Outage Probability for Multiple Relay Networks with Supplementary Non-Broadcast Links

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***Abstract*—Recent large system analyses have shown that the per-node throughput of purely broadcast wireless networks goes to zero as the size of the network increases. Importantly, the problem appears to be physical in nature, i.e., it cannot be overcome through clever protocols, codes, or modulations. This fundamental problem, along with the proliferation of wireless devices that are able to communicate over multiple independent communication modes (frequency bands, channels), motivates the study of a novel kind of communication network composed of conventional wireless nodes along with "multimodal" nodes that can communicate simultaneously over broadcast wireless and other *non-broadcast* point-point modes, e.g., wires, infrared, acoustic, or free-space optical links. Here, we provide an information outage probability analysis of multiple relay networks with an arbitrary number of supplementary non-broadcast links. We find that the non-broadcast links provide benefits over a wide range of broadcast/non-broadcast power allocations.**

***Index Terms*—Cooperative diversity, decode and forward, relay channel, multimodal networks, information outage probability.**

I. Introduction

THE challenges encountered when trying to extend point-point information theory to networks has motivated the study of large systems, where the capacity region collapses to a single point and order-accurate results are available. These so-called *scaling law* results have been largely pessimistic, i.e., per-node throughput goes to zero as the number of nodes increases [1], [2]. Ultimately, Franceschetti *et al* [3] considered the problem from a physical point of view, using Maxwell's physics of wave propagation with Shannon's cut-set bound. Unfortunately, the results indicate that *space itself* is a capacity bearing object and cannot support an arbitrary density of purely wireless nodes. In other words, vanishing throughput for purely wireless networks is a physical problem, not an engineering problem. The conclusion, it seems, is that the world will never be purely wireless.

With this in mind, along with the proliferation of devices that can communicate using multiple communication modes, e.g., wireless, wires (ethernet), infrared, free-space optical, near-field-magnetic-induction[1], or acoustic links, we consider here a novel kind of communication network. Our network contains, in addition to purely wireless nodes, "multimodal" nodes that can communicate simultaneously over broadcast wireless and one or more non-broadcast point-point communication modes. In contrast to conventional infrastructure-aided networks, e.g., cellular and ad hoc networks, where wireless and wired networks are designed and optimized separately followed by a joining of the networks, we are interested in joint coding and simultaneous communication across available broadcast and non-broadcast modes. We call these networks *multimodal networks* to distinguish them from conventional hybrid networks. Previous results on multimodal networks include traffic optimization over multimodal networks [5] and achievable rates for multiple-level multimodal relay networks [6] using a generalization of [7].

Our contribution here includes high-SNR information outage probability expressions for multimodal space-time coded multiple-relay cooperative communications across a flat-fading channel using and extension of the Laneman protocol [8]. This protocol is suboptimal in terms of achievable rate, but it permits an analytical treatment of outage probability and it is half-duplex and easier to implement than more complex coding schemes.

The rest of this paper is organized as follows. Section II contains the multimodal network model and the protocol. Section III provides a high-transmit-SNR outage probability expression for multimodal networks with an arbitrary number of supplementary source-relay and relay-destination non-broadcast communication channels. Section IV contains numerical results, and Section V concludes.

II. System Model

Our network and protocol models, along with notations, are extensions of [8]. There are $m+1$ total nodes, each capable of broadcast wireless transmission, including a source node s, a destination node d, and $m-1$ relay nodes $r_1, r_2, \ldots, r_{m-1}$, as shown in Fig. 1. Depending upon the particular topology under consideration, the source may have the additional ability to communicate independently with one or more relays through non-broadcast channels. Similarly, one or more relays may have the ability to communicate to the destination through non-broadcast channels. Of course, a total power constraint at each transmitting node must be satisfied.

We assume broadcast transmissions take place through independent, flat, Rayleigh fading channels, and the non-broadcast modes are finite bandwidth Gaussian channels[2]. All receiving nodes are able to accurately determine received

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[1]NFMI can be considered non-broadcast with some model constraints, e.g., when a minimum distance between nodes is maintained [4].

