

# Robustness of Deep Neural Networks to Occlusion

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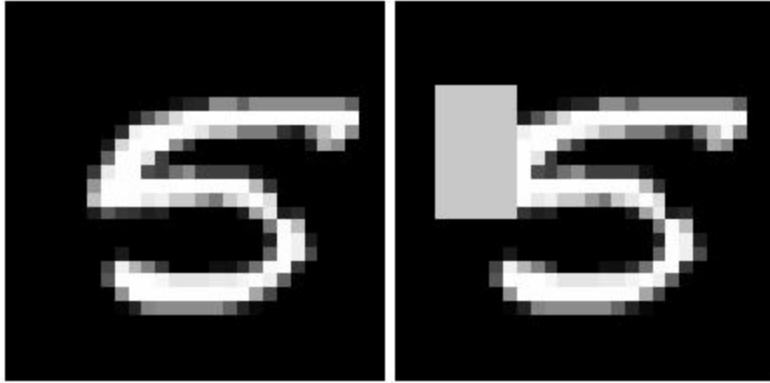
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# Occlusion



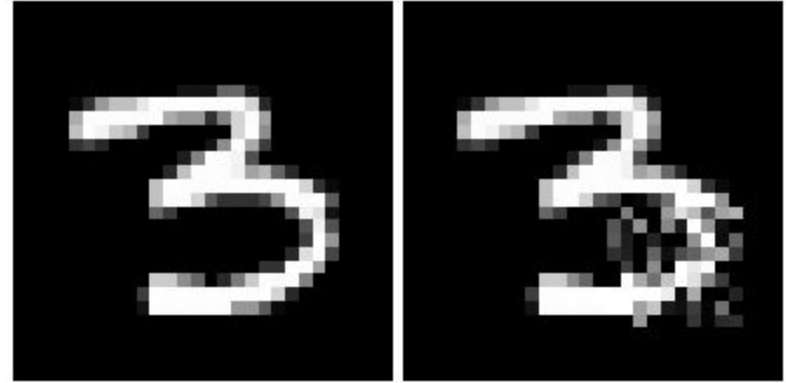
An occlusion is an event wherein parts of an image are blocked, either partially or completely by another object in the scene.

# Types of Occlusion



(a) Uniform

Colour is same in all occluded pixels.



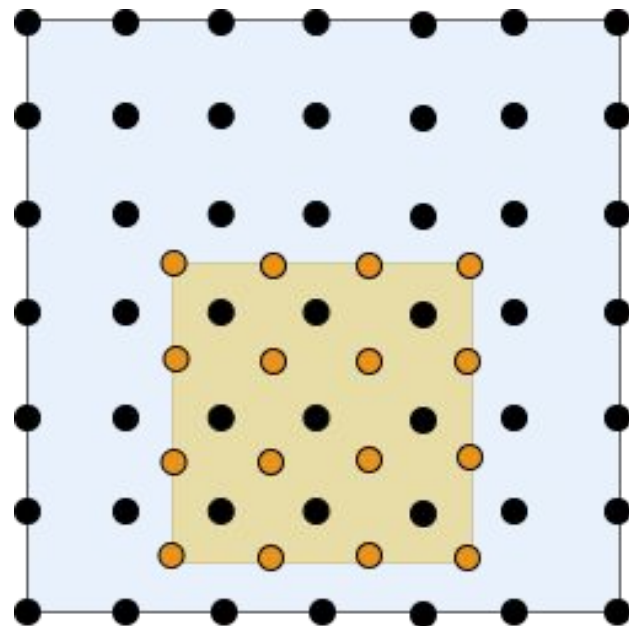
(b) Multiform

Colours vary from  $[-\epsilon, \epsilon]$  where  $\epsilon$  denotes the threshold between original pixel value and occluded pixel value.

