# Observer/Kalman Filter Identification for Online System Identification of Aircraft

John Valasek\* and Wei Chen<sup>†</sup>
Texas A&M University, College Station, Texas 77843-3141

The observer/Kalman filter identification method is applied to the problem of online system identification of accurate, locally linear, aircraft dynamic models of nonlinear aircraft. It is a time-domain technique that identifies a discrete input-output mapping from known input and output data samples, without user imposed a priori assumptions about model structure or model order. The basic formulation of observer/Kalman filter identification specific to the aircraft problem is developed and implemented in a nonlinear, six-degree-of-freedom simulation of an AV-8B Harrier. A similar simulation of a generic uninhabited combat aerial vehicle is also used. Numerical examples are presented, consisting of longitudinal and lateral/directional successive online identifications at different nonperfect trim conditions, identification with sensor noise on multiple channels, and identification with discrete gusts. Accuracy of the identified linear system models to the nonlinear plant is quantified with comparison of eigenvalues, the Vinnicombe gap metric, and time history matching. Results demonstrate that the observer/Kalman filter identification method is suitable for aircraft online identification of locally linear aircraft models and is generally insensitive to moderate intensity Gaussian white sensor noise and for light to moderate intensity discrete gusts.

### Introduction

THE capability to generate accurate, locally linear aircraft models, online and in real time, is particularly useful in autonomous/intelligent flight control applications, where the absence of a human operator shifts the burden of decision making in real time to the onboard systems. In this type of system, small perturbation linear models generated on demand find use in robust structured adaptive model inversion flight controllers, in online reference trajectory generators, and as in assisting intelligent flight decision agents with multimodel switching decisions, and as vehicle health monitoring mechanisms.

Besides autonomous/intelligent control applications, the recent literature contains examples and uses including identification of aircraft force and moment parameters<sup>7,8</sup> and identification of control effector characteristics.<sup>9,10</sup> These methods postprocess flight data to identify the parameters and generally possess the accuracy and data quality discussed in Refs. 11 and 12. In-flight system identification<sup>13</sup> uses current as opposed to postflight measurements of inputs and outputs and is particularly useful for flight control applications because the small perturbation linear aircraft models are available during flight. This technique has been used as a key element in a reconfigurable flight control system that has been successfully flight tested, <sup>14</sup> and it has also been integrated with optimization-based control allocation schemes.<sup>15</sup>

Regardless of the application, several types of linear system identification methods are available. Classical linear system identification methods explicitly approximate and simplify the nonlinear system dynamics in terms of a linear model, using either frequency-domain or time-domain methods. Three of the more common methods used are fast Fourier transforms, maximum likelihood estimation, and least squares. A survey of these traditional methods and their application to flight vehicles is provided in Ref. 16. Ho and

Presented as Paper 99-4173 at the AIAA Atmospheric Flight Mechanics Conference, Portland, OR, 10–12 August 1999; received 10 August 2000; revision received 13 July 2002; accepted for publication 19 July 2002. Copyright © 2003 by John Valasek and Wei Chen. Published by the American Institute of Aeronautics and Astronautics, Inc., with permission. Copies of this paper may be made for personal or internal use, on condition that the copier pay the \$10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code 0731-5090/03 \$10.00 in correspondence with the CCC.

Kalman had earlier introduced an alternative approach to system identification using the concept of minimum realization<sup>17</sup> or the smallest dimension state-space model among all possible realizable systems that have the same input-output relations. This result was subsequently extended by Juang and Pappa into the eigensystem realization algorithm (ERA). 18 ERA is a time-domain technique that directly solves for the system Markov parameters, or sampled pulse response histories, from the input and output data. It has been used with success on flexible structures<sup>19</sup> and for systems with sensor noise.<sup>20</sup> Two basic limitations of the ERA technique are that the initial conditions of the system states and controls must be zero and that the perturbed system must decay to zero in steady state. Because an input matrix must be inverted, the latter requirement results in input-output data records and storage requirements that can be excessive for lightly damped systems, such as elastic flight vehicles, because of the time required for responses to decay to zero. Additionally, the former requirement is especially limiting for flight vehicles. The trim state itself (zero accelerations, or the sum of all forces and moments equal to zero) is usually imperfect, especially in the presence of even small external atmospheric disturbances.

Certain states and controls are also nonzero and often not precisely

known in-flight. These limitations can preclude the use of ERA in a

real-time online system identification setting, although it has been achieved for certain types of systems when used with input-output

The observer/Kalman filter identification (OKID) technique is an extension of ERA that was developed<sup>22-24</sup> to permit efficient identi $fication of large flexible structures, particularly those of spacecraft. ^{25}$ Based on the concepts of stochastic estimation and deterministic Markov parameter identification techniques, OKID directly generates a discrete state-space locally linear model representation of the nonlinear system. For lightly damped systems, OKID artificially improves system damping by introducing an asymptotically stable deadbeat observer. This significantly reduces the required data record, storage space, and computation time. Nonzero initial conditions on the states and controls can be easily accommodated with OKID, so that the technique is applicable to flight vehicles. Online applications of system identification must often contend with external disturbances. For these cases, the objective is to identify the undisturbed system even though the output data also contains the unwanted response due to exogenous inputs. Versions of OKID that have been developed to account for this effect contain explicit periodic disturbance models,<sup>26</sup> partially known periodic disturbance models,<sup>27</sup> and completely unknown periodic disturbance models.<sup>28</sup> For the specific case of flight vehicle online system identification,

data correlations.21

<sup>\*</sup>Assistant Professor and Director, Flight Simulation Laboratory, Aerospace Engineering Department. Associate Fellow AIAA.