[2]We consider frequency flat non-broadcast channels, but the results can easily be extended to frequency-selective channels by decomposing them into parallel frequency flat sub-channels.

fading coefficients which incorporate small and large scale fading effects and pathless. We avoid the effects of interference at the relays by assuming the frequency band is orthogonalized across transmitting nodes, i.e., if the total broadcast wireless bandwidth available is $B_{w\ell}$, the bandwidth occupied by each transmitting terminal is $B_{w\ell}/m$.

The complex Gaussian broadcast channel gain between transmitting node x and receiving node y, $x \in \{s, r_1, r_2, \ldots, r_{m-1}\}$, $y \in \{r_1, r_2, \ldots, r_{m-1}, d\}$ is $a_{x,y}$, with variance $1/\lambda_{x,y}$. The non-broadcast channel gains are defined as $h_{x,y}$ and are fixed and deterministic. Wireless transmission takes place over two orthogonal time phases to accommodate half-duplex constraints. During Phase 1, the source transmits and the relays do not transmit wirelessly. During Phase 2, the source is silent and those relays that are able to decode the source message forward the message to the destination using the same subchannel using an ideal space-time code, as in [8] (decode-and-forward relaying). Non-broadcast transmissions do not need to be synchronized with wireless transmissions. The set of relays that are able to decode the source for a particular set of channel gains is $\mathcal{D}$. The protocol does not allow feedback or communication between relays. We divide the set of all relays into two (generally overlapping) subsets: $\mathcal{R}_s$ contains all relays that are connected to the source by a non-broadcast link, and $\mathcal{R}_d$ contains all relays that are connected to the destination by a non-broadcast link.

We assume a continuous-time power constraint for each node of P_c J/second. In general, a fraction of that power may be allocated to one (for relays) or more (for the source) non-broadcast modes. Assuming all of the power is allocated to broadcast wireless transmission, the discrete-time power constraint is $P = P_c/(B_{w\ell}/m)$ J/symbol since each terminal occupies bandwidth $B_{w\ell}/m$. Then we define the transmit SNR as [8] $\mathsf{SNR} = \frac{P_c}{N_0(B_{w\ell}/m)} = \frac{P}{N_0}$. Let $P_w \in [0, 1)$ be the fraction of the total power P allocated across all non-broadcast links emanating from the source, and let P_{s,r_j} be fraction of the total power allocated to the non-broadcast link between the source and the relay r_j, so that $P_w = \sum_j P_{s,r_j}$. If no such link exists, we set $P_{s,r_j} = 0$. For the space-time protocol of [8], each terminal occupies 1/2 of the total degrees of freedom[3], so the received SNR across the wireless channel at relay r_j is

$$\frac{2}{m}(1 - P_w)\mathsf{SNR}|a_{s,r_j}|^2 = \frac{P_c(1 - P_w)|a_{s,r_j}|^2}{N_0(B_{w\ell}/2)}, \qquad (1)$$

while the received SNR over the non-broadcast mode is

$$\frac{2}{m}\frac{P_{s,r_j}B_{w\ell}}{B_w}\mathsf{SNR}|h_{s,r_j}|^2 = \frac{P_c P_{s,r_j}|h_{s,r_j}|^2}{N_0(B_w/2)} \qquad (2)$$

where B_w is the bandwidth of the non-broadcast mode. For simplicity, we assume that all non-broadcast modes have the same bandwidth. Generalizing the results is straightforward. The received SNR at the destination can be described similarly, where we let $P_{r_k,d}$ be the fraction of the total power used by relay r_k for the non-broadcast link between relay r_k and the destination. If no such link exists, we set $P_{r_k,d} = 0$. Our

[3]Here we focus our discussion on a single source, though in general each relay could also be a source, so every node, including the source, occupies 1/2 of the total degrees of freedom. Our result also holds for every relay node that is also a source.

