P8160 Group Project 3: Bayesian Modeling of Hurricane Trajectories

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Objective

In this project, we firstly built a Bayesian model based on the track data of 703 hurricanes in the North Atlantic area since 1950. A Markov Chain Monte Carlo (MCMC) algorithm was designed to generate the distribution of corresponding parameters. With the start time and type of each hurricane, the estimated coefficients from the Bayesian model were used to explore the seasonal differences and wind speed changes over years. Finally, we explored the characteristics of hurricanes associated with the damage and deaths.

Background

Hurricanes are large rotating tropical storms with winds in excess of 119 kilometers per hour (74 mph). They usually form between June 1 and November 30 in the Atlantic Ocean but can develop in other oceans as well. They are known as typhoons in the western Pacific and cyclones in the Indian Ocean[1].

When a hurricane approaches land, tremendous damage can occur to the nearby cities. Therefore, scientists continue to improve their ability to forecast hurricanes. The sooner they can access accurate information about a hurricane's location and intensity, the better the chances to minimize the its impacts.

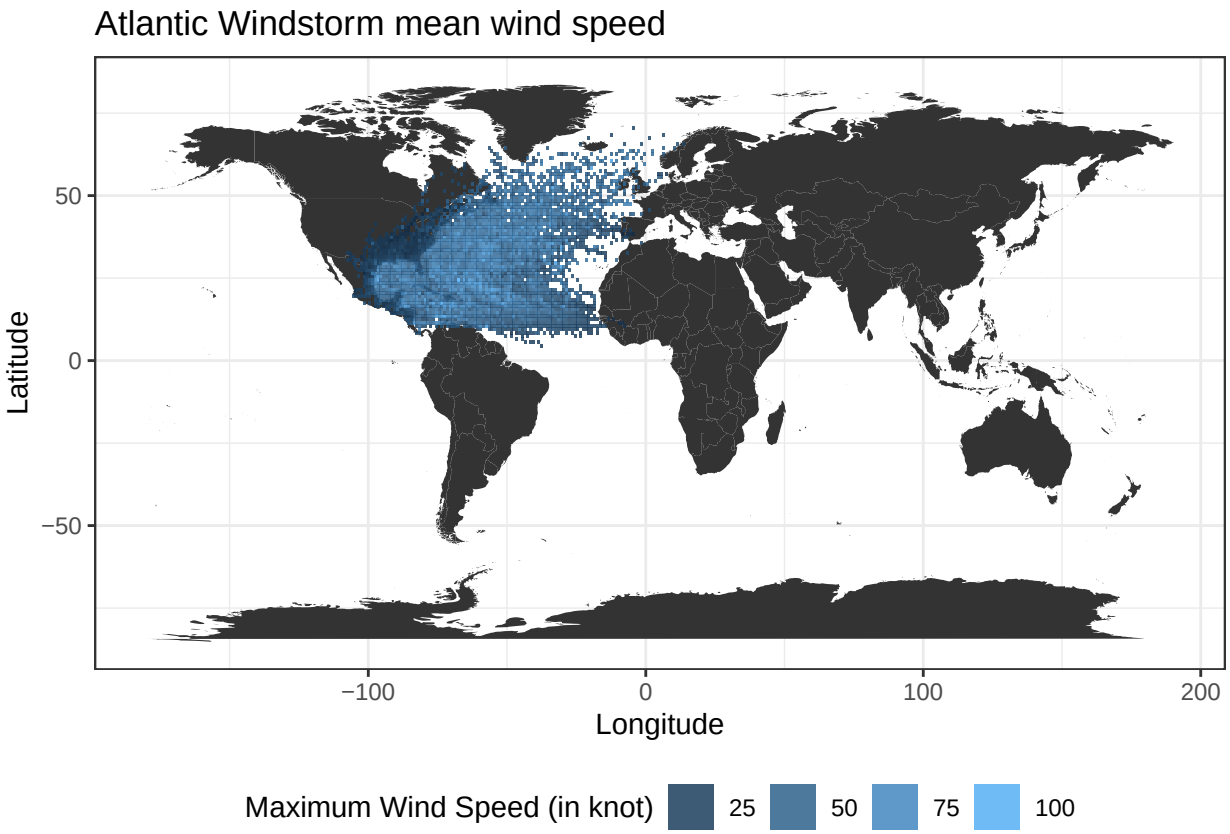
Data Description and Preprocessing

The first data `hurrican703.csv` collected the track data of 703 hurricanes in the North Atlantic area from 1950 to 2013. For all the hurricanes, their locations (longitude & latitude) and maximum wind speed were recorded every 6 hours. The variables include `ID`, `Season`, `Month`, `Nature`, `time`, `Latitude`, `Longitude`, and `Wind.kt`. Main data processing steps for this data are listed as follows:
1) Created 3 new variables including the changes of latitude and longitude, as well as the wind speed between the time t and $t-6$;
2) Removed 9 hurricanes with observations less than 5 to ensure the data partition;
3) To explore the seasonal differences, we converted the start month for each hurricane into the variable `season`, which includes Spring, Summer, Fall, and Winter. Finally, we have totally **691** hurricanes in the updated dataset.

