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Identification of Modal Parameters from Ambient Vibration Data by Modified Eigensystem Realization Algorithm*

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ABSTRACT

In the present paper, we propose a modification to the Eigensystem Realization Algorithm with Data Correlation (ERA/DC) for modal-parameter identification of structural systems subjected to stationary white-noise ambient vibration. By setting up a correlation-function matrix of stationary responses and introducing an appropriate factorization of the matrix, modal parameters of a system could then be effectively identified through the singular-value decomposition and eigenvalue analysis. Through numerical simulations, effectiveness and robustness of the proposed identification method is demonstrated.

Keywords: Eigensystem Realization Algorithm with Data Correlation (ERA/DC), Correlation technique, Stationary ambient vibration

I. INTRODUCTION

Experimental identification of modal parameters of a structure is usually carried out by measuring both its input and corresponding output. Some modal testing techniques use free or impulse response of structures so that the input (excitation) need not be measured. However, there are situations where controlled excitation or initial excitation may not be available, such as the case of in-operation testing or in-flight measurement. Consequently, it is desirable to develop techniques for modal-parameter identification directly from ambient vibration data, i.e., without the need of input measurement.

Numerous papers have been presented on system identification, which acquires its estimation of important parameters from the measured data. Ho and Kalman [1] proposed the important principles of minimal realization theory, which uses a sequence of real matrices known as Markov parameters (impulse response functions) to construct a state-space representation of a linear system. The error characteristics due to noise, however, are yet to be estimated. Zeiger and McEwen [2] proposed a concept combining singular-value decomposition (SVD) and minimal realization algorithm. Among follow-up developments on SVD and minimal realization algorithm, Juang and Pappa [3] proposed Eigensystem Realization Algorithm (ERA), which has been applied to identify modal parameters from impulse response of a system. Juang et al. also proposed a modification of ERA, generally known as ERA/DC (Eigensystem Realization Algorithm with Data Correlation) [4]. ERA/DC uses data correlations to reduce the noise effect in the process of modal-parameter identification. Ibrahim et al. [5, 6] presented a time-domain modal-identification method, usually known as ITD. ITD uses free-decay responses of a structure to identify its modal parameters. The method is effective in identifying complex modes even with closely spaced modes [7]. James et al. [8] developed the so-called Natural Excitation Technique (NExT), which uses correlation functions of measured response in combination with a time-domain parameter extraction scheme under the assumption of white-noise input. Chiang and Cheng [9] presented a correlation technique for modal-parameter identification of a linear, complex-mode system subjected to stationary ambient excitation. Chiang and Lin [10] further extended the correlation technique for modal-parameter identification of a system excited by non-stationary white noise in the form of a product model.

In the present paper, a modification to ERA/DC is presented for linear systems excited by stationary

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white-noise ambient vibration. By setting up a correlation-function matrix of the stationary ambient responses and introducing an appropriate factorization of the matrix, modal parameters of a system could then be identified using the singular-value decomposition (SVD) and eigenvalue analysis. Through numerical simulation, the applicability and robustness is examined of the proposed method for identifying modal parameters of a linear system from measurements contaminated with noise.

II. CORRELATION TECHNIQUE

James et al. [8] developed the so-called Natural Excitation Technique (NExT) using the correlation technique. It was shown that the cross-correlation between two response signals of a linear system with classical normal modes and subjected to white-noise inputs is of the same form as free vibration decay or impulse response. In combination with a time-domain parameter extraction scheme, such as the ITD method, this concept becomes a powerful tool for the identification analysis of structures under stationary ambient vibration.

When a system is excited by stationary white noise, the cross-correlation function $R_{ij}(\tau)$ between two stationary response signals $x_i(t)$ and $x_j(t)$ can be shown to be [9]

$$R_{ij}(\tau)=\sum_{r=1}^{n}\frac{\phi_{ir}A_{jr}}{m_r\omega_{dr}}\exp(-\zeta_r\omega_r\tau)\sin(\omega_{dr}\tau+\theta_r) \tag{1}$$

where ϕ_{ir} denotes the i-th component of the r-th mode shape, A_{jr} a constant, and m_r the r-th modal mass. The result above shows that $R_{ij}(\tau)$ in Eqn. (1) is a sum of complex exponential functions (modal responses), which is of the same mathematical form as the free vibration decay or the impulse response of the original system. Thus, the cross-correlation functions evaluated of responses data can be used as free vibration decay or as impulse response in time-domain modal extraction schemes so that measurement of white-noise inputs can be avoided. It is remarkable that the term $\phi_{ir}A_{jr}$ in Eqn. (1) will be identified as the mode-shape components. In order to eliminate the A_{jr} term and retain the true mode-shape components ϕ_{ir}, all the measured channels are correlated against a common reference channel, say x_j. The identified components then all possess the common A_{jr} component, which can be normalized out to obtain the desired mode shape ϕ_{ir}.

