Linear Algebra

Practice Problems for Quiz III

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Introduction

Herein we will discuss the solutions for problems relating to the third assignment in linear algebra. The goal of this is to become proficient in the skills required to succeed in both the class and the practical applications of it.

**The book from which the problems are taken is Steve Leon's Linear Algebra with Applications, 9e.*

Problem Set

§	**EXERCISES**
3.6	1, 4, 5, 12, 22, 25
4.2	1, 2, 5, 6, 14, 17
4.3	1, 2, 5, 7, 9, 11
5.2	1, 2, 3, 4, 13

3.6 Row Space and Column Space

Problem 1. For each of the following matrices, find a basis for the row space, a basis for the column space, and a basis for the null space.

(a) $\begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 1 & 3 & 4 \\ 1 & 2 & -1 & -2 \\ -3 & 8 & 4 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 3 & -2 & 1 \\ 2 & 1 & 3 & 2 \\ 3 & 4 & 5 & 6 \end{bmatrix}$

Solution. The problems will be broken down into the basis for the **row space** ($RS(A)$), **column space** ($CS(A)$), and **null space** ($N(A)$), in that order.

(a) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 1 & 4 \\ 4 & 7 & 8 \end{bmatrix}$

I) $RS(A)$ can be found by first reducing the matrix to row echelon form, to eliminate linearly dependent rows.

The row echelon form of $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

This leaves two linearly independent rows, $(1, 3, 2)$, $(0, 1, 0)$. These two rows form the basis of the row space of A, as all other rows of A can be obtained through addition or scalar multiplication of these rows.

$\therefore RS(A) = \{(1, 3, 2), (0, 1, 0)\}$

II)

Problem 2. In each of the following, determine the dimension of the subspace of $\mathbb{R}^3$ spanned by the given vectors.

(a) $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$

4.2 Matrix Representations of Linear Transformations

4.3 Similarity

5.2 Orthogonal Subspaces