



CHEN 401: UNIT OPERATIONS II

Lecture Note

By

Dr A. Abubakar

**Department of Chemical Engineering
Ahmadu Bello University, Zaria**

■ **COURSE OUTLINE**

- Drying
- Solid-Liquid Extraction (or Leaching)
- Liquid-Liquid Extraction
- Evaporation
- Crystallization
- Membrane Processes

■ **REFERENCE TEXT BOOK**

- *Coulson and Richardson's Chemical Engineering*, Vol. 2.
- *Transport Processes and Unit Operations* by Christie J. Geankoplis.

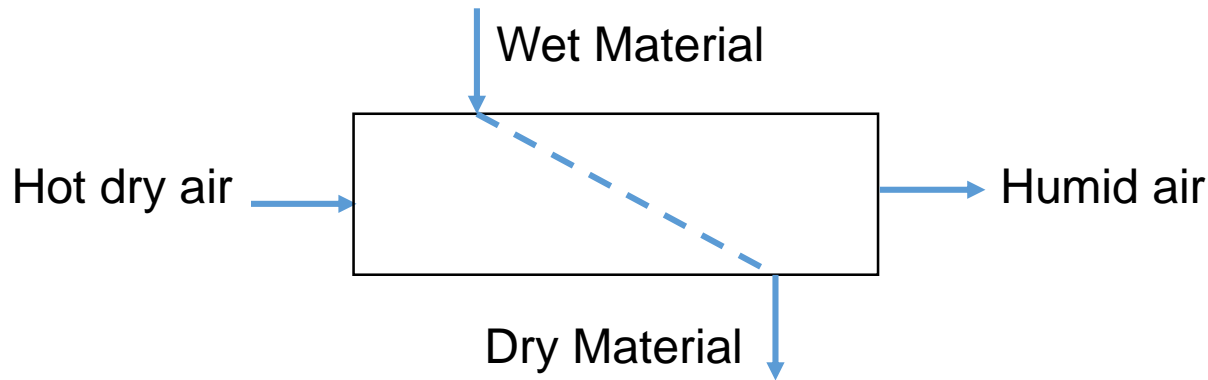
TOPIC 1

DRYING



1.1 Introduction

- Drying refers to the removal of relatively small amount of water or organic liquids from solids (process materials) to reduce the content of residual liquid to an acceptably low value.



- It is often the final operation in a manufacturing process as it is carried out immediately prior to packaging or dispatch.
- In some cases, drying is an essential part of the manufacturing process, for instance in paper making.

- Drying is a common yet a costly unit operation. It accounts for about 12 % of the manufacturing cost.
- **Reasons for carrying out drying:**
 - To reduce weight for shipping or reduce the cost of transport
 - To make a material more suitable for handling. E.g. Dry product like soap powder is much easier to package than moist/wet product
 - To provide definite desirable properties e.g. flowability, crispiness
 - To preserve product from bacterial growth. E.g. Biological materials pharmaceuticals.
 - To stabilize flavour and prolong shelf-life especially in foods
 - To reduce corrosion.

1.1.1 General methods of drying

Batch: The material is inserted into the drying equipment and drying proceeds for a given period of time.

Continuous: The material is continuously added to the dryer and dried material is continuously removed.

1.1.2 Classification of dry processes

Drying is categorized according to physical conditions used to add the heat and remove water vapour. They are direct contact drying, vacuum drying and freeze drying.

- **Direct contact drying:** The material is heated by direct contact with heated air (i.e. by convection) at the atmospheric pressure, and the water vapour is removed by the hot air.

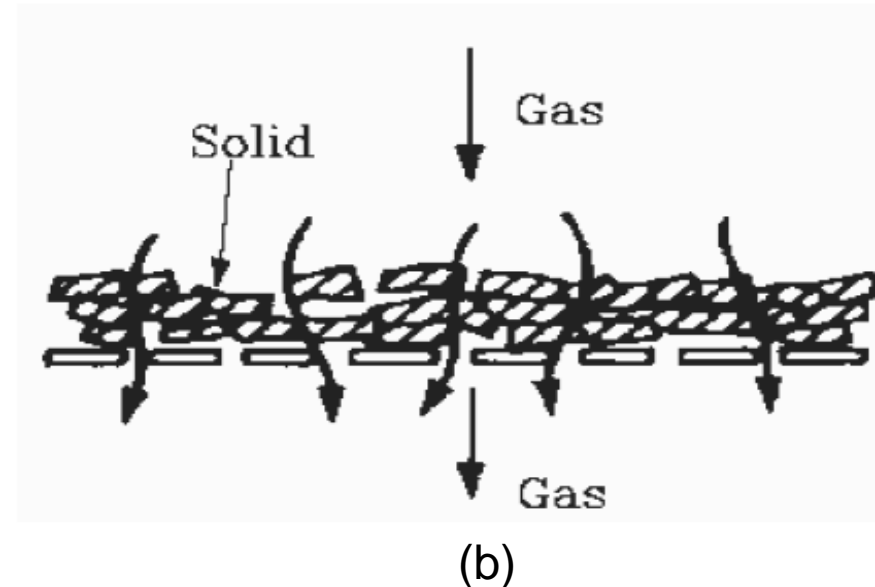
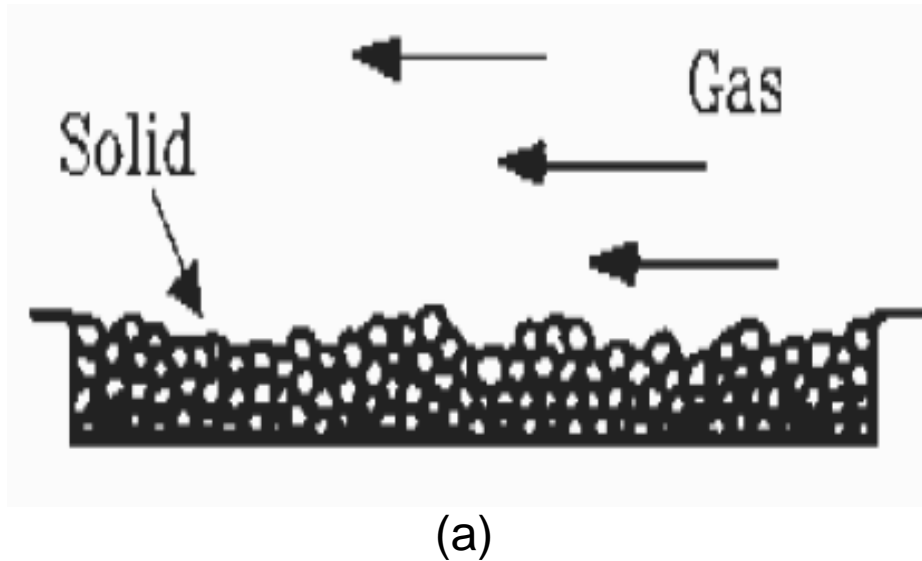


Fig. 1.1: Direct contact drying: (a) Parallel gas flow to surface; (b) Perpendicular gas flow to surface.

- **Vacuum drying:** The heat is added indirectly by contact with a metal wall or by radiation.

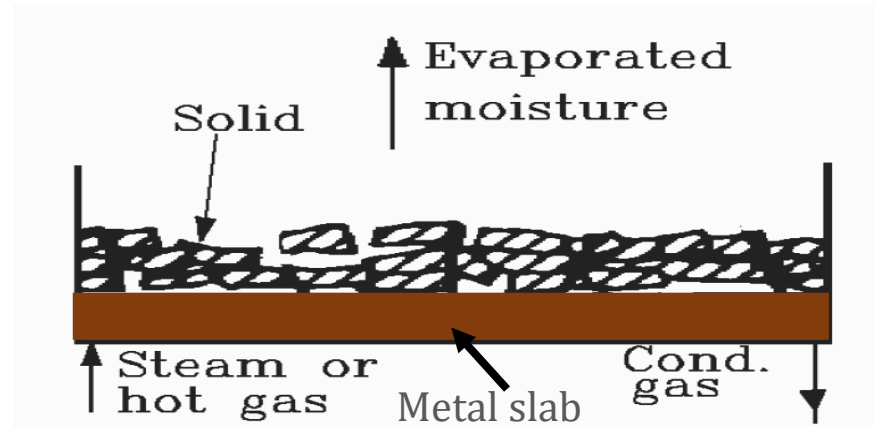


Fig. 1.2: Vacuum drying through metal wall.

- **Freeze drying:** The water is sublimed from frozen material in order to preserve the material. It is done by rapidly freezing the material and then subjecting it to a high vacuum which removes ice by sublimation.

1.2 Equipment for Drying

There are several types of dryers:

- **Tray dryers:** They are also called shelf, cabinet and compartment dryers. In these dryers, the materials to be dried are spread uniformly on a metal tray. Steam-heated air is circulated by a fan over and parallel to the surface of the trays.

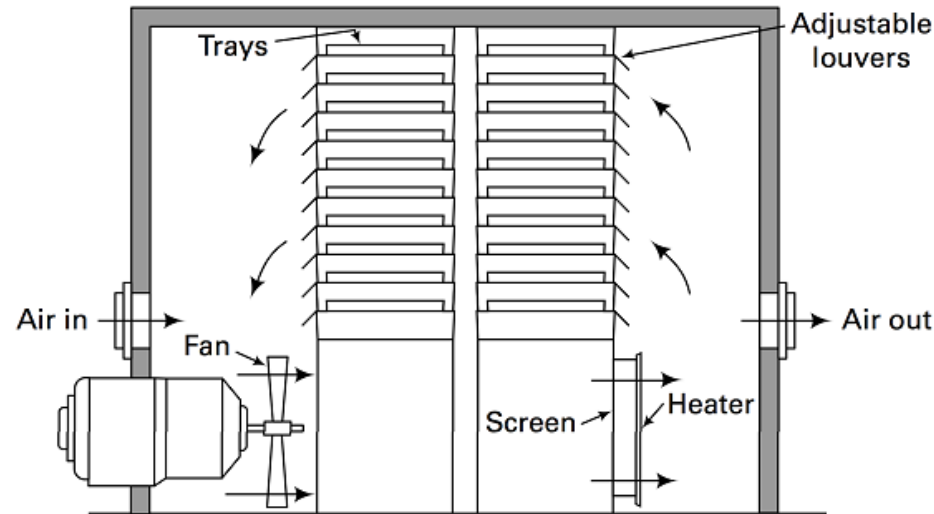


Fig. 1.3: Tray or shelf dryer

- **Vacuum-shelf indirect dryers:** They are directly heated batch dryer similar to the tray dryers but consist of cabinets made up of cast iron or still plates fitted with tightly fitted doors so that it can be operated under vacuum.
- **Continuous tunnels dryers:** They are batch truck or tray compartments operated in series (Fig. 1.4). The solids are placed on trays or on trucks which move continuously through a tunnel with hot gases passing over the surface of each tray.

