MATHCOUNTS C&P

Samyok Nepal

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1 Permutation Problems

Some problems are taken from Competition Math for Middle School or the Brookings Math Circle Archives.

1. Calcualte 3!, 4!, 5!, 6!, and 7!.

2. What is 0!?

3. Compute $8! - 7!$ without a calculator.

4. What is $\dfrac{25!}{24!}$? $\dfrac{5!}{10}$? What about $\dfrac{20 \cdot 19}{20!}$?

5. What is the number of ways you can reoganize the letters of the word BEAST? What about the word SAMYOK? What about a word with n distinct letters?

6. How many ways are there to organize the letters of the word OOF? ALGEBRA? WORD? COOKBOOK?

7. How many different ways can techguy2 order books from left to right?

8. Compute P(8,3).

9. Compute P(20,4).

10. Compute P(31,1).

11. What is P(n,1) in terms of n?

12. What is P(n,n) in terms of n?

13. Write a general formula for permutations.

14. How many ways are there to organize the letters of your AoPS username? What about the numbers? What about symbols?

15. How many ways are there to organize the characters of your AoPS username if all your numbers and symbols (if you have any) must come at the begining or end of your username?

16. Samyok is designing a program that follows these instructions:

    (a) START
    (b) Let x be equal to 10.
    (c) If $P(x, 10)$ is odd then go to line (c). If not, go to line (e).
    (d) Print x. Add one to x. Go to line 2.
    (e) Print DONE.
    (f) STOP.

What is the biggest number the program would print out before it prints DONE?

17. In a group of Mrs. Renkly, Wendy, Samyok, Serena, and you, we need to delegate three jobs: Leader, Writer, and Speaker. In how many combinations of officers would there be?

18. Same thing as above, but what if you are not allowed to be leader? Then how many combinations of officers would there be?

19. How many ways can we reorganize the letters in COUNTING if each combination must have a double N?

20. How many reorganizations of the letters in the word FOOT have a double O?

21. How many reorganizations of the letters in the word RECIPE have a double E?

22. How many reorganizations of the letters in the word ALPSAM have a double A?

23. How many reorganizations of the letters in the word VOWEL start with a vowel?

24. How many reorganizations of the letters in the word BEGINNING have an N in the beginning?

25. How many reorganizations of the digits in the number 123456 start with an even number? What about a prime number?

26. What is the probability that a randomly selected reorganization of the digits in the number $1111^2 = 1234321$ is a palindrome? What about the number 1234554321?

27. There are 10 parking spots in front of the building. If Samyok always gets one of the 4 shady ones, how many ways can the cars of all 7 people in the super-awesome-club be parked?

28. How many different ways can Samyok place his 5 AoPS books (each with a seperate Solutions book) if each solutions book must be next its corresponding book? (List of books: I2A, I2A Solutions, I2CP, I2CP Solutions, PC, PC Solutions, V2, V2 Solutions, ACPS, ACPS Solutions.)