III. MODIFIED EIGENSYSTEM REALIZATION ALGORITHM WITH DATA CORRELATION

Juang et al. [4] proposed a modification of ERA called the Eigensystem Realization Algorithm with Data Correlation (ERA/DC), which uses data correlations to reduce the noise effect for modal-parameter identification. It has been shown in the previous section that the correlation functions between two stationary response signals of a structure subjected to white-noise inputs can be treated as free vibration decay or as impulse response for further extraction of modal parameters. The correlation functions can thus be used to extract modal parameters of a system in the case of stationary ambient vibration. In the present paper, a modification to ERA/DC is presented for linear systems excited by stationary white-noise ambient vibrations. By setting up a correlation-function matrix of the stationary responses and introducing a proper factorization of the matrix, modal parameters of a system could then be identified through the singular-value decomposition (SVD) and eigenvalue analysis.

Consider an n-dimensional, discrete-time, linear, time-invariant dynamic system subjected to the excitation of a zero-mean stationary white noise $\boldsymbol{u}_k=\{u_i, i=1\sim k\}$. For finite but sufficiently long records, say of l time steps, we define the correlation-function matrix $\boldsymbol{T}_\tau$, which is composed of measured responses from every channel with different time-delayed signals, can be approximately expressed as

$$\boldsymbol{T}_\tau\equiv\boldsymbol{T}(\tau)\approx\frac{1}{l}\left[\boldsymbol{Y}_{k+\tau}\boldsymbol{Y}_k^T\right]_{p\times p} \tag{2}$$

where $\boldsymbol{Y}_k=\{y_{ij}, i=1\sim p, j=k-l\sim k\}$ signifies the measured responses of the system at the time-step k, and p is the number of the channel of the measured response. Note that the correlation-function matrix $\boldsymbol{T}_\tau$ of the measured response of the system could be obtained from a single sample function of time through use of the ergodic property of stationary random processes. Define the data-correlation matrix $\overline{\boldsymbol{R}}(k)$ to be formed using the correlation-function matrix $\boldsymbol{T}_\tau$, then in view of Eqn.(2), $\overline{\boldsymbol{R}}(k)$ can be therefore written as

$$\overline{\boldsymbol{R}}(k)=\begin{bmatrix}\boldsymbol{T}_k & \boldsymbol{T}_{k-1} & \cdots & \boldsymbol{T}_{k-\alpha+1}\\ \boldsymbol{T}_{k+1} & \boldsymbol{T}_k & \cdots & \boldsymbol{T}_{k-\alpha+2}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{T}_{k+\alpha-1} & \boldsymbol{T}_{k+\alpha-2} & \cdots & \boldsymbol{T}_k\end{bmatrix}_{\alpha p\times\alpha p} \tag{3}$$

where α is the number of the expansion channels in $\overline{\boldsymbol{R}}(k)$. Note that the correlation-function matrix $\boldsymbol{T}_\tau$ can be viewed as the impulse functions (also known as the Markov parameters) in the conventional ERA/DC process and can be expressed as [11]

$$\boldsymbol{T}_\tau\approx\boldsymbol{C}\boldsymbol{A}^{\tau-1}\boldsymbol{G} \tag{4}$$

where $\boldsymbol{C}$ is the $p\times n$ output influence matrix for the

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state vector, $\boldsymbol{A}$ is the $n \times n$ state matrix with dynamic characteristics of the system, and $\boldsymbol{G} \approx \frac{1}{l}\left[\boldsymbol{X}_{k+1}\boldsymbol{Y}_k^T\right]_{n\times p}$ is the so-called covariance matrix, where $\boldsymbol{X}_{k+1}$ is the $n \times l$ state-vectors matrix of the system. With this factorization, Eqn.(3) can be formulated as

