Lecture 2:
Image Classification with Linear Classifiers

Administrative: Assignment 1

Out tomorrow, Due 4/15 11:59pm

- K-Nearest Neighbor
- Linear classifiers: SVM, Softmax
- Two-layer neural network
- Image features

Administrative: Course Project

Project proposal due 4/18 (Monday) 11:59pm

Find your teammates on Ed (the pinned “Search for Teammates” post)

“Is X a valid project for 231n?” --- Ed private post / TA Office Hours

More info on the website

Administrative: Discussion Sections

This Friday 1:30pm-2:30 pm (recording will be made available)

Python / Numpy, Google Colab

Presenter: Manasi Sharma (TA)

Syllabus

Deep Learning Basics → Convolutional Neural Networks → Computer Vision Applications

Data-driven approaches
Linear classification & kNN
Loss functions
Optimization
Backpropagation
Multi-layer perceptrons
Neural Networks

Convolutions
PyTorch / TensorFlow
Activation functions
Batch normalization
Transfer learning
Data augmentation
Momentum / RMSProp / Adam
Architecture design

RNNs / Attention / Transformers
Image captioning
Object detection and segmentation
Style transfer
Video understanding
Generative models
Self-supervised learning
3D vision
Human-centered AI
Fairness & ethics

Image Classification

A Core Task in Computer Vision

Today:

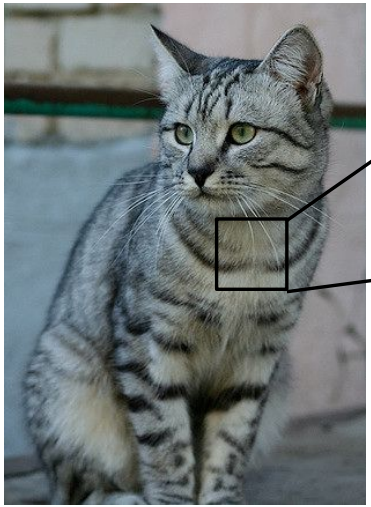
- The image classification task
- Two basic data-driven approaches to image classification
  - K-nearest neighbor and linear classifier

Image Classification: A core task in Computer Vision

(assume given a set of possible labels)
{dog, cat, truck, plane, ...}

cat

The Problem: Semantic Gap




```
[[105 112 108 111 104  99 106  99  96 103 112 119 104  97  93  87]
 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 128 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
 [114 108  85  55  55  69  64  54  64  87 112 129  98  74  84  91]
 [133 137 147 103  65  81  80  65  52  54  74  84 102  93  85  82]
 [128 137 144 140 109  95  86  70  62  65  63  63  60  73  86 101]
 [125 133 148 137 119 121 117  94  65  79  80  65  54  64  72  98]
 [127 125 131 147 133 127 126 131 111  96  89  75  61  64  72  84]
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 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```
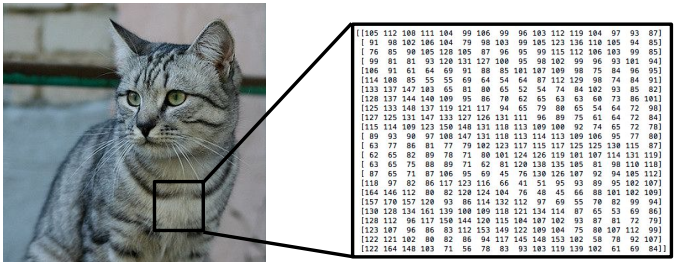

What the computer sees

An image is a tensor of integers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



All pixels change when the camera moves!

Challenges: Illumination

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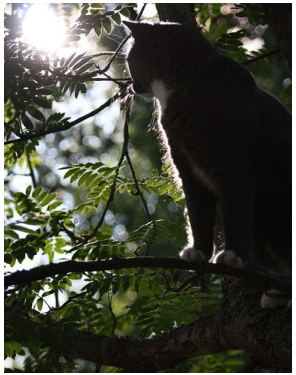

This image is CC0 1.0 public domain

Challenges: Background Clutter

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Challenges: Occlusion

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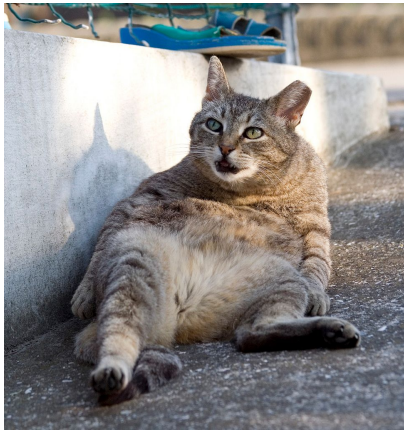
Challenges: Deformation



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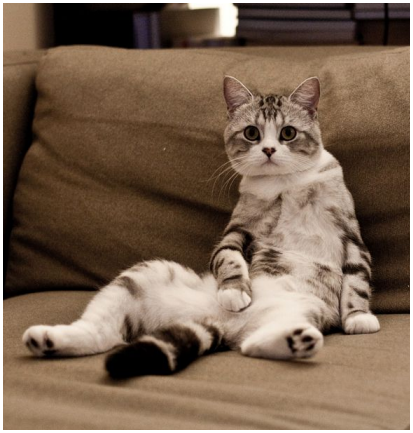

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Challenges: Intraclass variation

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Challenges: Context

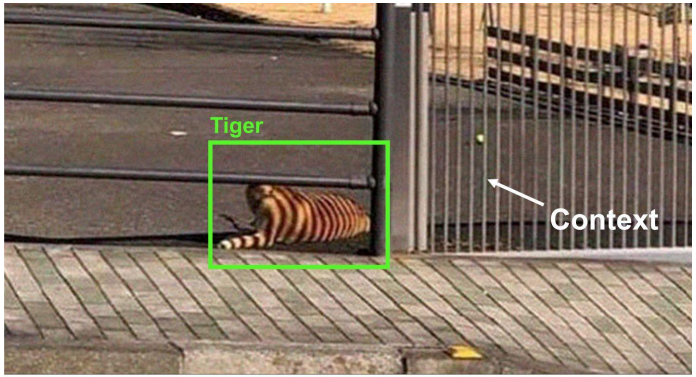
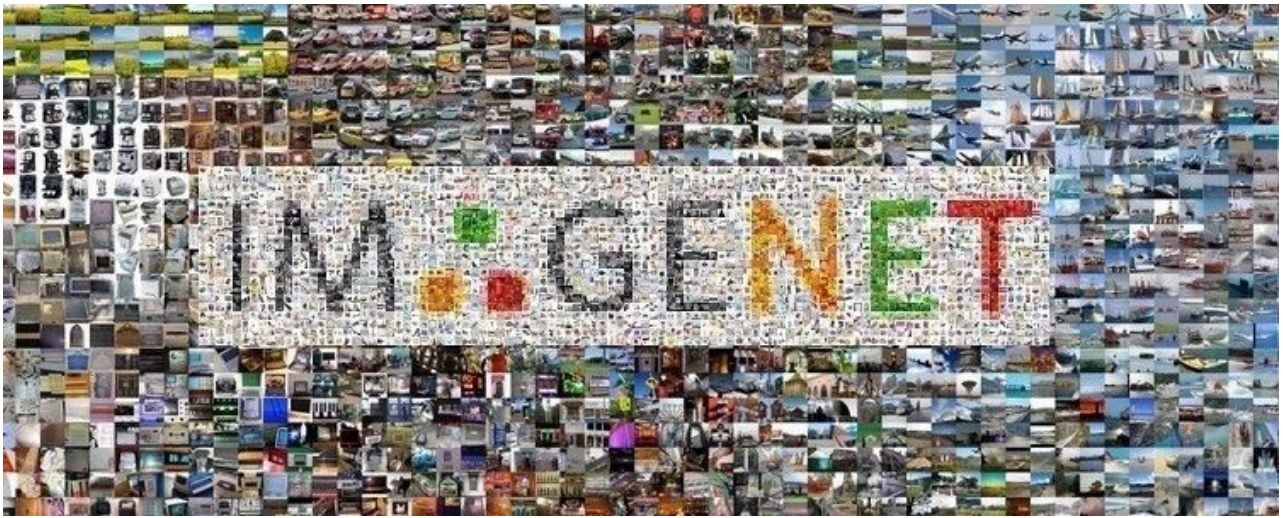


Image source:
https://www.linkedin.com/posts/ralph-aboujaoude-diaz-40838313_technology-artificialintelligence-computervision-activity-6912446088364875776-h-lq?utm_source=linkedin_share&utm_medium=member_desktop_web

Modern computer vision algorithms



An image classifier