The second data `hurricanoutcome2.csv` recorded the damages and death caused by 46 hurricanes in the United States, and some features extracted from the above hurricane records. To better explore the characteristics related with death and damage, we combined this data with the coefficients obtained from the first model by the hurricane ID. For this data, we also converted different start months into the corresponding seasons.

Exploratory Data Analysis

To generally understand the distribution of wind speed across the North Atlantic area, we created the following figure to show the mean maximum wind speed within each knot based on the longitude and latitude from the original data `hurrican703.csv`.



Statistical Methods

Likelihood

For each hurricane i and k_i's time points, we have the following Bayesian model:

$$Y_i(t+6) = \beta_{0i} + \beta_{1i}Y_i(t) + \beta_{2i}\Delta_{i1}(t) + \beta_{3i}\Delta_{i2}(t) + \beta_{4i}\Delta_{i3}(t) + \varepsilon_i(t),$$

where $Y_i(t)$ is the wind speed at time t, Δ_{i1}, Δ_{i2}, and Δ_{i3} are the changes of latitude, longitude, and the wind speed between time point t and $t-6$, respectively. $\varepsilon_i(t)$ follows a normal distributions with mean zero and variance σ^2. The above Bayesian model can be simplified as:

$$Y_i(t+6) = x_i(t) + \varepsilon_i(t),$$

where $\beta_i = (\beta_{0i}, \beta_{1i}, ..., \beta_{4i}) \sim N\left(0, \sigma^2\right)$. Based on the property of the multivariate linear regression model, for each hurricane i, we have:

$$Y_i \mid X_i \sim N_{k_i}\left(x_i\beta_i, \sigma^2 I_{k_i}\right),$$

where I_{k_i} is an identity matrix with k_i dimensions.

Thus, we can consider the following distribution of each hurricane i:

$$f\left(y_i \mid \beta_i, \sigma^2\right) = \left[(2\pi)^{k_i} \cdot \det\left(\sigma^2 I_{k_i}\right)\right]^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2}\left(y_i - x_i\beta_i\right)^\top \left(\sigma^2 I_{k_i}\right)^{-1}\left(y_i - x_i\beta_i\right)\right\}$$

.

From above, we derive the following likelihood function:

$$\begin{aligned} f\left(y \mid B, \sigma^2\right) &= \prod_{i=1}^{n} f\left(y_i \mid \beta_i, \sigma^2\right) \\ &= \prod_{i=1}^{n} \left(\left[(2\pi)^{k_i} \cdot \det\left(\sigma^2 I_{ki}\right)\right]^{-\frac{1}{2}} \cdot \exp\left\{ -\frac{1}{2} \left(y_i - x_i\beta_i\right)^\top \left(\sigma^2 I_{k_i}\right)^{-1} \left(y_i - x_i\beta_i\right) \right\} \right) \end{aligned}$$

Prior distributions

We assume the following non-informative prior distributions:

$$\beta_i = (\beta_{0i}, \beta_{1i}, \ldots, \beta_{4i}) \sim N_5(\mu, \Sigma),$$

where $B = \left(\beta_1^\top, \beta_2^\top, \ldots, \beta_n^\top\right)^\top$ and n is the number of hurricanes. So,

$$\pi(B|\mu, \Sigma^{-1}) = \prod_{i=1}^{n} f\left(\beta_i\right) \propto \det(\Sigma)^{-n/2} \cdot \exp\left\{ -\frac{1}{2} \sum_i \left[\left(\beta_i - \mu\right)^\top \left(\Sigma\right)^{-1} \left(\beta_i - \mu\right) \right] \right\}.$$

Also, $\pi\left(\sigma^2\right) \propto \frac{1}{\sigma^2}$; $\pi(\mu) \propto 1$; $\pi\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \cdot \exp\left(-\frac{1}{2}\Sigma^{-1}\right)$.

