Principal Components and Exploratory Factor Analysis

J. Kyle Roberts
Southern Methodist University

1 Principal Components

1.1 Single Factor Solution

First we need to obtain the HS.data dataset from the MBESS library. This is a famous dataset for talking about factor analysis and principal components. To see all of the information about the items, type ?HS.data at the command prompt after you load the library. The psych library contains the commands we will be using to run the principal components.

```
> library(MBESS)
> data(HS.data)
> library(psych)
```

First we will run on analysis with just five items. Specifically, visual, cubes, paper, flags and general. In our first example, we will only extract one principal component.

```
> pc <- principal(HS.data[, 7:11])
> pc
```

```
Principal Components Analysis
Call: principal(r = HS.data[, 7:11])
Standardized loadings based upon correlation matrix
         PC1   h2   u2
visual  0.77 0.60 0.40
cubes   0.62 0.38 0.62
paper   0.66 0.43 0.57
flags   0.70 0.50 0.50
general 0.48 0.23 0.77

                PC1
SS loadings    2.14
Proportion Var 0.43

Test of the hypothesis that 1 factor is sufficient.

The degrees of freedom for the null model are  10  and the objective function was  0.67
The degrees of freedom for the model are 5  and the objective function was  0.17
The number of observations was  301  with Chi Square =  51.4  with prob <  7.2e-10

Fit based upon off diagonal values = 0.75
```

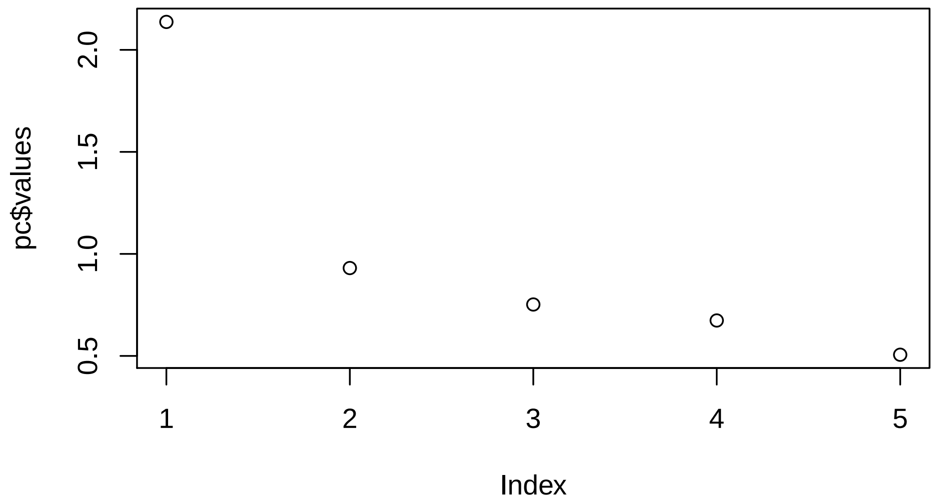
The eigenvalues can be see with:

```
> pc$values
```

```
[1] 2.1372113 0.9308580 0.7522036 0.6737686 0.5059585
```

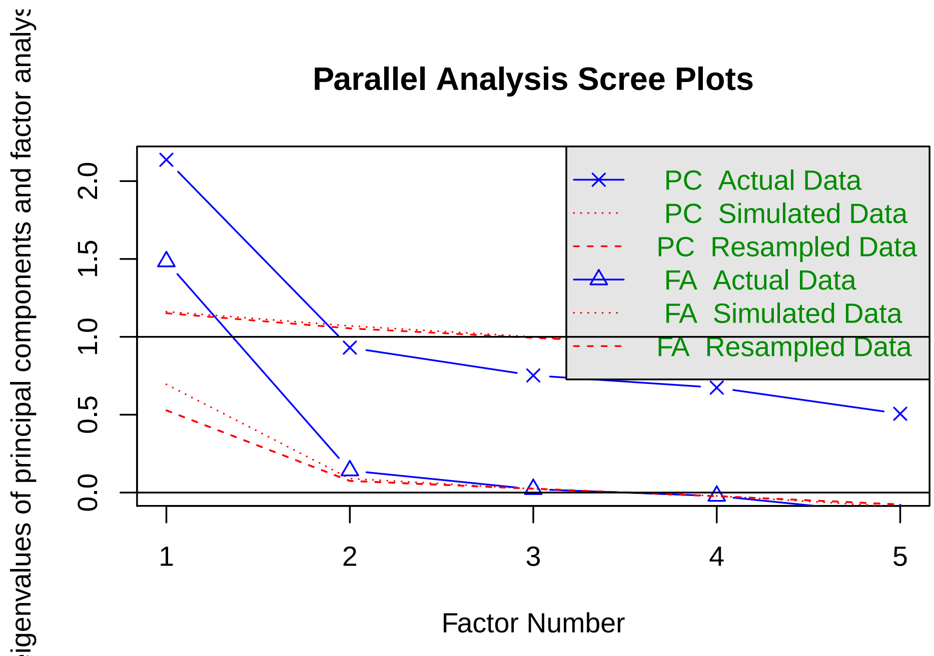
Likewise, the Scree Plot helps us determine the number of factors to retain in our analysis.

```
> plot(pc$values)
```