```
def classify_image(image):
  # Some magic here?
  return class_label
```

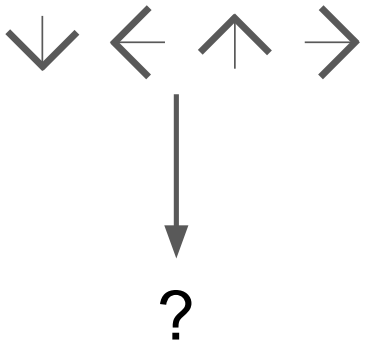
Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for recognizing a cat, or other classes.

Attempts have been made

Find edges

Find corners



John Canny, "A Computational Approach to Edge Detection", IEEE TPAMI 1986

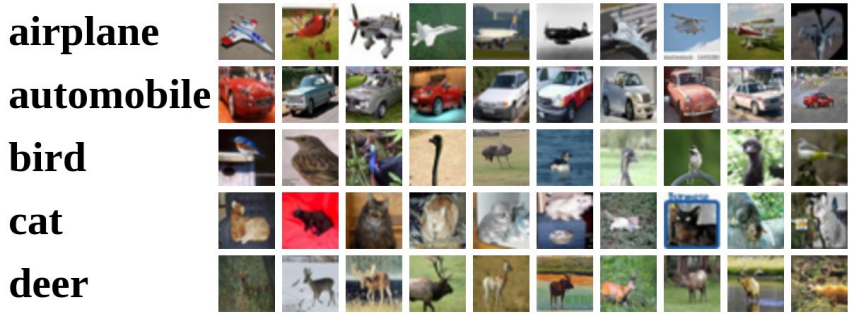
Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning algorithms to train a classifier
3. Evaluate the classifier on new images

```
def train(images, labels):
  # Machine learning!
  return model
```

```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

Example training set



Nearest Neighbor Classifier

First classifier: **Nearest Neighbor**

```
def train(images, labels):
  # Machine learning!
  return model
```

→ Memorize all data and labels

```
def predict(model, test_images):
  # Use model to predict labels
  return test_labels
```

→ Predict the label of the most similar training image

First classifier: **Nearest Neighbor**

Training data with labels

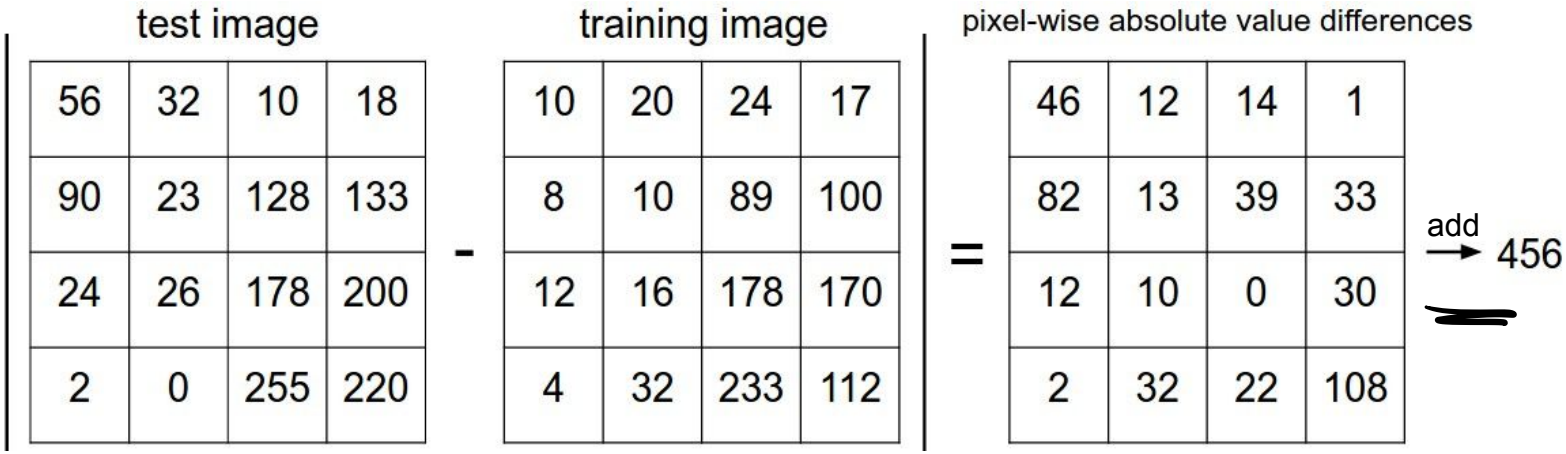
?

query data

Distance Metric $\left| \quad , \quad \right| \rightarrow \mathbb{R}$

Distance Metric to compare images

L1 distance: $$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



```python
import numpy as np

class NearestNeighbor:
  def __init__(self):
    pass

  def train(self, X, y):
    """ X is N x D where each row is an example. Y is 1-dimension of size N """
    # the nearest neighbor classifier simply remembers all the training data
    self.Xtr = X
    self.ytr = y

  def predict(self, X):
    """ X is N x D where each row is an example we wish to predict label for """
    num_test = X.shape[0]
    # lets make sure that the output type matches the input type
    Ypred = np.zeros(num_test, dtype = self.ytr.dtype)

    # loop over all test rows
    for i in xrange(num_test):
      # find the nearest training image to the i'th test image
      # using the L1 distance (sum of absolute value differences)
      distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
      min_index = np.argmin(distances) # get the index with smallest distance
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Nearest Neighbor classifier

Nearest Neighbor classifier

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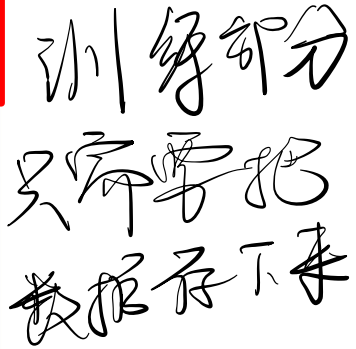
```

Memorize training data



Nearest Neighbor classifier

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      Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

    return Ypred
```

For each test image:
- Find closest train image
- Predict label of nearest image

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```

Nearest Neighbor classifier

Q: With N examples, how fast are training and prediction?

Ans: Train O(1),
predict O(N)

This is bad: we want classifiers that are **fast** at prediction; **slow** for training is ok

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      Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

    return Ypred
```