$$\begin{aligned}\overline{\boldsymbol{R}}(k) &= \begin{bmatrix} \boldsymbol{CA}^{k-1}\boldsymbol{G} & \boldsymbol{CA}^{k-2}\boldsymbol{G} & \cdots & \boldsymbol{CA}^{k-\alpha}\boldsymbol{G} \\ \boldsymbol{CA}^{k}\boldsymbol{G} & \boldsymbol{CA}^{k-1}\boldsymbol{G} & \cdots & \boldsymbol{CA}^{k-\alpha+1}\boldsymbol{G} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{CA}^{k+\alpha-2}\boldsymbol{G} & \boldsymbol{CA}^{k+\alpha-3}\boldsymbol{G} & \cdots & \boldsymbol{CA}^{k-1}\boldsymbol{G} \end{bmatrix}_{\alpha p\times\alpha p} \\ &= \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA} \\ \vdots \\ \boldsymbol{CA}^{\alpha-1} \end{bmatrix} \boldsymbol{A}^k \begin{bmatrix} \boldsymbol{A}^{-1}\boldsymbol{G} & \boldsymbol{A}^{-2}\boldsymbol{G} & \cdots & \boldsymbol{A}^{-\alpha}\boldsymbol{G} \end{bmatrix} \\ &\equiv \overline{\boldsymbol{P}}_\alpha \boldsymbol{A}^k \overline{\boldsymbol{Q}}_\alpha \end{aligned} \tag{5}$$

To further reduce the effect of noise, and to make the parameters estimation more accurate, we construct a generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ as follows:

$$\overline{\boldsymbol{U}}(k) = \begin{bmatrix} \overline{\boldsymbol{R}}(k) & \overline{\boldsymbol{R}}(k+1) & \cdots & \overline{\boldsymbol{R}}(k+b-1) \\ \overline{\boldsymbol{R}}(k+1) & \overline{\boldsymbol{R}}(k+2) & \cdots & \overline{\boldsymbol{R}}(k+b) \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\boldsymbol{R}}(k+a-1) & \overline{\boldsymbol{R}}(k+a) & \cdots & \overline{\boldsymbol{R}}(k+a+b-2) \end{bmatrix}_{\alpha pa\times\alpha pb} \tag{6}$$

where the integers a and b define how many correlation lags of $\overline{\boldsymbol{R}}(k)$ are included in the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ analysis. To make use of the property of consistency in the theory of system identification, we tend to use more measurement data in the analysis. Eqn.(6) signifies the use of expanded measurement samples through constructing a generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$. This will reduce the effect of noise. Substituting Eqn.(5) into Eqn.(6), the following equation can be derived:

$$\begin{aligned}\overline{\boldsymbol{U}}(k) &= \begin{bmatrix} \overline{\boldsymbol{P}}_\alpha \\ \overline{\boldsymbol{P}}_\alpha \boldsymbol{A} \\ \vdots \\ \overline{\boldsymbol{P}}_\alpha \boldsymbol{A}^{a-1} \end{bmatrix} \boldsymbol{A}^k \begin{bmatrix} \overline{\boldsymbol{Q}}_\alpha & \boldsymbol{A}\overline{\boldsymbol{Q}}_\alpha & \cdots & \boldsymbol{A}^{b-1}\overline{\boldsymbol{Q}}_\alpha \end{bmatrix} \\ &= \overline{\mathbb{P}}_a \boldsymbol{A}^k \overline{\mathbb{Q}}_b \end{aligned} \tag{7}$$

where $\overline{\mathbb{P}}_a$ and $\overline{\mathbb{Q}}_b$ are defined as generalized observability and controllability matrices, respectively. In order to identify system modes effectively, the sum of α and a should be greater than the number of modes to be identified.

Based on system realization theory, ERA/DC constructs a discrete state-space model of minimal order using the singular-value decomposition (SVD) analysis. To determine the system order, which is defined as the number of structural modes involved in the responses, SVD of the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ is performed. The rank of the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ is the system order, which is just the number of the obvious non-zero n singular values. Therefore, the state matrix $\boldsymbol{A}$ can then be obtained by retaining the first n sub-vectors as follows:

$$\boldsymbol{A} = \overline{\boldsymbol{S}}_n^{-1/2}\overline{\boldsymbol{V}}_n^T\overline{\boldsymbol{U}}(1)\overline{\boldsymbol{W}}_n\overline{\boldsymbol{S}}_n^{-1/2} \tag{8}$$