# Occlusion Position

- L1 distance between image pixel and surrounding occluding pixels is less than 1.
- Contributions from all four surrounding pixels is summed up.

$$s_{ij} = \max(0, \sum_{i' \in \mathbb{I}_n} (|i - i' + 1|) + \sum_{j' \in \mathbb{I}_n} (|j - j' + 1|) - 1)$$



# Occlusion Operation

$$x'_{ij} = x_{ij} - s_{ij} \times (x_{ij} - \zeta(x, i, j))$$

$x_{i,j}$  : Original pixel value

$x'_{i,j}$  : Updated pixel value

$s_{i,j}$  : Coefficient of occlusion

$\zeta(x, i, j)$  : Colour of occlusion at  $(i, j)$

Uniform occlusion:  $\zeta(x, i, j) = \mu.$

Multiform occlusion:  $\zeta(x, i, j) = x_{ij} + \Delta_p \quad \Delta_p \in [-\epsilon, \epsilon]$

# Objective

To explore various occlusion encodings for benchmark datasets like MNIST, CIFAR-10 and GTSRB and verify the robustness of neural networks to these perturbations.

# Random Erasing

- Randomly picks upper left corner of occlusion rectangle.
- Replaces or “erases” occlusion rectangle with random colour.

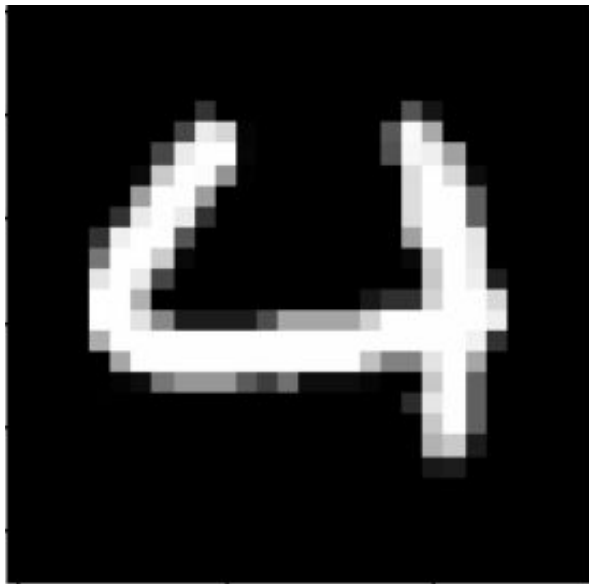
**Limitation:** Only picks integer coordinates  
We need to consider real-valued coordinates.

## Algorithm 1: Random Erasing Procedure

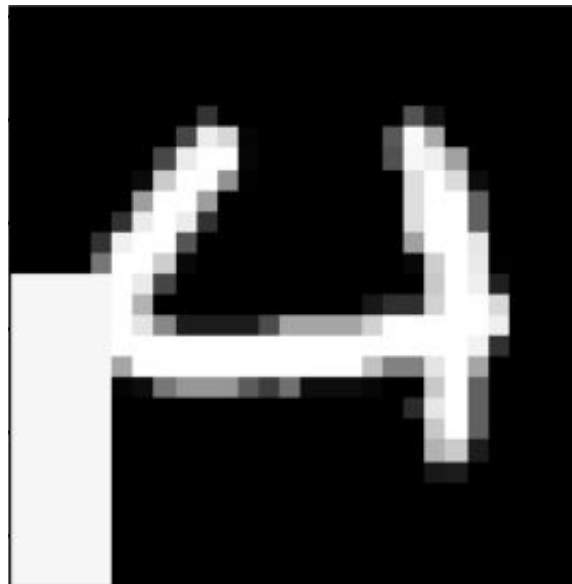
**Input** : Input image  $I$ ; Image size  $W$  and  $H$ ; Area of image  $S$ ; Erasing probability  $p$ ; Erasing area ratio range  $s_l$  and  $s_h$ ; Erasing aspect ratio range  $r_1$  and  $r_2$ .  
**Output**: Erased image  $I^*$ .  
**Initialization**:  $p_1 \leftarrow \text{Rand}(0, 1)$ .

```
1 if  $p_1 \geq p$  then
2    $I^* \leftarrow I$ ;
3   return  $I^*$ .
4 else
5   while True do
6      $S_e \leftarrow \text{Rand}(s_l, s_h) \times S$ ;
7      $r_e \leftarrow \text{Rand}(r_1, r_2)$ ;
8      $H_e \leftarrow \sqrt{S_e \times r_e}$ ,  $W_e \leftarrow \sqrt{\frac{S_e}{r_e}}$ ;
9      $x_e \leftarrow \text{Rand}(0, W)$ ,  $y_e \leftarrow \text{Rand}(0, H)$ ;
10    if  $x_e + W_e \leq W$  and  $y_e + H_e \leq H$  then
11       $I_e \leftarrow (x_e, y_e, x_e + W_e, y_e + H_e)$ ;
12       $I(I_e) \leftarrow \text{Rand}(0, 255)$ ;
13       $I^* \leftarrow I$ ;
14      return  $I^*$ .
15    end
16  end
17 end
```

