ = 0$

$R_{AH} = 42.43$ KN

Now, $R_A = \sqrt{R^2_{AH} + R^2_{AV}}$

$= \sqrt{(42.43)^2 + (35.33)^2}$

$= 55.21$ KN $(\rightarrow)$

$$\tan \theta = \frac{R_{AV}}{R_{AH}} = \frac{35.33}{42.43}$$

$\theta = (39.78)^\circ$