where $\overline{\boldsymbol{V}}_n$ and $\overline{\boldsymbol{W}}_n$ are the matrices with orthonormal property, i.e., $\overline{\boldsymbol{V}}_n^T\overline{\boldsymbol{V}}_n = \boldsymbol{I}_n = \overline{\boldsymbol{W}}_n^T\overline{\boldsymbol{W}}_n$, and $\overline{\boldsymbol{S}}_n$ is a diagonal matrix containing the n largest singular values. Denote the matrices of eigenvalues and eigenvectors of the n–order state matrix $\boldsymbol{A}$ as $\boldsymbol{Z} = diag\left(z_1, z_2, \cdots, z_n\right)$ and $\boldsymbol{\Psi} = \left[\psi_1, \psi_2, \cdots, \psi_n\right]$, respectively. The eigenvalues contain the information of modal frequencies and damping ratios. Thus, once the state matrix $\boldsymbol{A}$ is obtained from measured data, the modal parameters of the structure system of interest can then be determined by solving the eigenvalue problem associated with the state matrix $\boldsymbol{A}$. The state matrix $\boldsymbol{A}$ thus can be expressed as

$$\boldsymbol{A} = \boldsymbol{\Psi}\boldsymbol{Z}\boldsymbol{\Psi}^{-1} \tag{9}$$

Substituting Eqn.(9) into Eqn.(4), the following equation can be derived:

$$\boldsymbol{T}_\tau = \boldsymbol{C}_m\boldsymbol{Z}^{\tau-1}\boldsymbol{G}_m \tag{10}$$

where $\boldsymbol{C}_m = \boldsymbol{C\Psi}$ and $\boldsymbol{G}_m = \boldsymbol{\Psi}^{-1}\boldsymbol{G}$. Note that $\boldsymbol{C}_m$ contains the modal shapes of the system. The eigenvalues of the original vibration system in the continuous-time domain can be evaluated as follows [8]:

$$\lambda_i = \frac{\ln z_i}{\Delta t} = \lambda_i^R + \lambda_i^I = -\zeta_i\omega_i + i\omega_i\sqrt{1-\zeta_i^2} \tag{11}$$

where λ_i^R and λ_i^I are the real and imagery parts of λ_i, respectively, and Δt is the sampling interval. The natural frequencies and damping ratios of the system could then be obtained from Eqn.(11):

$$\omega_i = \sqrt{\left(\lambda_i^R\right)^2 + \left(\lambda_i^I\right)^2} \tag{12}$$

$$\zeta_i = \frac{-\lambda_i^R}{\sqrt{\left(\lambda_i^R\right)^2 + \left(\lambda_i^I\right)^2}} \tag{13}$$

It has been shown that a modification to the Eigensystem Realization Algorithm with Data Correlation (ERA/DC) is presented for modal-parameter identification from stationary ambient vibration data. By setting up a correlation-function matrix of ambient responses of structure subjected to stationary white noise and introducing an appropriate factorization of the matrix, modal parameters of a system could then be identified through SVD and the eigenvalue analysis.

IV. NUMERICAL SIMULATION AND DISCUSSION

To demonstrate the feasibility and effectiveness of the proposed method, we consider a linear 6-dof chain model with viscous damping. A schematic representation of this model is shown in Fig.1. The mass matrix $\boldsymbol{M}$, stiffness matrix $\boldsymbol{K}$, and the proportional damping matrix $\boldsymbol{C}$ of the system are given as follows:

$$\boldsymbol{M} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix} kg$$

$$\boldsymbol{K} = 600 \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 3 & -2 \\ 0 & 0 & 0 & 0 & -2 & 5 \end{bmatrix} {}^{N}\!/_{m}$$

$$\boldsymbol{D} = 0.1\boldsymbol{M} + 0.001\boldsymbol{K} \quad {}^{N \cdot \sec}\!/_{m}$$

Consider that the ambient vibration input can be modeled as stationary white noise. The stationary white noise is generated using the spectrum approximation method [12] as a zero-mean band-pass noise, whose standard deviation is $0.02\ \mathrm{N}^2 \cdot \sec\ / rad$ with a frequency range from 0 to 50 Hz. The sampling interval is chosen as $\Delta t = 0.01\ \sec$, and the sampling period is $T = N_t \cdot \Delta t = 1310.72 \sec$. The stationary white noise simulated serves as the excitation force acting on the 6th mass point of the system. The time signal of a simulated sample of the stationary white noise and the power spectrum are shown in Fig.2 and Fig.3, respectively. The displacement responses of the system were obtained using Newmark's method.

