P8160 Group Project 3: Bayesian Modeling of Hurricane Trajectories

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Objective

In this project, we firstly built a Bayesian model based on the track data of 703 hurricanes in the North Atlantic area since 1950. A Markov Chain Monte Carlo (MCMC) algorithm was designed to generate the distribution of corresponding parameters. With the start time and type of each hurricane, the estimated coefficients from the Bayesian model were used to explore the seasonal differences and wind speed changes over years. Finally, we explored the characteristics of hurricanes associated with the damage and deaths.

Background

Hurricanes are large rotating tropical storms with winds in excess of 119 kilometers per hour (74 mph). They usually form between June 1 and November 30 in the Atlantic Ocean but can develop in other oceans as well. They are known as typhoons in the western Pacific and cyclones in the Indian Ocean[1].

When a hurricane approaches land, tremendous damage can occur to the nearby cities. Therefore, scientists continue to improve their ability to forecast hurricanes. The sooner they can access accurate information about a hurricane's location and intensity, the better the chances to minimize the its impacts.

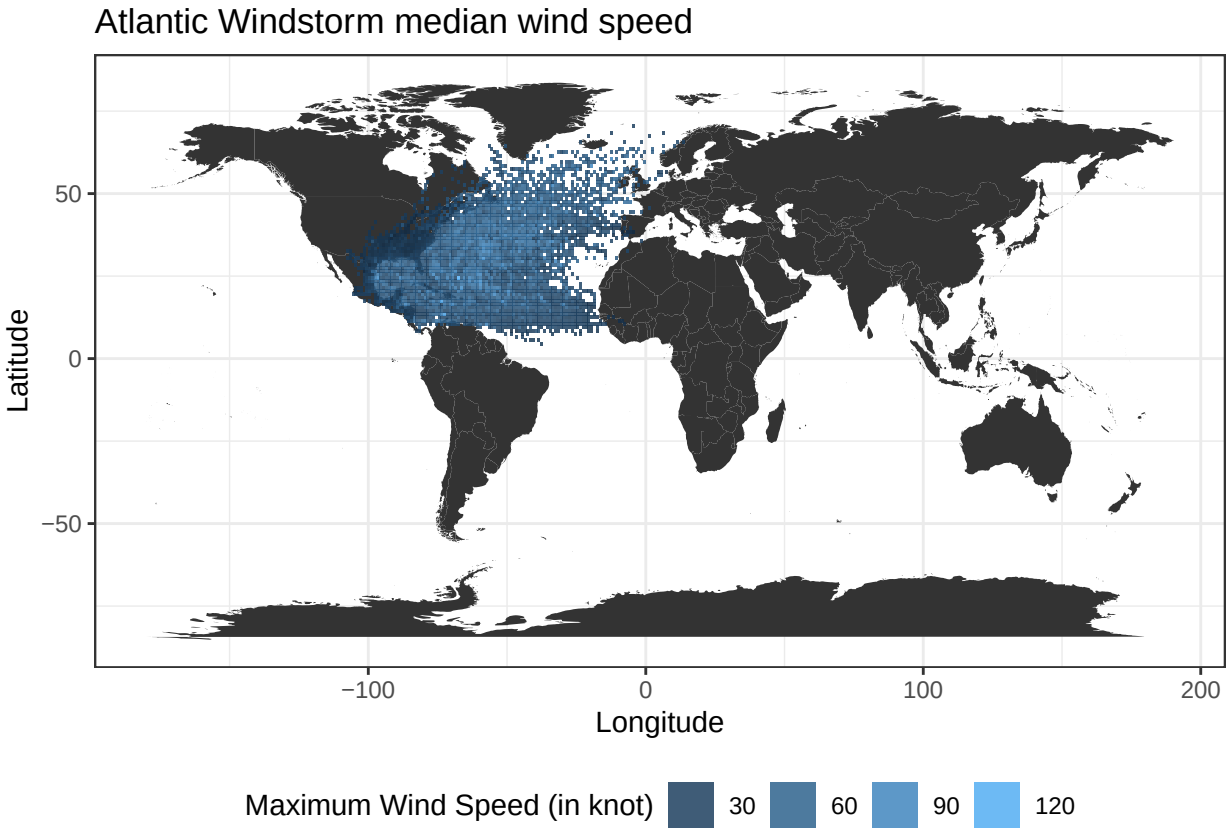
Data Description and Preprocessing

The first data `hurrican703.csv` collected the track data of 703 hurricanes in the North Atlantic area from 1950 to 2013. For all the hurricanes, their locations (longitude & latitude) and maximum wind speed were recorded every 6 hours (e.g. 0, 6, 12, etc. excluding the data points are not in the 6-hour interval, e.g. the time at 4 am.). The variables include `ID`, `Season`, `Month`, `Nature`, `time`, `Latitude`, `Longitude`, and `Wind.kt`. Main data processing steps for this data are listed as follows:
1) Created 3 new variables including the changes of latitude and longitude, as well as the wind speed between the time t and $t-6$;
2) Removed 9 hurricanes with observations less than 5 to ensure the data partition;
3) To explore the seasonal differences, we converted the start month for each hurricane into the variable `season`, which includes Spring, Summer, Fall, and Winter. Finally, we have totally **691** hurricanes in the updated dataset.

The second data `hurricanoutcome2.csv` recorded the damages and death caused by 46 hurricanes in the United States, and some features extracted from the above hurricane records. To better explore the characteristics related with death and damage, we combined this data with the coefficients obtained from the first model by the hurricane ID. For this data, we also converted different start months into the corresponding seasons.