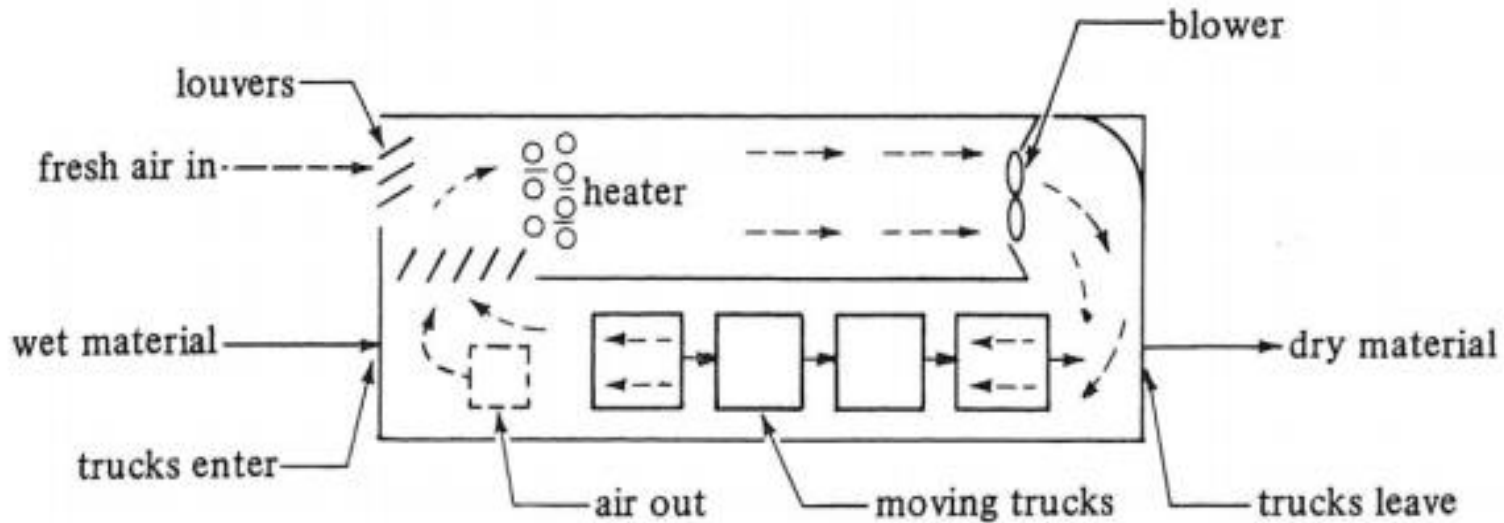


Fig. 1.4: Continuous tunnel dryer

- **Rotary dryers:** They consist of hollow cylinders which are rotated and usually slightly inclined toward the outlet (Fig. 1.5). The wet granular solids are fed at the high end and move through the shell as it rotates. The heating can be direct (with hot gases in countercurrent flow) or indirect (through the heated wall of the cylinders).
- **Drum dryers:** They consist of heated metal rolls shown in Fig. 1.6 on the outside of which a thin layer of liquid or slurry is evaporated to dryness. The final dry solid is scrapped off the roll, which is revolving slowly. They are suitable for handling slurries or pastes of solids in fine suspension and for solutions. They function partly as evaporators and also as dryers.

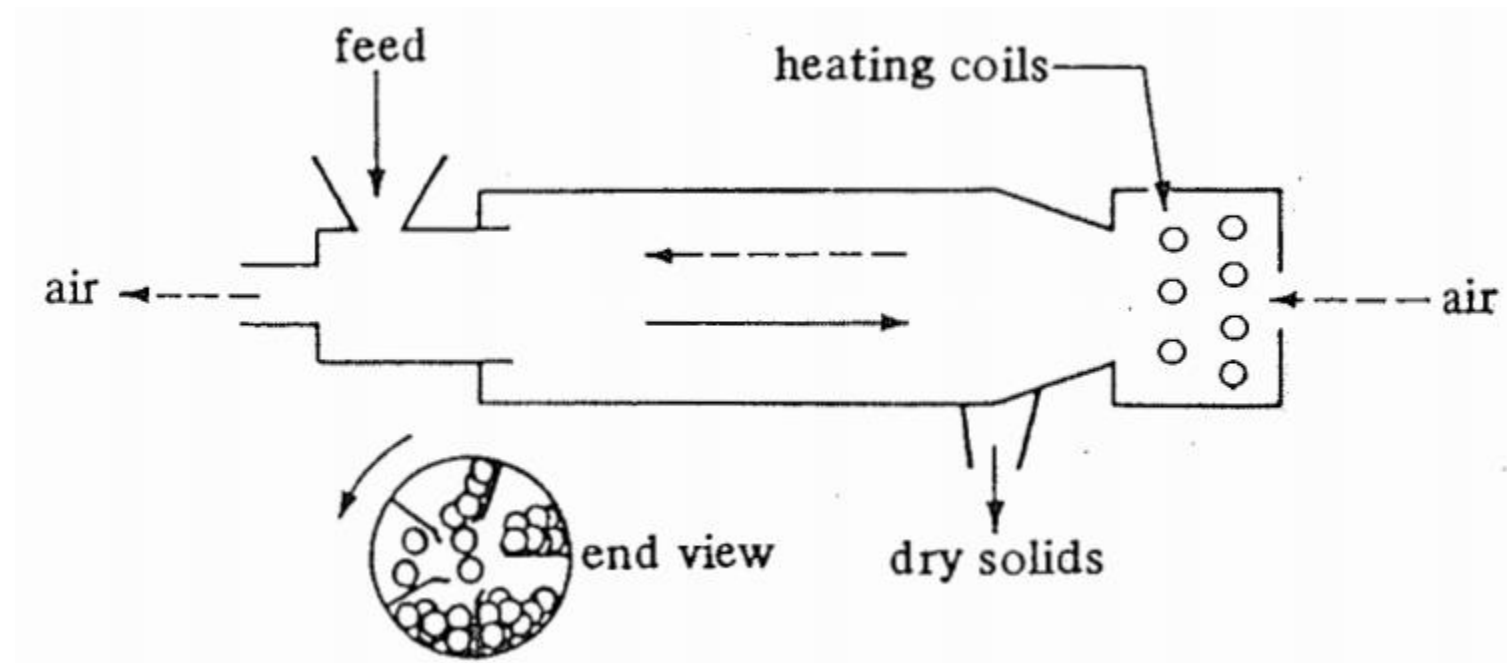


Fig. 1.5: Rotary dryer

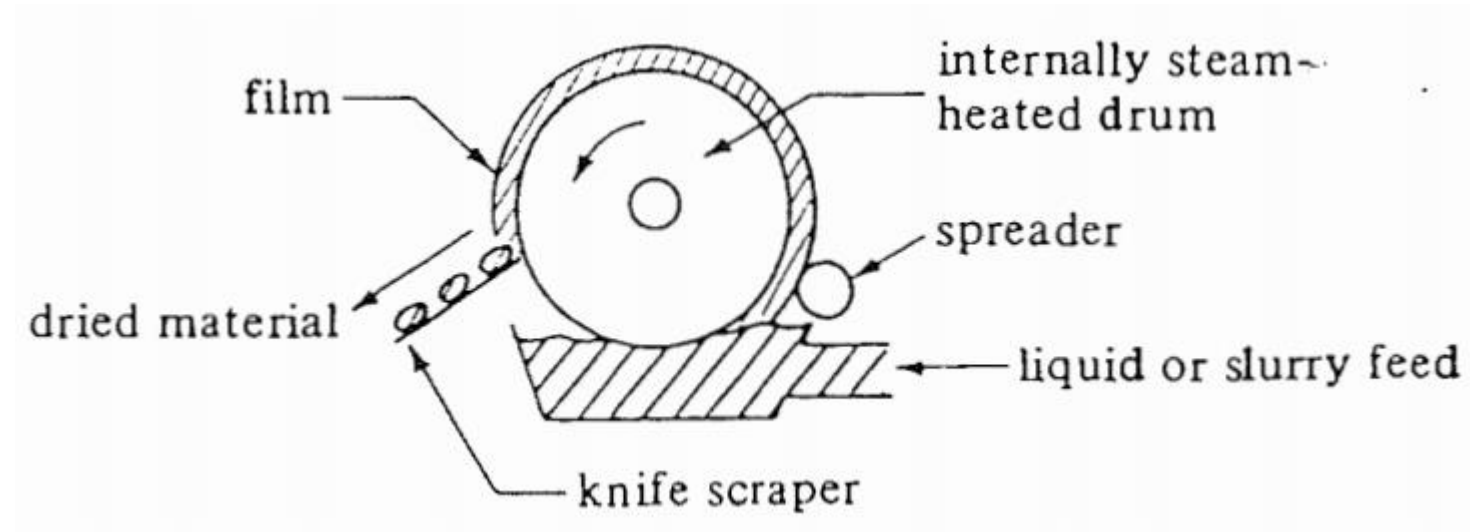


Fig. 1.6: Drum dryer

- **Spray dryers:** In these dryers, a liquid or slurry solution is sprayed into a hot gas stream in the form of a mist of fine droplets. The water is rapidly vaporized from the droplets, leaving particles of dry solid which are separated from the gas stream. The flow of gas and liquid in the spray chamber may be countercurrent, cocurrent, or a combination of both.