To examine the robustness of the proposed identification approach under noisy conditions, 10% noise is added to the simulated displacement response data. For data with sufficiently long duration, the correlation-function matrix $\boldsymbol{T}_\tau$ of the responses of the

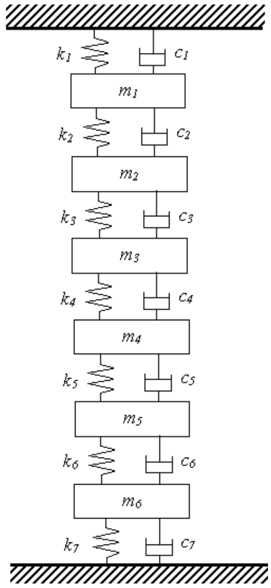

Figure 1 Schematic plot of the 6-dof chain system

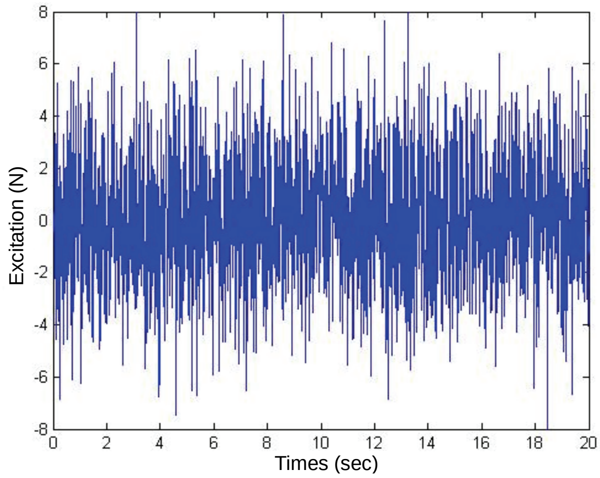

Figure 2 A sample function of stationary white noise in time domain

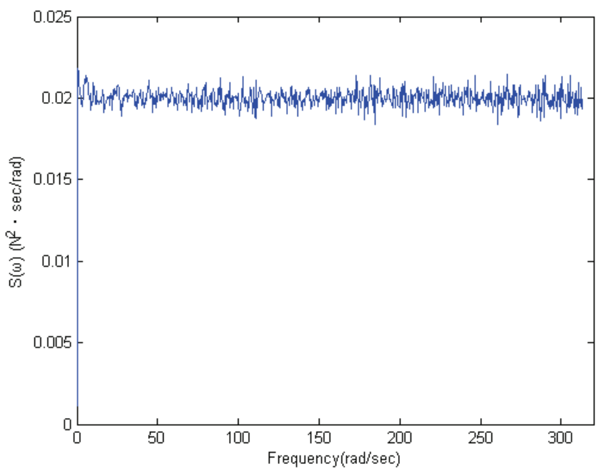

Figure 3 Power spectrum of the simulated stationary white noise

system could be estimated from a single sample function of time (ergodicity assumption). Once the correlation-function matrix $\boldsymbol{T}_\tau$ was evaluated from Eqn.(2), the data-correlation matrix $\overline{\boldsymbol{R}}(k)$ and a generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ could be further constructed. The state matrix $\boldsymbol{A}$ can thus be estimated through the SVD analysis and modal parameters of the system could then be identified by solving the eigenvalue problem associated with the state matrix $\boldsymbol{A}$.

In theory, a continuum structure has an infinite number of degrees of freedom and an infinite number of modes. In practice, we do not know in advance how many modes are required to describe the dynamical behavior of the observed structural system. However, the number of important modes of the system under consideration could be roughly found by using the singular-value decomposition analysis. The number of non-zero singular values is the rank of the generalized Hankel matrix $\overline{\boldsymbol{U}}(k)$ and is also the order of the system. By examining the distribution of the singular values associated with the responses of the system, we could determine the order, as well as the number of modes to be identified, of the system. The result is shown in Fig.4, from which the order of the system model was determined to be 12.

