#EngineeringPlus Online Course Series

ADAMAS SCHOOL OF ENGINEERING AND TECHNOLOGY

Artificial Intelligence

Module: 02
Lecture:

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Introduction

- First-order logic is another way of knowledge representation in artificial intelligence. It is an extension to propositional logic.
- FOL is sufficiently expressive to represent the natural language statements in a concise way.
- First-order logic is also known as **Predicate logic or First-order predicate logic**. First-order logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects
- First-order logic (like natural language) does not only assume that the world contains facts like propositional logic but also assumes the following things in the world:

Assumptions

- **Objects:** A, B, people, numbers, colors, wars, theories, squares, pits, wumpus, ……
- **Relations: It can be unary relation such as:** red, round, is adjacent, **or n-any relation such as:** the sister of, brother of, has color, comes between
- **Function:** Father of, best friend, third inning of, end of, ……

Assumptions

- As a natural language, first-order logic also has two main parts:
  - **Syntax**
  - **Semantics**

Basic Elements of First-order logic

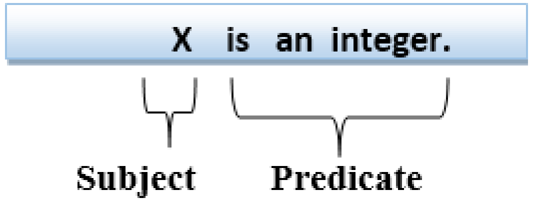
- **Constant**
- 1, 2, A, John, Mumbai, cat,....
- **Variables**
- x, y, z, a, b,....
- **Predicates**
- Brother, Father, >,....
- **Function**
- sqrt, LeftLegOf, ...
- **Connectives**
- $\land$, $\lor$, $\neg$, $\Rightarrow$, $\Leftrightarrow$
- **Equality**
- ==
- **Quantifier**
- $\forall$, $\exists$

Types of sentences

- **Atomic sentences**
- Atomic sentences are the most basic sentences of first-order logic. These sentences are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as **Predicate (term1, term2, ..., term n)**.
- **Example: Ravi and Ajay are brothers: => Brothers(Ravi, Ajay).**
  Chinky is a cat: => cat (Chinky).

Complex Sentences

- Complex sentences are made by combining atomic sentences using connectives.
- **First-order logic statements can be divided into two parts:**
- **Subject:** Subject is the main part of the statement.
- **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.
- **Consider the statement: "x is an integer.",**



Quantifiers in First-order logic

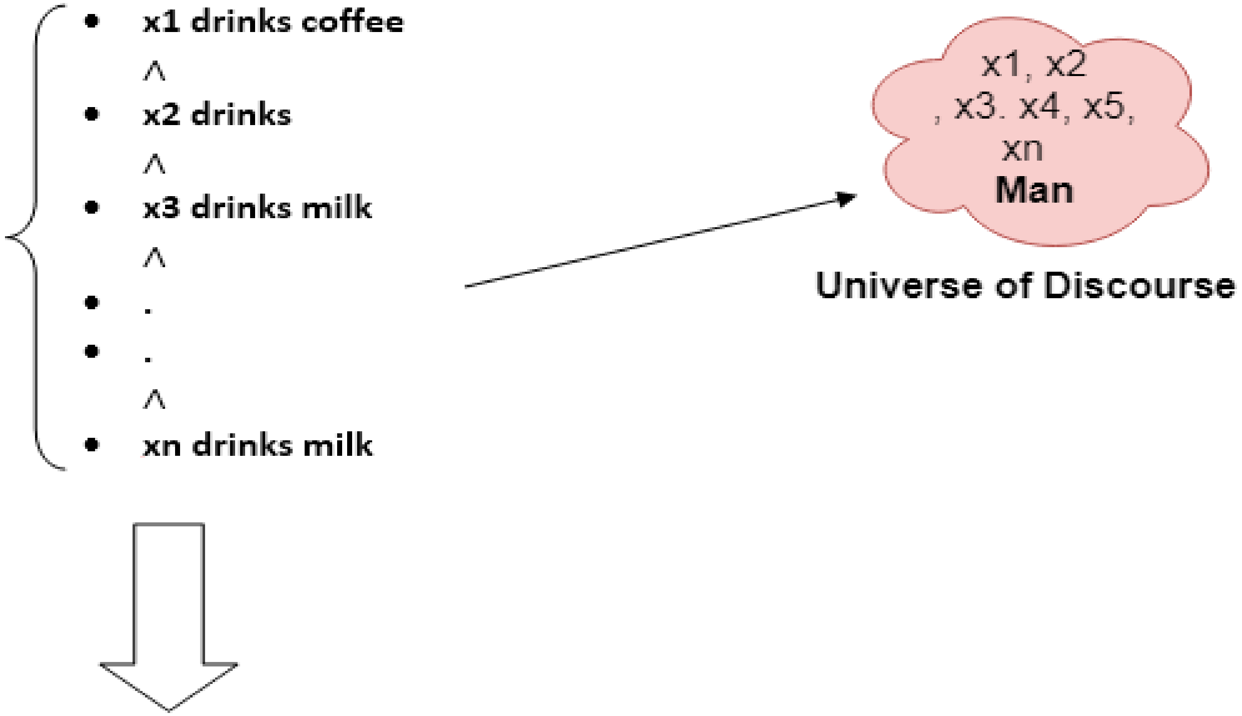
- A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- There are two types of quantifier:
  - **Universal Quantifier, (for all, everyone, everything)**
  - **Existential quantifier, (for some, at least one).**