<sup>&</sup>lt;sup>†</sup>Graduate Research Assistant, Aerospace Engineering Department. Student Member AIAA.

the external disturbances encountered are gusts and atmospheric turbulence. Although the former tend to be discrete in nature, and easily modeled as step or ramp functions, the latter type are stochastic, not periodic. The accuracy of models identified online using one of the disturbance OKID methods will depend upon how closely the stochastic disturbance approximates the modeled periodic disturbance.

The objective of this paper is to demonstrate the application and feasibility of the OKID method to the problem of online determination of accurate, locally linear aircraft models of nonlinear flight vehicles. The paper starts by reviewing the basic formulation of the OKID algorithm and the definition of system Markov parameters. It then discusses observer Markov parameters and their relationship to system identification. The formulation is then extended to flight vehicle systems with imprecisely known trim values, and examples of multiple online identifications. Identifications for systems with sensor noise, and identification in the presence of atmospheric gust disturbances are presented. Accuracy of the identified model of the linearized nonlinear dynamics are verified with the Vinnicombe gap metric<sup>29</sup> and time history matching comparisons. Results are presented for aircraft nonlinear six-degree-of-freedom simulations of an AV-8 Harrier and a generic vertical/standard takeoff and landing (V/STOL) uninhabited combat aerial vehicle (UCAV6).

#### **Basic Formulation**

The discrete-time, linear state-space small perturbation model of the linearized nonlinear dynamics of an aircraft in the trim state can be represented in the form

$$x(k+1) = Ax(k) + Bu(k),$$
  $y(k) = Cx(k) + Du(k)$  (1)

where  $x(k) \in \mathbb{R}^n$ ,  $y(k) \in \mathbb{R}^m$ , and  $u(k) \in \mathbb{R}^r$  are state, output, and control inputs vectors of dimension n, m, and r, respectively. The development follows from Ref. 22. Solving for the output y(k) with zero initial condition from Eqs. (1) in terms of the previous inputs u(i),  $k = 0, 1, 2, \ldots, k$ , yields

$$x(0) = 0$$

$$y(0) = Du(0)$$

$$x(1) = Bu(0)$$

$$y(1) = CBu(0) + Du(1)$$

$$x(2) = ABu(0) + Bu(1)$$

$$y(2) = CABu(0) + CBu(1) + Du(2)$$

$$\vdots$$

$$x(k) = \sum_{i=1}^{k} A^{i-1}Bu(k-i)$$

$$y(k) = \sum_{i=1}^{k} CA^{i-1}Bu(k-i) + Du(k)$$
(2)

Writing Eqs. (2) in matrix form, we have

$$y = [y(0) \quad y(1) \quad y(2) \quad \cdots \quad y(l-1)]$$
  
 $Y = [D \quad CB \quad CAB \quad \cdots \quad CA^{l-2}B]$  (3)

with Markov parameters  $D, CB, CAB, \ldots, CA^{k-1}B$ , which are commonly used as the basis to identify mathematical models for linear dynamic systems. Linear state-space models can then be generated from Markov parameters using the relation

$$\overset{m \times l}{\mathbf{y}} = \underset{m \times rl}{Y} \overset{rl \times l}{U} \tag{4}$$

with Hankel matrix (see Ref. 17) U defined as

$$U = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(l-1) \\ u(0) & u(1) & \cdots & u(l-2) \\ & & u(0) & \cdots & u(l-3) \\ & & & \ddots & \vdots \\ & & & u(0) \end{bmatrix}$$

For lightly damped systems, however, the slow decaying response may produce a large Hankel matrix and, therefore, long computation times. The OKID method overcomes this problem by adding an observer into the system and placing eigenvalues as desired. Adding and subtracting an observer term Gy(k) to the right-hand side of the state equation in Eqs. (1) yields

$$x(k+1) = Ax(k) + Bu(k) + Gy(k) - Gy(k)$$
$$= (A + GC)x(k) + (B + GD)u(k) - Gy(k)$$

which can be written compactly as

$$x(k+1) = \bar{A}x(k) + \bar{B}v(k) \tag{5}$$

where

$$\bar{A} = A + GC, \qquad \bar{B} = [B + GD, -G], \qquad v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}$$

G is an  $n \times m$  arbitrary matrix that can be used to make the matrix  $\bar{A}$  as stable as desired, that is, place the eigenvalues of  $\bar{A}$  to any desired values if the system is observable. This ensures that  $C\bar{A}^{k-1}\bar{B}=0$  for  $k \ge p$ , where p is the number of independent Markov parameters. For the identification problem, where A and C are unknown, the algorithm is normally set to place the eigenvalues at the origin by default, thereby, producing a deadbeat response. This also reduces the length of the data record, thereby relieving some of the computational burden. Note that v(k) is the input vector to the new observer augmented system Eq. (5) and is composed of the nominal system Eq. (1) inputs and outputs.