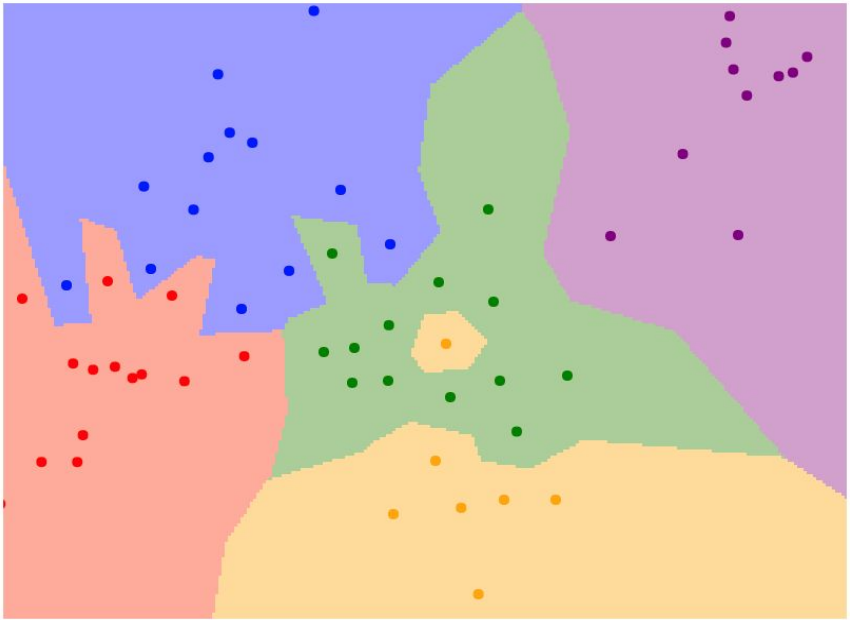
Nearest Neighbor classifier

Many methods exist for fast / approximate nearest neighbor (beyond the scope of 231N!)

A good implementation: https://github.com/facebookresearch/faiss

Johnson et al, “Billion-scale similarity search with GPUs”, arXiv 2017

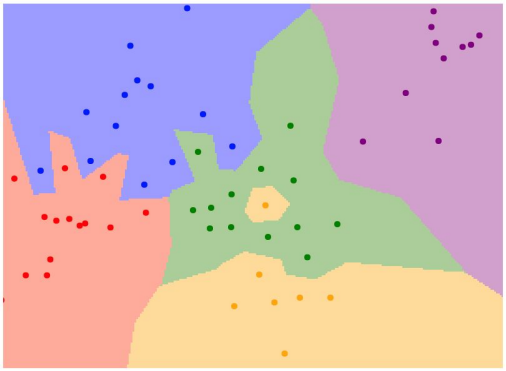
What does this look like?



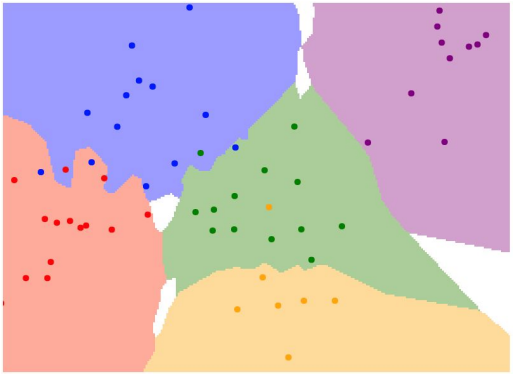
1-nearest neighbor

K-Nearest Neighbors

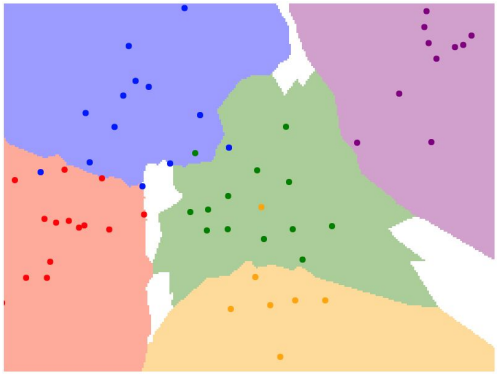
Instead of copying label from nearest neighbor,
take **majority vote** from K closest points



K = 1



K = 3

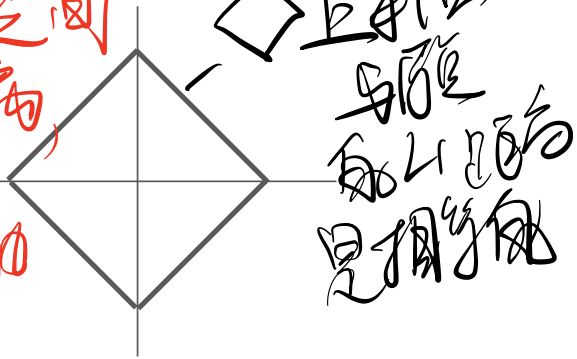


K = 5

K-Nearest Neighbors: Distance Metric

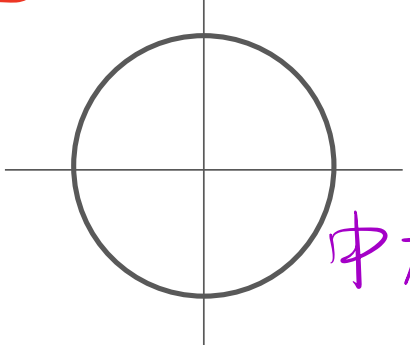
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$



K-Nearest Neighbors: Distance Metric

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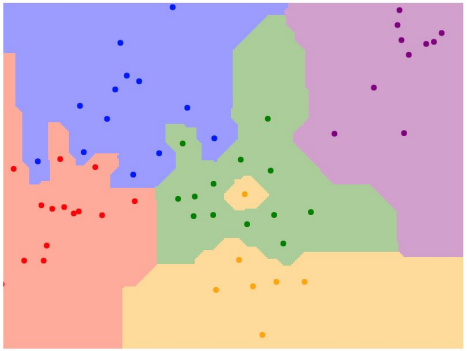

K = 1

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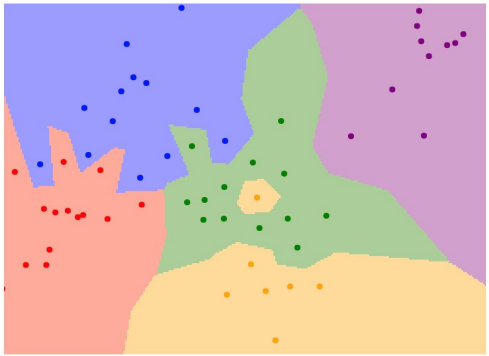
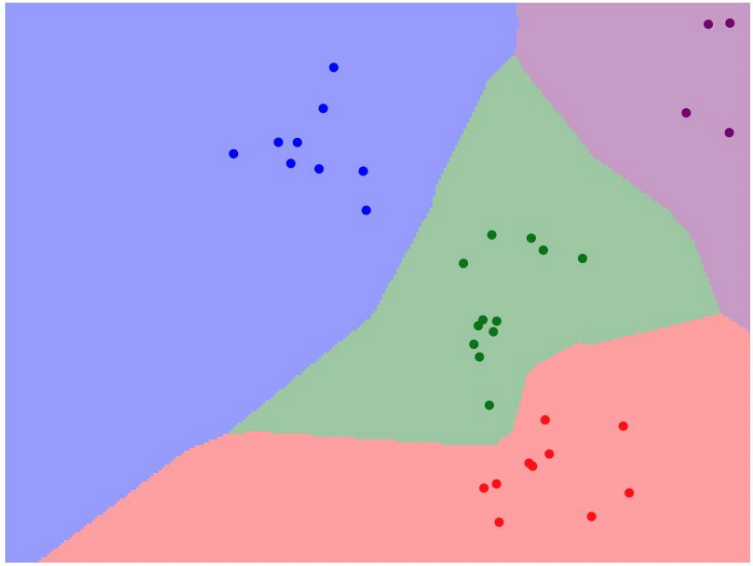


K = 1

K-Nearest Neighbors: try it yourself!



http://vision.stanford.edu/teaching/cs231n-demos/knn/

Hyperparameters

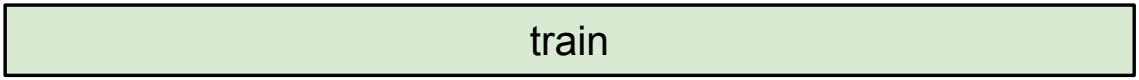
What is the best value of **k** to use?
What is the best **distance** to use?