Universal Quantifier

- Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.
- The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.
- If x is a variable, then ∀x is read as:
  - **For all x**
  - **For each x**
  - **For every x.**

Example

- **All man drink coffee.**



So in shorthand notation, we can write it as :

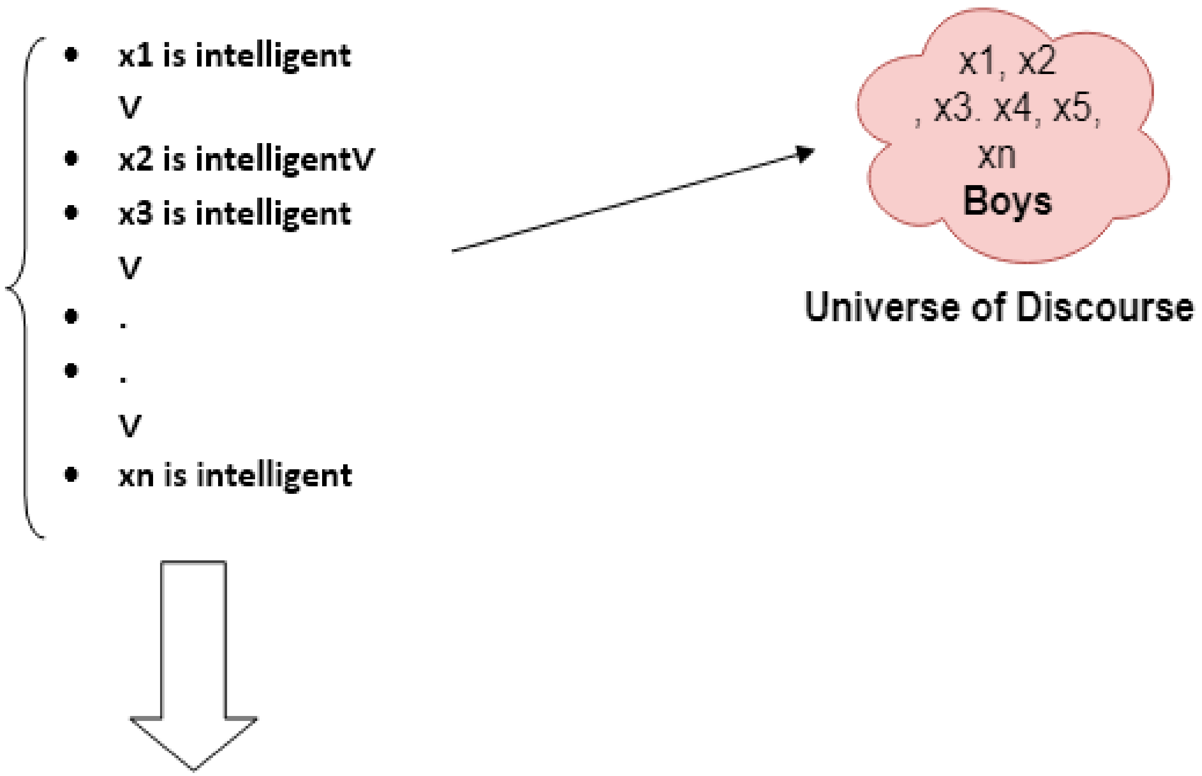
- **$\forall x$ man(x) → drink (x, coffee).**
- It will be read as: There are all x where x is a man who drink coffee.