For the online system identification of an aircraft, nonzero initial conditions on the states and controls must be assumed, for which Eq. (5) is extended into the following form:

$$x(k+1) = \bar{A}x(k) + \bar{B}v(k)$$

$$x(k+2) = \bar{A}x(k+1) + \bar{B}v(k+1)$$

$$= \bar{A}^{2}x(k) + \bar{A}\bar{B}v(k) + \bar{B}v(k+1)$$

$$\vdots$$

$$x(k+p) = \bar{A}x(k+p-1) + \bar{B}v(k+p-1)$$

$$= \bar{A}^{p}x(k) + \bar{A}^{p-1}\bar{B}v(k) + \bar{A}^{p-2}\bar{B}v(k+1)$$

$$+ \dots + \bar{B}v(k+p-1)$$

Using the measurement equation yields

$$y(k+p) = Cx(k+p) + Du(k+p)$$

$$= C\bar{A}^{p}x(k) + C\bar{A}^{p-1}\bar{B}v(k) + C\bar{A}^{p-2}\bar{B}v(k+1)$$

$$+ \dots + C\bar{B}v(k+p-1) + Du(k+p)$$
 (6)

The set of these equations for a sequence of k = 0, ..., l - 1 can be written as

$$\bar{\mathbf{v}} = C\bar{A}^P \mathbf{x} + \bar{Y}\bar{V} \tag{7}$$

where

$$\bar{\mathbf{y}} = [\mathbf{y}(p) \quad \mathbf{y}(p+1) \quad \cdots \quad \mathbf{y}(l-1)]$$

$$\bar{Y} = [D \quad C\bar{B} \quad C\bar{A}\bar{B} \quad \cdots \quad C\bar{A}^{(p-1)}\bar{B}]$$

where  $D, C\bar{B}, C\bar{A}\bar{B}, \ldots, C\bar{A}^{(p-1)}\bar{B}$  are observer Markov parameters and  $\bar{V}$  is a Hankel matrix:

$$\bar{V} = \begin{bmatrix}
 u(p) & u(p+1) & \cdots & u(l-1) \\
 v(p-1) & v(p) & \cdots & v(l-2) \\
 v(p-2) & v(p-1) & \cdots & v(l-3) \\
 \vdots & \vdots & \ddots & \vdots \\
 v(0) & v(1) & \cdots & v(l-p-1)
\end{bmatrix}$$

The first term in Eq. (7) represents the effect of the preceding p-1 time steps. When  $A^p$  is sufficiently small and all of the states in x are bounded, Eq. (7) can be approximated by neglecting the first term on the right-hand side, such that

$$\bar{\bar{\mathbf{y}}}^{m \times l} = \bar{\bar{\mathbf{Y}}}_{m \times [(m+r)p+r]}^{[(m+r)p+r] \times l} \bar{\mathbf{V}}$$
(8)

Equation (8) then has a least-squares solution

$$\bar{Y} = \bar{y}\bar{V}^+ \tag{9}$$

where  $\bar{V}^+$  is the pseudoinverse of the matrix V. The observer Markov parameters  $\bar{Y}$  can be partitioned out as

$$\bar{Y} = [D \quad C\bar{B} \quad C\bar{A}\bar{B} \quad \cdots \quad C\bar{A}^{(p-1)}\bar{B}]$$
$$= [\bar{Y}_0 \quad \bar{Y}_1 \quad \bar{Y}_2 \quad \cdots \quad \bar{Y}_p]$$

and solved for by inspection,

$$\bar{Y}_0 = D$$

$$\bar{Y}_k = C\bar{A}^{(k-1)}\bar{B}$$

$$= [C(A+GC)^{k-1}(B+GD) -C(A+GC)^{k-1}G]$$

$$= [\bar{Y}_k^{(-1)} - \bar{Y}_k^{(2)}], \qquad k = 1, 2, 3, ... \qquad (10)$$

However, it is the system Markov parameters that are desired, and these must be extracted from the observer Markov parameters. Even if the system matrix A has slow eigenvalues, large numbers of system Markov parameters are not required because the deadbeat response provided by the observer permits the number of required Markov parameters to be kept small. Likewise if the system is marginally unstable but observable. The general relationship between the system Markov parameters and the observer Markov parameters is

$$D = Y_0 = \bar{Y}_0$$

$$Y_k = Y_k^{(1)} - \sum_{i=1}^k \bar{Y}_i^{(2)} Y_{(k-1)} \quad \text{for} \quad k = 1, \dots, p$$

$$Y_k = -\sum_{i=1}^p \bar{Y}_i^{(2)} Y_{(k-1)} \quad \text{for} \quad k = p+1, \dots, \infty \quad (11)$$

With the system Markov parameters now determined, the ERA method is used to obtain the desired discrete system realization [A, B, C, D], using a singular value decomposition of the Hankel matrix:

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \cdots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \cdots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \cdots & Y_{k+\alpha+\beta-2} \end{bmatrix}$$

$$H(0) = R_n \sum S_n, \qquad \hat{A} = \sum_n^{-\frac{1}{2}} R_n^T H(1) S_n \sum_n^{-\frac{1}{2}}$$

$$\hat{B} = \sum_{n=1}^{\frac{1}{2}} S_{n}^{T} E_{r}, \qquad \hat{C} = E_{m}^{T} R_{n} \sum_{n=1}^{\frac{1}{2}}$$
(13)

where

$$E_m^T = [I_m \quad O_m \quad \cdots \quad O_m], \qquad E_r^T = [I_r \quad O_r \quad \cdots \quad O_r]$$

and the caret indicates identified system matrices, as opposed to the true system matrices. Because noise usually has lower energy than system signals, the ERA selects the signals that have larger singular values and considers them to be from the true signals. Identification in the presence of disturbances is not an advantage of this method because it is basically a least-squares method whose goal is to minimize the error between real output and identified output.