The Scree Plot is not the only way to determine the number of factors to retain. We should also consider the parallel analysis. In this type of analysis, we are looking from divergence from a "random set" of observations of the same size as your variable set.

```
> fa.parallel(HS.data[, 7:11])

Parallel analysis suggests that the number of factors =  2  and the number of components
```



This produces the individual factor loadings.

```
> pc$loadings

Loadings:
        PC1
visual  0.772
cubes   0.617
paper   0.659
flags   0.704
general 0.480

                 PC1
SS loadings    2.137
Proportion Var 0.427
```

This command produces the communality coefficients.

```
> pc$communality

   visual     cubes     paper     flags   general
0.5963138 0.3800894 0.4345012 0.4955644 0.2307425
```

To produce the component scores, we must specify a new model.

```
> pc2 <- principal(HS.data[, 7:11], scores = T)
> head(pc2$scores)

         PC1
1 -0.8892280
2 -0.4818657
3 -1.0665028
4  0.9664297
5 -1.0687487
6 -0.6700910
```

We can now use these component scores in a linear model. For example:

```
> m1 <- lm(pc2$scores ~ HS.data$numeric + HS.data$arithmet)
> summary(m1)

Call:
lm(formula = pc2$scores ~ HS.data$numeric + HS.data$arithmet)

Residuals:
     Min       1Q   Median       3Q      Max
-2.67415 -0.57759 -0.05913  0.55708  3.11089

Coefficients:
```

```
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      -1.88931    0.26854  -7.036 1.37e-11 ***
HS.data$numeric   0.08089    0.01252   6.461 4.22e-10 ***
HS.data$arithmet  0.03068    0.01208   2.540   0.0116 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Residual standard error: 0.8907 on 298 degrees of freedom
Multiple R-squared: 0.212,       Adjusted R-squared: 0.2067
F-statistic: 40.08 on 2 and 298 DF,  p-value: 3.831e-16
```

1.2 Multi-factor Solution

1.2.1 Extracting all 8 components

First we subset the dataset to include only 8 variables

```
> new.data <- subset(HS.data, select = c(deduct, numeric, problemr, arithmet, paragrap
+     wordc, wordm))
> cor(new.data)
```

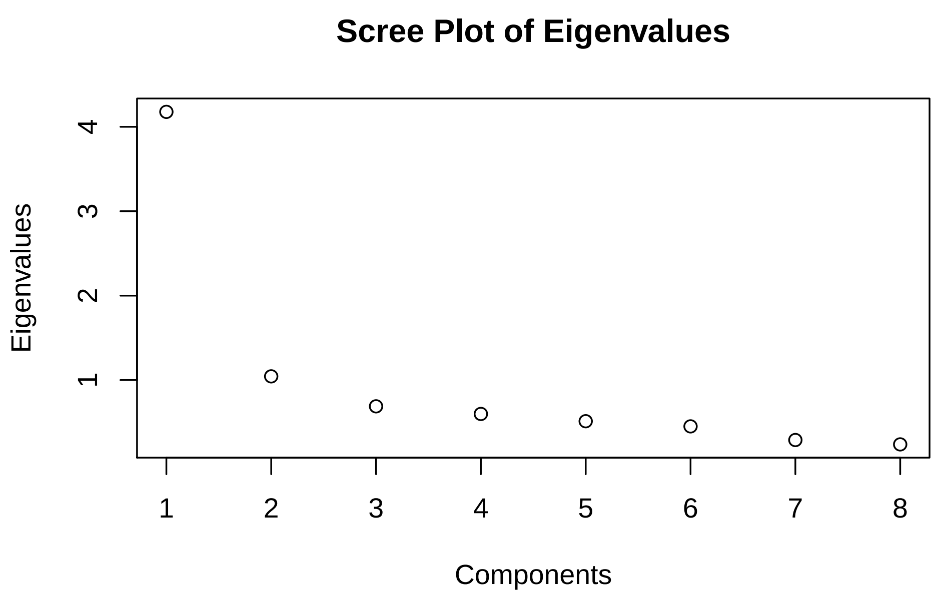
```
            deduct   numeric  problemr  arithmet  paragrap  sentence     wordc     wordm
deduct   1.0000000 0.3969520 0.3938008 0.3288406 0.3501101 0.3382241 0.3497083 0.3807184
numeric  0.3969520 1.0000000 0.3614647 0.4590460 0.3307238 0.3024186 0.3169175 0.3447537
problemr 0.3938008 0.3614647 1.0000000 0.3909644 0.4313540 0.4616993 0.4029156 0.5008951
arithmet 0.3288406 0.4590460 0.3909644 1.0000000 0.4073259 0.3787611 0.3978134 0.4329759
paragrap 0.3501101 0.3307238 0.4313540 0.4073259 1.0000000 0.7331702 0.5818947 0.7044802
sentence 0.3382241 0.3024186 0.4616993 0.3787611 0.7331702 1.0000000 0.6744079 0.7199555
wordc    0.3497083 0.3169175 0.4029156 0.3978134 0.5818947 0.6744079 1.0000000 0.5816137
wordm    0.3807184 0.3447537 0.5008951 0.4329759 0.7044802 0.7199555 0.5816137 1.0000000
```

This will extract ALL components (note that the rotation default is varimax).