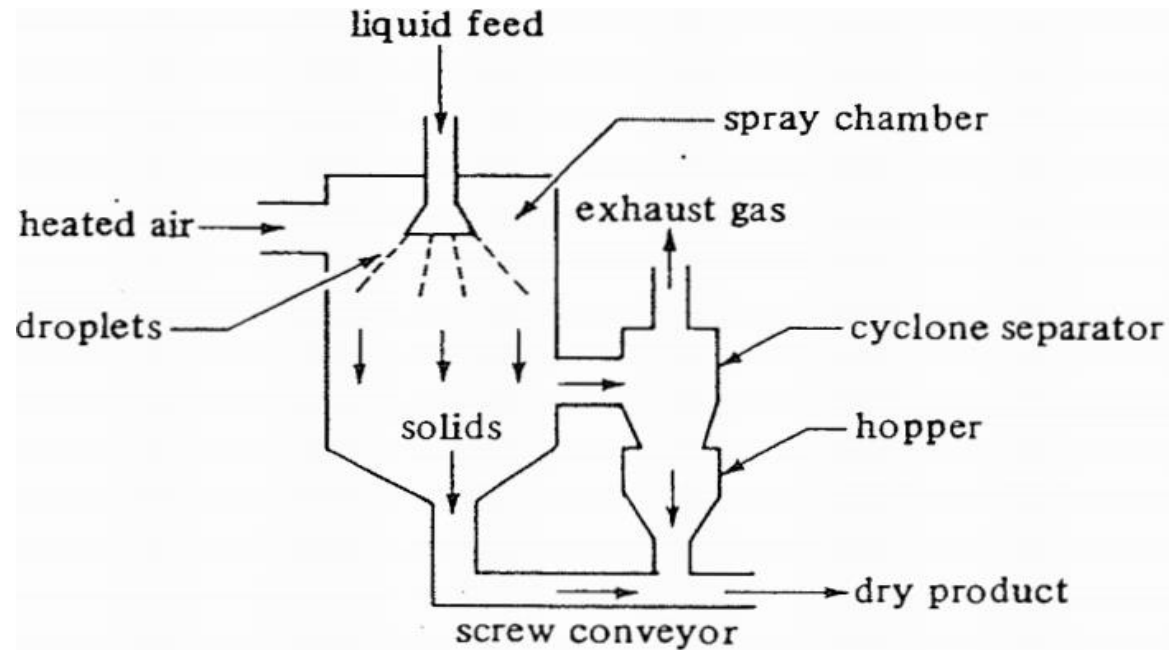


Fig. 1.7: Spray dryer

- **Fluidized bed dryers:** Here, heated air, or hot gas from a burner, is passed by way of a plenum chamber and a diffuser plate, fitted with suitable nozzles to prevent any back-flow of solids, into the fluidised bed of material, from which it passes to a dust separator.

Wet material is fed continuously into the bed through a rotary valve, and this mixes immediately with the dry charge. Dry material overflows through a downcomer to an integral after-cooler.

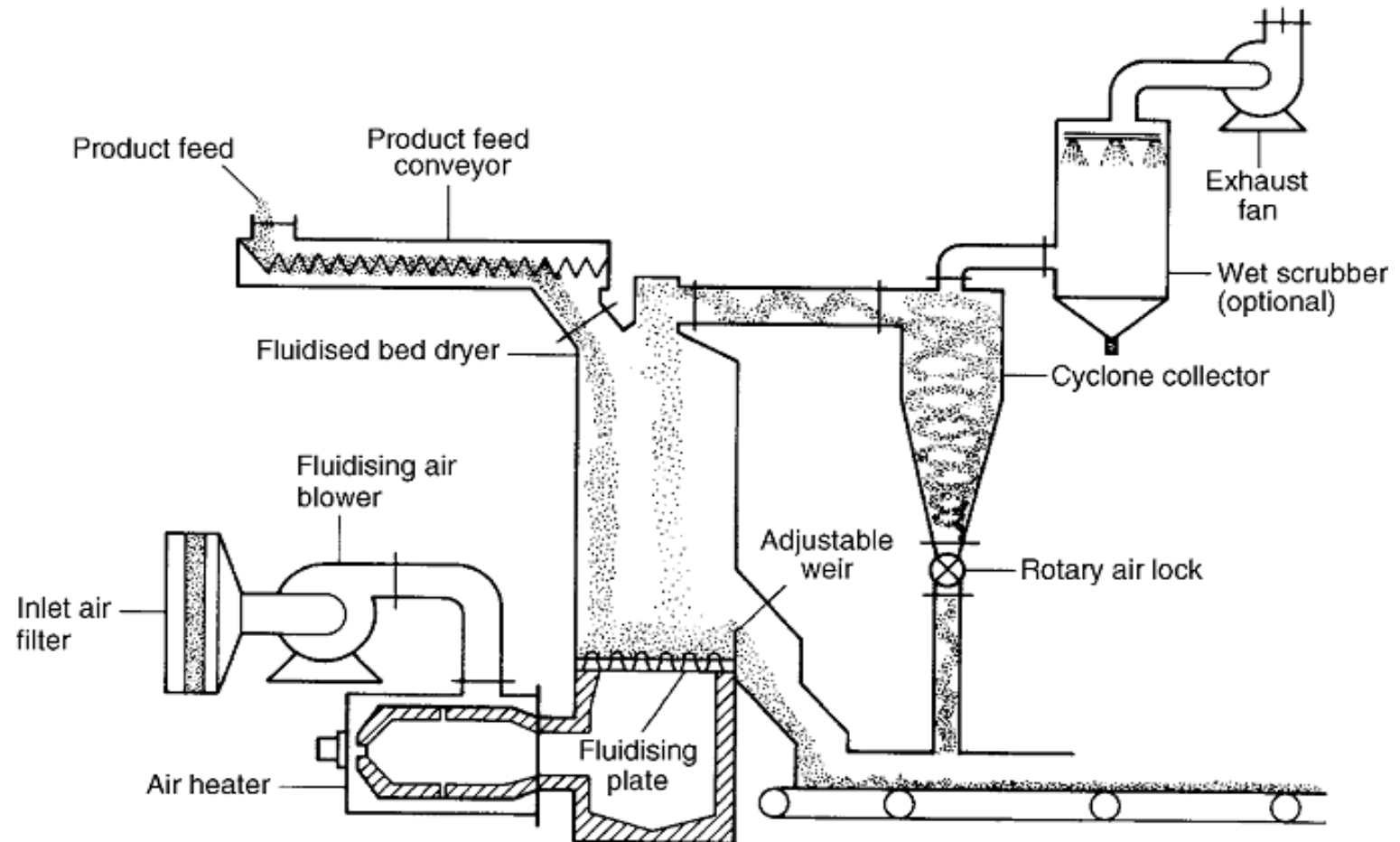


Fig. 1.8: Fluidized bed dryer

1.3 General Principles of Drying

- The moisture content of a material is usually expressed on dry basis (i.e. mass of water per mass of the dry material), though moisture content is sometimes expressed on a wet basis (mass of water per mass of the wet material).
- If a material is exposed to air at a given temperature and humidity, the material will either lose or gain water until an equilibrium condition is established. The moisture content at the equilibrium is called **equilibrium moisture content**, and it varies widely with the type of material and the temperature of the air. Variation of equilibrium moisture contents of some materials with humidity is shown in Fig. 1.9.

1.3.1 *The nature of moisture in solid materials*

- **Bound moisture:** This is the minimum moisture a material can carry. It is water retained so that it exerts a vapour pressure less than its partial pressure at the same temperature (See Fig. 1.10). Note that moisture evaporates from a wet solid only when its vapour pressure exceeds the partial pressure.

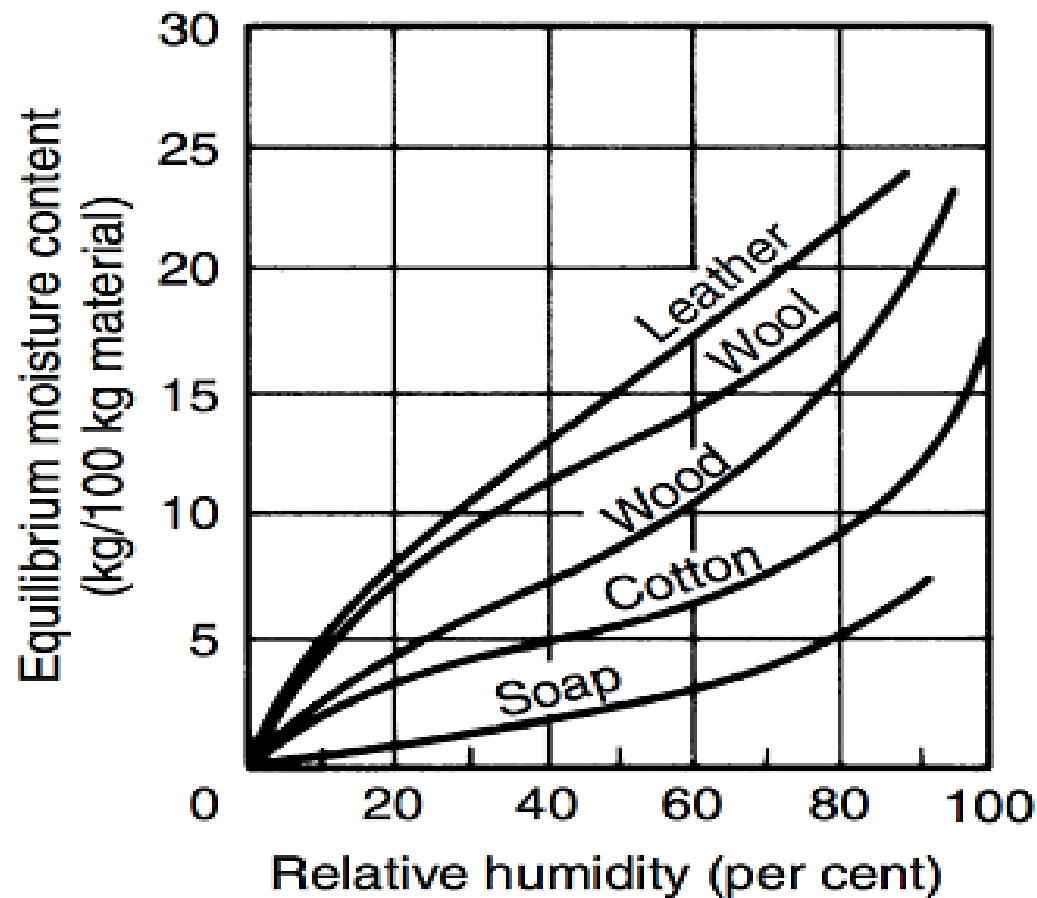


Fig. 1.9: Equilibrium moisture content of a solid as a function of relative humidity at 293 K