Conditional posteriors

The posterior distribution is the product of the likelihood and the prior:

$$g\left(B, \sigma^2, \mu, \Sigma^{-1} \mid y\right) \propto f\left(y \mid B, \sigma^2\right) \cdot \pi(B \mid \mu, \Sigma^{-1}) \cdot \pi\left(\sigma^2\right) \cdot \pi(\mu) \cdot \pi\left(\Sigma^{-1}\right),$$

so we have:

$$\begin{aligned} \pi\left(\sigma^2 \mid \cdot\right) &\propto \prod_{i=1}^{n} \det\left(\sigma^2 I_{k_i}\right)^{-\frac{1}{2}} \cdot \exp\left\{ -\frac{1}{2}\Sigma_i \left[\left(y_i - x_i\beta_i\right)^\top \left(\sigma^2 I_{ki}\right)^{-1} \left(y_i - x_i\beta_i\right) \right] \right\} \cdot \sigma^{-2} \\ &= \left(\sigma^2\right)^{-\frac{1}{2}\Sigma_i k_i} \cdot \exp\left\{ -\frac{1}{2\sigma^2}\Sigma_i \left[\left(y_i - x_i\beta_i\right)^\top \left(y_i - x_i\beta_i\right) \right] \right\} \cdot \sigma^{-2} \\ &= \left(\sigma^2\right)^{-1-\frac{1}{2}\Sigma_i k_i} \cdot \exp\left\{ -\frac{1}{2\sigma^2}\Sigma_i\Sigma_{ti} \left(y_{i,t} - x_{i,t}\beta_i\right)^2 \right\} \end{aligned}$$

Therefore, $\sigma^2 \sim$ Inverse Gamma $\left(\frac{1}{2}\Sigma_i k_i, \frac{1}{2}\Sigma_i\Sigma_{t_i} \left(y_{i,t} - x_{i,t}\beta_i\right)^2\right)$.

$$\begin{aligned} \pi(\Sigma^{-1} \mid \cdot) &\propto \det(\Sigma)^{-n/2} \cdot \exp\left\{ -\frac{1}{2}\Sigma_i \left(\beta_i - \mu\right)^\top \Sigma^{-1} \left(\beta_i - \mu\right) \right\} \cdot \det(\Sigma)^{-(d+1)} \cdot \exp\left\{ -\frac{1}{2}\Sigma^{-1} \right\} \\ &= \det(\Sigma)^{-(n/2+d+1)} \cdot \exp\left\{ -\frac{1}{2} \left[\Sigma^{-1} + \Sigma_i \left(\beta_i - \mu\right)^\top \Sigma^{-1} \left(\beta_i - \mu\right) \right] \right\} \\ &\propto \det(\Sigma^{-1})^{(n+2d+2)/2} \cdot \exp\left\{ -\frac{1}{2} \operatorname{tr}\left[\Sigma^{-1} \cdot \left(I + \Sigma_i \left(\beta_i - \mu\right) \left(\beta_i - \mu\right)^\top \right] \right\} \\ &\propto \det(\Sigma^{-1})^{(n+3d+3-d-1)/2} \cdot \exp\left\{ -\frac{1}{2} \operatorname{tr}\left[\Sigma^{-1} \cdot \left(I + \Sigma_i \left(\beta_i - \mu\right) \left(\beta_i - \mu\right)^\top \right] \right\} \end{aligned}$$

Thus $\Sigma^{-1} \sim$ Wishart $\left(n + 3d + 3, \left[I + \Sigma_i \left(\beta_i - \mu\right) \left(\beta_i - \mu\right)^\top\right]^{-1}\right)$, that is:

$$\Sigma \sim \text{Inverse Wishart}\left(n + 3d + 3, I + \Sigma_i \left(\beta_i - \mu\right) \left(\beta_i - \mu\right)^\top\right)$$

$$
\begin{aligned}
\pi(\mu \mid \cdot) &\propto \exp\left\{-\frac{1}{2}\Sigma_i\left[(\beta_i-\mu)^\top \Sigma^{-1}(\beta_i-\mu)\right]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i\left(\beta_i^\top\Sigma^{-1}\beta_i+\mu^\top\Sigma^{-1}\mu-2\beta_i^\top\Sigma^{-1}\mu\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\Sigma_i\beta_i^\top\Sigma^{-1}\beta_i+\mu^\top n\Sigma^{-1}\mu-2\Sigma_i\beta_i^\top\Sigma^{-1}\mu\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\mu^\top n\Sigma^{-1}\mu-2\Sigma_i\beta_i^\top\Sigma^{-1}\mu+\Sigma_i\beta_i^\top\Sigma^{-1}\beta_i\right)\right\} \\
&= \exp\left\{-\frac{1}{2}(\mu^\top \underbrace{n\Sigma^{-1}}_{M}\mu-2\mu^\top\underbrace{\Sigma_i\Sigma^{-1}\beta_i}_{N}+\Sigma_i\beta_i^\top\Sigma^{-1}\beta_i)\right\} \\
&= \exp\left\{-\frac{1}{2}\left[\left(\mu-M^{-1}N\right)^\top M\left(\mu-M^{-1}N\right)\right]\right\}.
\end{aligned}
$$