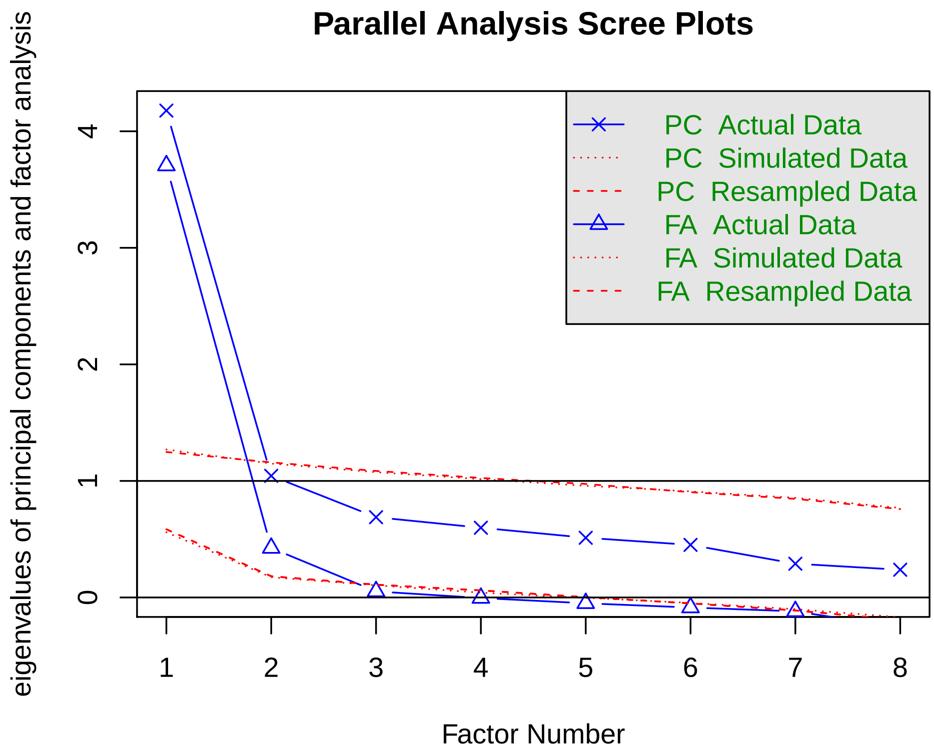
```
> pc8 <- principal(new.data, nfactors = 8)
> pc8
```

```
Principal Components Analysis
Call: principal(r = new.data, nfactors = 8)
Standardized loadings based upon correlation matrix
          RC6  RC4  RC3  RC5  RC2  RC1  RC7  RC8 h2       u2
deduct   0.12 0.15 0.95 0.12 0.17 0.10 0.11 0.08  1 -1.8e-15
numeric  0.10 0.13 0.17 0.20 0.94 0.09 0.09 0.07  1 -8.9e-16
problemr 0.13 0.92 0.16 0.15 0.14 0.13 0.16 0.13  1  1.2e-15
arithmet 0.14 0.15 0.12 0.93 0.21 0.13 0.13 0.09  1 -1.1e-15
paragrap 0.24 0.16 0.13 0.16 0.12 0.85 0.28 0.27  1 -4.4e-16
sentence 0.33 0.19 0.12 0.13 0.10 0.34 0.30 0.78  1  4.4e-16
wordc    0.89 0.15 0.13 0.15 0.11 0.21 0.19 0.23  1 -1.1e-15
```

```
wordm    0.24 0.21 0.15 0.17 0.12 0.31 0.82 0.26  1  1.1e-16

                 RC6  RC4  RC3  RC5  RC2  RC1  RC7  RC8
SS loadings    1.07 1.04 1.03 1.03 1.03 1.02 0.93 0.84
Proportion Var 0.13 0.13 0.13 0.13 0.13 0.13 0.12 0.10
Cumulative Var 0.13 0.26 0.39 0.52 0.65 0.78 0.90 1.00

Test of the hypothesis that 8 factors are sufficient.

The degrees of freedom for the null model are  28  and the objective function was  3.55
The degrees of freedom for the model are -8  and the objective function was  0
The number of observations was  301  with Chi Square =  0  with prob <  NA

Fit based upon off diagonal values = 1

> plot(pc8$values, main = "Scree Plot of Eigenvalues", xlab = "Components", ylab = "Ei
```




```
> fa.parallel(new.data)

Parallel analysis suggests that the number of factors =  2  and the number of components
```



Factor Loadings

```
> pc8$loadings

Loadings:
         RC6   RC4   RC3   RC5   RC2   RC1   RC7   RC8
deduct   0.116 0.153 0.945 0.116 0.168 0.102 0.107
numeric        0.133 0.170 0.197 0.940
problemr 0.135 0.924 0.164 0.150 0.141 0.133 0.159 0.128
arithmet 0.138 0.147 0.121 0.928 0.205 0.126 0.127
paragrap 0.238 0.159 0.130 0.156 0.119 0.846 0.275 0.266
sentence 0.331 0.187 0.118 0.129       0.341 0.297 0.782
wordc    0.890 0.146 0.135 0.153 0.112 0.211 0.191 0.227
wordm    0.236 0.214 0.149 0.171 0.122 0.306 0.819 0.260

                 RC6   RC4   RC3   RC5   RC2   RC1   RC7   RC8
SS loadings    1.074 1.043 1.035 1.030 1.025 1.024 0.932 0.837
Proportion Var 0.134 0.130 0.129 0.129 0.128 0.128 0.116 0.105
Cumulative Var 0.134 0.265 0.394 0.523 0.651 0.779 0.895 1.000
```

