1. $P_1=\frac{2\cdot C_{18}^{8}}{C_{20}^{10}}=\frac{9}{19}$ $\qquad$ $P_2=\frac{2\cdot C_{18}^{9}}{C_{20}^{10}}=\frac{10}{19}$

2. $P_1=\frac{9}{14}$ $\qquad$ (2) "取到甲袋"记为 A_1，"取到乙袋"记为 A_2

B: "取到红球" $\quad P(A_1)=P(A_2)=\frac{1}{2} \quad P(B|A_1)=\frac{5}{7} \quad P(B|A_2)=\frac{4}{7}$

$P_2=\frac{1}{2}\times\frac{5}{7}+\frac{1}{2}\times\frac{4}{7}=\frac{9}{14}$

(3) B_1: "从甲袋取了一个红球" $\quad B_2$: "从甲袋取了一个白球"

C: 从乙袋取了一个红球 $\quad P(B_1)=\frac{5}{7} \quad P(B_2)=\frac{2}{7}$

$P(C|B_1)=\frac{5}{8} \qquad P(C|B_2)=\frac{4}{8}$

$$P_3=P(B_1|C)=\frac{\frac{5}{7}\times\frac{5}{8}}{\frac{5}{7}\times\frac{5}{8}+\frac{2}{7}\times\frac{4}{8}}=\frac{25}{33}$$

3. (1) $F(0-)=F(0)\Rightarrow A=0$, $\quad F(1-)=F(1)\Rightarrow B=C-\frac{1}{2}-1$

$F(2-)=F(2)\Rightarrow 2C-\frac{1}{2}\times 2^2-1=1 \Rightarrow C=2 \qquad B=\frac{1}{2}$

(2) $$\frac{dF(x)}{dx}=f(x)=\begin{cases} x, & 0\le x<1, \\ 2-x, & 1\le x<2, \\ 0 & \text{其他}; \end{cases}$$

(3) $$P(X>\tfrac{3}{2}|X>\tfrac{1}{2})=\frac{P(X>\frac{3}{2})}{P(X>\frac{1}{2})}=\frac{1-F(\frac{3}{2})}{1-F(\frac{1}{2})}=\frac{1-\frac{7}{8}}{1-\frac{1}{2}(\frac{1}{2})^2}=\frac{1}{7}$$

4. (1) $F_Y(y)=P(\min\{X,2\}\le y)=1-P(X>y,2>y)$

$$=\begin{cases} 0 & y<0 \\ P(X\le y) & 0\le y<2 \\ 1 & y\ge 2 \end{cases} = \begin{cases} 0, & y<0, \\ 1-e^{-\frac{1}{2}y}, & 0\le y<2, \\ 1, & y\ge 2. \end{cases}$$

(2) $P(Y=2)=P(X\ge 2)=1-F_X(2)=e^{-\frac{1}{2}\times 2}=e^{-1}$

(3) $$P(X>3|Y=2)=\frac{P(X>3,X\ge 2)}{P(Y=2)}=\frac{P(X>3)}{P(Y=2)}=\frac{e^{-\frac{1}{2}\times 3}}{e^{-1}}=e^{-\frac{1}{2}}$$

5. (1) $P(XY\neq 0)=a+c+0.2=0.4$ ① $\dfrac{P(Y\le 0, X\le 0)}{P(X\le 0)}=\dfrac{a+b+0.1}{a+b+0.3}=\dfrac{2}{3}$ ② $a+b+c+0.6=1$ ③

$\Rightarrow a=0.1 \quad b=0.2 \quad c=0.1$

(2)

X	-1	0	1
	0.2	0.4	0.4

Y	-1	0	1
	0.3	0.4	0.3

(3)

$X+Y$	-2	-1	0	1	2
	0.1	0.1	0.4	0.3	0.1

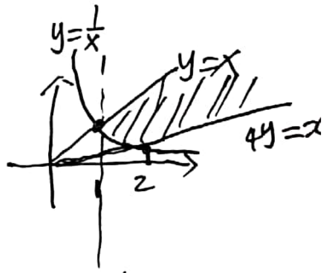
6. $Y=1 \ = P(X=0)P(Y=1)$

$P(X=0, Z=0)=P(X=0,\ X+Y\text{为奇})=P(X=0, \cancel{Y=0}\ Y=1)\ \cancel{+P(X=0, X+Y=2)}=\cancel{(1-p)^2}\ p(1-p)$

$P(X=0)P(Z=0)=(1-p)[P(X=0,Y=1)+P(X=1,Y=0)]=(1-p)[p(1-p)+(1-p)p]$

$p(1-p)\ \cancel{(1-p)^2}=(1-p)[2p(1-p)] \xRightarrow[p(1-p)>0]{} \cancel{2p}\ 2(1-p)=1 \Rightarrow p=\frac{1}{2}$

7. (1) $\iint_{R^2} f(x,y)\,dxdy=1 \Rightarrow A\int_1^{+\infty}\left(\int_{\frac{1}{x}}^{x}\frac{1}{y}dy\right)\frac{1}{x^2}dx=\frac{2}{A} \Rightarrow A=2$