Therefore, $\mu \sim MVN\left(M^{-1}N, M^{-1}\right)$.

$$
\begin{aligned}
&\pi(B \mid \cdot) \propto \exp\left\{-\frac{1}{2}\Sigma_i\left[(y_i-x_i\beta_i)^\top\left(\sigma^2 I_{ki}\right)^{-1}(y_i-x_i\beta_i)\right]\right\}\cdot\exp\left\{-\frac{1}{2}\Sigma_i\left[(\beta_i-\mu)^\top(\Sigma)^{-1}(\beta_i-\mu)\right]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i\left[(y_i-x_i\beta_i)^\top\left(\sigma^2 I_{ki}\right)^{-1}(y_i-x_i\beta_i)+(\beta_i-\mu)^\top\Sigma^{-1}(\beta_i-\mu)\right]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i[y_i^\top\sigma^{-2}I_{ki}y_i+\beta_i^\top x_i^\top\sigma^{-2}I_{k_i}x_i\beta_i-2y_i^\top\sigma^{-2}I_{ki}x_i\beta_i+\beta_i^\top\Sigma^{-1}\beta_i+\mu^\top\Sigma^{-1}\mu-2\mu^\top\Sigma^{-1}\beta_i]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i[y_i^\top\sigma^{-2}I_{ki}y_i+\mu^\top\Sigma^{-1}\mu+\beta_i^\top\left(\Sigma^{-1}+x_i^\top\sigma^{-2}I_{ki}x_i\right)\beta_i-2\left(y_i^\top\sigma^{-2}I_{ki}x_i+\mu^\top\Sigma^{-1}\right)\beta_i]\right\}
\end{aligned}
$$

We can define the following terms:

$$
\begin{aligned}
R &= y_i^\top\sigma^{-2}I_{k_i}y_i+\mu^\top\Sigma^{-1}\mu \\
V &= \Sigma^{-1}+x_i^\top\sigma^{-2}I_{ki}x_i \\
M &= \sigma^{-2}x_i^\top y_i+\Sigma^{-1}\mu
\end{aligned}
$$

Thus, $\pi(B \mid \cdot) \propto \left(\beta_i - V^{-1}M\right)^\top V\left(\beta_i - V^{-1}M\right) \sim MVN\left(V^{-1}M, V^{-1}\right)$

Gibbs Sampling

Since directly generating the above jointly density is rather complicated, we can implement the gibbs sampler to sample each variable in turn. Based on the previous conditional distributions, we updated the four parameters including B, σ^2, $\sum$, and μ in sequence. Finally, we selected 10,000 iterations. Note that we ignored the first 8,000 iterations to ensure the stationary distribution of the Markov chain was reached.

Regression Models

To explore seasonal and annual difference in the wind speed, we took the β coefficients from the Bayesian model as the response and built the following linear regression model:

$$\beta_j \sim Season + Year + Nature, j = 0, ..., 4,$$

where season and nature are categorical, and year is continuous variable.

To predict the hurricane-induced damage, we incorporated the coefficients from the Bayesian model and the new predictors in the second data, selected some featured variables by lasso, and finally fit a regression model. The variables β_0, Year, max speed, total affected population, and the affected population reside in the United States were selected in the model.

$$Damage \sim \beta_0 + Year + Maxspeed + Total.Pop + Percent.USA$$

We built a poisson regression model to evaluate characteristics related to deaths.

$$\begin{aligned} Death \sim \beta_4 + MonthSummer + Maxspeed + Maxpressure \\ + Meanpressure + Hours + Total.Pop + Percent.Poor \end{aligned}$$

Results

Gibbs Sampling

Task 5

Task 6

Discussion and Limitations

Contributions

We contributed to this project evenly.

Reference

[1] https://www.whoi.edu/know-your-ocean/ocean-topics/ocean-human-lives/natural-disasters/hurricanes/
?gclid=Cj0KCQjwsdiTBhD5ARIsAIpW8CIO0bvwlo16Pisxe_WV7owA4KdbMfpAhMxqa-z8Ir3fsQyfiHPa2MAaAlmZEALw
wcB

Appendix

For codes please click *here*