- **Unbound moisture:** This is excess water held primarily in the voids of the solid. It is the water contained by a substance which exerts a vapour pressure as high as that of free water at the same temperature.

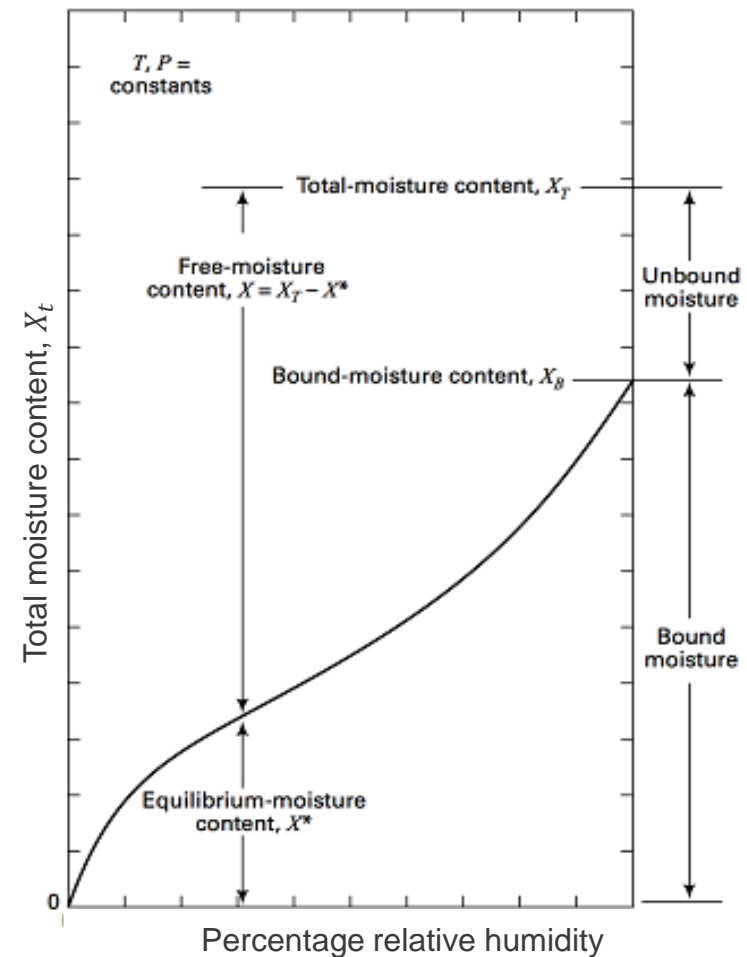
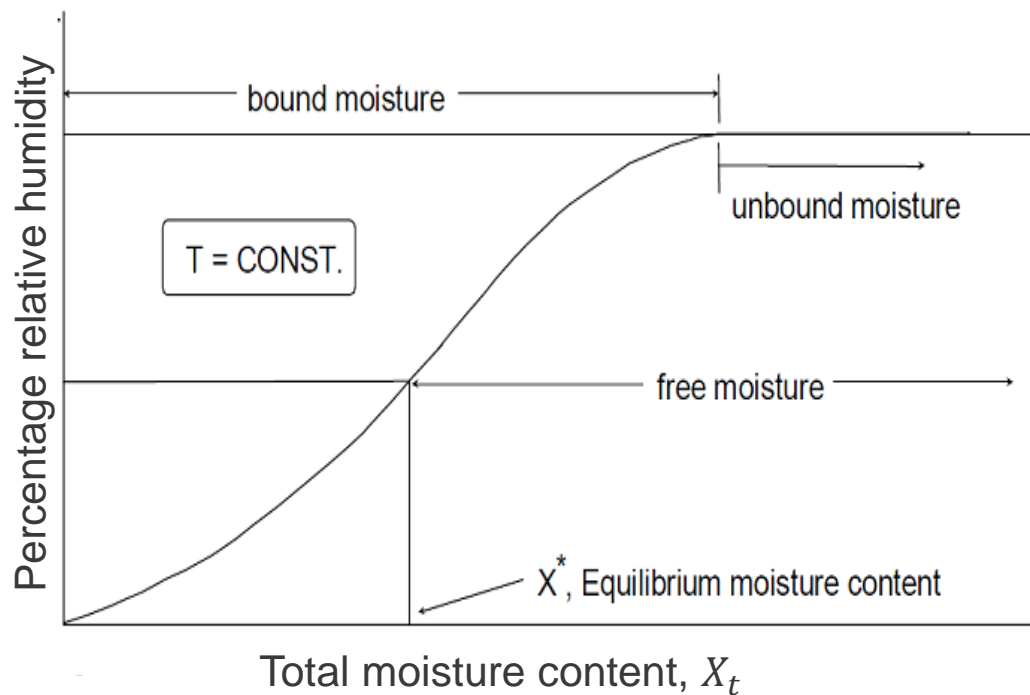


Fig. 1.10: Nature of moisture in solids

- **Free moisture content (X):** This is moisture above the equilibrium moisture content. It is the water that can be removed by drying.
- **Equilibrium moisture content (X^*):** This is the portion of the water in the wet solid which can not be removed by drying.

Note: Equilibrium moisture content cannot be predicted and therefore, it has to be determined experimentally.

1.3.2 Humidification terms

Since drying processes strongly depend on the humidity of air as shown in Fig. 1.8, the following humidification terms must be familiar with.

- **Humidity, \mathcal{H} :** It is the mass of water vapour per unit mass of dry air.

$$\text{Mass of water vapour} = \frac{P_w M_w}{RT}$$

$$\text{Mass of dry gas} = \frac{(P - P_w) M_A}{RT}$$

$$\mathcal{H} = \frac{P_w}{P - P_w} \left(\frac{M_w}{M_A} \right) \quad (1.1)$$

P_w = Partial pressure of the vapour; P = Total pressure; T = Temperature; R = gas constant; M_w and M_A = Molecular weights of the water vapour and dry air.



$$\mathcal{H} = \frac{18}{29} \frac{P_w}{P - P_w} = 0.622 \frac{P_w}{P - P_w} \quad (1.2)$$

- **Saturation humidity, \mathcal{H}_s :** It is the humidity of the air when it is saturated with water vapour at a given temperature. Therefore,

$$\mathcal{H}_s = \frac{P_{ws}}{P - P_{ws}} \left(\frac{M_w}{M_A} \right) \quad (1.3)$$

P_{ws} = Partial pressure of the vapour at saturated air.

- **Percentage humidity, \mathcal{H}_A :** It is ratio of the actual humidity, \mathcal{H} to saturation humidity, \mathcal{H}_s based percentage. It is written as;

$$\mathcal{H}_A = 100 \left(\frac{\mathcal{H}}{\mathcal{H}_s} \right) \quad (1.4)$$

- **Percentage relative humidity, \mathcal{H}_R :** It is the amount of vapour in air relative to the maximum amount of water vapour the air can contain at the same temperature and total pressure. Alternatively, it is the ratio of the partial pressure of water vapour to its vapour pressure at the same temperature i.e.