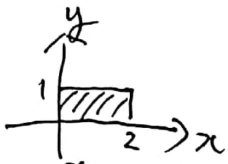
(2) $P(4Y>X)=\frac{1}{2}\int_1^2\frac{dx}{x^2}\int_{\frac{1}{x}}^{x}\frac{1}{y}dy+\frac{1}{2}\int_2^{+\infty}\frac{dx}{x^2}\int_{\frac{x}{4}}^{x}\frac{1}{y}dy=\frac{1}{2}$

(3) $f_X(x)=\begin{cases}\int_{\frac{1}{x}}^{x}\frac{1}{2x^2}\frac{1}{y}dy & x>1\\ 0 & x\le 1\end{cases}=\begin{cases}\frac{\ln x}{x^2} & x>1\\ 0 & x\le 1\end{cases}$

$f_Y(y)=\begin{cases}0 & y\le 0\\ \int_{\frac{1}{y}}^{+\infty}\frac{1}{2y}\frac{1}{x^2}dx & 0<y<1\\ \int_{y}^{+\infty}\frac{1}{2y}\frac{1}{x^2}dx & y\ge 1\end{cases}=\begin{cases}0, & y\le 0,\\ \frac{1}{2}, & 0<y<1,\\ \frac{1}{2y^2}, & y\ge 1.\end{cases}$

(4) 不独立.

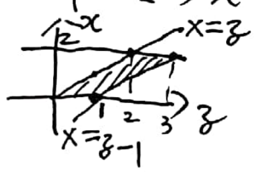
8. $f(x,y)=\begin{cases}\frac{1}{2}, & 0\le x\le 2,\ 0\le y\le 1,\\ 0, & \text{其他}.\end{cases}$



(1) $f_U(z)=\int_{-\infty}^{+\infty}f(x,z-x)dx$

$=\begin{cases}\int_0^z\frac{1}{2}dx & 0<z<1\\ \int_{z-1}^{z}\frac{1}{2}dx & 1\le z<2\\ \int_{z-1}^{2}\frac{1}{2}dx & 2\le z<3\\ 0 & \text{其他}\end{cases}=\begin{cases}\frac{z}{2} & 0<z<1\\ \frac{1}{2} & 1\le z<2\\ \frac{3-z}{2} & 2\le z<3\\ 0 & \text{其他}\end{cases}$

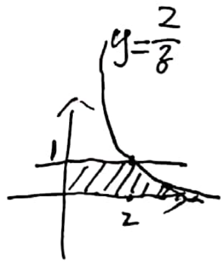
$\begin{cases}0\le x\le 2\\ 0\le z-x\le 1\end{cases}\rightarrow$



(2) $f_V(z)=\int_{-\infty}^{+\infty}|y|f(zy,y)dy$

$=\begin{cases}\int_0^1 y\frac{1}{2}dy & 0<z\le 2\\ \int_0^{\frac{2}{z}}y\frac{1}{2}dy & 2\le z<+\infty\\ 0 & z\le 0\end{cases}=\begin{cases}\frac{1}{4} & 0<z<2\\ \frac{1}{z^2} & 2\le z\\ 0 & z\le 0\end{cases}$

$\begin{cases}0\le zy\le 2\\ 0\le y\le 1\end{cases}$



9. (1) $P_k = P(Y=k \mid X=n) = C_n^k p^k (1-p)^{n-k} \xlongequal{p=0.8} C_n^k \, 0.8^k \, 0.2^{n-k}$

(2) $P(X=n, Y=k) = P(X=n)P(Y=k \mid X=n) = \frac{\lambda^n e^{-\lambda}}{n!} C_n^k \, 0.8^k \, 0.2^{n-k}$

$$P(Y=k) = \sum_n P(X=n, Y=k) = \sum_{n=k}^{\infty} \frac{\lambda^n e^{-\lambda}}{\cancel{n!}} \frac{\cancel{n!}}{k!(n-k)!} 0.8^k \, 0.2^{n-k}$$

$$= \frac{(0.8\lambda)^k e^{-\lambda}}{k!} \sum_{n=k}^{\infty} \frac{(0.2\lambda)^{n-k}}{(n-k)!} \xlongequal{n-k=j} \frac{(0.8\lambda)^k e^{-\lambda}}{k!} \sum_{j=0}^{\infty} \frac{(0.2\lambda)^j}{j!} = \frac{(0.8\lambda)^k e^{-\lambda}}{k!} \cdot e^{0.2\lambda}$$

$$= \frac{(0.8\lambda)^k}{k!} e^{-0.8\lambda} \qquad Y \sim \Pi(0.8\lambda)$$

(3) $P(Z=m \mid Y=k) = \frac{P(X-Y=m, Y=k)}{P(Y=k)} = \frac{P(X=m+k)P(Y=k \mid X=m+k)}{P(Y=k)}$

$$= \frac{\frac{\lambda^{m+k} e^{-\lambda}}{(m+k)!} \cdot \frac{(m+k)!}{k!m!} 0.8^k \, 0.2^{n-k}}{\frac{(0.8\lambda)^k e^{-0.8\lambda}}{k!}} = \frac{(0.2\lambda)^m e^{-0.2\lambda}}{m!}$$

(4) 如(2)所得，可知 $Z \sim \Pi(0.2\lambda) \Rightarrow Z$ 与 Y 独立.