## Random Erasing



Original Image

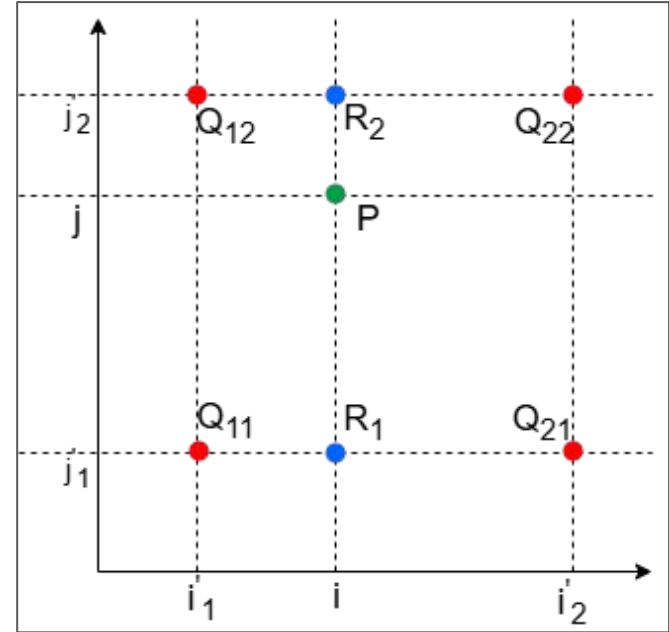


Occluded image



# Bilinear Interpolation

- Calculates pixel intensities when image is mapped to another geometry.
- Bilinear interpolation considers 4 nearest neighbors of interpolated point.
- Image pixel is affected by occlusion pixel if they are less than  $\sqrt{2}$  units apart.