These are **hyperparameters**: choices about the algorithms themselves.

Very problem/dataset-dependent.
Must try them all out and see what works best.

Setting Hyperparameters

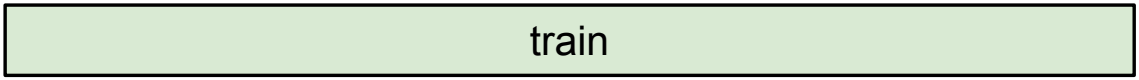
Idea #1: Choose hyperparameters
that work best on the **training data**



Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: K = 1 always works perfectly on training data



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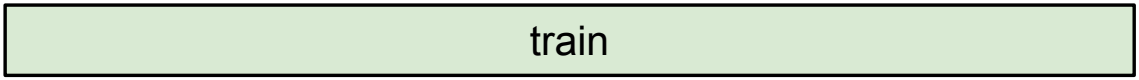
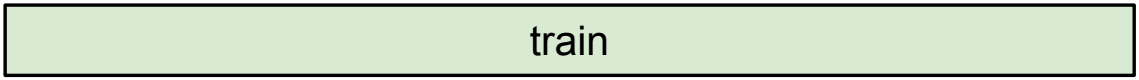


Idea #2: choose hyperparameters that work best on **test** data

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: K = 1 always works perfectly on training data



Idea #2: choose hyperparameters that work best on **test** data

BAD: No idea how algorithm will perform on new data

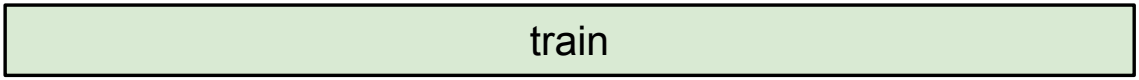
Never do this!

在训练集上训练，在一个名test上最好的，也不行，因为test并不能代表所有的 unseen data

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: K = 1 always works perfectly on training data



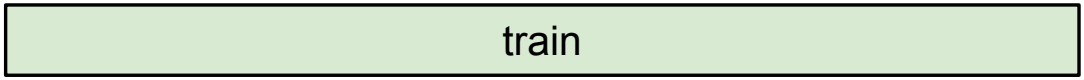
Idea #2: choose hyperparameters that work best on **test** data

BAD: No idea how algorithm will perform on new data

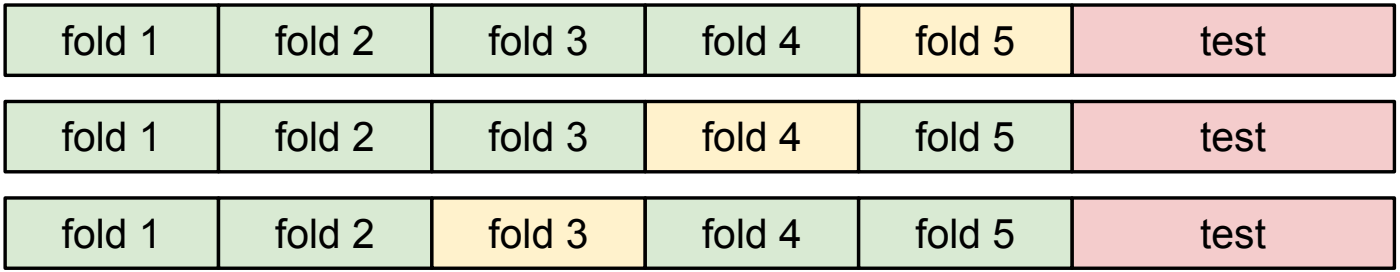
Idea #3: Split data into **train**, **val**; choose hyperparameters on val and evaluate on test

Better!

Setting Hyperparameters



Idea #4: **Cross-Validation**: Split data into **folds**,
try each fold as validation and average the results



Useful for small datasets, but not used too frequently in deep learning

Example Dataset: **CIFAR10**

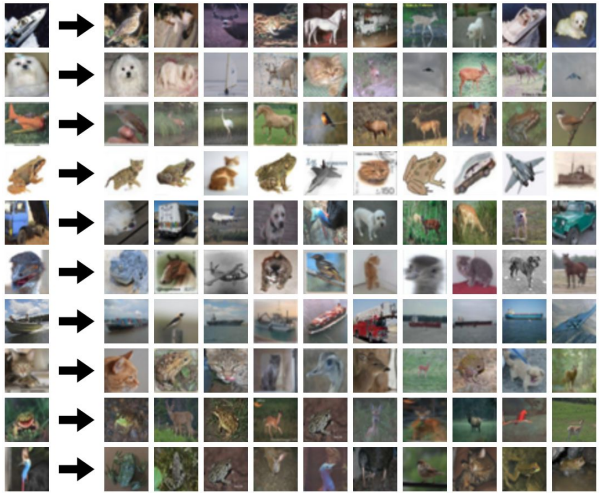
10 classes
50,000 training images
10,000 testing images

Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Example Dataset: **CIFAR10**

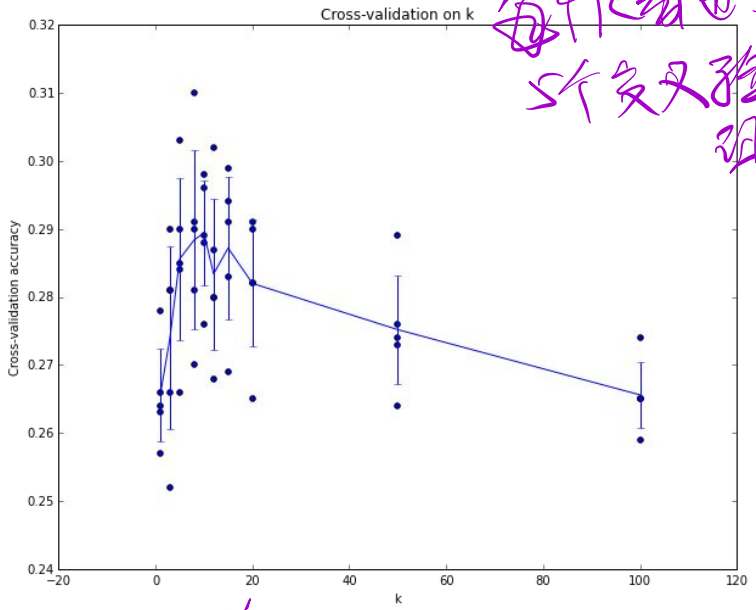
10 classes
50,000 training images
10,000 testing images

Test images and nearest neighbors



Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Setting Hyperparameters



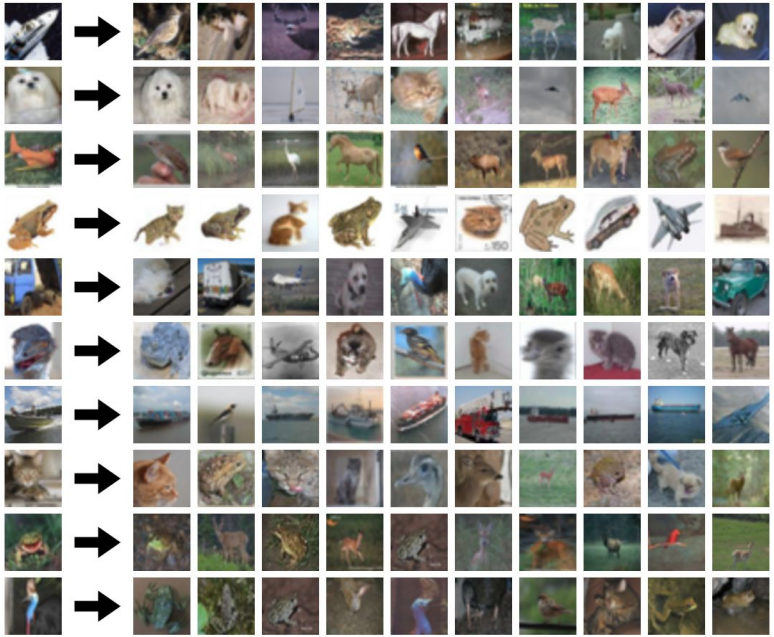
Example of 5-fold cross-validation for the value of **k**.