$$\mathcal{H}_R = \frac{P_w}{P'_w} \times 100 \quad (1.5)$$

P'_w = vapour pressure of water

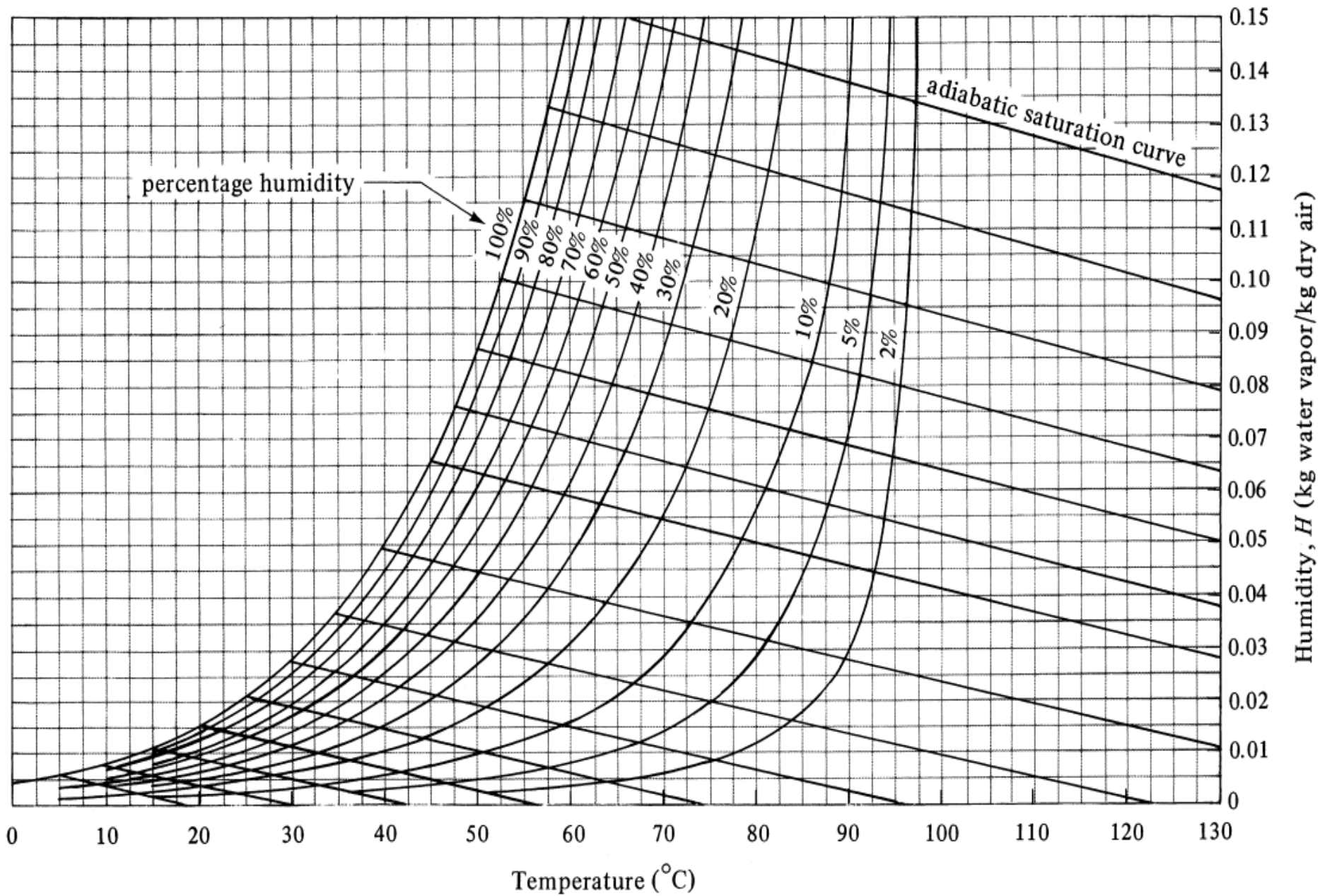


Fig. 1.11: Humidity chart of air-water at 1 atm

- **Humid heat, c_s :** It is the heat required to raise unit mass of dry air and its associated water vapour by unit temperature at constant pressure. i.e.

$$c_s = 1.005 + 1.88\mathcal{H} \quad \text{kJ/kg.K} \quad (1.6)$$

- **Humid volume, v_H :** It is the volume occupied by unit mass of dry air and its associated water vapour. It is equivalent to the inverse density $1/\rho$ of moist air. In SI units, humid volume is given as;

$$v_H = \frac{22.4T}{273} \left(\frac{1}{M_A} + \frac{\mathcal{H}}{M_w} \right)$$

$$v_H = [2.83 \times 10^{-3} + 4.56 \times 10^{-3}\mathcal{H}]T_{db} \quad (1.7)$$

T_{db} = Dry bulb temperature or temperature of air measured by ordinary thermometer.

- **Saturated volume:** It is the humid volume of saturated air.
- **Total enthalpy, H :** It is the enthalpy of a unit mass of dry air and its associated water vapour.

$$H = c_s(T - T_o) + \mathcal{H}\lambda_o$$

$$H = (1.005 + 1.88\mathcal{H})(T - T_o) + \mathcal{H}\lambda_o \quad (1.8)$$

T = Humid air temperature, T_o = Reference temperature, λ_o = Latent heat of vaporisation of water at T_o

- **Wet-bulb temperature (T_{wb}):** It is the temperature an air would have if it were cooled to saturation by evaporation of water into it, with the latent heat being supplied by the air. Measurements of dry-bulb and wet-bulb temperatures using thermometers are shown in Fig. 1.12.

- **Dew point:** The temperature to which moist air must be cooled for water vapour to start to condense out of it. It is the temperature at which air becomes completely saturated.

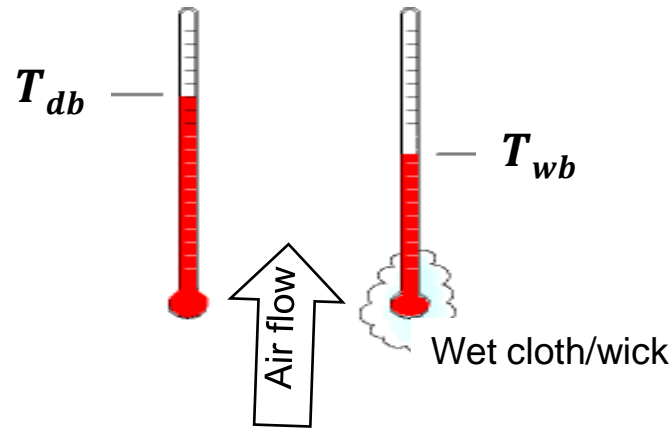


Fig. 1.12: Dry-bulb and wet-bulb thermometers

- **Adiabatic saturation temperature (T_s):** It is the temperature at which saturation water vapour is reached in a gas stream in an adiabatic process. Consider a stream of air at temperature T_{db} and humidity \mathcal{H} . It adiabatically contacts fine water droplets long enough to reach equilibrium (i.e. saturation) and comes out at T_s and \mathcal{H}_s .

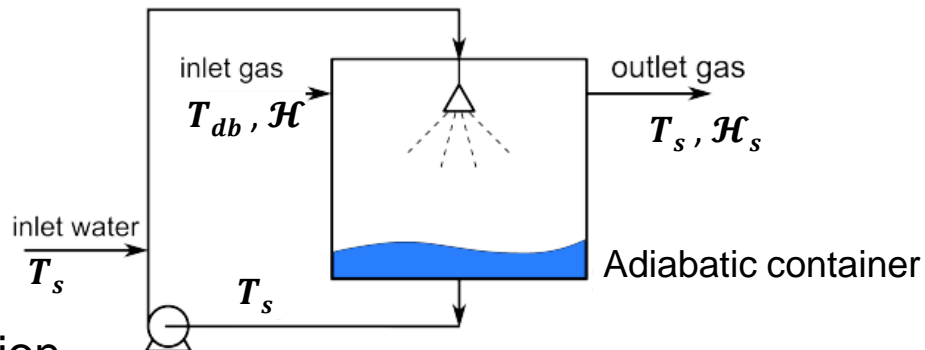
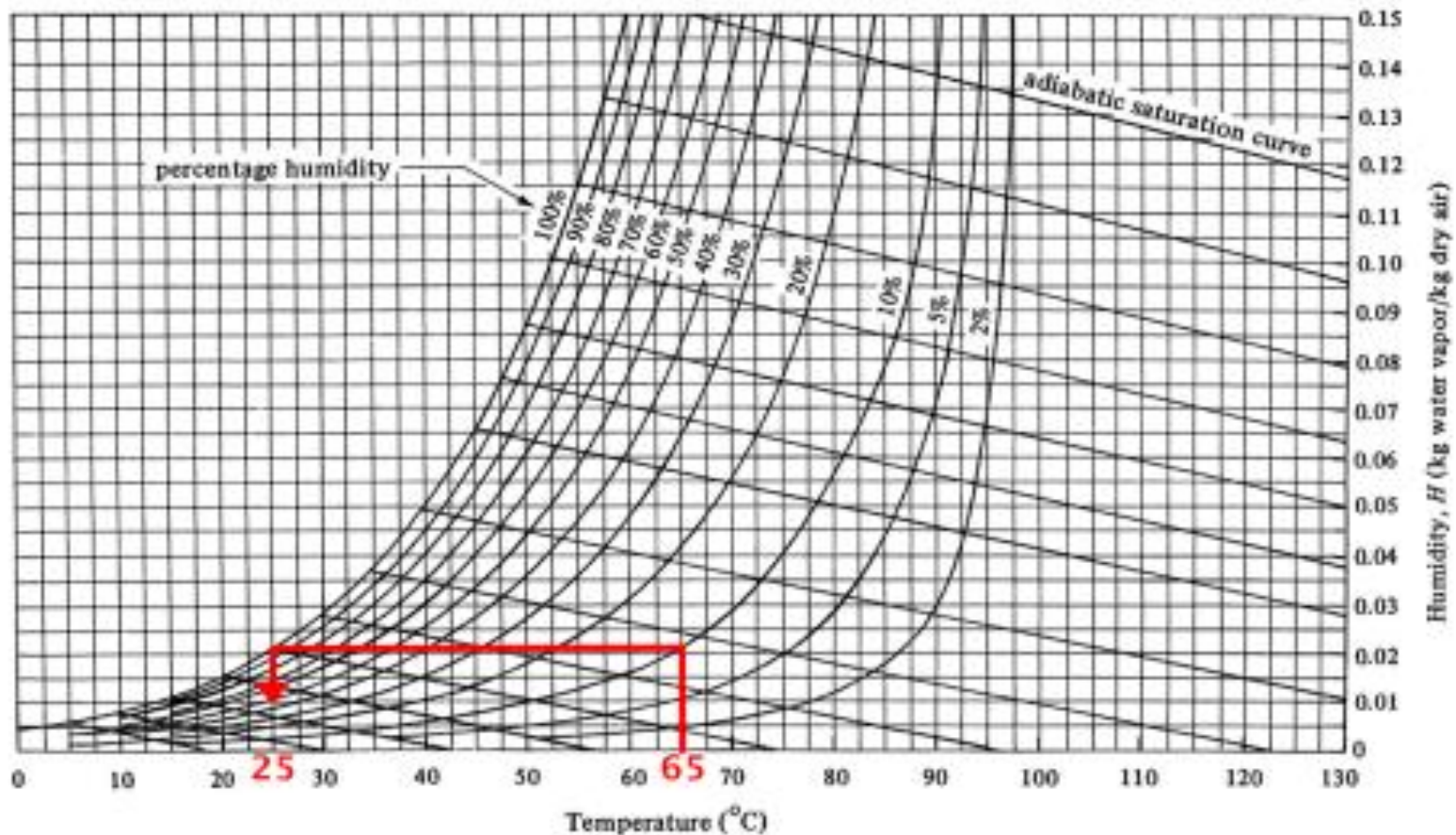


Fig. 1.13: Adiabatic saturation

- For air–water vapour mixtures at atmospheric pressure, wet-bulb temperature, T_{wb} is approximately equal to the adiabatic saturation temperature, T_s .