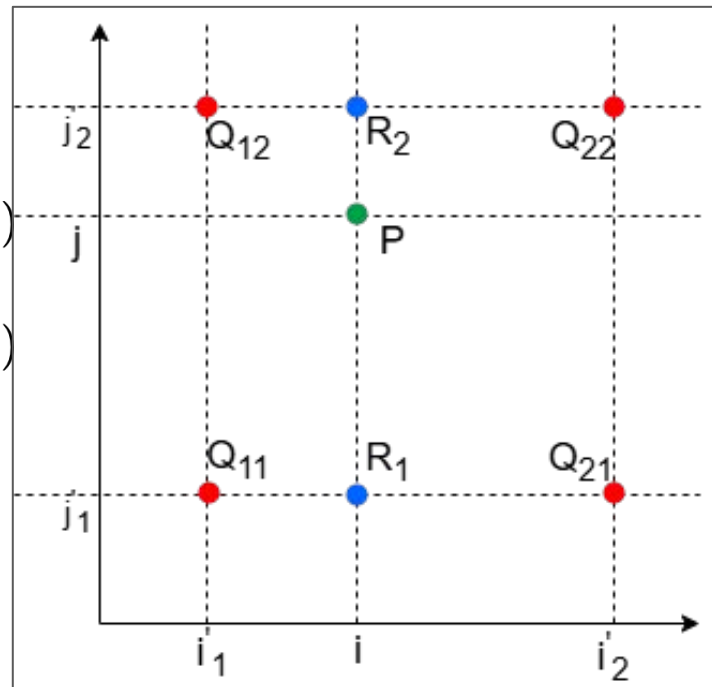


# Bilinear Interpolation

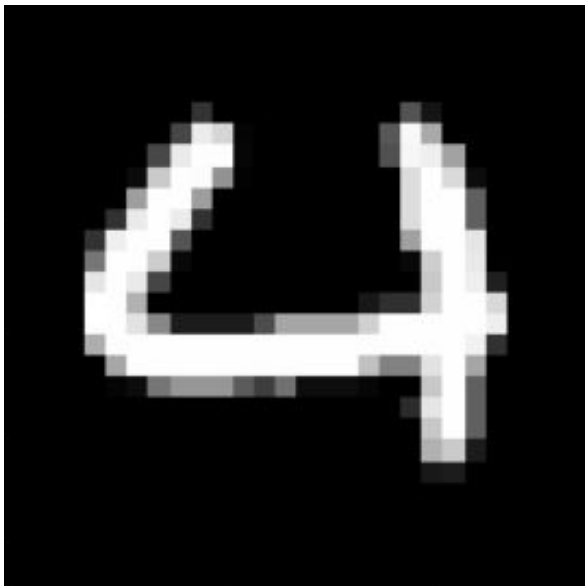
$$R_1(x, y) = Q_{11}(x_2 - x)/(x_2 - x_1) + Q_{21}(x - x_1)/(x_2 - x_1)$$

$$R_2(x, y) = Q_{12}(x_2 - x)/(x_2 - x_1) + Q_{22}(x - x_1)/(x_2 - x_1)$$

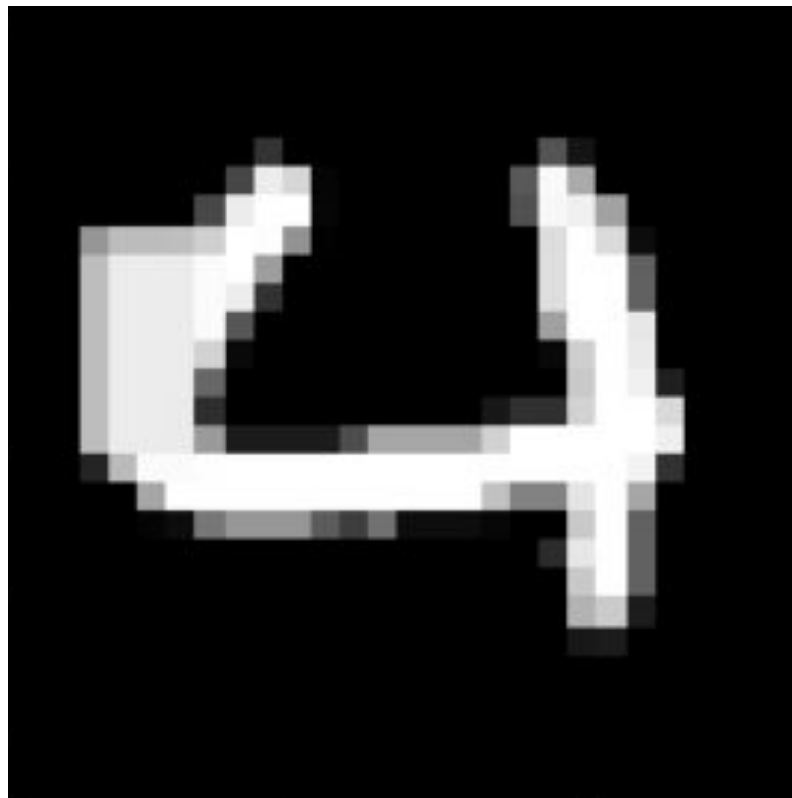
$$P(x, y) = R_1(y_2 - y)/(y_2 - y_1) + R_2(y - y_1)/(y_2 - y_1)$$