Each point: single outcome.

The line goes through the mean, bars indicated standard deviation

(Seems that k ~= 7 works best for this data)

What does this look like?

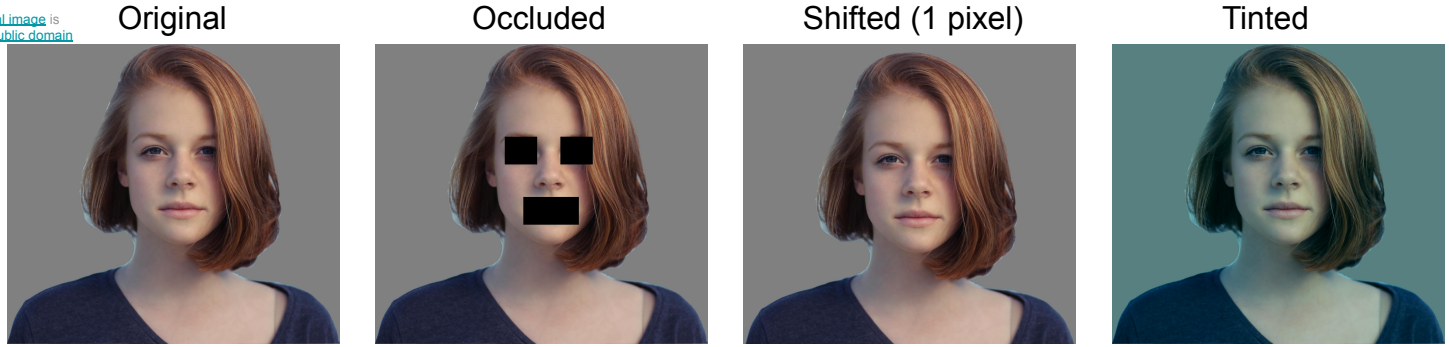


What does this look like?

L1, L2

k-Nearest Neighbor with pixel distance **never used.**

- Distance metrics on pixels are not informative



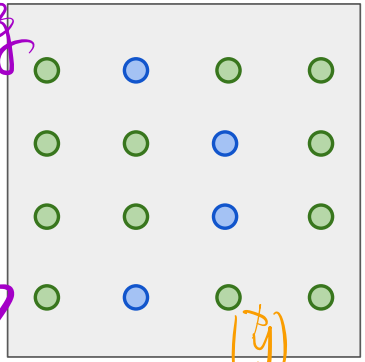
(All three images on the right have the same pixel distances to the one on the left)

k-Nearest Neighbor with pixel distance **never used.**

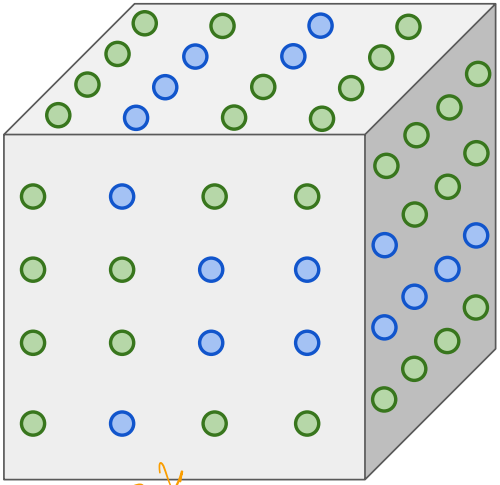
- Curse of dimensionality

Dimensions = 1
Points = 4

Dimensions = 2
Points = 4^2



Dimensions = 3
Points = 4^3



K-Nearest Neighbors: Summary

In **image classification** we start with a **training set** of images and labels, and must predict labels on the **test set**

The **K-Nearest Neighbors** classifier predicts labels based on the K nearest training examples

Distance metric and K are **hyperparameters**

Choose hyperparameters using the **validation set**;

Only run on the test set once at the very end!

Linear Classifier

Parametric Approach

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving
class scores

W
parameters
or weights

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving class scores

W
parameters or weights

Parametric Approach: Linear Classifier

$$f(x, W) = Wx$$

3072x1

10x1

10x3072

Image

Array of **32x32x3** numbers
(3072 numbers total)

f(**x**,**W**)

10 numbers giving class scores

W

parameters or weights

Parametric Approach: Linear Classifier

$$f(x, W) = Wx + b$$

3072x1 (x)

10x1 (b)

10x1 (f(x,W))

10x3072 (W)

Image

Array of **32x32x3** numbers
(3072 numbers total)

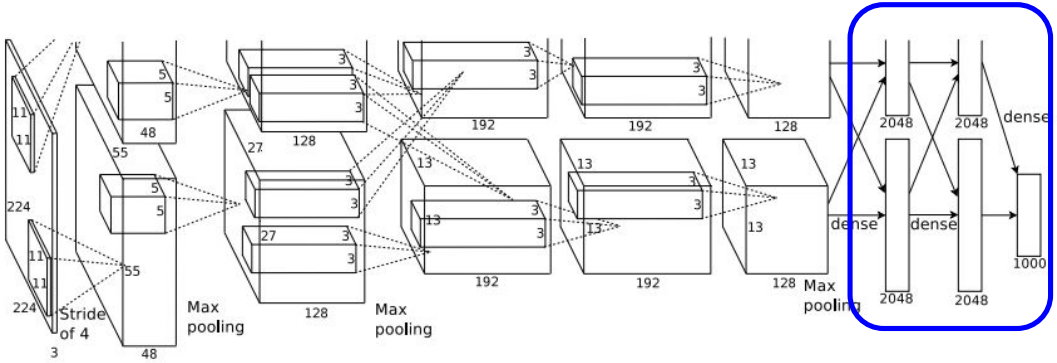
f(**x**,**W**)

10 numbers giving class scores

W
parameters
or weights

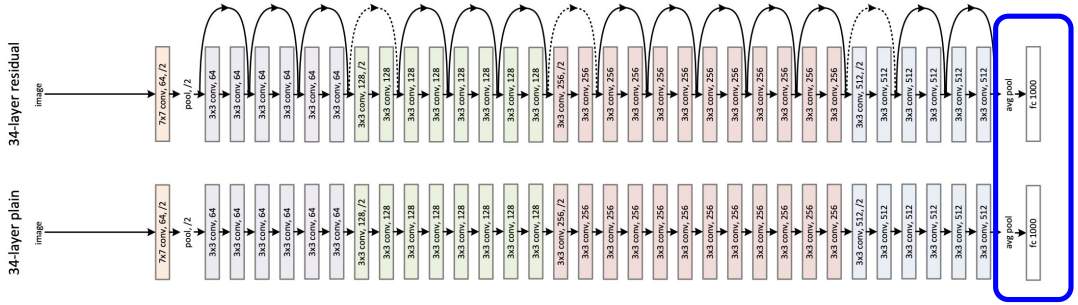
Neural Network

Linear classifiers



[Krizhevsky et al. 2012]