Communality coefficients

```
> pc8$communality
```

```
  deduct  numeric problemr arithmet paragrap sentence    wordc    wordm 
       1        1        1        1        1        1        1        1 
```

To produce the component scores, we must specify a new model

```
> pc8.2 <- principal(new.data, nfactors = 8, scores = T)
> head(pc8.2$scores)
```

```
          RC6         RC4        RC3         RC5        RC2        RC1        RC7
1 -1.00890630  1.25979445 -1.4462749  0.09640643  0.1602572 -0.8966346 -1.3100434  2.591
2 -0.36722281  1.47215193 -1.3738650  0.18185916  0.2033640 -1.0564072 -0.4724874 -0.731
3 -2.03018937 -0.09995563 -0.7384961 -0.13374965 -0.6068631 -0.8265910 -0.4892902 -0.931
4 -0.91346335 -0.15504069 -1.2028911 -0.90668964 -0.5506871 -0.2084395  0.9137091  0.723
5 -0.06297032 -0.88157520  0.2487514 -0.99681502  0.4488598 -0.3829034  0.9932680 -0.237
6  0.74536606 -0.56698396 -0.3510692  0.40943397 -2.5899584 -1.5912043 -0.4819236 -0.330
```

1.2.2 Extracting only 2 components

This will extract 2 components

```
> pc2 <- principal(new.data, nfactors = 2, rotate = "varimax")
> pc2
```

```
Principal Components Analysis
Call: principal(r = new.data, nfactors = 2, rotate = "varimax")
Standardized loadings based upon correlation matrix
          RC1  RC2   h2   u2
deduct   0.22 0.68 0.52 0.48
numeric  0.11 0.82 0.69 0.31
problemr 0.45 0.54 0.49 0.51
arithmet 0.30 0.67 0.54 0.46
paragrap 0.84 0.24 0.76 0.24
sentence 0.89 0.19 0.83 0.17
wordc    0.76 0.25 0.65 0.35
wordm    0.81 0.30 0.75 0.25

                RC1  RC2
SS loadings    3.09 2.13
Proportion Var 0.39 0.27
Cumulative Var 0.39 0.65

Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  28  and the objective function was  3.55
The degrees of freedom for the model are 13  and the objective function was  0.36
The number of observations was  301  with Chi Square =  104.87  with prob <  1.9e-16

Fit based upon off diagonal values = 0.97
```

Factor Loadings

```
> pc2$loadings

Loadings:
         RC1   RC2
deduct   0.223 0.685
numeric  0.110 0.821
problemr 0.446 0.536
arithmet 0.300 0.675
paragrap 0.836 0.238
sentence 0.889 0.192
wordc    0.764 0.253
wordm    0.815 0.301

                 RC1   RC2
SS loadings    3.087 2.134
Proportion Var 0.386 0.267
Cumulative Var 0.386 0.653
```

Communality coefficients

```
> pc2$communality

   deduct   numeric  problemr  arithmet  paragrap  sentence     wordc     wordm
0.5188747 0.6868129 0.4865909 0.5449207 0.7550064 0.8278737 0.6473761 0.7541154
```

To produce the component scores, we must specify a new model

```
> pc2.2 <- principal(new.data, nfactors = 2, scores = T)
> head(pc2.2$scores)

          RC1        RC2
1 -0.05145814 -0.3682112
2 -1.02685693  0.0587834
3 -2.05411320 -0.8077327
4  0.18772949 -1.4701100
5 -0.16809807 -0.2944662
6 -0.66099977 -1.7714983
```

2 Exploratory Factor Analysis

2.1 Single Factor Solution

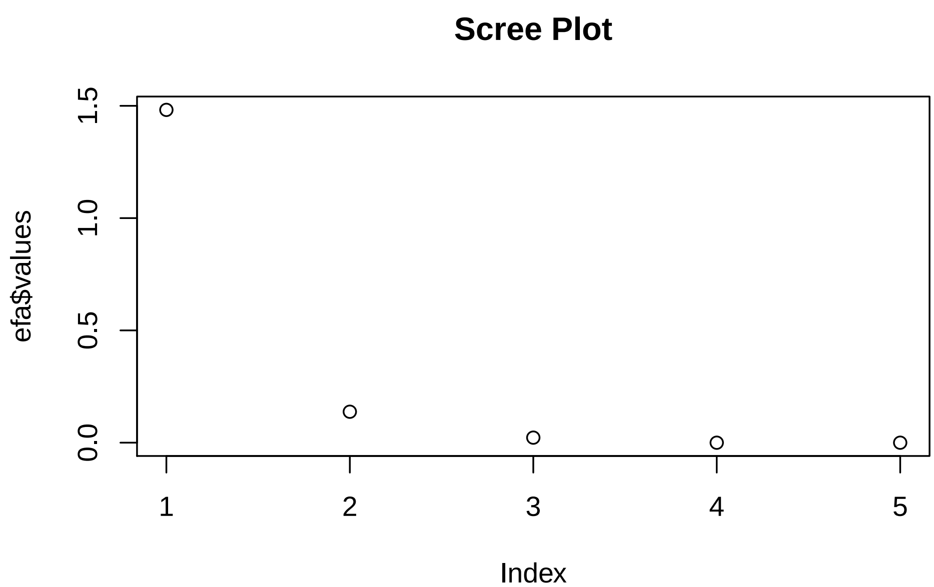
In EFA, we are solving a correlated latent factor structure rather than an orthogonal (as in PCA).

```
> efa <- factor.pa(HS.data[, 7:11])
> efa

Factor Analysis using method =  pa
Call: factor.pa(r = HS.data[, 7:11])
Unstandardized loadings based upon covariance matrix
         PA1   h2   u2   H2   U2
visual  0.72 0.52 0.48 0.52 0.48
cubes   0.47 0.22 0.78 0.22 0.78
paper   0.52 0.27 0.73 0.27 0.73
flags   0.60 0.36 0.64 0.36 0.64
general 0.34 0.12 0.88 0.12 0.88

                PA1
SS loadings    1.48
Proportion Var 0.30

 Standardized loadings
        V  PA1   h2   u2
visual  1 0.72 0.52 0.48
cubes   2 0.47 0.22 0.78
paper   3 0.52 0.27 0.73
flags   4  0.6 0.36 0.64
general 5 0.34 0.12 0.88

                PA1
SS loadings    1.48
Proportion Var 0.30

Test of the hypothesis that 1 factor is sufficient.

The degrees of freedom for the null model are  10  and the objective function was  0.67
The degrees of freedom for the model are 5  and the objective function was  0.04

The root mean square of the residuals is  0.03
The df corrected root mean square of the residuals is  0.06
The number of observations was  301  with Chi Square =  12.12  with prob <  0.033

Tucker Lewis Index of factoring reliability =  0.925
RMSEA index =  0.07  and the 90 % confidence intervals are  0.07 0.077
BIC =  -16.41
Fit based upon off diagonal values = 0.98
Measures of factor score adequacy
                                                PA1
Correlation of scores with factors             0.84
```