For the aircraft linear perturbation model, longitudinal perturbed state variables (outputs) are body-axis forward velocity, angle of attack, body-axis pitch rate, and pitch attitude angle and perturbed control inputs are stabilator, and nozzle:

$$x = \mathbf{y} = \begin{bmatrix} u & \alpha & q & \theta \end{bmatrix}^T, \qquad \mathbf{u} = \begin{bmatrix} \delta_s & \delta_n \end{bmatrix}^T$$
 (14)

Likewise, for the lateral/directional model, perturbed state variables (outputs) are sideslip angle, body-axis roll and yaw rates, and bank angle, and perturbed control inputs are aileron and rudder:

$$\mathbf{x} = \mathbf{y} = [\beta \quad p \quad r \quad \phi]^T, \qquad \mathbf{u} = [\delta_a \quad \delta_r]^T \qquad (15)$$

Using data records consisting of the perturbed control inputs and perturbed outputs, the assumed form of the input system to the OKID algorithm is a discrete state-space representation of the form,

$$x(k+1) = Ax(k) + Bu(k),$$
  $y(k) = x(k)$  (16)

# **Unspecified Trim Values**

Because aircraft are not always precisely trimmed even in the so-called trimmed condition, values for the trim states and controls may be undetermined or inaccurate. This is especially true for intelligent autonomously controlled uninhabited aerial vehicles and UCAVs because it may not always be possible (due to time constraints) or desirable (due to mission constraints) to command these aircraft to precise, steady-state (trim) conditions. When this situation occurs, the desired perturbed state and control variables that have a linear relationship to the nonlinear system cannot be separated from the true trim states and controls. Ideally, a formulation that does not require the trim values to be specified is desired. To solve this problem, it is shown that we can change the form of the discrete linear equation to make the true input and output satisfy a modified linear model. It is well known that only small perturbations of a linear system around a steady-state or trim value have a linear relationship, that is,

$$x(k+1) = Ax(k) + Bu(k)$$

$$X(k+1) - X_1(k+1) = A[X(k) - X_1(k)] + B[U(k) - U_1(k)]$$

$$X(k+1) = AX(k) + BU(k) + [X_1(k+1) - AX_1(k) - BU_1(k)]$$

where X and U are total states and controls, x and u are perturbed states and controls, and  $X_1$  and  $U_1$  are trim states and controls. Considering the trim values to be constant for a given flight condition and noting that  $X_1(k+1) - AX_1(k) - BU_1(k) = -B^T$  permits Eq. (17) to be written

$$X(k+1) = AX(k) + BU(k) - B^{T}$$

$$= AX(k) + \begin{bmatrix} B & B^{T} \end{bmatrix} \begin{bmatrix} U(k) \\ -1 \end{bmatrix}$$
(18)

Similarly, with perturbed outputs y and trim outputs  $y_1$ , the outputs can be converted to

$$y(k) = Y(k) - Y_1(k)$$

$$= x(k)$$

$$= X(k) - X_1(k)$$

$$Y(k) = X(k) + [Y_1(k) - X_1(k)]$$

$$= X(k) + D^T$$
(19)

where

$$D^T = Y_1 - X_1$$

This new linear system, Eqs. (18) and (19), has the same matrices A and C as the perturbation model, but enlarged input and output matrices B and C.

#### **Linear Model Verification**

Accuracy of the identified plant models to the true plant models is verified in this paper using the Vinnicombe gap metric.<sup>29</sup> It is a modified gap metric that provides a measure of the distance on an operator space between a nominal linear, time-invariant system  $P_1$  and its perturbed linear, time-invariant system  $P_2$ . The Vinnicombe metric is a scalar value defined as

$$\delta_{V}(P_{1}, P_{2}) := \begin{cases} \left\| \left( I + P_{1} P_{1}^{*} \right)^{-\frac{1}{2}} (P_{1} - P_{2}) \left( I + P_{2} P_{2}^{*} \right)^{-\frac{1}{2}} \right\|_{\infty} \le 1 \\ 1 \quad \text{otherwise} \end{cases}$$
(20)

with identity matrix I. The norm in Eq. (20) requires that a winding number condition be satisfied, which for rational plants is

$$\det (I + P_2^* P_1)(j\omega) \neq 0 \forall \omega$$
WNO \det \left(I + P\_2^\* P\_1\right) + \eta(P\_1) - \eta(P\_2) = 0 (21)

where the winding number WNO is evaluated on the standard Nyquist contour indented around any imaginary axis poles of  $P_1$  and  $P_2$ ,  $\eta(P)$  is the number of open right-hand poles of P, and  $\ast$  denotes complex-conjugate transpose. If two systems  $P_1$  and  $P_2$  are close in the Vinnicombe metric, then their closed-loop performance will be the same, for the same controller C. When the stability margin is defined as

$$b_{P,C} = \|T_{P,C}\|_{\infty}^{-1} \tag{22}$$

where

$$T_{P,C} = \begin{bmatrix} I \\ P \end{bmatrix} (I + CP)^{-1} [I \quad C]$$

then if  $[P_1,C]$  is stable, then  $\delta_V(P_1,P_2) < b_{P_1,C}$  and  $[P_2,C]$  is stable. Additionally,  $b_{P,C}$  is zero if  $T_{P,C}$  is unstable. Furthermore, if  $[P_1,C]$  is stable and  $\delta_V(P_1,P_2) < \eta b_{P_1,C}$  where  $\eta < 1$ , then performance is close. Reference 29 contains an explicit expression for an upper bound. Although originally created to quantify the robust stabilization of closed-loop systems, the Vinnicombe metric is a functional on open-loop models and, therefore, applicable to identified open-loop linear dynamic systems.