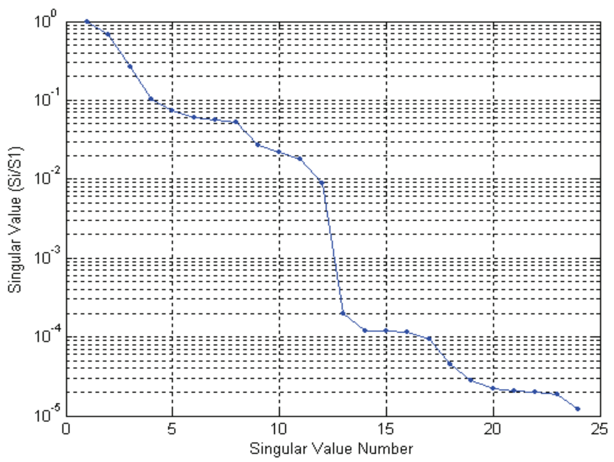

Figure 4 Singular values associated with the responses corresponding to stationary white noise input

The modal frequencies and damping ratios can be obtained via Eqns.(11), (12) and (13) following the eigen-analysis of the state matrix $\boldsymbol{A}$, and the mode shapes can be obtained from $\boldsymbol{C}_m$ as given in Eqn.(10). The results of modal-parameter identification are summarized in Table 1 and Table2, respectively, from which we observe that in general the errors in natural frequencies are less than 1 % and the errors in damping ratios are less than 10%. The identified mode shapes are also compared with the exact mode shapes in Fig.5. The errors of identified damping ratios and mode shapes are somewhat higher due to the fact that the response of the system generally has lower sensitivity to these modal parameters than to the modal frequencies. It is seen that good results of identification have been obtained from the simulated ambient response data contaminated with 10% noise. The results of modal-parameter identification through the conventional ITD [5,6] and the ERA [3] are also summarized in Table 1, Table 2, and Fig.5. It should be mentioned that the conventional ITD or the ERA method uses only free-vibration or impulse response of a structure to identify modal parameters. Therefore, in order to convert the original forced responses into free-vibration data, we need additional treatments, such as employing the correlation technique. Furthermore, as to the correlation technique, the choice of a reference channel for computing correlation functions is significant to the result of identification of modal parameters. The reference channel is usually chosen as the response channel whose Fourier spectrum has rich frequency content around the modes of interest [10]. If we choose a reference channel with poor frequency content, the identification results will also be poor. In contrast, in the present method, we directly compute the correlation-function matrix $\boldsymbol{T}_\tau$, each row of which is treated as the impulse response corresponding to each DOF. We could thus take advantage of the computation of $\boldsymbol{T}_\tau$ to get rid of the additional treatment of converting forced responses into the free-vibration data and the selection of reference channel.

V. CONCLUSIONS

A modification to the Eigensystem Realization Algorithm with Data Correlation (ERA/DC) is presented in this paper for modal-parameter identification from stationary white-noise ambient vibration data. By setting up a correlation-function matrix $\boldsymbol{T}_\tau$ of stationary ambient responses and introducing an appropriate factorization of the correlation-function matrix, modal parameters of a system could then be identified via the singular-value decomposition and eigenvalue analysis. Through numerical simulations, the effectiveness and robustness of the proposed identification method is demonstrated in a moderately noisy condition. By taking advantage of the computation of correlation-function matrix $\boldsymbol{T}_\tau$, we avoid the additional treatment of converting the original forced-vibration data into the free-vibration data and the selection of reference channel for computing correlation.