Linear layers



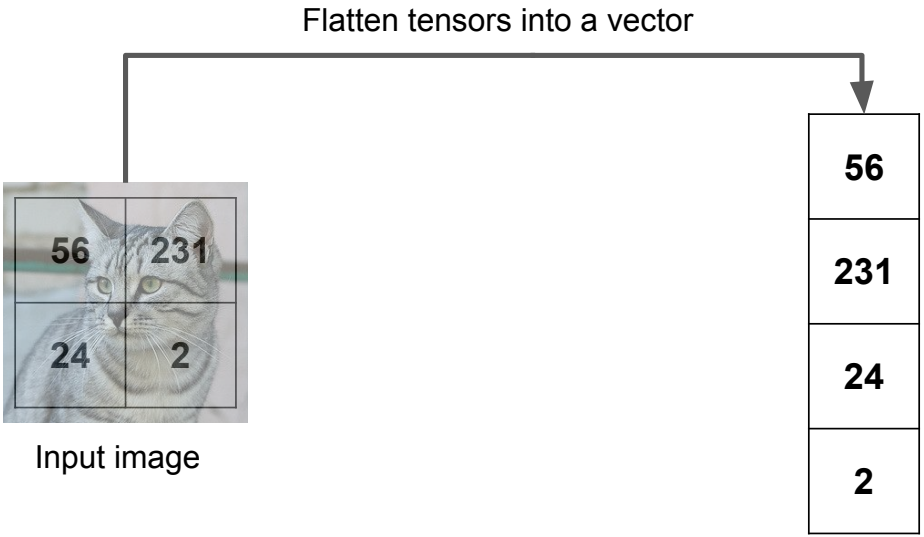
[He et al. 2015]

Recall CIFAR10

50,000 training images
each image is **32x32x3**

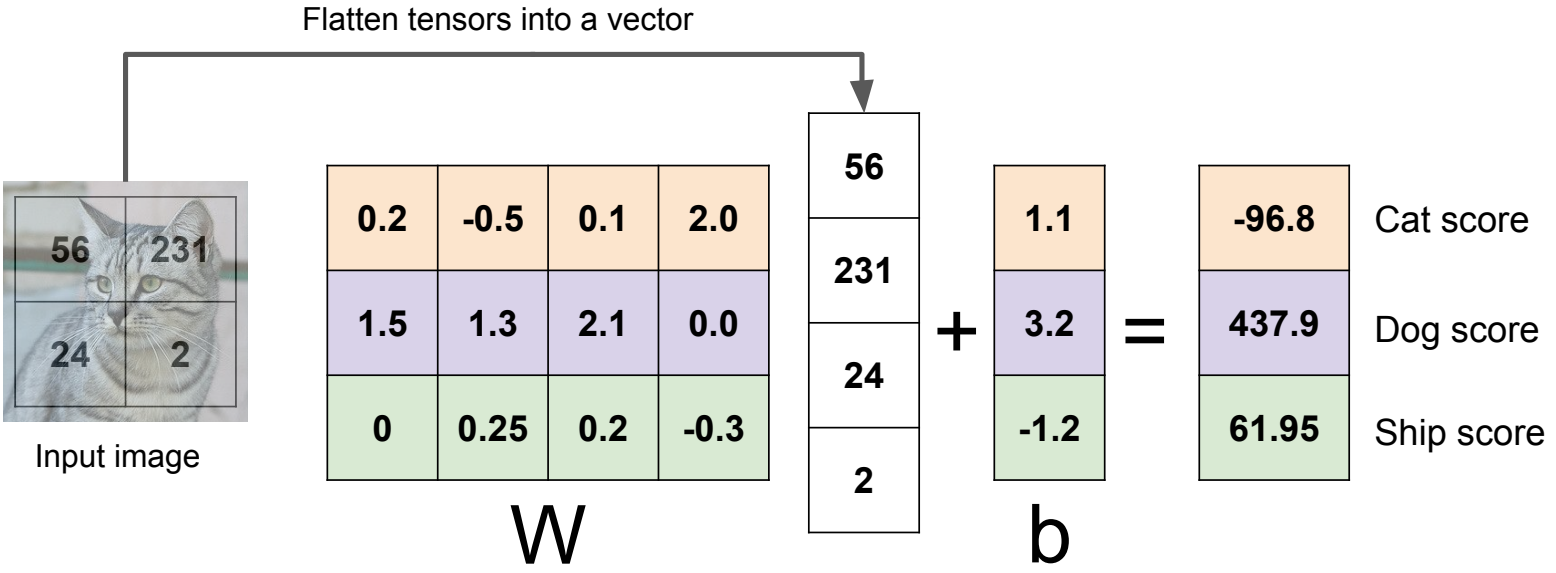
10,000 test images.