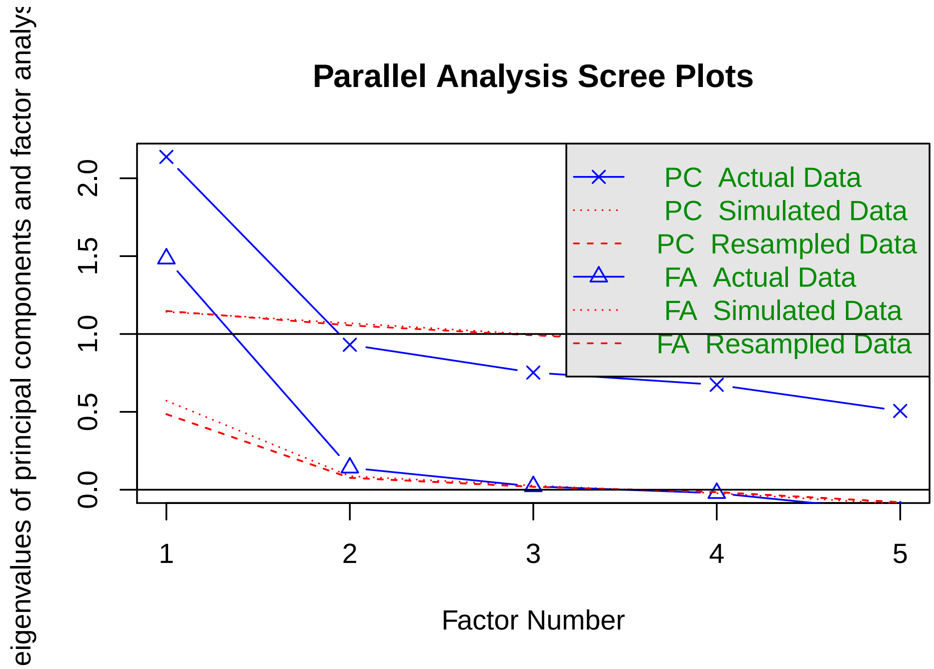
```
Multiple R square of scores with factors       0.71
Minimum correlation of possible factor scores  0.41
```

Eigenvalues

```
> efa$values

[1] 1.482127e+00 1.378552e-01 2.248116e-02 2.220446e-16 2.220446e-16

> plot(efa$values, main = "Scree Plot")
```




```
> fa.parallel(HS.data[, 7:11])

Parallel analysis suggests that the number of factors =  2  and the number of components
```



Factor Loadings

```
> efa$loadings

Loadings:
        PA1
visual  0.718
cubes   0.472
paper   0.523
flags   0.596
general 0.341

                 PA1
SS loadings    1.482
Proportion Var 0.296
```

Communality coefficients

```
> efa$communality

 visual   cubes   paper   flags general
   0.52    0.22    0.27    0.36    0.12
```

To produce the component scores, we must specify a new model

```
> efa2 <- factor.pa(HS.data[, 7:11], scores = T)
> head(efa2$scores)
```

```
         PA1
1 -0.9521456
2 -0.2385255
3 -0.7288804
4  0.7300731
5 -0.7699293
6 -0.3502616
```

We can now use these factor scores in a regression equation

```
> efa1 <- lm(efa2$scores ~ HS.data$numeric + HS.data$arithmet)
> summary(efa1)

Call:
lm(formula = efa2$scores ~ HS.data$numeric + HS.data$arithmet)

Residuals:
     Min       1Q   Median       3Q      Max
-2.46787 -0.50928 -0.03704  0.44861  2.39699

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)      -1.52556    0.22777  -6.698 1.05e-10 ***
HS.data$numeric   0.06591    0.01062   6.207 1.80e-09 ***
HS.data$arithmet  0.02442    0.01024   2.385   0.0177 *
---
Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ 1

Residual standard error: 0.7555 on 298 degrees of freedom
Multiple R-squared: 0.1973,	Adjusted R-squared: 0.1919
F-statistic: 36.62 on 2 and 298 DF,  p-value: 6.027e-15
```