# **Numerical Examples**

The first example demonstrates two successive online identifications. The flight vehicle based applications of Refs. 1-6 only require identification on demand, not identification at each time step using persistent excitations. Therefore, this example demonstrates two successive online identifications, on demand. A nonlinear, nonreal-time, six-degree-of-freedom simulation of the AV-8B Harrier is used as the nonlinear aircraft model.<sup>30</sup> There is no closed-loop controller in this simulation, so that all results are open loop (bare airframe). None of the identification for this first example is done offline or a posteriori. The OKID algorithm is embedded directly in the simulation code itself and can be invoked multiple times online during the simulation run, at times selected by the user. The length of the data stream record for use in the identification is selected by the user and, in this example, is 15 s in duration. The initial flight condition is Mach 0.56 at an altitude of 10,000 ft. Figure 1 shows longitudinal states and controls throughout the 70-s maneuver, and Fig. 2 shows lateral/directional states and controls. The first online identification sequence is selected by the user to begin at time equals 5 s, at which point excitations consisting of doublets (and in some cases triplets) are introduced through all longitudinal and lateral/directional controls. After a simulator-generated linear model and an OKID-identified linear model is generated at this

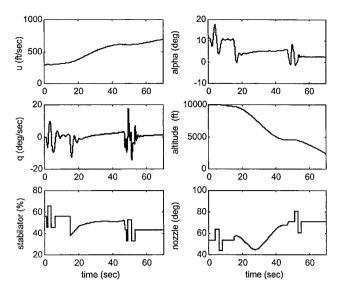


Fig. 1 Longitudinal states and controls, nonlinear AV-8B simulation.

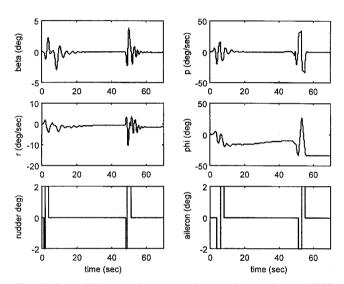


Fig. 2 Lateral/directional states and controls, nonlinear AV-8B simulation.

initial flight condition, the aircraft is commanded to descend, capture an altitude of 5000 ft, and retrim. At this new and different flight condition, the second online identification sequence begins at time equals 40 s with the same excitation magnitudes as used in the first identification sequences. The altitude loss after the second identification is due to a small residual nonzero bank angle that is not commanded back to zero. A simulator-generated linear model is not obtained at the second trim point because this open-loop batch simulation does not precisely trim the aircraft except at time equals zero, with an iterative numerical trim algorithm.

Figures 3 and 4 show the accuracy of the identified OKID linear models obtained in the example just described. Comparisons of the OKID-identified linear model are made to a simulator-generated linear model using a standard small perturbation differencing technique and to the nonlinear simulation itself. The same inputs that generate the nonlinear simulation responses are used as the inputs to both linear models for the comparison. Results from the first lateral/directional identification (Fig. 3) demonstrate that the OKID local linear model faithfully approximates the nonlinear model at this condition, and the Vinnicombe<sup>29</sup> metric had a value of 0.234. The second online identification sequence occurs from time equals 40-70 s during the same simulation run, but at a different flight condition. Note that although the aircraft is not perfectly trimmed at the second flight condition, Fig. 4 shows that the online-generated OKID local linear longitudinal model exhibits good accuracy to the nonlinear simulation model. The Vinnicombe metric for the second longitudinal identification had a value of 0.173. Because the

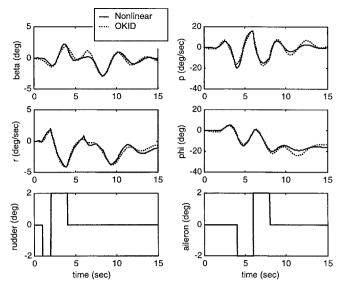


Fig. 3 Comparison of first lateral/directional model identification to nonlinear AV-8B simulation.

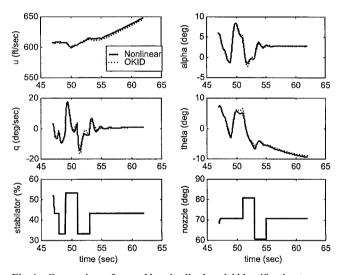


Fig. 4 Comparison of second longitudinal model identification to nonlinear AV-8B simulation.

second flight condition is at a higher dynamic pressure than the first flight condition and the magnitude of control excitations was the same as for the initial lower dynamic pressure flight condition, significantly larger and in fact perhaps excessively large perturbations were introduced. This can adversely affect the fidelity of the identified local linear model. The intent of this example is to demonstrate the basic feasibility of online system identification using the OKID method. Clearly, improved results can be achieved by proper scaling and scheduling of excitation magnitudes with flight condition. This consideration is beyond the scope of the present paper.