Exploratory Data Analysis

To generally understand the distribution of wind speed across the North Atlantic area, we created the following figure to show the mean maximum wind speed within each knot based on the longitude and latitude from the original data `hurrican703.csv`.



Statistical Methods

Likelihood

For each hurricane i and k_i's time points, we have the following Bayesian model:

$$Y_i(t+6) = \beta_{0i} + \beta_{1i}Y_i(t) + \beta_{2i}\Delta_{i1}(t) + \beta_{3i}\Delta_{i2}(t) + \beta_{4i}\Delta_{i3}(t) + \varepsilon_i(t),$$

where $Y_i(t)$ is the wind speed at time t, Δ_{i1}, Δ_{i2}, and Δ_{i3} are the changes of latitude, longitude, and the wind speed between time point t and $t-6$, respectively. $\varepsilon_i(t)$ follows a normal distributions with mean zero and variance σ^2. The above Bayesian model can be simplified as:

$$Y_i(t+6) = x_i(t) + \varepsilon_i(t),$$

where $\beta_i = (\beta_{0i}, \beta_{1i}, ..., \beta_{4i}) \sim N\left(0, \sigma^2\right)$. Based on the property of the multivariate linear regression model, for each hurricane i, we have:

$$Y_i \mid X_i \sim N_{k_i}\left(x_i\beta_i, \sigma^2 I_{k_i}\right),$$

where I_{k_i} is an identity matrix with k_i dimensions.

Thus, we can consider the following distribution of each hurricane i:

$$f\left(y_i \mid \beta_i, \sigma^2\right) = \left[(2\pi)^{k_i} \cdot \det\left(\sigma^2 I_{k_i}\right)\right]^{-\frac{1}{2}} \cdot \exp\left\{-\frac{1}{2}\left(y_i - x_i\beta_i\right)^\top \left(\sigma^2 I_{k_i}\right)^{-1}\left(y_i - x_i\beta_i\right)\right\}$$

.

From above, we derive the following likelihood function:

$$\begin{aligned} f\left(y \mid B, \sigma^2\right) &= \prod_{i=1}^{n} f\left(y_i \mid \beta_i, \sigma^2\right) \\ &= \prod_{i=1}^{n} \left(\left[(2\pi)^{k_i} \cdot \det\left(\sigma^2 I_{ki}\right)\right]^{-\frac{1}{2}} \cdot \exp\left\{ -\frac{1}{2} \left(y_i - x_i\beta_i\right)^\top \left(\sigma^2 I_{k_i}\right)^{-1} \left(y_i - x_i\beta_i\right) \right\} \right) \end{aligned}$$

Prior distributions

We assume the following non-informative prior distributions:

$$\beta_i = (\beta_{0i}, \beta_{1i}, \ldots, \beta_{4i}) \sim N_5(\mu, \Sigma),$$

where $B = \left(\beta_1^\top, \beta_2^\top, \ldots, \beta_n^\top\right)^\top$ and n is the number of hurricanes. So,

$$\pi(B|\mu, \Sigma) = \prod_{i=1}^{n} f\left(\beta_i\right) \propto \det(\Sigma)^{-n/2} \cdot \exp\left\{ -\frac{1}{2} \sum_i \left[\left(\beta_i - \mu\right)^\top \left(\Sigma\right)^{-1} \left(\beta_i - \mu\right) \right] \right\}.$$