2.2 Multi-factor Solution

2.2.1 Extracting all 8 factors

This will extract ALL components (note that the rotation default is varimax, but we will change this to oblimin)

```
> library(GPArotation)
> efa8 <- factor.pa(new.data, nfactors = 8, rotate = "oblimin")
> efa8

Factor Analysis using method =  pa
Call: factor.pa(r = new.data, nfactors = 8, rotate = "oblimin")
Standardized loadings based upon correlation matrix
         PA2 PA3 PA5 PA4 PA6 PA1 PA7 PA8 h2 u2
```

```
deduct     0  1  0  0  0  0  0  0 1 0
numeric    1  0  0  0  0  0  0  0 1 0
problemr   0  0  0 -1  0  0  0  0 1 0
arithmet   0  0  1  0  0  0  0  0 1 0
paragrap   0  0  0  0  0  1  0  0 1 0
sentence   0  0  0  0  0  0  0  1 1 0
wordc      0  0  0  0  1  0  0  0 1 0
wordm      0  0  0  0  0  0 -1  0 1 0

                PA2  PA3  PA5  PA4  PA6  PA1  PA7  PA8
SS loadings    1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00
Proportion Var 0.13 0.13 0.13 0.12 0.12 0.12 0.12 0.12
Cumulative Var 0.13 0.25 0.38 0.50 0.63 0.75 0.88 1.00

 With factor correlations of
      PA2   PA3   PA5   PA4   PA6   PA1   PA7   PA8
PA2  1.00  0.40  0.46 -0.36  0.32  0.33 -0.34  0.30
PA3  0.40  1.00  0.33 -0.39  0.35  0.35 -0.38  0.34
PA5  0.46  0.33  1.00 -0.39  0.40  0.41 -0.43  0.38
PA4 -0.36 -0.39 -0.39  1.00 -0.40 -0.43  0.50 -0.46
PA6  0.32  0.35  0.40 -0.40  1.00  0.58 -0.58  0.67
PA1  0.33  0.35  0.41 -0.43  0.58  1.00 -0.70  0.73
PA7 -0.34 -0.38 -0.43  0.50 -0.58 -0.70  1.00 -0.72
PA8  0.30  0.34  0.38 -0.46  0.67  0.73 -0.72  1.00

Test of the hypothesis that 8 factors are sufficient.

The degrees of freedom for the null model are  28  and the objective function was  3.55
The degrees of freedom for the model are -8  and the objective function was  0

The root mean square of the residuals is  0
The number of observations was  301  with Chi Square =  0  with prob <  NA

Tucker Lewis Index of factoring reliability =  1.028
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                               PA2 PA3 PA5 PA4 PA6 PA1 PA7 PA8
Correlation of scores with factors               1   1   1   1   1   1   1   1
Multiple R square of scores with factors         1   1   1   1   1   1   1   1
Minimum correlation of possible factor scores    1   1   1   1   1   1   1   1

 WARNING, the factor score fit indices suggest that the solution is degenerate. Try a di
```

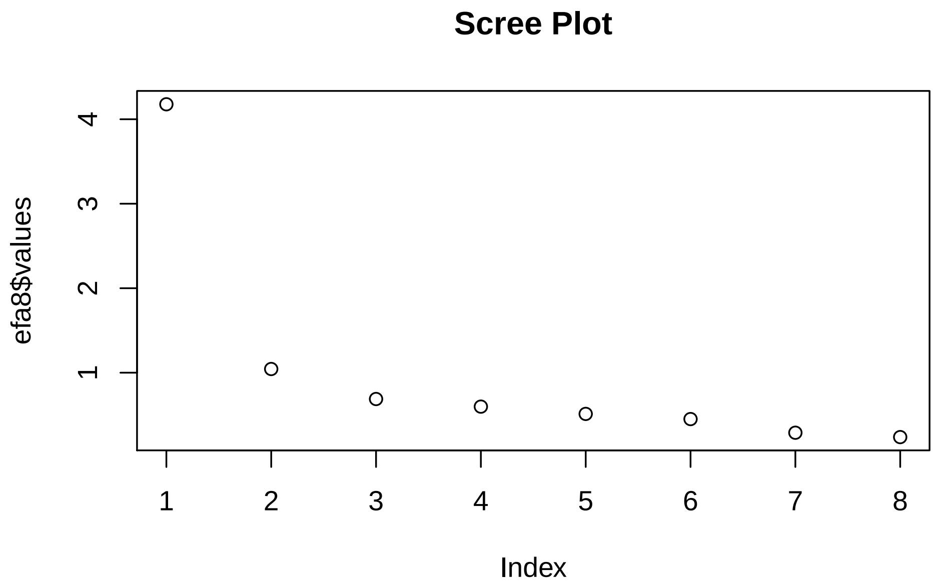
Eigenvalues

```
> efa8$values
```

```
[1] 4.1776307 1.0439401 0.6887955 0.5983585 0.5123791 0.4513061 0.2895678 0.2380223
```