The examples that follow are sensor noise and external disturbance cases, using data from the UCAV6 nonlinear, non-real-time, six-degree-of-freedom flight simulation program of Ref. 31 as the nonlinear aircraft model. It is a hypothetical V/STOL-capable UCAV, based on a 70% scaled-down, uninhabited version of the AV-8B Harrier. Unlike the earlier results for the successive online identification example, the OKID linear models for the following examples are generated a posteriori in MATLAB®, using the m-file codes of the System/Observer/Controller Identification Toolbox for MATLAB of Ref. 32. The first case considers zero mean, Gaussian, white additive noise. The noise is injected into both the roll rate channel (0.005 variance), and body-axis side velocity channel (0.0004 variance). The latter results in noise in the sideslip angle measurement because sideslip angle is approximated in the linear model as the ratio of body-axis side velocity to total velocity. For

Table 1 Lateral/directional eigenvalue comparison in sensor noise test case

| OKID                                                                          | Linearizer                                                                     |
|-------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| $\lambda_{1,2} = -2.087 \pm 3.938j$ $\lambda_3 = -8.532$ $\lambda_4 = 0.0328$ | $\lambda_{1,2} = -1.413 \pm 4.578j$ $\lambda_3 = -7.364$ $\lambda_4 = 0.00835$ |

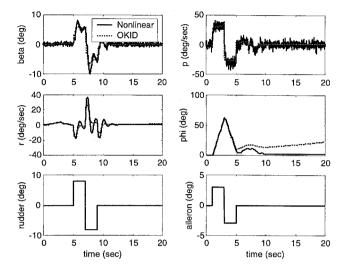


Fig. 5 Comparison of lateral/directional model identification in white noise to nonlinear UCAV6 simulation, N(0,0.005) roll rate, and N(0,1) side velocity.

the light noise characteristics considered in this test case, Fig. 5 indicates that, in a qualitative sense, the OKID method is relatively insensitive to additive sensor noise. Comparison of the eigenvalues in Table 1 shows that there are numerical differences between the true and identified models, particularly in the spiral mode, but the modal composition of the OKID linear model is correct. The Vinnicombe gap metric has a large value of 0.996, which is most likely due to the presence of noise in the system, which the metric is not specifically designed to cope with. Although a conservative result, use of some form of gap metric is desirable for autonomous applications. This is because time histories cannot be judged visually onboard the vehicle, and calculation of a gap metric is faster than simulating several seconds and comparing magnitudes or norms of tracking errors. With data sampling of 50 Hz and using MATLAB on a Pentium 120-MHz personal computer, the CPU time for this case is 5.7 s. When it is considered that four states and two controls are being identified in the presence of sensor noise and that an algorithm is used implemented in a software interpreter environment, this result indicates that with a more powerful processor and a hardware implemented algorithm, the identification might be achieved in near real time.

To evaluate OKID performance in the presence of moderate and severe discrete atmospheric disturbances, vertical gusts of 5 and 10 ft/s were introduced during the excitation input sequence. The vertical gusts were timed to be of as much disruption to the identification process as possible and resulted in an additional component in the body-axis vertical velocity w. Note that a positive vertical gust results is a nose-down motion because w is defined as positive downward in the north-east-down body-axis system. Figure 6 shows the result for a positive in sign, moderate vertical gust of 5 ft/s. To aid comparison of the results, the nonlinear time history shown in Fig. 6 does not contain the gust disturbance because the OKID algorithm is attempting to identify the undisturbed nonlinear model, as desired. Table 2 indicates that OKID correctly identified the modal composition, but overall the accuracy is seen to be mediocre, particularly for the frequency of the short-period mode. This result is not unexpected because the short-period mode tends to be strongly exhibited in body-axis pitch rate and in angle of attack, and a disturbance in w will be exhibited as a disturbance in angle of attack. The OKID algorithm has no way of knowing that the output data record

Table 2 Longitudinal eigenvalue comparison in 5-ft/s discrete vertical gust test case

| OKID                                                                      | Linearizer                                                                   |
|---------------------------------------------------------------------------|------------------------------------------------------------------------------|
| $\lambda_{1,2} = -3.1702 \pm 2.877j$ $\lambda_{3,4} = -0.0658 \pm 0.027j$ | $\lambda_{1,2} = -2.187 \pm 7.777j$<br>$\lambda_{3,4} = -0.0287 \pm 0.0815j$ |

Table 3 Longitudinal eigenvalue comparison in 10-ft/s discrete vertical gust test case

| OKID                                | Linearizer                             |
|-------------------------------------|----------------------------------------|
| $\lambda_{1,2} = -2.847 \pm 3.812j$ |                                        |
| $\lambda_3 = -0.175$                | $\lambda_{1,2} = -2.187 \pm 7.777j$    |
| $\lambda_4 = -0.0631$               | $\lambda_{3,4} = -0.0287 \pm 0.0815 j$ |

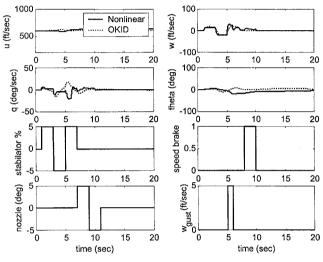


Fig. 6 Comparison of longitudinal model identification under 5-ft/s vertical gust to undisturbed nonlinear UCAV6 simulation.

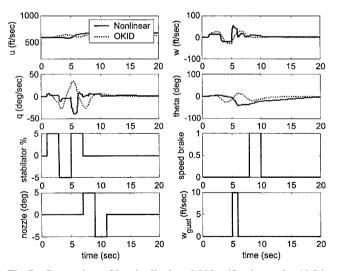


Fig. 7 Comparison of longitudinal model identification under 10-ft/s vertical gust to undisturbed nonlinear UCAV6 simulation.

contains the aircraft response to a disturbance, and so it identifies the excitation plus disturbance linear model. A Vinnicombe<sup>29</sup> gap metric value of 1 corroborates this conclusion.