Also, $\pi\left(\sigma^2\right) \propto \frac{1}{\sigma^2}$; $\pi(\mu) \propto 1$; $\pi\left(\Sigma^{-1}\right) \propto |\Sigma|^{-(d+1)} \cdot \exp\left(-\frac{1}{2}\Sigma^{-1}\right)$.

Conditional posteriors

The posterior distribution is the product of the likelihood and the prior:

$$g\left(B, \sigma^2, \mu, \Sigma^{-1} \mid y\right) \propto f\left(y \mid B, \sigma^2\right) \cdot \pi(B \mid \mu, \Sigma^{-1}) \cdot \pi\left(\sigma^2\right) \cdot \pi(\mu) \cdot \pi\left(\Sigma^{-1}\right),$$

so we have:

For σ^2,

$$\begin{aligned} \pi\left(\sigma^2 \mid \cdot\right) &\propto \prod_{i=1}^{n} \det\left(\sigma^2 I_{k_i}\right)^{-\frac{1}{2}} \cdot \exp\left\{ -\frac{1}{2}\Sigma_i \left[\left(y_i - x_i\beta_i\right)^\top \left(\sigma^2 I_{ki}\right)^{-1} \left(y_i - x_i\beta_i\right) \right] \right\} \cdot \sigma^{-2} \\ &= \left(\sigma^2\right)^{-\frac{1}{2}\Sigma_i k_i} \cdot \exp\left\{ -\frac{1}{2\sigma^2}\Sigma_i \left[\left(y_i - x_i\beta_i\right)^\top \left(y_i - x_i\beta_i\right) \right] \right\} \cdot \sigma^{-2} \\ &= \left(\sigma^2\right)^{-1-\frac{1}{2}\Sigma_i k_i} \cdot \exp\left\{ -\frac{1}{2\sigma^2}\Sigma_i\Sigma_{ti} \left(y_{i,t} - x_{i,t}\beta_i\right)^2 \right\} \end{aligned}$$

Therefore, $\sigma^2 \sim$ Inverse Gamma $\left(\frac{1}{2}\Sigma_i k_i, \frac{1}{2}\Sigma_i\Sigma_{t_i} \left(y_{i,t} - x_{i,t}\beta_i\right)^2\right)$.

For Σ,

$$\begin{aligned} \pi(\Sigma^{-1} \mid \cdot) &\propto \det(\Sigma)^{-n/2} \cdot \exp\left\{ -\frac{1}{2}\Sigma_i \left(\beta_i - \mu\right)^\top \Sigma^{-1} \left(\beta_i - \mu\right) \right\} \cdot \det(\Sigma)^{-(d+1)} \cdot \exp\left\{ -\frac{1}{2}\Sigma^{-1} \right\} \\ &= \det(\Sigma)^{-(n/2+d+1)} \cdot \exp\left\{ -\frac{1}{2} \left[\Sigma^{-1} + \Sigma_i \left(\beta_i - \mu\right)^\top \Sigma^{-1} \left(\beta_i - \mu\right) \right] \right\} \\ &\propto \det(\Sigma^{-1})^{(n+2d+2)/2} \cdot \exp\left\{ -\frac{1}{2} \operatorname{tr}\left[\Sigma^{-1} \cdot \left(I + \Sigma_i \left(\beta_i - \mu\right) \left(\beta_i - \mu\right)^\top \right] \right\} \\ &\propto \det(\Sigma^{-1})^{(n+3d+3-d-1)/2} \cdot \exp\left\{ -\frac{1}{2} \operatorname{tr}\left[\Sigma^{-1} \cdot \left(I + \Sigma_i \left(\beta_i - \mu\right) \left(\beta_i - \mu\right)^\top \right] \right\} \end{aligned}$$

Thus $\Sigma^{-1} \sim \text{Wishart}\left(n+3d+3, \left[I+\Sigma_i\left(\beta_i-\mu\right)\left(\beta_i-\mu\right)^\top\right]^{-1}\right)$, that is:

$$\Sigma \sim \text{Inverse Wishart}\left(n+3d+3, I+\Sigma_i\left(\beta_i-\mu\right)\left(\beta_i-\mu\right)^\top\right)$$