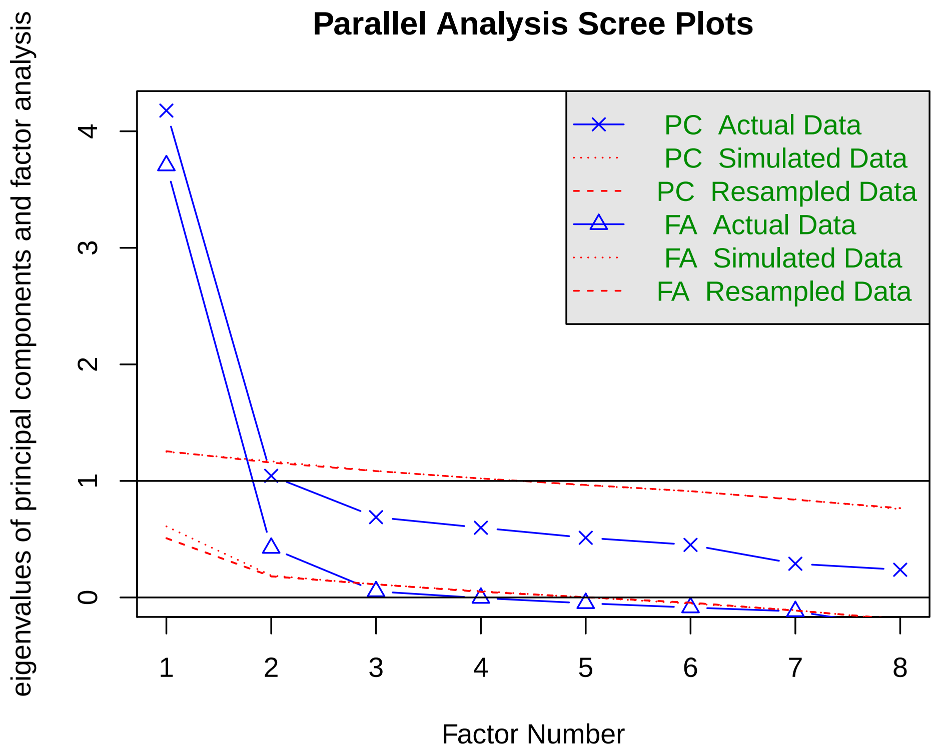
Scree plot

```
> plot(efa8$values, main = "Scree Plot")
```




```
> fa.parallel(new.data)
```

```
Parallel analysis suggests that the number of factors =  2  and the number of components
```



Factor Loadings

```
> efa8$loadings

Loadings:
         PA2 PA3 PA5 PA4 PA6 PA1 PA7 PA8
deduct        1
numeric   1
problemr            -1
arithmet          1
paragrap                      1
sentence                              1
wordc                     1
wordm                           -1

                PA2   PA3   PA5   PA4   PA6   PA1   PA7   PA8
SS loadings    1.000 1.000 1.000 1.000 1.000 1.000 1.000 1.000
Proportion Var 0.125 0.125 0.125 0.125 0.125 0.125 0.125 0.125
Cumulative Var 0.125 0.250 0.375 0.500 0.625 0.750 0.875 1.000
```

Communality coefficients

```
> efa8$communality
```

```
  deduct  numeric problemr arithmet paragrap sentence    wordc    wordm
       1        1        1        1        1        1        1        1
```

To produce the component scores, we must specify a new model

```
> efa8.2 <- factor.pa(new.data, nfactors = 8, scores = T, rotate = "oblimin")
> head(efa8.2$scores)
```

```
         PA2        PA3        PA5         PA4         PA6        PA1        PA7
1  0.4204280 -1.6393363  0.3499192 -1.71144656 -2.02403792 -1.8853926  2.6016561  5.2172
2  0.3632796 -1.6542005  0.3039945 -2.15724270  0.02072663 -1.0956368  0.2352110 -0.7416
3 -0.3866321 -0.4499127  0.3679465 -0.42008623 -2.04521492 -0.3152520 -0.1553631 -0.5730
4 -0.2452212 -1.2155830 -0.8817477  0.07324125 -1.40892274 -0.6572090 -1.5278005  1.3087
5  0.7507221  0.3408962 -1.2630964  1.14673268  0.03191360 -0.7168585 -1.9084986 -0.4683
6 -2.9096464  0.1201468  1.1910502  0.27281322  1.57901713 -2.0652525  0.0900083 -0.0105
```