# Bilinear Interpolation

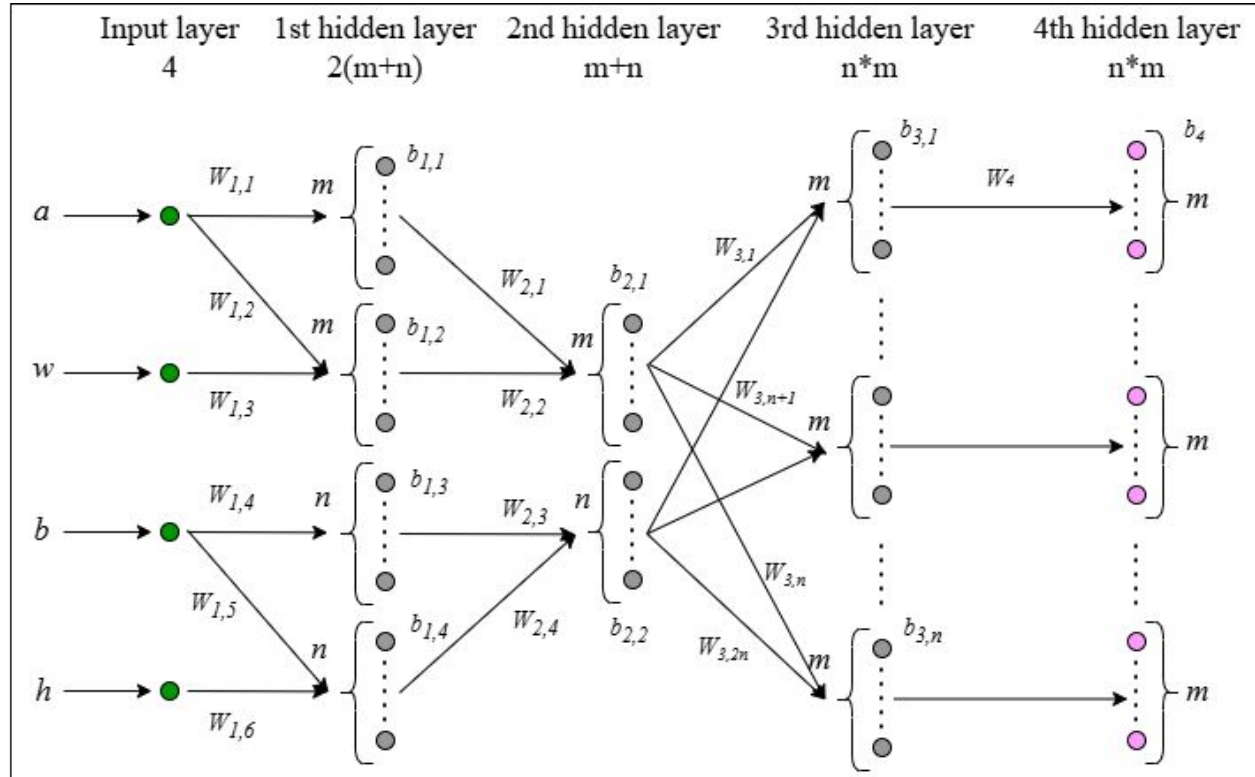


Original Image



Occluded image

# Occlusion Encoding with Neural Networks

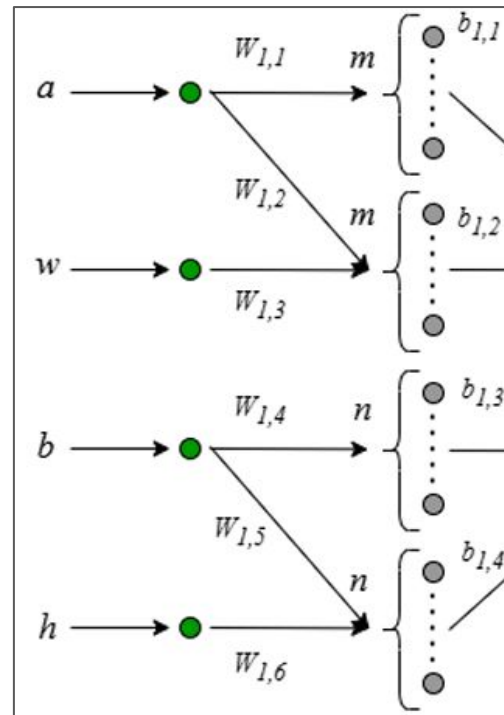


# Occlusion Encoding with Neural Networks

First layer encodes the input (a, w, b, h)

$$W_{1,1} = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ 1 \end{bmatrix}_{m \times 1}, W_{1,2} = \begin{bmatrix} -1 \\ -1 \\ \cdot \\ -1 \end{bmatrix}_{m \times 1}, W_{1,3} = \begin{bmatrix} -1 \\ -1 \\ \cdot \\ -1 \end{bmatrix}_{m \times 1};$$