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

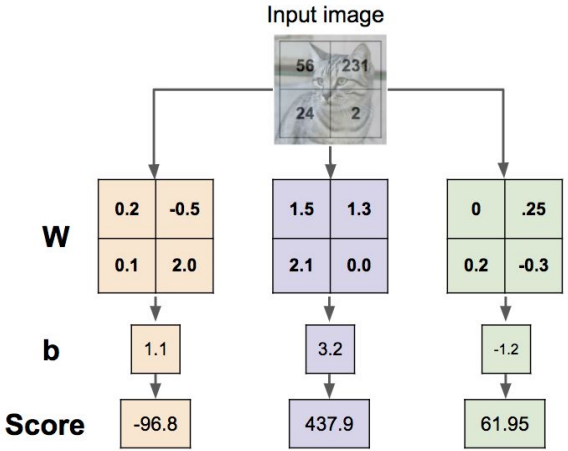


Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

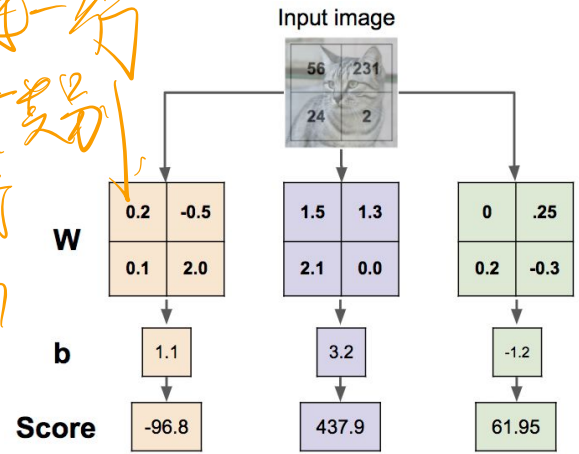
Algebraic Viewpoint



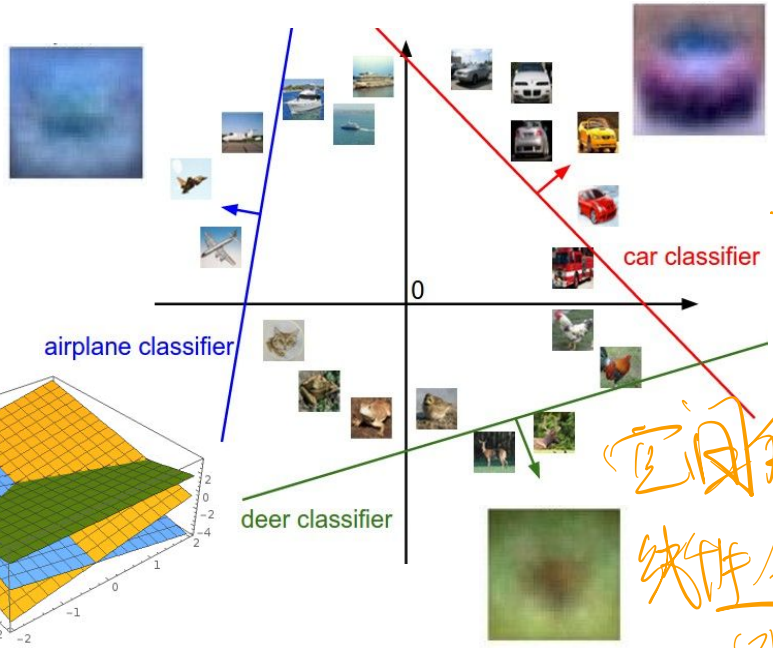
Interpreting a Linear Classifier



Interpreting a Linear Classifier: Visual Viewpoint



Interpreting a Linear Classifier: Geometric Viewpoint



$$f(x,W) = Wx + b$$

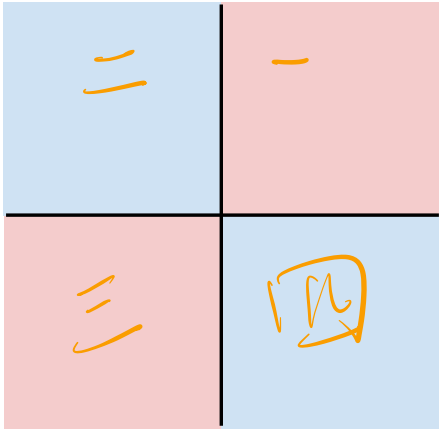
Array of **32x32x3** numbers
(3072 numbers total)

Plot created using Wolfram Cloud

Hard cases for a linear classifier

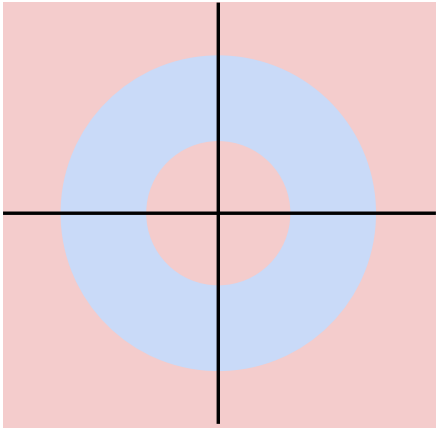
Class 1:
First and third quadrants

Class 2:
Second and fourth quadrants



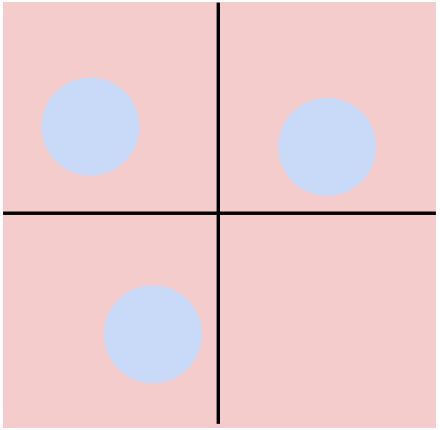
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

Class 2:
Everything else



Linear Classifier – Choose a good W



airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.

2. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

A **loss function** tells how good our current classifier is



cat	**3.2**	1.3	2.2
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A **loss function** tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and y_i is (integer) label

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A **loss function** tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and
y_i is (integer) label

Loss over the dataset is a average of loss over examples:

$$L = \frac{1}{N} \sum_i L_i(f(x_i, W), y_i)$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x,W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
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frog	-1.7	2.0	**-3.1**

正确类别的得分比其他类别大1，就不产生损失

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

这意思是，将s算出来的结果当中，比正确那个类值小的都取0；其余取不同的了

Suppose: 3 training examples, 3 classes.
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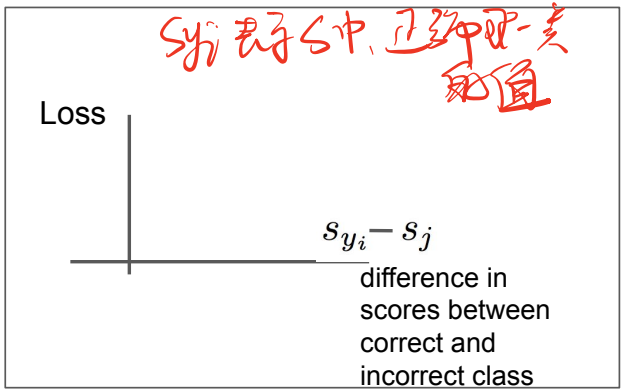
Interpreting Multiclass SVM loss:



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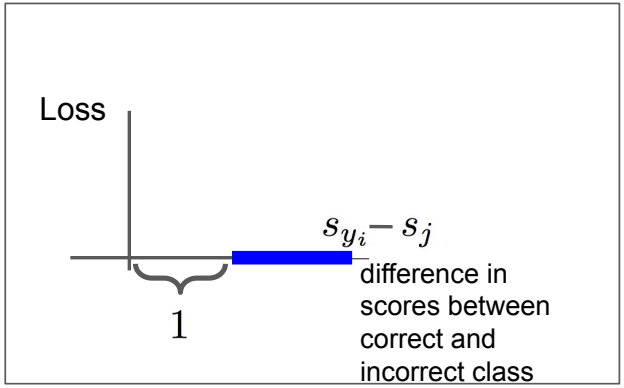

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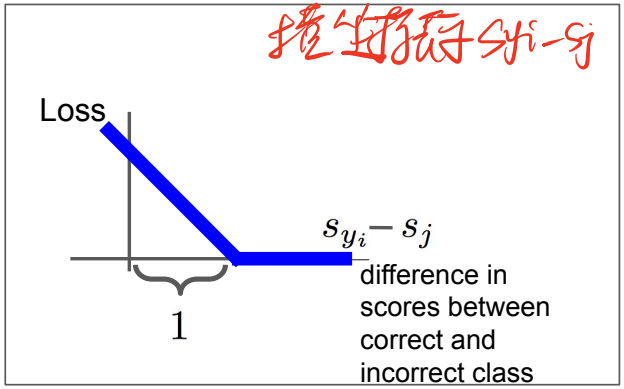

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$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

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Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9		

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

Multiclass SVM loss:

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and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
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and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Loss over full dataset is average:

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i$$

L = (2.9 + 0 + 12.9)/3
= 5.27

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

Multiclass SVM loss:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

cat	1.3
car	**4.9**
frog	2.0
Losses:	0

Q1: What happens to loss if car scores decrease by 0.5 for this training example?

Q2: what is the min/max possible SVM loss L_i?

Q3: At initialization W is small so all s ≈ 0. What is the loss L_i, assuming N examples and C classes?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q4: What if the sum was over all classes? (including j = y_i)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q5: What if we used mean instead of sum?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q6: What if we used

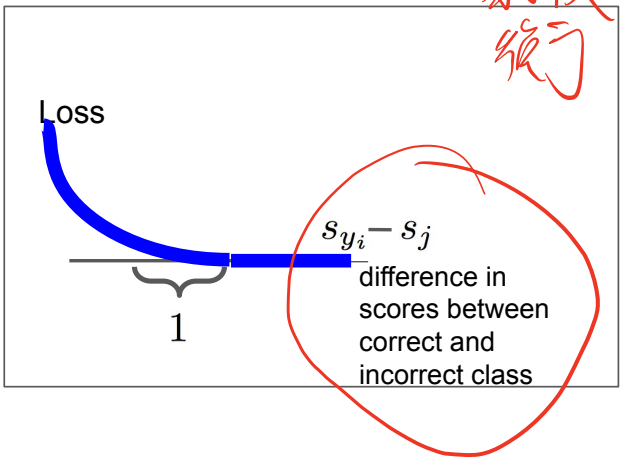
$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:



Q6: What if we used

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Multiclass SVM Loss: Example code

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

```
def L_i_vectorized(x, y, W):
  scores = W.dot(x)
  margins = np.maximum(0, scores - scores[y] + 1)
  margins[y] = 0
  loss_i = np.sum(margins)
  return loss_i
```

First calculate scores
Then calculate the margins $s_j - s_{yi} + 1$
only sum j is not y_i, so when $j = y_i$, set to zero.
sum across all j

有的上 L=0 及 W不唯一，2W, 4W ...