Figure 7 shows the results for a positive in sign, severe vertical gust of 10 ft/s. For this stronger gust, Table 3 shows that the OKID algorithm is unable to predict even the correct modal composition of the aircraft dynamics. The Vinnicombe<sup>29</sup> gap metric reports a value of 1 for this case. Although the order of the model is correct, and the modes are correctly identified as being stable, and the short-period mode is predicted fairly well, the OKID algorithm incorrectly predicts first-order modes, whereas the linearizer-generated model

of the undisturbed aircraft contains the standard short-period and phugoid modes. Incorrect modal composition of an identified linear model, as occurred in this example, in general renders the identified model useless. Most system identification techniques have some degree of difficulty handling external disturbances. The OKID technique is no different; as both disturbances examples indicate, it is not reliable for linear system identification of nonlinear systems subjected to anything more than mild discrete external disturbances. Because the basic formulation of OKID does not explicitly account for external disturbances, this result is not unexpected.

#### **Conclusions**

The OKID methodology, a time-domain input-output mapping technique originally intended for flexible structure a posteriori system identification, has been applied to the problem of on demand, online linear model system identification of nonlinear, rigid-body aircraft dynamics. A formulation suitable for nonzero initial conditions (suitable for aircraft trim) was developed and embedded to run online in a nonlinear, non-real-time, six-degree-of-freedomsimulationprogramof an AV-8B Harrier. An example using this system was presented for consecutive online longitudinal and lateral/directional identifications at different trim conditions, and additional examples for the cases of sensor noise and discrete external disturbances (gusts) were generated using a nonlinear, non-real-time, six-degree-of-freedom simulation program of a UCAV. Results in the examples were compared using eigenvalues, time histories, and the Vinnicombe<sup>29</sup> gap metric.

Based on the results presented, it is concluded that the OKID identification method is entirely suitable, without major modification, for the problem of generating online linear system models of nonlinear, rigid-body aircraft dynamics. As with any identification method, accuracy of OKID results is strongly dependent on the ability to excite system modes sufficiently, without driving the system unstable. This entails proper selection of which control effectors to use for injecting excitation inputs and the magnitudes of these inputs. It is particularly important for aircraft applications because the aircraft response to control inputs varies strongly with flight condition.

The nonzero initial condition feature of OKID allows the online identification of accurate linear aircraft models at various flight conditions, even when the aircraft is not perfectly trimmed, that is, when the system was not in steady state.

Values for the Vinnicombe<sup>29</sup> gap metric tended to be very conservative for the noise and disturbance cases. This does not necessarily mean that an identified model was totally unsuitable, but another candidate measure to consider would be the frequency-by-frequency equivalent of the Nu gap metric, which is the chordal distance of the generalized stability margin as a function of frequency.

Finally, for the class of system studied, OKID is relatively insensitive to moderate-intensity Gaussian white sensor noise on multiple channels, and light-to-moderate-intensity discrete gust disturbances. However, identification accuracy degrades significantly in the presence of large magnitude, high-intensity discrete gust disturbances.

#### Acknowledgments

This research is sponsored the U.S. Navy, Office of Naval Research under Grant Number N00014-97-1-0938. The Project Monitor is Allen Moshfegh. This support is gratefully appreciated by the authors. The authors also thank the reviewers and Associate Editor for their insightful comments and suggestions which improved the paper.

## References

<sup>1</sup>Akella, M., Subbarao, K., and Junkins, J. L., "Nonlinear Adaptive Auto-Pilot for Uninhabited Aerial Combat Vehicles," *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA, Reston, VA, 1999, pp. 1353–1363.

<sup>2</sup>Verma, A., Subbarao, K., and Junkins, J. L., "Novel Trajectory Tracking Methodology Using Structured Adaptive Model Inversion for Uninhabited Aerial Vehicles," *Proceedings of the American Control Conference*, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, 2000, pp. 859–863.

<sup>3</sup>Verma, A., and Junkins, J. L., "Inverse Dynamics Approach for Real-Time Determination of Feasible Aircraft Reference Trajectories," *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA, Reston, VA, 1999, pp. 545–554.

<sup>4</sup>Krishnamurthy, K., and Ward, D. T., "Intelligent Inference Engine for Autonomous Aerial Vehicles," *Proceedings of the AIAA Atmospheric Flight Mechanics Conference and Exhibit*, AIAA, Reston, VA, 1999, pp. 650–660.

<sup>5</sup>Krishnamurthy, K., and Ward, D. T., "Intelligent Flight Director for Autonomous Aircraft," *Proceedings of the Aerospace Sciences Meeting and Conference*, AIAA, Reston, VA, 2000, pp. 321–330.

<sup>6</sup>Krishnamurthy, K., and Ward, D. T., "Intelligent Systems for Autonomous Aircraft," *Proceedings of the IEEE Systems, Man and Cybernetics Meeting*, Vol. 4, Inst. of Electrical and Electronics Engineers, Piscataway, NJ, 2000, pp. 2369–2374.

<sup>7</sup>Morelli, E. A., "Real-Time Parameter Estimation in the Frequency Domain," *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit, AIAA, Reston, VA, 1999, pp. 477–487.* 

<sup>8</sup>Lee, D.-C., and Nagati, M. G., "Improved Coupled Force and Moment Parameter Estimation for Aircraft," *Proceedings of the AIAA Atmospheric Flight Mechanics Conference and Exhibit*, AIAA, Reston, VA, 1999, pp. 484–492.

<sup>9</sup>Elgersma, M., Enns, D., Shald, S., and Voulgaris, P., "Parameter Identification for Systems with Redundant Actuators," *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit*, AIAA, Reston, VA, 1998, pp. 109–117.