For μ,

$$\begin{aligned}
\pi(\mu \mid \cdot) &\propto \exp\left\{-\frac{1}{2}\Sigma_i\left[\left(\beta_i-\mu\right)^\top\Sigma^{-1}\left(\beta_i-\mu\right)\right]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i\left(\beta_i^\top\Sigma^{-1}\beta_i+\mu^\top\Sigma^{-1}\mu-2\beta_i^\top\Sigma^{-1}\mu\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\Sigma_i\beta_i^\top\Sigma^{-1}\beta_i+\mu^\top n\Sigma^{-1}\mu-2\Sigma_i\beta_i^\top\Sigma^{-1}\mu\right)\right\} \\
&= \exp\left\{-\frac{1}{2}\left(\mu^\top n\Sigma^{-1}\mu-2\Sigma_i\beta_i^\top\Sigma^{-1}\mu+\Sigma_i\beta_i^\top\Sigma^{-1}\beta_i\right)\right\} \\
&= \exp\left\{-\frac{1}{2}(\mu^\top\underbrace{n\Sigma^{-1}}_{M}\mu-2\mu^\top\underbrace{\Sigma_i\Sigma^{-1}\beta_i}_{N}+\Sigma_i\beta_i^\top\Sigma^{-1}\beta_i)\right\} \\
&= \exp\left\{-\frac{1}{2}\left[\left(\mu-M^{-1}N\right)^\top M\left(\mu-M^{-1}N\right)\right]\right\}.
\end{aligned}$$

Therefore, $\mu \sim MVN\left(M^{-1}N, M^{-1}\right)$.

And for B,

$$\begin{aligned}
&\pi(B \mid \cdot) \propto \exp\left\{-\frac{1}{2}\Sigma_i\left[\left(y_i-x_i\beta_i\right)^\top\left(\sigma^2 I_{ki}\right)^{-1}\left(y_i-x_i\beta_i\right)\right]\right\}\cdot\exp\left\{-\frac{1}{2}\Sigma_i\left[\left(\beta_i-\mu\right)^\top(\Sigma)^{-1}\left(\beta_i-\mu\right)\right]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i\left[\left(y_i-x_i\beta_i\right)^\top\left(\sigma^2 I_{ki}\right)^{-1}\left(y_i-x_i\beta_i\right)+\left(\beta_i-\mu\right)^\top\Sigma^{-1}\left(\beta_i-\mu\right)\right]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i[y_i^\top\sigma^{-2}I_{ki}y_i+\beta_i^\top x_i^\top\sigma^{-2}I_{k_i}x_i\beta_i-2y_i^\top\sigma^{-2}I_{ki}x_i\beta_i+\beta_i^\top\Sigma^{-1}\beta_i+\mu^\top\Sigma^{-1}\mu-2\mu^\top\Sigma^{-1}\beta_i]\right\} \\
&= \exp\left\{-\frac{1}{2}\Sigma_i[y_i^\top\sigma^{-2}I_{ki}y_i+\mu^\top\Sigma^{-1}\mu+\beta_i^\top\left(\Sigma^{-1}+x_i^\top\sigma^{-2}I_{ki}x_i\right)\beta_i-2\left(y_i^\top\sigma^{-2}I_{ki}x_i+\mu^\top\Sigma^{-1}\right)\beta_i]\right\}
\end{aligned}$$

We can define the following terms:

$$\begin{aligned}
R &= y_i^\top\sigma^{-2}I_{k_i}y_i+\mu^\top\Sigma^{-1}\mu \\
V &= \Sigma^{-1}+x_i^\top\sigma^{-2}I_{ki}x_i \\
M &= \sigma^{-2}x_i^\top y_i+\Sigma^{-1}\mu
\end{aligned}$$

Thus, $\pi(B \mid \cdot) \propto \left(\beta_i-V^{-1}M\right)^\top V\left(\beta_i-V^{-1}M\right) \sim MVN\left(V^{-1}M, V^{-1}\right)$

Gibbs Sampling

Since directly generating the above parameters from the jointly density is rather complicated, we implement the gibbs sampler to sample each variable in turn. Based on the previous conditional distributions, we updated the four parameters including B, σ^2, Σ, and μ in sequence. Finally, we determined 10,000 iterations. Note that we ignored the first 8,000 iterations so that the stationary distribution of the Markov chain was reached. To evaluate how well the estimated Bayesian model tracks the individual hurricanes, we divided the observations within each hurricane ID by 8:2 randomly to create a training and testing set.