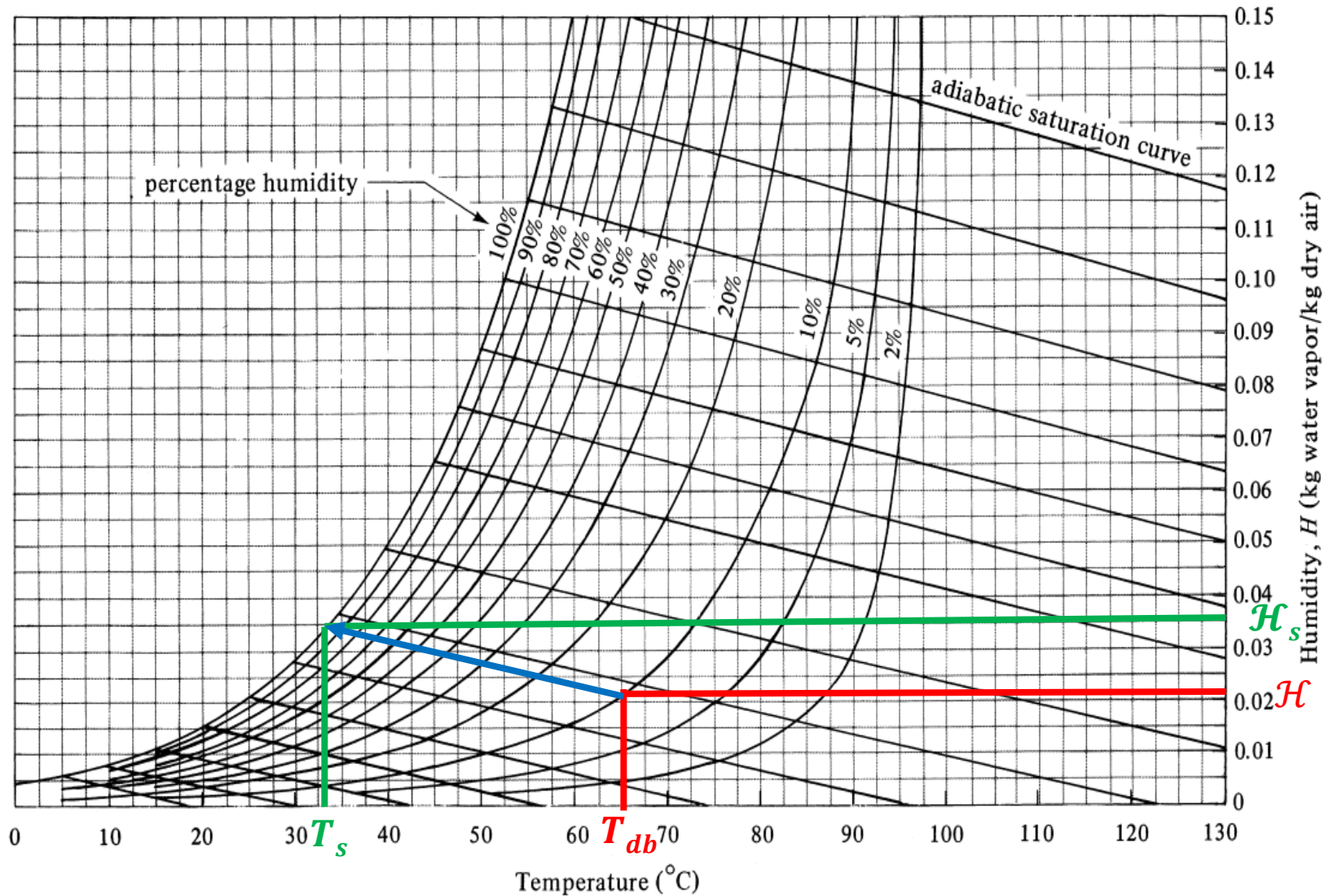
Example 1.1: Air at 65 °C has percentage humidity of 10 %. What are the dew point temperature and humidity?

Solution: As shown below, dew point temperature = 25 °C; Humidity = 0.021 kg/kg



Example 1.2: Use the information in example 1.1 to determine adiabatic saturation temperature and saturation humidity.

Solution: As shown below, adiabatic saturation temperature, $T_s (\approx T_{wb}) = 33\text{ }^{\circ}\text{C}$; saturation humidity, $\mathcal{H}_s = 0.035\text{ kg/kg}$. (NOTE: $T = T_{db}$)



Example 1.3: An air stream at 70 °C and carrying 55 g water per kg dry air is adiabatically contacted with liquid water until it reaches equilibrium. The process is continuous and operating at steady-state.

- a. What is the percentage humidity of the incoming air stream?
- b. What is the percentage humidity of the air stream leaving?
- c. What is the humidity [mass/mass] of the air stream leaving?
- d. What is the temperature of the air stream leaving?

Solution:

Exercise 1.1: Air at 55 °C and 1 atm enters a dryer with a humidity of 0.03 kg water per kg dry air. What are values for the:

- a. Recorded dry-bulb temperature
- b. Percentage humidity
- c. Dew point temperature
- d. Humid heat
- e. Humid volume
- f. Wet-bulb temperature

1.4 Rate of Drying for Batch Process

1.4.1 Rate of drying curves

- Total moisture content (on dry basis) in solid material is calculated as follows:

$$X_t = \frac{W - W_s}{W_s} \quad (1.9)$$

where X_t = total moisture content (kg of water/kg of solid); W = total weight of the wet solid (kg); W_s = weight of dry solid (kg)

- As stated earlier, drying process can only remove the free moisture content (X). According to the definitions already given, the following relation exists :

$$X = X_t - X^* \quad (1.10)$$

where X^* = equilibrium moisture content

- Therefore, the amount of the free moisture content (X) decreases with time during the drying process. This is shown in Fig. 1.14a.
- The rate of drying (R) is expressed in terms of flux (i.e. mass per time per area) as;

$$R = -\frac{W_s}{A} \frac{dX}{dt} \quad (1.11)$$

where R = rate of drying (kg/s m²); A = surface area of solid exposed for drying (m²). 24

- The rate of drying is obtained as the slope of tangent of moisture content curve at different time. The plot of the rate of drying against free moisture content is shown in Fig. 1.14b.

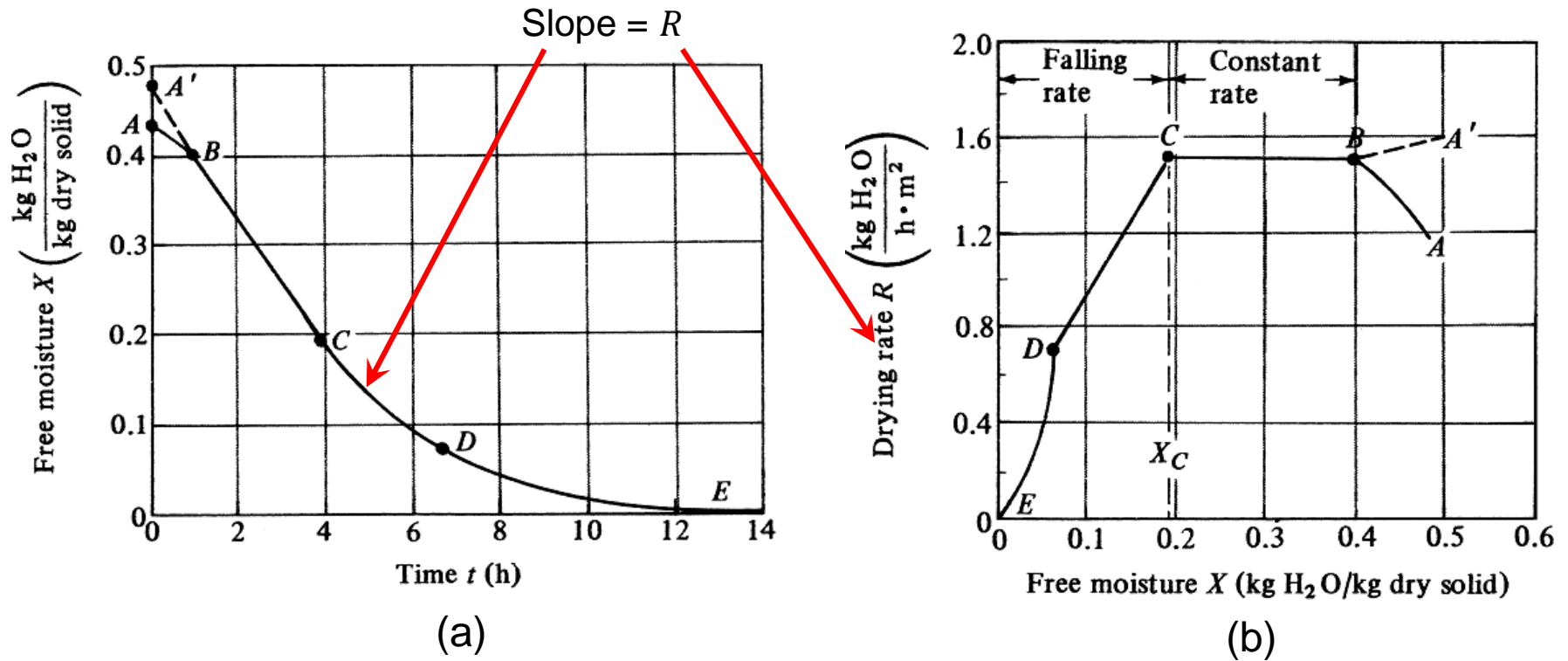


Fig. 1.14: Drying curve: (a) Free moisture content against time; (b) Rate of drying against free moisture content

1.4.2 Drying periods

There are basically two types of drying periods as shown in Fig. 1.13: **Constant rate period** and **falling rate period**. These periods and other important points are represented in Fig. 1.13 as follows:

A → B: Warming up (unsteady) period where the solid surface conditions come into equilibrium with the drying air.