<sup>10</sup>Oelker, H.-C., and Friehmelt, H., "Aerodynamic System Identification of Nonlinear Rudder Characteristics Based on Flight Test Data," *Proceedings of the AIAA Atmospheric Flight Mechanics Conference and Exhibit*, AIAA, Reston, VA, 1998, pp. 248–258.

<sup>11</sup>Morelli, E. A., and Klein, V., "Accuracy of Aerodynamic Model Parameters Estimated from Flight-Test Data," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 1, 1997, pp. 74–80.

<sup>12</sup> Friehmelt, H., "Quality Effects on Parameter Identification: An X-31A Study Case," *Proceedings of the AIAA Atmospheric Flight Mechanics Conference and Exhibit*, 1997, pp. 784–790.

<sup>13</sup>Morelli, E. A., "In-Flight System Identification," *Proceedings of the AIAA Atmospheric Flight Mechanics Conference and Exhibit*, AIAA, Reston, VA, 1998, pp. 1746–1756.

<sup>14</sup>Ward, D. G., Monaco, J. F., and Bodson, M., "Development and Flight Testing of a Parameter Identification Algorithm for Reconfigurable Control," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 6, 1998, pp. 948– 956

<sup>15</sup>Buffington, J., Chandler, P., and Pachter, M., "Integration of On-Line System Identification and Optimization-Based Control Allocation," Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit, AIAA, Reston, VA, 1998, pp. 1746–1756.

<sup>16</sup>Iliff, K. W., "Parameter Estimation for Flight Vehicles," *Journal of Guidance Control and Dynamics*, Vol. 12, No. 5, 1989, pp. 609–622.

ance, Control, and Dynamics, Vol. 12, No. 5, 1989, pp. 609–622.

<sup>17</sup>Ho, B. L., and Kalman, R. E., "Effective Construction of Linear

State-Variable Models from Input-Output Data," *Proceedings of the 3rd Annual Allerton Conference on Circuit and System Theory*, Univ. of Illinois, Monticello, IL, 1965, pp. 449–459.

<sup>18</sup> Juang, J.-N., and Pappa, R. S., "An Eigensystem Realization Algorithm for Modal Parameter Identification and Model Reduction," *Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 5, 1985, pp. 620–627.

<sup>19</sup>Pappa, R. S., Schenk, A., and Noll, C., "Some Experiences with the Eigensystem Realization Algorithm," *Sound and Vibration*, Vol. 22, No. 1, 1988, pp. 30–35.

<sup>20</sup>Juang, J.-N., and Pappa, R. S., "Effects of Noise on Modal Parameters Identified by the Eigensystem Realization Algorithm," *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 3, 1986, pp. 294–303.

<sup>21</sup>Cooper, J. E., "On-Line Version of the Eigensystem Realization Algorithm Using Data Correlations," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 1, 1997, pp. 137–142.

<sup>22</sup>Phan, M. Q., Juang, J.-N., and Longman, R. W., "Identification of Lin-

<sup>22</sup>Phan, M. Q., Juang, J.-N., and Longman, R. W., "Identification of Linear Multivariable System by Identification of Observer with Assigned Real Eigenvalues," *Journal of the Astronautical Sciences*, Vol. 40, No. 2, 1992, pp. 261–279.

pp. 261–279.

<sup>23</sup>Phan, M. Q., Horta, L. G., Juang, J.-N., and Longman, R. W., "Identification of Linear System by an Asymptotically Stable Observer," NASA TP 3164, June 1992.

<sup>24</sup>Juang, J.-N., Phan, M. Q., Horta, L. G., and Longman, R. W., "Identification of Observer/Kalman Filter Markov Parameters: Theory and Experiments," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 2, 1993, pp. 320–329.

pp. 320–329.

<sup>25</sup>Chen, C.-W., Huang, J.-K., Phan, M. Q., and Juang, J.-N., "Integrated System Identification and State Estimation for Control of Flexible Space Structures," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 1, pp. 88–95.

pp. 88–95.

<sup>26</sup>Phan, M. Q., Goodzeit, N. E., and Juang, J.-N., "Identification of System and Periodic Disturbances," *Proceedings of the 1997 ASME Design Engineering Technical Conferences*, American Society of Mechanical Engineers, Fairfield, NJ, 1997, pp. 1–10.

<sup>27</sup>Juang, J.-N., and Eure, K. W., "Predictive Feedback and Feedforward Control for Systems with Unknown Disturbances," NASA TM 208744, Dec. 1908

<sup>28</sup>Goodzeit, N. E., and Phan, M. Q., "System Identification in the Presence of Completely Unknown Periodic Disturbances," *Journal of Guidance, Control, and Dynamics*, Vol. 23, No. 2, 2000, pp. 251–259.

<sup>29</sup>Vinnicombe, G., "Frequency Domain Uncertainty and the Graph Topology," *IEEE Transactions on Automatic Control*, Vol. 38, No. 9, 1993, pp. 1371–1383.

pp. 1371–1383.

30 Ziegler, G., "AV-8B Batch Simulator User's Guide," Aerospace Engineering, Texas A&M Univ., College Station, TX, Sept. 1997.

<sup>31</sup>Smith, D. M., "UCAV6 Batch Simulator User's Guide," Aerospace Engineering, Texas A&M Univ., College Station, TX, June 1999.

<sup>32</sup>Juang, J.-N., Horta, L.G., and Phan, M. Q., "User's Guide for System/Observer/Controller Identification Toolbox," NASA TM 107566, Feb. 1992.