Regression Models

To explore the seasonal and annual difference in the wind speed, we took the β coefficients obtained from the Bayesian model as the response and built the following linear regression model:

$$\beta_j \sim Season + Year + Nature, j = 0, ..., 4,$$

where season and nature are categorical variables, and year is continuous.

To further predict the hurricane-induced damage, we incorporated the coefficients from the Bayesian model and the new predictors in the second data, selected some features by lasso, and finally fit a regression model. Inside lasso regression, we applied 'leave-one-out' validation. We applied linear regression model for damage prediction. The variables β_0, Year, max speed, total affected population, and the affected population reside in the United States were selected in the model. That is,

$$Damage \sim \beta_0 + Year + Maxspeed + Total.Pop + Percent.USA$$

Then we also select some variables by lasso and built the following Poisson regression model to evaluate the characteristics related to deaths since death is recorded as count, and we offset the Hours variable.

$$\begin{aligned} log(Death) \sim \beta_4 + MonthSummer + Maxspeed + Maxpressure \\ + Meanpressure + Total.Pop + Percent.Poor + offset(Hours), \end{aligned}$$

where the above β_0 and β_4 are the coefficients from the Bayesian model.

Results

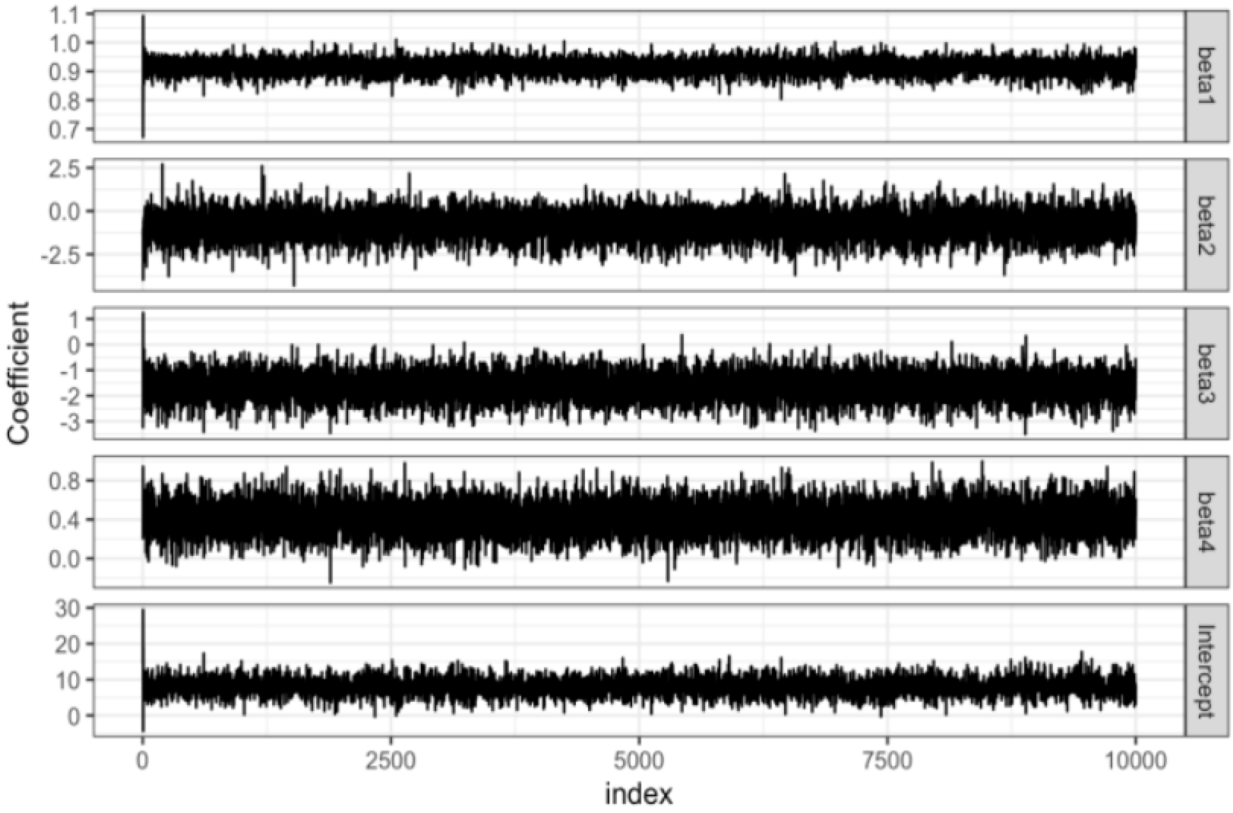
MCMC-Gibbs Sampling

The initial values for μ, σ^2 and Σ we set are (50,0,0,0,0), 0.5 and `diag(0.5, 5, 5)` respectively. In the case of Gibbs results, we set 10000 iterations and record the $\beta_{i,j}^n$ in each iteration with a linear model (from the least square method) to demonstrate how similar the coefficients are for each hurricane's train data and μ^n.

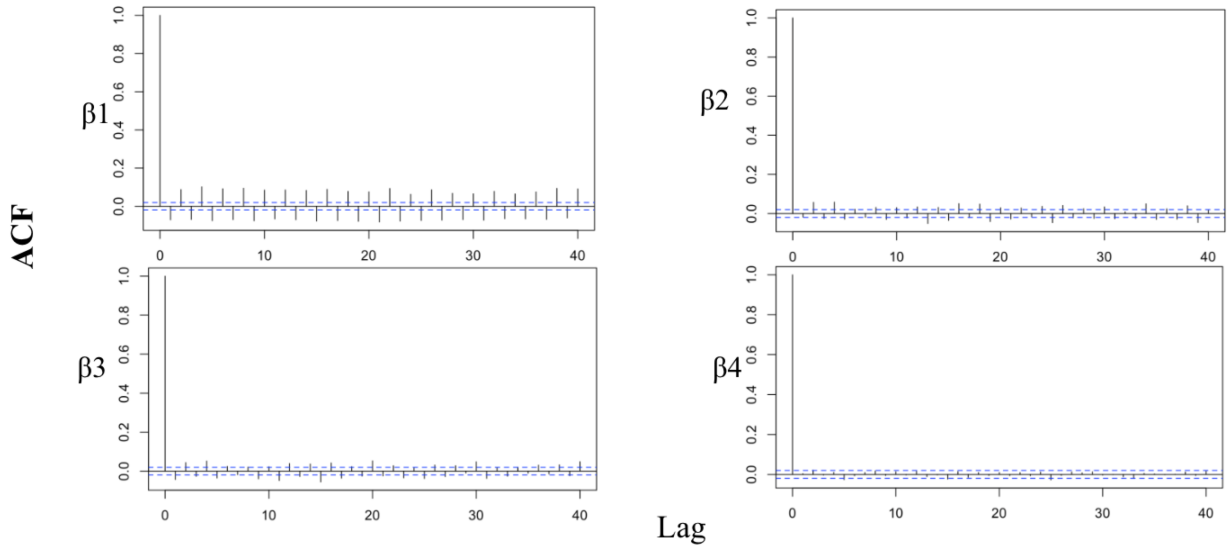
To minimize the variance generated randomly by Gibbs sampling and avoid the non convergent situation at the beginning of the iteration, we only take the coefficients value from 8000 to 10000 times where we take the mean value for each coefficients $\beta_{i,j}$ as the final results. To evaluate how well the estimated Bayesian model tracks the individual hurricanes, we divided the observations within each hurricane ID by 8:2 randomly to create a training and testing set. Then we calculated RMSE for each different hurricane by taking the value of test dataset for each hurricane and the coefficients $\hat{\beta_{i,j}}$ to predict the wind speed at $t + 6$.