A': Hot solid.

B → C: Constant rate period. This is a period in which the surface of the solid remains saturated with liquid because the rate of water vapour from interior of the solid to the surface equals the rate of evaporation of water vapour into the air stream. Thus the temperature remains constant and the rate of drying depends on the rate of heat transfer to the drying surface.

C: Critical free moisture content, X_c after which the drying rate starts falling and surface temperature rises.

C → D: First falling rate period. This is a period in which, the surface is no longer capable of supplying sufficient free moisture to saturate the air in contact with it. i.e. The rate of water vapour to surface is less than its rate of evaporation from surface. Under these conditions, the rate of drying depends very much on the mechanism by which the moisture from inside the material is transferred to the surface.

D → E: Second falling-rate period. This is a period in which evaporation of water vapour is from inside of solid. i.e. The plane of separation has moved into the solid because the solid surface is dry.

E: Equilibrium moisture content, X^* from which no further drying occur.

1.4.3 Time of drying

In most case drying calculations involves the determination time required to dry the material from a given initial free moisture content X_1 to a final moisture content X_2 .

Time of drying for constant rate period

The time required for drying from X_1 to X_2 for constant rate period can be determined by the following methods:

1. Using drying curve which is constructed from experimental data.

Example 1.4: A solid whose drying curve is represented by Fig 1.14a is to be dried from free moisture content $X_1 = 0.38$ kg water/ kg dry solid to $X_2 = 0.25$ kg water/ kg dry solid. Estimate the time required.

Solution:

From the graph, $t_1 = 1.28$ h and $t_2 = 3.08$ h

Time for drying = $3.08 - 1.28 = 1.80$ h

2. Using the rate of drying equation [i.e. Eq. (1.11)] if the value of the drying rate is known.

The integration of Eq. (1.11) gives;

$$\int_0^t dt = -\frac{W_s}{A} \int_{X_1}^{X_2} \frac{dX}{R}$$

$$t_c = \frac{W_s}{AR_c} (X_1 - X_2) \quad (1.12)$$

where R_c , t_c = rate and time of drying for constant rate period respectively

Example 1.5: A solid is to be dried from free moisture content $X_1 = 0.38$ kg water/ kg dry solid to $X_2 = 0.25$ kg water/ kg dry solid. Estimate the time required given that $W_s/A = 21.5$ kg/m² and $R_c = 1.51$ kg/h.m²

Solution:

$$t = \left(\frac{21.5}{1.51} \right) (0.38 - 0.25) = 1.85 \text{ h}$$

3. Using predicted heat transfer coefficient if the value of the drying rate is not known is drying curve.

As stated earlier, the rate of drying is in form of mass flux. In constant-rate drying period, the wet surface continually supplies moisture. Now, assuming all the heat provided is only used to evaporate liquid, then

$$(\text{Moisture flux})(\lambda_{wb}) = \text{Heat Flux}$$

$$R_c \lambda_o = \frac{\text{Driving force}}{\text{Resistance}} = \frac{dT}{1/h}$$

where λ_{wb} = Latent heat of vaporization at T_{wb} ; h = heat transfer coefficient

$$-\frac{W_s}{A} \frac{dX}{dt} = h(T_{air} - T_{solid\ surface}) = h(T_{db} - T_{wb})$$

Note: air temperature, T_{air} = dry bulb temperature; Surface temperature, $T_{solid\ surface}$ = wet-bulb temperature

$$\int_0^t dt = -\frac{\lambda_{wb} W_s}{Ah(T_{db} - T_{wb})} \int_{X_1}^{X_2} dX$$

$$t_c = \frac{\lambda_{wb} W_s}{Ah(T_{db} - T_{wb})} (X_1 - X_2) \quad (1.13)$$

Comparing Eqs. (1.12) and (1.13) gives;

$$R_c = \frac{h}{\lambda_{wb}} (T_{db} - T_{wb}) \quad (1.14)$$

Eq. (1.14) can be approximately written in terms of mass transfer coefficient as;

$$R_c = k_y M_A (\mathcal{H}_{wb} - \mathcal{H}) \quad (1.15)$$

where \mathcal{H} = humidity at T_{wb} ; \mathcal{H}_{db} = humidity at T_{db} ; k_y = mass transfer coefficient; M_A = molecular weight of air (28).

Now, the following are correlations for heat transfer coefficient, h

<p>Parallel air flow to drying surface</p> <ul style="list-style-type: none"> ▪ Air temperature between 45-150 °C ▪ Mass velocity G of 2450-29300 kg/h.m² (or a velocity of 0.61-7.6 m/s). 	$h = 0.0204G^{0.8} \quad [\text{SI unit}] \quad (1.16)$
<p>Perpendicular air flow to drying surface</p> <ul style="list-style-type: none"> ▪ Air temperature between 45-150 °C ▪ Mass velocity G of 3900-19500 kg/h.m² (or a velocity of 0.9-4.6 m/s). 	$h = 1.17G^{0.37} \quad [\text{SI unit}] \quad (1.17)$

The gas mass velocity is given as;

$$G = 3600\rho v \quad (1.18)$$

where v and ρ = velocity and density in SI units and G is [kg/h.m²]

Example 1.6: An insoluble wet granular material is dried in a pan 0.457 x 0.457 m and 25.4 mm deep. The material is 25.4 mm deep in the pan, and the sides and bottom can be considered to be insulated. Heat transfer is by convection from an air stream flowing parallel to the surface at a velocity of 6.1 m/s. The air is at 65.6°C and has a humidity of 0.010 kg water/kg dry air. Estimate the rate of drying and total evaporation rate for the constant-rate period.

Solution:

Basis: 1 kg of dry air

$$T_{db} = 65.6 \text{ }^{\circ}\text{C}; \mathcal{H} = 0.010 \text{ kg water/kg dry air}; v = 6.1 \text{ m/s}$$

$$A = 0.457 \times 0.457 = 0.209 \text{ m}^2$$

Using Eq. (1.7),

$$v_H = [2.83 \times 10^{-3} + 4.56 \times 10^{-3} \mathcal{H}] T_{db}$$

$$\begin{aligned} v_H &= [2.83 \times 10^{-3} + 4.56 \times 10^{-3} \times 0.01](273 + 65.6) \\ &= 0.974 \text{ m}^3/\text{kg of air} \end{aligned}$$

$$\begin{aligned} \text{Mass of air (i.e. moist air)} &= 1.0 \text{ kg of dry air} + 0.01 \text{ kg of water} \\ &= 1.01 \text{ kg of air} \end{aligned}$$

$$\begin{aligned} \rho &= \frac{\text{Mass of air}}{v_H} = \frac{1.01}{0.974} \\ &= 1.037 \text{ kg/m}^3 \end{aligned}$$

$$G = 3600 \rho v = 3600 \times 1.037 \times 6.1 = 22770 \text{ kg/h.m}^2$$

Using Eq. (1.16),

$$h = 0.0204 G^{0.8} = 0.0204 (22770)^{0.8} = 62.45 \text{ W/m}^2.\text{K}$$

Using the humidity chart (Fig. 1.11), at $T_{db} = 65.6\text{ }^{\circ}\text{C}$ and $\mathcal{H} = 0.010\text{ kg water/kg dry air}$, $T_{wb} = 28.9\text{ }^{\circ}\text{C}$ and $\mathcal{H}_{wb} = 0.026\text{ kg water/kg dry air}$.

At $T_{wb} = 28.9\text{ }^{\circ}\text{C}$, $\lambda_{wb} = 2433\text{ kJ/kg}$ from **steam table**.

Therefore, substituting into Eq. (1.14) gives;

$$\begin{aligned} R_c &= \frac{h}{\lambda_{wb}} (T_{db} - T_{wb}) = \frac{62.45}{2433 \times 1000} (65.6 - 28.9) \\ &= 9.42 \times 10^{-4} \text{ kg/s.m}^2 \\ &= 9.42 \times 10^{-4} \times 3600 = 3.39 \text{ kg/h.m}^2 \end{aligned}$$

Total evaporation rate = $R_c A = 3.39 \times 0.209 = 0.708\text{ kg water/h}$.

Time of drying for falling rate period

The time required for drying from X_1 to X_2 for falling drying period can be determined by the following methods:

1. Graphical integration method.

In the falling rate period, R varies with X . So it can not be integrated as in the constant rate period. However, it can be graphically integrated by plotting $1/R$ against X and determining the area under the curve according to Eq. (1.11) as recalled below.

$$R = -\frac{W_s}{A} \frac{dX}{dt}$$

$$\int_0^t dt = -\frac{W_s}{A} \int_{X_1}^{X_2} \frac{dX}{R}$$

$$t_f = \frac{W_s}{A} \int_{X_2}^{X_1} \frac{dX}{R_f} \quad (1.19)$$

where R_f , t_f = rate and time of drying for falling rate period respectively

Example 1.7: A batch of wet solid whose graphical drying-rate curve is represented by Fig. 1.14b is to be dried from a free moisture content of $X_1 = 0.38$ kg water/kg dry solid to $X_2 = 0.04$ kg water/kg dry solid. The weight of the dry solid is $W_s = 399$ kg dry solid and $A = 18.58$ m² of top drying surface. Calculate the time for drying.