2.2.2 Extracting only 2 factors

```
> efa2 <- factor.pa(new.data, nfactors = 2, rotate = "oblimin")
> efa2
```

```
Factor Analysis using method =  pa
Call: factor.pa(r = new.data, nfactors = 2, rotate = "oblimin")
Unstandardized loadings based upon covariance matrix
           PA1   PA2   h2   u2   H2   U2
deduct    0.13  0.49 0.33 0.67 0.33 0.67
numeric  -0.07  0.74 0.49 0.51 0.49 0.51
problemr  0.30  0.39 0.39 0.61 0.39 0.61
arithmet  0.15  0.54 0.41 0.59 0.41 0.59
paragrap  0.80  0.04 0.67 0.33 0.67 0.33
sentence  0.96 -0.10 0.82 0.18 0.82 0.18
wordc     0.66  0.11 0.53 0.47 0.53 0.47
wordm     0.76  0.11 0.69 0.31 0.69 0.31

                PA1  PA2
SS loadings    2.88 1.47
Proportion Var 0.36 0.18
Cumulative Var 0.36 0.54

 Standardized loadings
         item   PA1   PA2   h2   u2
deduct      1  0.13  0.49 0.33 0.67
numeric     2 -0.07  0.74 0.49 0.51
problemr    3  0.30  0.39 0.39 0.61
arithmet    4  0.15  0.54 0.41 0.59
paragrap    5  0.80  0.04 0.67 0.33
```

```
sentence    6  0.96 -0.10 0.82 0.18
wordc       7  0.66  0.11 0.53 0.47
wordm       8  0.76  0.12 0.69 0.31

                 PA1  PA2
SS loadings    2.87 1.46
Proportion Var 0.36 0.18
Cumulative Var 0.36 0.54

 With factor correlations of
    PA1 PA2
PA1 1.0 0.6
PA2 0.6 1.0

Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  28  and the objective function was  3.55
The degrees of freedom for the model are 13  and the objective function was  0.05

The root mean square of the residuals is  0.01
The df corrected root mean square of the residuals is  0.03
The number of observations was  301  with Chi Square =  15.54  with prob <  0.27

Tucker Lewis Index of factoring reliability =  0.995
RMSEA index =  0.027  and the 90 % confidence intervals are  0.027 0.034
BIC =  -58.65
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                 PA1  PA2
Correlation of scores with factors              0.96 0.86
Multiple R square of scores with factors        0.91 0.74
Minimum correlation of possible factor scores  0.83 0.48
```

Eigenvalues

```
> efa2$values

[1] 3.766484e+00 5.745630e-01 8.001942e-02 5.789766e-02 1.304531e-02 2.220446e-16 2.2204
```

Factor Loadings

```
> efa2$loadings

Loadings:
         PA1    PA2
deduct    0.125  0.493
```

```
numeric           0.742
problemr  0.305  0.394
arithmet  0.148  0.544
paragrap  0.797
sentence  0.959
wordc     0.657  0.111
wordm     0.756  0.115

                 PA1   PA2
SS loadings    2.693 1.282
Proportion Var 0.337 0.160
Cumulative Var 0.337 0.497
```

Communality coefficients

```
> efa2$communality

  deduct  numeric problemr arithmet paragrap sentence    wordc    wordm
    0.33     0.49     0.39     0.41     0.67     0.82     0.53     0.69
```

To produce the component scores, we must specify a new model

```
> efa2.2 <- factor.pa(new.data, nfactors = 2, scores = T, rotate = "oblimin")
> head(efa2.2$scores)

         PA1        PA2
1  0.5402841 -0.7278641
2 -1.2792800  0.4743499
3 -1.8733252 -0.3088065
4  0.6727311 -1.4215953
5 -0.1033492 -0.1743156
6 -0.3729431 -1.4735704
```

Run the analysis with Promax rotation

```
> efa2.promax <- factor.pa(new.data, nfactors = 2, rotate = "promax")
> efa2.promax

Factor Analysis using method =  pa
Call: factor.pa(r = new.data, nfactors = 2, rotate = "promax")
Unstandardized loadings based upon covariance matrix
           PA1   PA2   h2   u2   H2   U2
deduct    0.03  0.55 0.33 0.67 0.33 0.67
numeric  -0.22  0.84 0.49 0.51 0.49 0.51
problemr  0.23  0.44 0.39 0.61 0.39 0.61
arithmet  0.04  0.61 0.41 0.59 0.41 0.59
paragrap  0.80  0.03 0.67 0.33 0.67 0.33
```

```
sentence  0.99 -0.13 0.82 0.18 0.82 0.18
wordc     0.65  0.11 0.53 0.47 0.53 0.47
wordm     0.75  0.11 0.69 0.31 0.69 0.31

                PA1  PA2
SS loadings    2.71 1.63
Proportion Var 0.34 0.20
Cumulative Var 0.34 0.54

 Standardized loadings
         item   PA1   PA2   h2   u2
deduct      1  0.03  0.55 0.33 0.67
numeric     2 -0.22  0.84 0.49 0.51
problemr    3  0.23  0.44 0.39 0.61
arithmet    4  0.04  0.61 0.41 0.59
paragrap    5  0.80  0.03 0.67 0.33
sentence    6  0.99 -0.13 0.82 0.18
wordc       7  0.65  0.11 0.53 0.47
wordm       8  0.75  0.11 0.69 0.31

                PA1  PA2
SS loadings    2.71 1.63
Proportion Var 0.34 0.20
Cumulative Var 0.34 0.54

 With factor correlations of
     PA1  PA2
PA1 1.00 0.71
PA2 0.71 1.00

Test of the hypothesis that 2 factors are sufficient.

The degrees of freedom for the null model are  28  and the objective function was  3.55
The degrees of freedom for the model are 13  and the objective function was  0.05

The root mean square of the residuals is  0.01
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The number of observations was  301  with Chi Square =  15.54  with prob <  0.27

Tucker Lewis Index of factoring reliability =  0.995
RMSEA index =  0.027  and the 90 % confidence intervals are  0.027 0.034
BIC =  -58.65
Fit based upon off diagonal values = 1
Measures of factor score adequacy
                                                PA1  PA2
```

```
Correlation of scores with factors             0.96 0.89
Multiple R square of scores with factors       0.91 0.79
Minimum correlation of possible factor scores  0.83 0.58
```