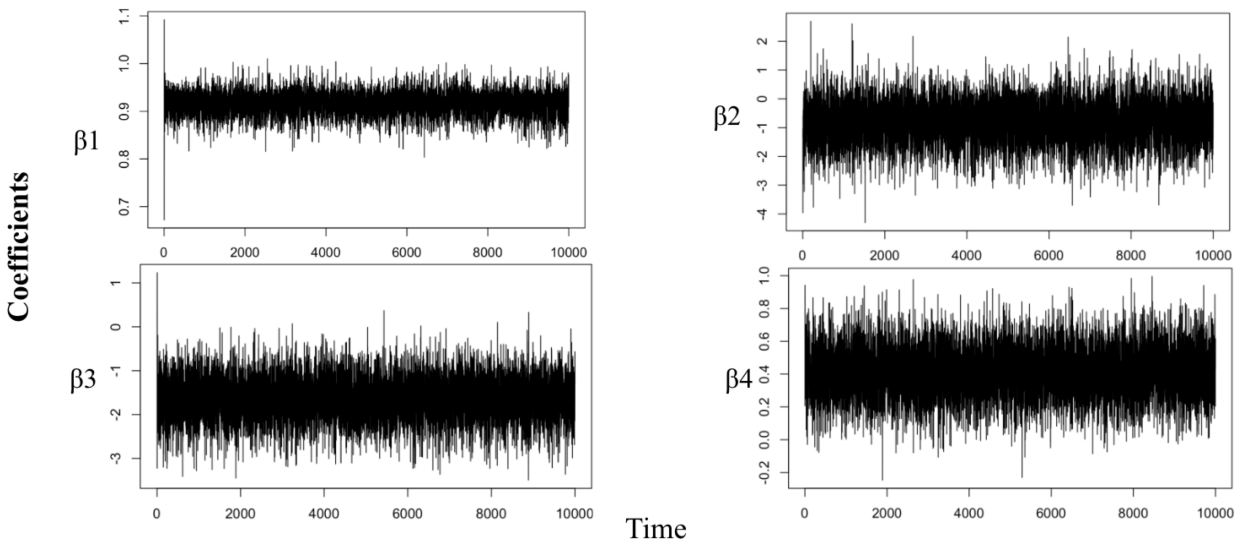
For our final results obtained from Gibbs sampling, you could jump to our *github*(Column2: RMSE for the test dataset, Column3-7: Mean coefficients $\beta_{i,j}$ for each hurricane by Gibbs sampling, Column8-12: coefficients $\beta_{i,j}$ for each hurricane by lm() function, Column13-17: Mean μ for B)

In order to further elucidate the results from our Gibbs methods, we take `ABLE.1950` as an example. Firstly, we trace the Gibbs step for only extracting the $\beta_{1,j}^n$ from B^n by plotting the value.



Then to look deeply into how the coefficients change with the increasing of iteration times, we also plot the autocorrelation plot(ACF) and time series plot (TS) where we could tell that the correlation for coefficient values between zero and blue line are not statistically significant.





For the case of `ABLE.1950`, the RMSE for the remaining 20% test dataset is 4.25 and the comparison of coefficients $\beta_{1,j}$ is in the table below.

Table 1: Compare the Coefficients from Gibbs Sampling and lm function

model	Intercept	Beta1	Beta2	Beta3	Beta4
Gibbs sampling	7.601	0.919	-0.788	-1.644	0.411
lm	8.833	0.912	-1.394	-2.013	2.983

Seasonal and Annual Difference

There are three significant variables in the regression model for β_1, which represent current wind speed. Season spring and summer, year all have negative coefficients, indicating wind speed tend to decrease in spring and summer. There is no evidence of increasing trend in β_1 with the increase in year.

Table 2: Beta 1 Coefficients

	Estimate	Std. Error	t value	Pr(> |t|)
(Intercept)	2.3394	0.3366	6.9498	0.0000
seasonSpring	−0.0636	0.0195	−3.2599	0.0012
seasonSummer	−0.0161	0.0062	−2.6198	0.0090
seasonWinter	−0.0476	0.0303	−1.5721	0.1164
year	−0.0008	0.0002	−4.4804	0.0000
natureET	0.0247	0.0233	1.0597	0.2897
natureNR	−0.0174	0.0367	−0.4743	0.6355
natureSS	0.0027	0.0160	0.1703	0.8648
natureTS	−0.0086	0.0125	−0.6900	0.4904

There are no significant variables in the regression model for β_2 and β_3. Season spring is the only significant variable in the regression model for β_4, indicating seasonal difference in acceleration of wind speed.