Solution:

From Fig. (1.14b), the critical moisture content, $X_c = 0.195$ kg water/kg dry solid while $X_1 = 0.38$ kg water/kg dry solid and $X_2 = 0.04$ kg water/kg dry solid. Therefore, $X_2 < X_c < X_1$, meaning that drying is in constant-rate and falling-rate periods.

Note the following:

- When X_1 and $X_2 > X_c$, the drying is completely in **constant rate period**.
- When X_1 and $X_2 < X_c$, the drying is completely in **falling rate period**.
- When $X_2 < X_c < X_1$, the drying is in both **constant rate period** from X_1 to X_c and **falling rate period** from X_c to X_2 .

Now, for the constant rate period, $X_1 = 0.38$ and $X_2 = X_c = 0.195$. from Fig. 1.14b, $R_c = 1.51$ kg water/h.m². Substituting into Eq. (1.12), gives;

$$t_c = \frac{W_s}{AR_c} (X_1 - X_c) = \frac{399}{18.58 \times 1.51} (0.38 - 0.195) \\ = 2.63 \text{ h.}$$

For the falling rate period, the reading values of R_f (and then $1/R_f$) in Fig. 1.14b for various values of X , which range from $X_1 = X_c = 0.195$ to $X_2 = 0.04$, are presented in the table below.

X	R_f	$1/R_f$	X	R_f	$1/R_f$
0.195	1.51	0.663	0.065	0.71	1.41
0.150	1.21	0.826	0.050	0.37	2.70
0.100	0.90	1.11	0.040	0.27	3.70

In Fig. (1.15), a plot of $1/R_f$ against X is made and the area under the curve from $X_1 = 0.195$ (point C) to $X_1 = 0.04$ is determined as follows:

$$\int_{X_2}^{X_1} \frac{dX}{R_f} = A_1 + A_2 + A_3 = (2.5 \times 0.024) + (1.18 \times 0.056) + (0.84 \times 0.075) \\ = 0.189 \text{ m}^2$$

Substituting into Eq. (1.19) gives;

$$t_f = \frac{W_s}{A} \int_{X_2}^{X_1} \frac{dX}{R_f} = \frac{399}{18.58} (0.189) = 4.06 \text{ h}$$

Total time of drying, $t = t_c + t_f = 2.63 + 4.06 = 6.69 \text{ h}$

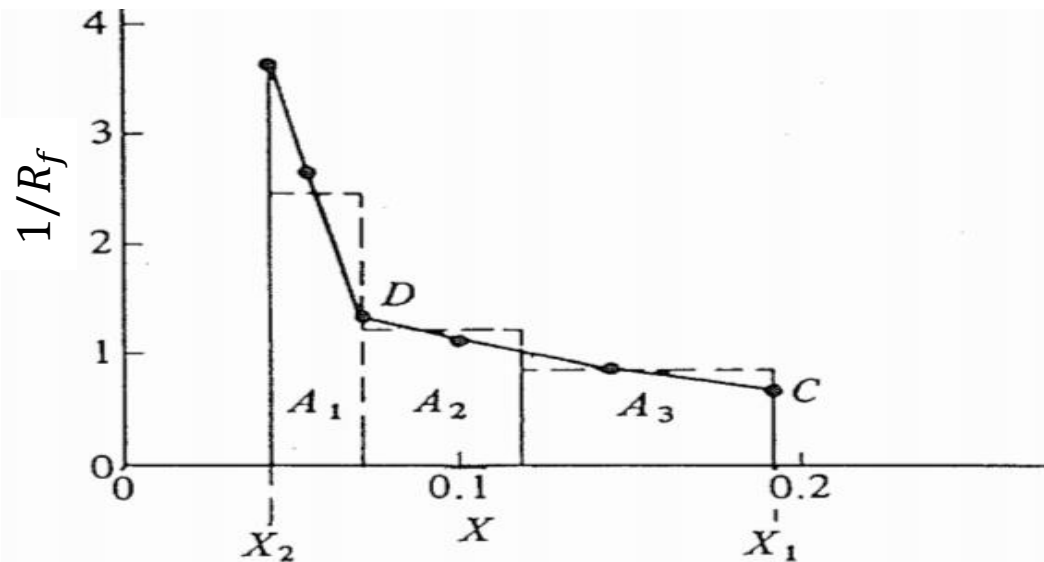


Fig. 1.15: Graphical integration for falling rate period

2. Method for special cases.

(A). Drying rate, R is a linear function of X : In this case, X_1 and $X_2 < X_c$ and R is expressed as follows:

$$R = aX + b \quad (1.20)$$

where a = slope; b = constant

Differentiating Eq. (1.20) and substituting it into Eq.(1.11) gives;

$$\begin{aligned} dR &= adX \\ R &= -\frac{W_s}{aA} \frac{dR}{dt} \xrightarrow{\text{green arrow}} \int_0^t dt = \frac{W_s}{aA} \int_{R_2}^{R_1} \frac{dX}{R} \\ t_f &= \frac{W_s}{aA} \ln \frac{R_1}{R_2} \end{aligned} \quad (1.21)$$

Since $R_1 = aX_1 + b$ and $R_2 = aX_2 + b$, then

$$a = \frac{R_1 - R_2}{X_1 - X_2} \quad (1.22)$$

Substituting Eq. (1.22) into Eq.(1.21) gives

$$t_f = \frac{W_s(X_1 - X_2)}{A(R_1 - R_2)} \ln \frac{R_1}{R_2} \quad (1.23)$$

(B). Drying rate, R is a linear function of X through the origin: In some cases, a straight line from the critical moisture content, X_c , passing through the origin adequately represents the whole falling rate period. i.e.

$$R = aX \quad (1.24)$$

Differentiating Eq. (1.24) and substituting it into Eq.(1.11) gives;

$$dR = a dX$$

$$R = -\frac{W_s}{aA} \frac{dR}{dt} \longrightarrow \int_0^t dt = \frac{W_s}{aA} \int_{R_2}^{R_1} \frac{dX}{R}$$

$$t_f = \frac{W_s}{aA} \ln \frac{R_1}{R_2}$$

The slope a of the line is R_c/X_c , and for $X_1 = X_c$ at $R_1 = R_c$, it becomes;

$$t_f = \frac{W_s X_c}{A R_c} \ln \frac{R_c}{R_2}$$

Noting also that $R_c/R_2 = X_c/X_2$. Therefore,

$$t_f = \frac{W_s X_c}{A R_c} \ln \frac{X_c}{X_2} \quad (1.25)$$

Or

$$R = R_c \frac{X}{X_c} \quad (1.26)$$

Example 1.8: Repeat Example 1.7, but as an approximation assume a straight line of the rate R versus X through the origin from point X_c to $X = 0$ for the falling rate period.

Solution:

$R_c = 1.51$ kg water/h.m², $X_c = 0.195$ kg water/kg dry solid and $X_2 = 0.04$ kg water/kg dry solid

Substituting into Eq. (1.25) gives;

$$t_f = \frac{W_s X_c}{AR_c} \ln \frac{X_c}{X_2} = \frac{399(0.195)}{18.58(1.51)} \ln \left(\frac{0.195}{0.04} \right) \\ = 4.39 \text{ h}$$

Example 1.9: A wet solid is dried from 25 to 10 per cent moisture under constant drying conditions in 15 ks (4.17 h). If the critical and the equilibrium moisture contents are 15 and 5 per cent respectively, how long will it take to dry the solid from 30 to 8 per cent moisture under the same conditions?

Solution:

Note:

- Moisture contents are presented on dry basis unless otherwise stated.

- Also, the various moisture contents given in this example include the equilibrium moisture content. Since drying only removes free moisture contents, the equilibrium moisture content needs to be subtracted from the various moisture contents given according to Eq. (1.10).

$$X^* = 0.05$$

$$X_c = 0.15 - 0.05 = 0.10$$

For the first drying operation:

$$X_1 = 0.25 - 0.05 = 0.20; X_2 = 0.10 - 0.05 = 0.05; t = 15 \text{ ks}$$

Since $X_2 < X_c < X_1$, the drying is in both constant rate period from X_1 to X_c and falling rate period from X_c to X_2 .

Combining Eqs. (1.12) and (1.25) gives;

$$t = t_c + t_f = \frac{W_s}{AR_c} (X_1 - X_c) + \frac{W_s X_c}{AR_c} \ln \frac{X_c}{X_2}$$

$$15 = \frac{W_s}{AR_c} \left[(X_1 - X_c) + X_c \ln \frac{X_c}{X_2} \right]$$

$$\frac{W_s}{AR_c} = \frac{15}{\left[(X_1 - X_c) + X_c \ln \frac{X_c}{X_2}\right]} = \frac{15}{\left[(0.20 - 0.10) + 0.10 \ln \frac{0.10}{0.05}\right]}$$

$$\frac{W_s}{AR_c} = 88.59$$

For the second drying operation:

$$X_1 = 0.30 - 0.05 = 0.25; X_2 = 0.08 - 0.05 = 0.03; t = ?$$

Again, $X_2 < X_c < X_1$. Therefore, the drying is in both constant rate period from X_1 to X_c and falling rate period from X_c to X_2 .

Combining Eqs. (1.12) and (1.25) gives;

$$t = t_c + t_f = \frac{W_s}{AR_c} (X_1 - X_c) + \frac{W_s X_c}{AR_c} \ln \frac{X_c}{X_2}$$

$$t = \frac{W_s}{AR_c} \left[(X_1 - X_c) + X_c \ln \frac{X_c}{X_2} \right]$$

$$t = 88.59 \left[(0.25 - 0.10) + 0.10 \ln \frac{0.10}{0.03} \right]$$

$$t = 23.95 \text{ ks (6.65 h)}$$