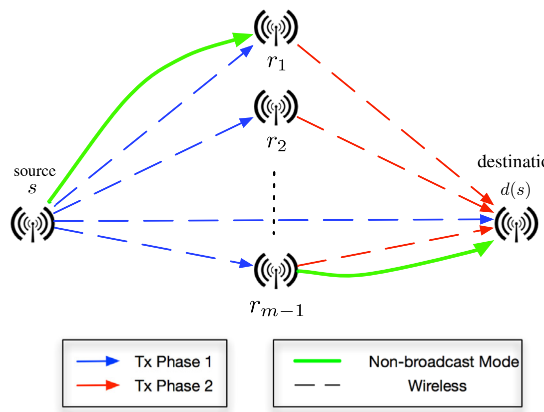

Fig. 1. The network topology introduced here, as an extension of the space-time protocol in [8]. We use two orthogonal time phases and we allow non-broadcast communication modes between the source and a subset of relays, and between another subset of relays and the destination.

analytical figure of merit is information outage probability, i.e., the probability that the realized mutual information at destination, I falls below a target rate R. This serves as a lower bound on the codeword error probability of a practical code operating at spectral efficiency R.

III. Main Results: Information Outage Probability

In this section we present high transmit SNR expressions for information outage probability for the network in Fig. 1. Note our result applies to any number of relay nodes/sources and any number of supplemental non-broadcast modes. In general, an additional non-broadcast channel can guarantee a deterministic rate based on the channel's characteristics and the power allocation. The total mutual information received at relay $r \in \mathcal{R}_s$, for example, is

$$\begin{aligned} I_{s,r} &= \frac{B_{w\ell}}{2}\log\left(1 + (1 - P_w)\frac{2}{m}\mathsf{SNR}\,|a_{s,r}|^2\right) + \\ &\quad \frac{B_w}{2}\log\left(1 + \frac{2P_{s,r}B_{w\ell}}{mB_w}\mathsf{SNR}\,|h_{s,r}|^2\right) \text{ bits/sec.} \end{aligned}$$

We normalize by $B_{w\ell}$ to obtain

$$\begin{aligned} I_{s,r} &= \frac{1}{2}\log\left(1 + (1 - P_w)\frac{2}{m}\mathsf{SNR}\,|a_{s,r}|^2\right) + \\ &\quad + \frac{B_w}{2B_{w\ell}}\log\left(1 + \frac{2P_{s,r}B_{w\ell}}{mB_w}\mathsf{SNR}\,|h_{s,r}|^2\right) \\ &\quad \text{bits/sec/wireless Hz.} \end{aligned} \qquad (3)$$

The first term is random. The second term is deterministic and fixed and serves to increase the probability that $I_{s,r}$ is larger than the target rate R, i.e., the probability that $r \in \mathcal{D}$. We define for relay r,

$$R_{s,r}^{-} = \left(R - \frac{B_w}{2B_{w\ell}}\log\left(1 + \frac{2P_{s,r}B_{w\ell}}{mB_w}\mathsf{SNR}\,|h_{s,r}|^2\right)\right)^+$$

as the rate[4] that must be supported by the *broadcast wireless* channel between the source and relay r for r to be in

[4]We use the common notation $(x)^+ = \max\{0, x\}$.

the decoding set. Clearly, if $r \notin \mathcal{R}_s$, then $P_{s,r} = 0$ and $R_{s,r^-} = R$. Note that $R_{s,r}^-$ cannot be negative, i.e., if the non-broadcast mode supports the rate R at the power allocation $P_{s,r}$, the relay does not need to try to decode the wireless received signal. Because of the nature of this protocol, there is no reason to use more power on any non-broadcast mode than is required to support rate R.

On the other hand, the mutual information at the destination received over the two time phases is

$$\begin{aligned} I \;=\; & \frac{1}{2}\log\left(1+\frac{2}{m}\left(1-P_w\right)\mathsf{SNR}\left|a_{s,d}\right|^2\right)+ \\ & \frac{1}{2}\log\left(1+\frac{2}{m}\mathsf{SNR}\sum_{r\in\mathcal{D}}(1-P_{r,d})|a_{r,d}|^2\right)+ \\ & \frac{B_w}{2B_{w\ell}}\sum_{r\in\mathcal{D}}\log\left(1+\frac{2P_{r,d}B_{w\ell}}{mB_w}\mathsf{SNR}\left|h_{r,d}\right|^2\right), \end{aligned} \tag{4}$$

and we define

$$R_d^- = \left(R - \frac{B_w}{2B_{w\ell}}\sum_{r\in\mathcal{D}}\log\left(1+\frac{2P_{r,d}B_{w\ell}}{mB_w}\mathsf{SNR}\left|h_{r,d}\right|^2\right)\right)^+$$

as the rate that must be supported by the broadcast wireless transmissions from the source and relays to avoid an outage at the destination.