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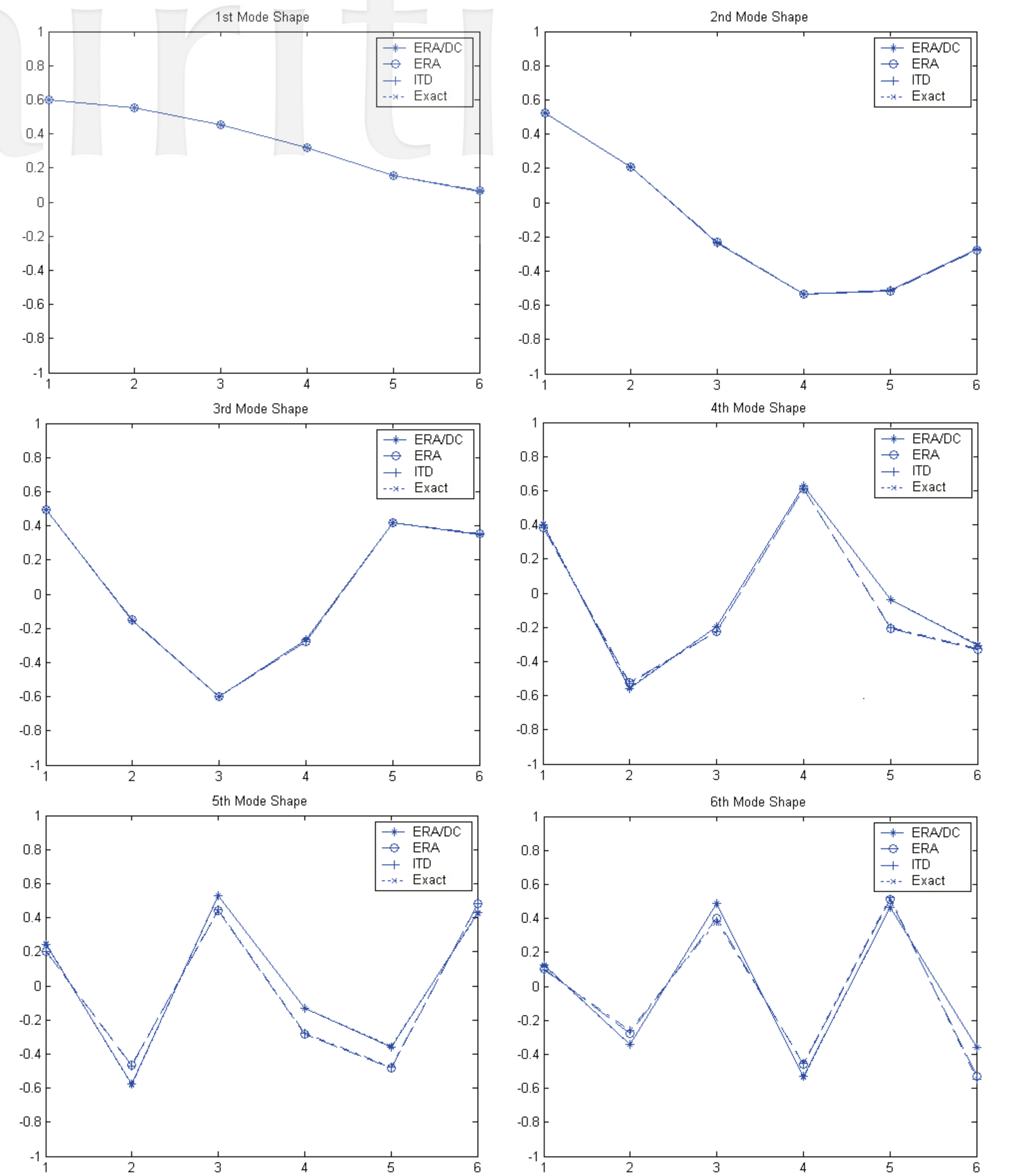

Figure 5 Comparison between the identified mode shapes and the exact mode shapes of the 6-dof system subjected to stationary white noise input

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Table 1 Results of natural frequency identification of the 6-dof. system subjected to stationary white noise input

Mode	Natural Frequency (rad/sec)						
	Exact	ERA/DC	Error(%)	ERA	Error(%)	ITD	Error(%)
1	5.03	5.02	0.31	5.03	0.00	5.03	0.02
2	13.45	13.42	0.21	13.42	0.20	13.43	0.17
3	19.80	19.71	0.44	19.71	0.42	19.72	0.40
4	26.69	26.49	0.73	26.50	0.68	26.51	0.64
5	31.66	31.37	0.91	31.38	0.89	31.39	0.85
6	33.73	33.34	1.14	33.36	1.10	33.40	0.98

Table 2 Results of damping ratio identification of the 6-dof. system subjected to stationary white noise input

Mode	Damping Ratio (%)						
	Exact	ERA/DC	Error(%)	ERA	Error(%)	ITD	Error(%)
1	1.25	1.27	2.06	1.32	6.25	1.31	5.43
2	1.04	1.08	3.57	1.08	3.75	1.07	2.67
3	1.24	1.30	4.62	1.31	5.31	1.29	4.19
4	1.52	1.62	6.54	1.58	4.07	1.53	0.68
5	1.74	1.80	3.43	1.77	1.46	1.70	2.31
6	1.83	1.95	6.27	1.73	5.92	1.83	0.40

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