Existential Quantifier

- Existential quantifiers are the type of quantifiers, which express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.
- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:
  - **There exists a 'x.'**
  - **For some 'x.'**
  - **For at least one 'x.'**

Example

- **Some boys are intelligent.**



So in short-hand notation, we can write it as:

Example: contd.

- **$\exists x$: boys(x) $\wedge$ intelligent(x)**
- It will be read as: There are some x where x is a boy who is intelligent.

Points to remember

- The main connective for universal quantifier ∀ is implication →.
- The main connective for existential quantifier ∃ is and ∧.

Examples of FOL using Quantifier

- **All birds fly.**
  In this question the predicate is "**fly(bird)**."
  And since there are all birds who fly so it will be represented as follows.
  $$\forall x\ \mathbf{bird(x)} \rightarrow \mathbf{fly(x)}.$$
- **Every man respects his parent.**
  In this question, the predicate is "**respect(x, y)," where x=man, and y= parent**.
  Since there is every man so will use ∀, and it will be represented as follows:
  $$\forall x\ \mathbf{man(x)} \rightarrow \mathbf{respects\ (x, parent)}.$$

Examples of FOL using Quantifier: contd.

- **Some boys play cricket.**
  In this question, the predicate is "**play(x, y)**," where x= boys, and y= game. Since there are some boys so we will use **∃, and it will be represented as**:
  ∃x boys(x) → play(x, cricket).
- **Not all students like both Mathematics and Science.**
  In this question, the predicate is "**like(x, y)," where x= student, and y= subject**.
  Since there are not all students, so we will use **∀ with negation, so** following representation for this:
  ¬∀ (x) [student(x) → like(x, Mathematics) ∧ like(x, Science)].

FOL inference rules for quantifier

- Universal Generalization
- Universal Instantiation
- Existential Instantiation
- Existential introduction

Universal Generalization

- Universal generalization is a valid inference rule which states that if premise P(c) is true for any arbitrary element c in the universe of discourse, then we can have a conclusion as ∀ x P(x).
- It can be represented as:
- .

$$\frac{P(c)}{\forall x\, P(x)}$$

Example: Let's represent, P(c): "**A byte contains 8 bits**", so for **∀ x P(x)** "**All bytes contain 8 bits**.", it will also be true.

Universal Instantiation

- Universal instantiation is also called as universal elimination or UI is a valid inference rule. It can be applied multiple times to add new sentences.
- It can be represented as:

$$\frac{\forall x\, P(x)}{P(c)}$$

IF "Every person like ice-cream"=> $\forall x\, P(x)$ so we can infer that
"John likes ice-cream" => $P(c)$

Existential Instantiation

- Existential instantiation is also called as Existential Elimination, which is a valid inference rule in first-order logic.
- It can be applied only once to replace the existential sentence.
- It can be represented as:

$$\frac{\exists x \, P(x)}{P(c)}$$

Example:
From the given sentence: **∃x Crown(x) ∧ OnHead(x, John),**
So we can infer: **Crown(K) ∧ OnHead(K, John),** as long as K does not appear in the knowledge base.

Existential introduction

- An existential introduction is also known as an existential generalization, which is a valid inference rule in first-order logic.
- It can be represented as:

$$\frac{P(c)}{\exists x P(x)}$$

Example: Let's say that,
“John got good marks in English."
"Therefore, someone got good marks in English."

References

- Artificial Intelligence, Russell & Norvig, Pearson