For simplicity of notation, we define new effective channel parameters

$$\lambda_{s,r}^- \;=\; \lambda_{s,r}\left[\frac{2^{2R_{s,r}^-}-1}{2^{2R}-1}\right] \le \lambda_{s,r}, \tag{5}$$

$$\lambda_{s,d}^- \;=\; \lambda_{s,d}\left[\frac{2^{2R_d^-}-1}{2^{2R}-1}\right] \le \lambda_{s,d}, \tag{6}$$

$$\lambda_{r,d}^- \;=\; \lambda_{r,d}\left[\frac{2^{2R_d^-}-1}{2^{2R}-1}\right] \le \lambda_{r,d}. \tag{7}$$

Our main result provides a high-SNR accurate expression for information outage probability.

Theorem 1: For the network in Fig. 1, the information outage probability at high SNR behaves as

$$\begin{aligned} P_{\text{out}}(R,\mathsf{SNR}) \;\sim\; & \left[\frac{2^{2R}-1}{(1-P_w)2\mathsf{SNR}/m}\right]^m \times \\ & \sum_{\mathcal{D}}\left[\lambda_{s,d}^-\prod_{r\notin\mathcal{D}}\lambda_{s,r}^-\prod_{r\in\mathcal{D}}\frac{(1-P_w)}{(1-P_{r,d})}\lambda_{r,d}^- \times\right. \\ & \left. A_{|\mathcal{D}|}(2^{2R_d^-}-1)\right], \end{aligned} \tag{8}$$

where the sum is over all possible decoding sets and where $A_n(t) = 1/(n-1)!\int_0^1 (w^{(n-1)}(1-w))/(1+tw)\mathrm{d}w$. The proof appears in the appendix. Note that because the effective channel parameters, e.g., $\lambda_{s,d}^-$, are smaller than their purely wireless versions, the impact of the non-broadcast modes is to replace the broadcast channels with new effective broadcast channels with larger power. Despite the transmit SNR penalty of $(1-P_w)$, this improves performance relative to [8] as we will see in the next section.

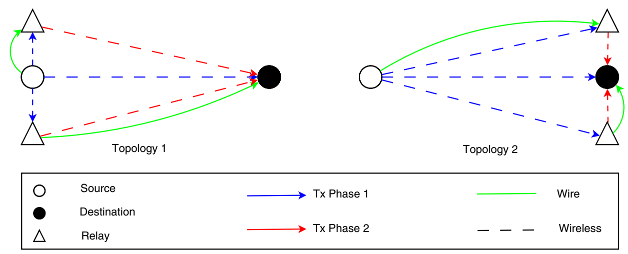

Fig. 2. The network topologies under consideration.

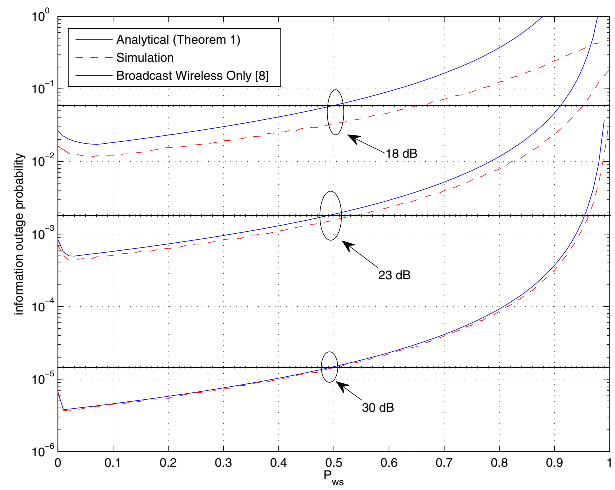

Fig. 3. Information outage probability for Topology 1 from in Fig. 2. The analytical results of Theorem 1 are compared with simulation for $B_w = B_{w\ell}$, $h_{s,r_1} = h_{r_2,d} = 1$, $P_{r_2,d} = 0.25$, $\lambda_{s,d} = 8$, $\lambda_{s,r_1} = \lambda_{s,r_2} = 2$, and $\lambda_{r_1,d} = \lambda_{r_2,d} = 9$. The SNR is fixed at 18dB, 23dB, or 30dB as indicated on the figure. The results from [8] are included for comparison purposes.