Table 3: Beta 4 Coefficients

	Estimate	Std. Error	t value	Pr(> |t|)
(Intercept)	2.3357	1.3705	1.7043	0.0888
seasonSpring	−0.2316	0.0794	−2.9177	0.0036
seasonSummer	−0.0024	0.0251	−0.0938	0.9253
seasonWinter	−0.0587	0.1233	−0.4758	0.6343
year	−0.0009	0.0007	−1.3816	0.1676
natureET	−0.0984	0.0947	−1.0391	0.2991
natureNR	−0.1024	0.1495	−0.6846	0.4938
natureSS	−0.0971	0.0650	−1.4927	0.1360
natureTS	−0.0365	0.0509	−0.7176	0.4733

Hurricane-induced damage and deaths prediction

After variable selection by lasso regression, the linear regression model for hurricane-induced damage has positive coefficients for Year, β_0, Maxspeed, Total.Pop, and Percent.USA, indicating positive correlation between these hurricane characteristics and damage. Among these, year is the only significant variable.

Table 4: Coefficients of regression model for Damage

	Estimate	Std. Error	t value	Pr(> |t|)
(Intercept)	11.155	3.067	3.638	0.000833
Year	7.918	3.229	2.452	0.019035
beta0	3.728	3.158	1.181	0.245323
Maxspeed	4.787	3.257	1.470	0.150046
Total.Pop	2.128	3.240	0.657	0.515240
Percent. USA	5.913	3.205	1.845	0.073077

The variables selected by lasso regression for hurricane-induced deaths include Summer, β_4, Maxspeed, Maxpressure, Hours, Total.Pop, and Percent Poor. All the selected variables are significant except for total population in the Poisson regression model. β_4, hours, and Percent.Poor has positive correlation with deaths while the other variables have negative coefficients.

Table 5: Coefficients of regression model for Deaths

	Estimate	Std. Error	z value	Pr(> |z|)
(Intercept)	6.654e + 01	2.937e + 00	22.652	< 2e − 16
beta 4	4.019e + 00	7.226e − 02	55.614	< 2e − 16
MonthSummer	−1.172e + 00	2.275e − 02	−51.494	< 2e − 16
Maxspeed	−5.448e − 03	3.541e − 04	−15.386	< 2e − 16
Maxpressure	−6.729e − 02	2.891e − 03	−23.279	< 2e − 16
Meanpressure	−1.522e − 04	3.094e − 05	−4.920	8.67e − 07
Total.Pop	1.736e − 08	1.246e − 08	1.393	0.164
Percent.Poor	3.463e − 02	1.587e − 04	218.230	< 2e − 16

Discussion and Limitations

MCMC algorithm could return converged results whatever the initial values are, but some initial values that allow for faster convergence. Since our likelihood are actually based on normal assumption, `lm` regression coefficients might be warm start. At the same time, MCMC returns consistent results with hypothesis. We supposed coefficients from 700 hurricanes follow multivariate normal distribution with common mean mu. In our results, mean of coefficients beta are very close to mu.

There are some limitations for our project. For mcmc, when calculating the variance of beta coefficients, we apply the sum square of residuals based on current beta values and corresponding x values, i.e. previous wind speed, latitude, longitude and previous wind speed variation. However, some specific hurricanes only have records no more than 10 and almost zero wind speed variation, which might cause anomalies in iteration. To be specific, zero predictor values force the product of coefficient and predictor to be zero and there is an extremely weak regularization on coefficients. For these types of hurricanes, mcmc might not work well and return large RMSE. RMSE also depended on how we split training and testing data in cross validation.

We hope mcmc and ordinary linear regression have similar results, but the amount of some specific hurricanes is too small and some predictors values are highly correlated within such a limited time span. If more than two predictors of one hurricane are highly correlated, the determinant of the predictors matrix for this particular hurricane seems to be zero which indicates that coefficients are not unique. In this case, the lm function in R simply returns NA for constant predictors, so we could not make a comparison with mcmc predictors easily.

Contributions

We contributed to this project evenly.

Reference

[1] https://www.whoi.edu/know-your-ocean/ocean-topics/ocean-human-lives/natural-disasters/hurricanes/?gclid=Cj0KCQjwsdiTBhD5ARIsAIpW8CIO0bvwlo16Pisxe_WV7owA4KdbMfpAhMxqa-z8Ir3fsQyfiHPa2MAaAlmZEALwwcB

Appendix

For codes please click *here*