Softmax classifier

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

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Softmax Function

Probabilities must be >= 0

cat	**3.2**	exp →	**24.5**
car	5.1		164.0
frog	-1.7		0.18

unnormalized probabilities

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Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

		exp		normalize	
cat	**3.2**	→	**24.5**	→	**0.13**
car	5.1	→	164.0	→	0.87
frog	-1.7	→	0.18	→	0.00

unnormalized probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	Probabilities must be >= 0	normalize	Probabilities must sum to 1	
cat	**3.2**		**24.5**		**0.13**	→ L_i = -log(0.13) = **2.04**
car	5.1		164.0		0.87	
frog	-1.7		0.18		0.00	
	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities	

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities
	Probabilities must be >= 0				Probabilities must sum to 1
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13) = **2.04**

Maximum Likelihood Estimation
Choose weights to maximize the likelihood of the observed data (See CS 229 for details)

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

$$L_i = -\log P(Y = y_i|X = x_i)$$

	Unnormalized log-probabilities / logits	exp	Probabilities must be >= 0	normalize	Probabilities must sum to 1	compare	Correct probs
cat	**3.2**		**24.5**		**0.13**		**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00
	Unnormalized log-probabilities / logits		unnormalized probabilities		probabilities		Correct probs

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

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$$L_i = -\log P(Y = y_i|X = x_i)$$

Probabilities must be >= 0

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	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities	compare	Correct probs
cat	**3.2**		**24.5**		**0.13**		**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Kullback–Leibler divergence

$$D_{KL}(P||Q) = \sum_y P(y) \log \frac{P(y)}{Q(y)}$$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

$$L_i = -\log P(Y = y_i|X = x_i)$$

Probabilities must be >= 0

Probabilities must sum to 1

	Unnormalized log-probabilities / logits	exp	unnormalized probabilities	normalize	probabilities		Correct probs
cat	**3.2**		**24.5**		**0.13**	compare	**1.00**
car	5.1		164.0		0.87		0.00
frog	-1.7		0.18		0.00		0.00

Cross Entropy

$$H(P, Q) = H(p) + D_{KL}(P||Q)$$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

$$L_i = -\log P(Y = y_i|X = x_i)$$

越大，说明分类正确越好

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

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Maximize probability of correct class

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Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

cat	**3.2**
car	5.1
frog	-1.7

Q1: What is the min/max possible softmax loss L_i?

Q2: At initialization all s_j will be approximately equal; what is the softmax loss L_i, assuming C classes?

Softmax Classifier (Multinomial Logistic Regression)

cat **3.2**

car 5.1

frog -1.7

Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

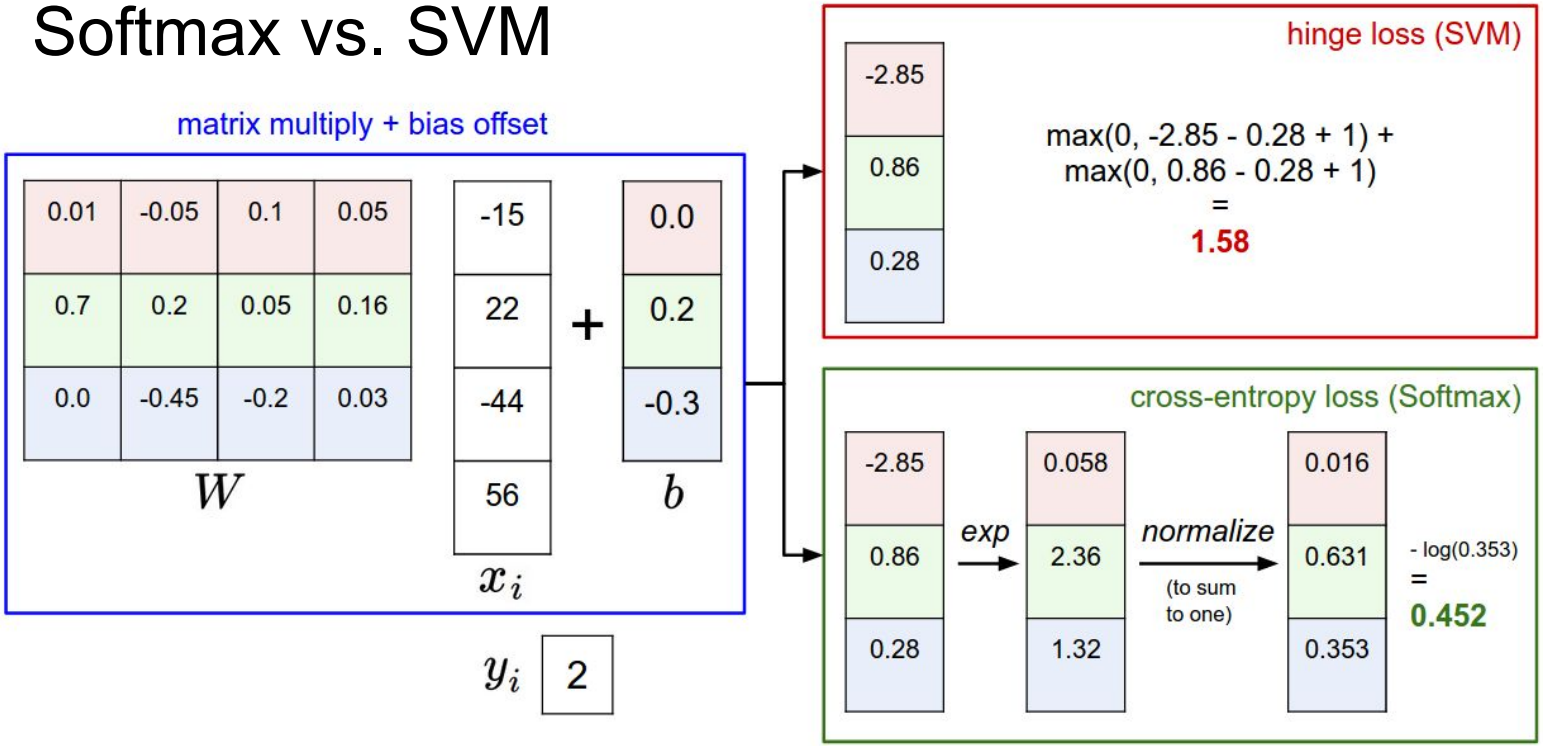
$$L_i = -\log P(Y = y_i|X = x_i)$$

Putting it all together:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

Q2: At initialization all s will be approximately equal; what is the loss?
A: -log(1/C) = log(C),
If C = 10, then L_i = log(10) ≈ 2.3

Softmax vs. SVM



Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right) \qquad L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:
[10, -2, 3]
[10, 9, 9]
[10, -100, -100]
and $y_i = 0$

Q: What is the **softmax loss** and the **SVM** loss?

Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:
[20, -2, 3]
[20, 9, 9]
[20, -100, -100]
and $y_i = 0$

Q: What is the **softmax loss** and the **SVM** loss **if I double the correct class score from 10 -> 20?**

Coming up:

- Regularization
- Optimization

$$f(x,W) = Wx + b$$