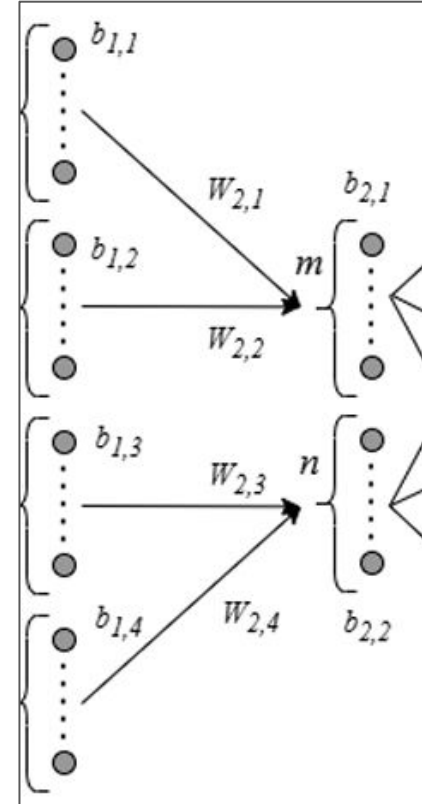
$$b_{1,1} = \begin{bmatrix} -1 \\ -2 \\ \cdot \\ \cdot \\ -m \end{bmatrix}_{m \times 1}, b_{1,2} = \begin{bmatrix} 2 \\ 3 \\ \cdot \\ \cdot \\ m+1 \end{bmatrix}_{m \times 1}$$



# Occlusion Encoding with Neural Networks

Second layer: if  $i^{\text{th}}$  neuron in the first  $m$  neurons is 1 and  $j^{\text{th}}$  neuron in the next  $n$  neurons is 1 then  $(i,j)$  is occluded.

$$W_{2,i} = \begin{bmatrix} -1 & & & \\ & -1 & & \\ & & \ddots & \\ & & & -1 \end{bmatrix}_{m \times m}$$

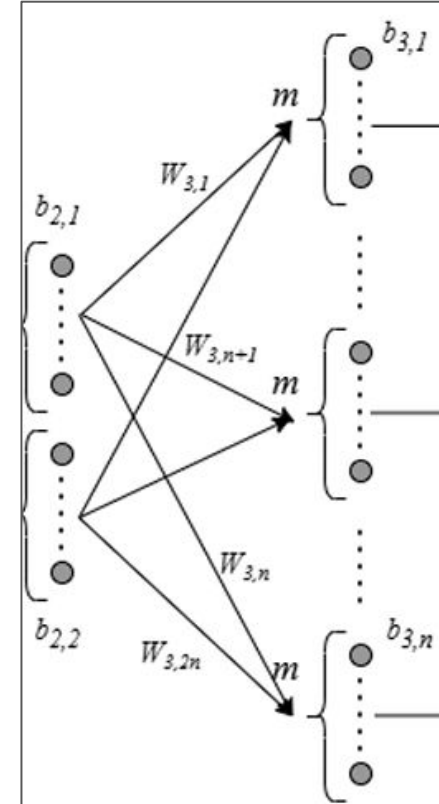


# Occlusion Encoding with Neural Networks

Third layer: Outputs  $m \times n$  neurons, each neuron has the occlusion factor  $s_{ij}$  of each pixel.

$$W_{3,i} = \begin{bmatrix} 1 & 0 & \cdot & \cdot & 0 \\ 1 & 0 & \cdot & \cdot & 0 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ 1 & 0 & \cdot & \cdot & 0 \\ i \end{bmatrix}_{m \times m}, W_{3,n+i} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \cdot & \\ & & & \cdot \\ & & & & 1 \end{bmatrix}_{m \times n}$$

$$b_{3,i} = \begin{bmatrix} -1 \\ -1 \\ \cdot \\ \cdot \\ -1 \end{bmatrix}_{1 \times m}$$