IV. Numerical Results

We compare the outage probability result of Theorem 1 with Monte Carlo simulation of (3) and (4) and for the 2-relay scenario where there are non-broadcast modes between the source and relay r_1 and between relay r_2 and the destination. Fig. 2 describes two topologies of interest, and Fig. 3 and Fig. 4 contain outage probability results versus $P_w \in [0,1)$ for various fixed values of SNR. The power of the non-broadcast mode between r_2 and the destination is fixed at $P_{r_2,d} = 0.25$, but, in general, all transmit powers should be jointly optimized.

We see close correspondence between simulation and analysis at high SNR (the "lower left" parts of the figures). We also see that performance improves rapidly with increasing transmit power along the source-relay non-broadcast mode until the point of minimum outage probability is reached. This occurs when the non-broadcast mode has enough power to support rate R. Increasing P_w beyond this point only serves to decrease the probability that r_2 is in the decoding set. We note that the diversity gain for all plots is $m = 3$.

V. Conclusions

We have developed high-SNR expressions for outage probability for wireless networks that are supplemented by non-broadcast links. Instead of separately optimizing wired

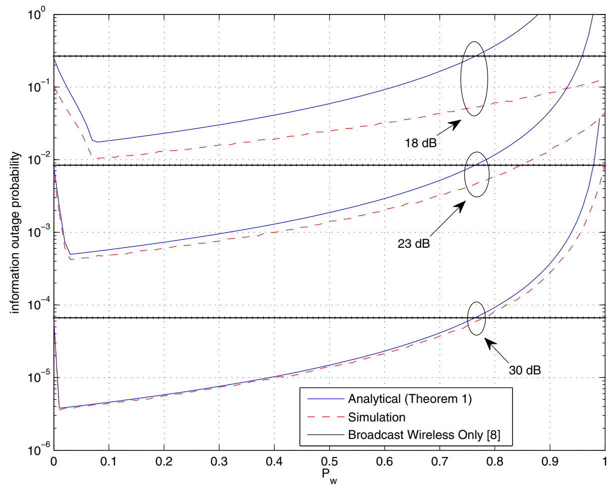

Fig. 4. Information outage probability for Topology 2 from in Fig. 2. The analytical results of Theorem 1 are compared with simulation for $B_w = B_{w\ell}$, $h_{s,r_1} = h_{r_2,d} = 1$, $P_{r_2,d} = 0.25$, $\lambda_{s,d} = 8$, $\lambda_{s,r_1} = \lambda_{s,r_2} = 9$, and $\lambda_{r_1,d} = \lambda_{r_2,d} = 2$. The SNR is fixed at 18dB, 23dB, or 30dB as indicated on the figure. The results from [8] are included for comparison purposes.

and wireless networks as in conventional infrastructure-aided networks, we have considered joint coding and simultaneous communication across multiple communication modes. Results indicate significant performance gains over a wide range of transmit powers and network topologies. Future work includes dealing with more sophisticated protocols and developing practical coding schemes.

VI. Appendix

Here we prove Theorem 1. We have

$$P_{\text{out}}(R, \mathsf{SNR}) = \sum_{\mathcal{D}} \mathsf{P}\{\mathcal{D}\}\mathsf{P}\{I < R|\mathcal{D}\}. \quad (9)$$