# Occlusion Encoding with Neural Networks

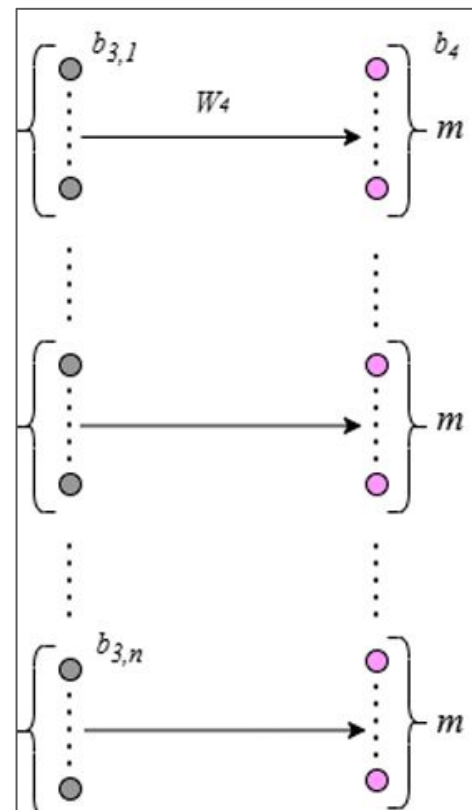
Fourth layer: Encodes the pixel values of original image and occlusion colours.

$$W_4 = \begin{bmatrix} \mu - x_1 & & & \\ & \mu - x_2 & & \\ & & \ddots & \\ & & & \mu - x_{m \times n} \end{bmatrix}_{(mn) \times (mn)}$$

$$W_4 = \begin{bmatrix} \Delta_1 & & & \\ & \Delta_2 & & \\ & & \ddots & \\ & & & \Delta_{mn} \end{bmatrix}_{(mn) \times (mn)}$$

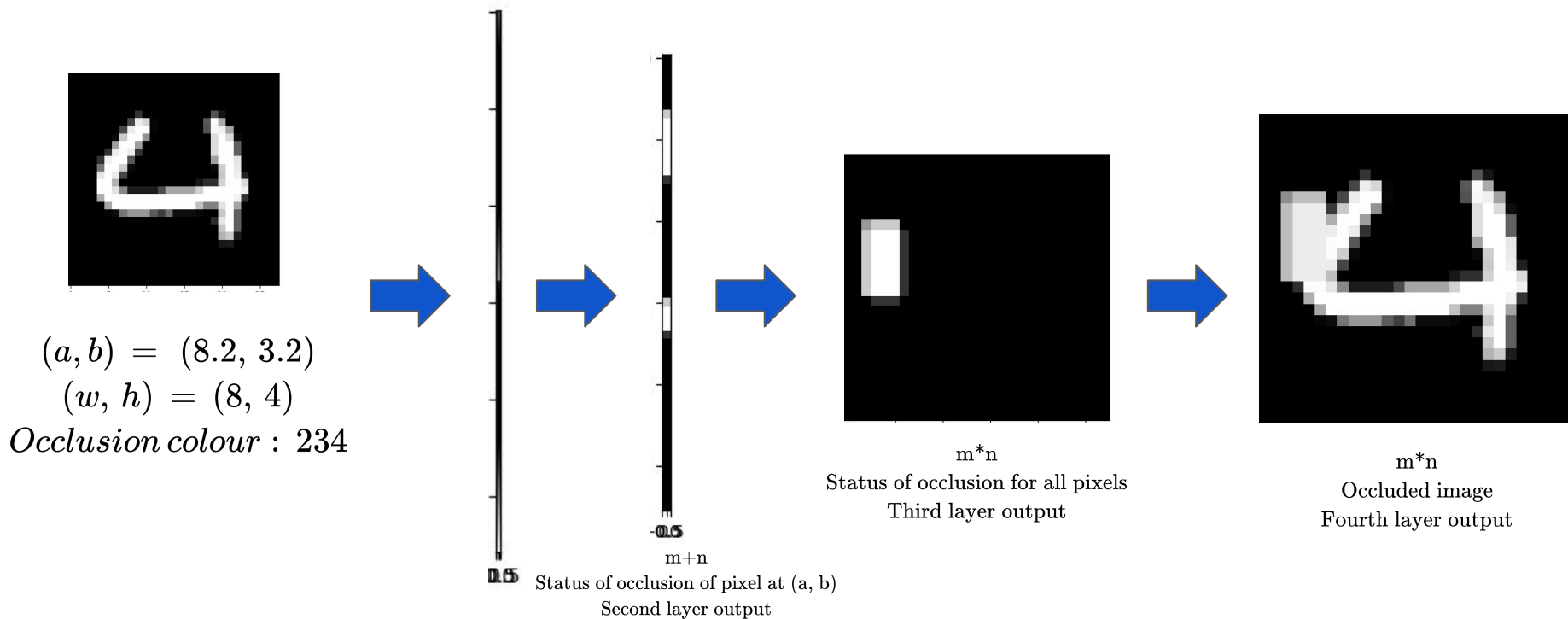
$$b_4 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_{mn} \end{bmatrix}_{mn \times 1}$$

$$W_4 \cdot O_3 + b_4 = (\zeta(x, i, j) - x_{ij}) \times s_{ij} + x_{ij}$$

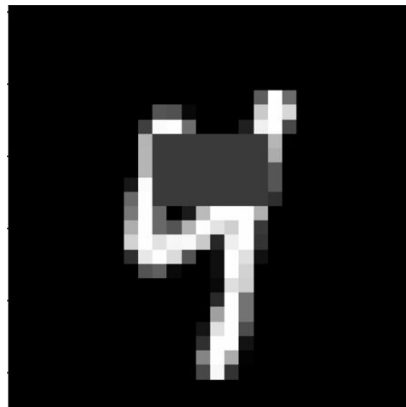
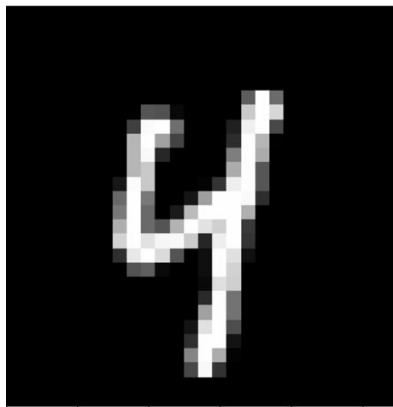




# Occlusion Encoding with Neural Networks



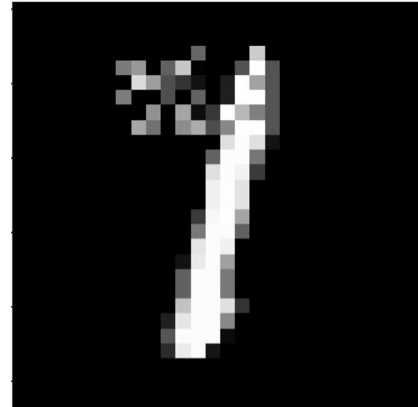
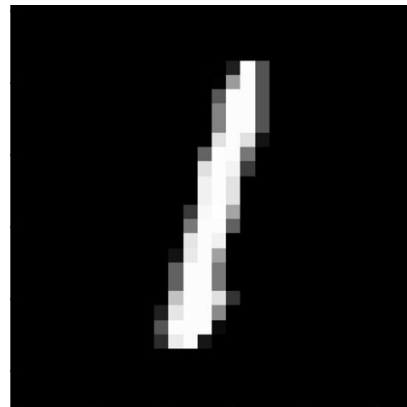
## Results



Original image

Occluded Image

"4" classified as "9"



Original image

Occluded Image

"1" classified as "8"

# Results



Original image

Occluded Image

“deer” classified as “bird”



Original image

Occluded Image

“truck” classified as “ship”

## Results



Original image

Occluded Image

“50” classified as “80”



Occluded image

Original Image

“70” classified as “30”

# Future Work

- Extend encoding to handle more complex shapes like parallelograms, triangles and circles.
- Estimate a range for the threshold  $\epsilon$ .

# References

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**Thank You**