Now, for any relay r,

$$\begin{aligned}\mathsf{P}\{r \in \mathcal{D}\} &= \mathsf{P}\left\{\frac{1}{2}\log\left(1 + (1-P_w)\frac{2}{m}\mathsf{SNR}\,|a_{s,r}|^2\right) > R^-_{s,r}\right\} \\ &= \mathsf{P}\left\{|a_{s,r}|^2 > \frac{2^{2R^-_{s,r}} - 1}{(1-P_w)2\mathsf{SNR}/m}\right\} \\ &= \exp\left[-\lambda_{s,r}\frac{2^{2R^-_{s,r}} - 1}{(1-P_w)2\mathsf{SNR}/m}\right],\end{aligned}$$

where $R^-_{s,r} = R$ if there is no non-broadcast link between the source and relay r. Then, considering all relays,

$$\begin{aligned}\mathsf{P}\{\mathcal{D}\} &= \prod_{r\in\mathcal{D}} \exp\left[-\lambda_{s,r}\frac{2^{2R^-_{s,r}} - 1}{(1-P_w)2\mathsf{SNR}/m}\right] \times \\ &\quad \prod_{r\notin\mathcal{D}}\left(1 - \exp\left[-\lambda_{s,r}\frac{2^{2R^-_{s,r}} - 1}{(1-P_w)2\mathsf{SNR}/m}\right]\right) \\ &= \prod_{r\in\mathcal{D}} \exp\left[-\lambda^-_{s,r}\frac{2^{2R} - 1}{(1-P_w)2\mathsf{SNR}/m}\right] \times \\ &\quad \prod_{r\notin\mathcal{D}}\left(1 - \exp\left[-\lambda^-_{s,r}\frac{2^{2R} - 1}{(1-P_w)2\mathsf{SNR}/m}\right]\right).\end{aligned}$$

Using a power series expansion, we can write an accurate high-SNR approximation as

$$\mathsf{P}\{\mathcal{D}\} \sim \left[\frac{2^{2R} - 1}{(1-P_w)2\mathsf{SNR}/m}\right]^{m-|\mathcal{D}|-1} \prod_{r\notin\mathcal{D}} \lambda^-_{s,r}. \quad (10)$$

Thus, compared to the purely broadcast wireless case [8, Equation (21)] the supplementary non-broadcast channels between the source and the relays create *virtual* broadcast channels that have increased channel power at the cost of a decreased transmit SNR.

Now, conditioning on the decoding set, we have

$$\begin{aligned}\mathsf{P}\{I < R|\mathcal{D}\} &= \\ \mathsf{P}\Bigg\{&\frac{1}{2}\log\left(1 + \frac{2}{m}(1-P_w)\,\mathsf{SNR}\,|a_{s,d}|^2\right) + \\ &\frac{1}{2}\log\left(1 + \frac{2}{m}\mathsf{SNR}\sum_{r\in\mathcal{D}}(1-P_{r,d})|a_{r,d}|^2\right) < R^-_d\Bigg\}.\end{aligned}$$

Notice that this form is similar to [8, Equation (49)] where, here, $u_m = (1-P_w)|a_{s,d}|^2$ is exponentially distributed with parameter $\lambda_{s,d}/(1-P_w)$, $u_r = (1-P_{r,d})|a_{r,d}|^2, r \in \mathcal{D}$ is similarly distributed with parameter $\lambda_{r,d}/(1-P_{r,d})$, and R is replaced with R^-_d. Thus we can write a high-SNR approximation as

$$\begin{aligned}\mathsf{P}\{I < R|\mathcal{D}\} &\sim \left[\frac{2^{2R^-_d} - 1}{2\mathsf{SNR}/m}\right]^{|\mathcal{D}|+1} \times \\ &\frac{\lambda_{s,d}}{(1-P_w)}\prod_{r\in\mathcal{D}}\frac{\lambda_{r,d}}{(1-P_{r,d})} \times A_{|\mathcal{D}|}(2^{2R^-_d} - 1) \\ &= \left[\frac{2^{2R} - 1}{(1-P_w)2\mathsf{SNR}/m}\right]^{|\mathcal{D}|+1} \times \\ &\lambda^-_{s,d}\prod_{r\in\mathcal{D}}\frac{(1-P_w)}{(1-P_{r,d})}\lambda^-_{r,d} \times A_{|\mathcal{D}|}(2^{2R^-_d} - 1). \quad (11)\end{aligned}$$

Using (10) and (11) in (9) yields the desired result.

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