



**Optimization of Healthcare Facilities:  
Allocating Hospital Facilities to COVID-19 Patients**

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## Abstract

The novel coronavirus disease (COVID-19) has caused an exponential rise in the amount of healthcare facilities required for patients. A deep knowledge in operations research is key when it comes to developing more efficient and effective approaches to optimize a problem statement. This project aims to perform optimization by building an assignment model to maximize the treatments for COVID-19 infected patients to best meet the supply and demand constraints. The data to build the model is collected and produced, based on which a mathematical model is built. Sensitivity analysis is then performed on the model to examine the change in the objective function as the demand and supply were altered by resizing the number of healthcare equipment. AMPL and Python programming languages were used to accurately and mathematically optimize the objective function to efficiently assign treatment plans to patients based on their priority levels.

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## Introduction

The world was heavily impacted by the coronavirus disease (COVID-19) in the year 2020. COVID-19 caused mild to severe respiratory illness in the bodies of the people who were exposed to the coronavirus disease. The deterioration in the health of an individual enforces them to be taken care of under medical facilities. The exponential growth in the number of COVID-19 infected patients caused an increase in the demand on healthcare facilities.

This project analyzes the factors necessary for the optimization of beds/ventilators/wards to COVID-19 infected patients in Boston, Massachusetts to save as many lives as possible. To solve this problem, priority rules and categories were formed. A risk category was created that classified each patient into an age group, those aged 65 or younger, those aged between 65 to 85 and those aged 85 or above. Another layer under the risk category was the pre-existing conditions category, patients were divided into groups, one where the patients did have a pre-existing condition and another where the patients did not have any pre-existing condition. An illness category, consisting of a mild and severe category was also used to aid the priority rules, these rules help determine which group of patients should receive what kind of treatment.

To cover the demands of the treatment, supply data was generated. This supply data consisted of the number of beds, wards, ventilators and ICUs available in the hospitals. Another layer was added to the project to extend the supply data, a treatment plan was developed. The treatment plan was divided into six different categories which determined what type of treatment will be offered to a patient along with the total supply for the treatment. Treatment plans consisted of the following, hospitalization without supplemental oxygen, hospitalization with supplemental oxygen, hospitalization with intensive therapy ward, ICU with a ventilator, ICU without a ventilator, and self isolation at home.

An assignment model was developed, where supply constraints were defined, these constraints were the number of facilities available for different treatment plans, based on the model, the objective function was also formed to maximize the number of assignments of patients with respect to their priority levels to the respective treatment plan. The data for this model was created based on our understanding of optimization and assignment problems to best fit our model, the data was formed according to a weekly basis. This suggests, the numbers displayed in our final model are the average taken for a particular week.

Programming languages Python were used to import the data from our final model to accurately determine how well can the optimization of treatment plans be done with respect to the defined constraints so that the demand of treatments are met in the most efficient manner.

## Literature Review

### Hospitalization Demand in Massachusetts

There are around 680K number of Covid-19 confirmed cases along with the number of deaths reached 17,528 in Massachusetts up until today. The number of treatments available never meet up the demand in Boston. Simply because the number of the increasing patients of Covid-19 was out of expectation in Massachusetts.

### Hospitalization Supply in Massachusetts

For this project, We have defined some treatment plans in order to fulfil the needs of patients. There are several treatments that need to be explained for this project.

- a. The major treatment provided would be hospitalization. But there are various treatment cares which were considered when involving the hospitalization.
  1. The first category is normal hospitalization care. Normal hospitalization care would be defined as remaining in hospital under observation for future determination or inpatient admission. Hospitals could assign the patient beds in order to keep the management.
  2. The second category for hospitalization would be a normal ward with supplemental oxygen. The patients who belong to this category have been identified as confirmed cases of Covid-19. But their health condition is still mild which means the only necessary treatment for them would be isolation since the Covid-19 is highly contagious. Maybe supplemental oxygen would be provided as well for this situation.
  3. The third category would be the intensive therapy ward. It is similar to ICU(intensive care unit) but it is designed to take care of the seriously ill patients who are confirmed on Covid-19. Those patients could have ventilators or special equipment as well as 24 hours nurse support for this category.
  4. The fourth category would be an intensive care unit or ICU without ventilators. This type of treatment was meant to take care of patients with severe or life-threatening illnesses of Covid-19. It requires constant care, close supervision from life support equipment and medication in order to ensure their normal bodily functions.
  5. The last category would be an intensive care unit with ventilators. This treatment is similar to the previous treatments but ventilators are required to have in order to take care of the critically ill patients.

- b. Since not all patients with COVID-19 require hospital admission. Only the patients whose clinical presentation warrants in-patient clinical management for supportive medical care should be admitted to the hospital under appropriate isolation precautions. People who are not hospitalized would be isolated at home without receiving any medical treatment. But quarantine would also be considered in this project

### **Types of patients as parameters with different medical conditions**

As mentioned earlier in this section, there are 18 groups of patients with various medical conditions. We included only 3 types of elements when grouping the patients.

- A. First one is the illness category, we have included two types of illness: mild and severe simply indicate the severity of illness. The severity rate of patients who are 65 and older is assumed to be 50%, while the severity rate of patients who are 65 and under is assumed to be 20%.
- B. Second is the risk category, patients would have three types of risk category: Pre-existing condition, without pre-existing condition, lung disease. With/without pre-existing condition means the patients who have symptoms of heart diseases or diabetes, those symptoms have been included within the definition of pre-existing condition in our project since it is not part of the main reason when the patients have Covid-19. Covid-19 is a respiratory disease which means that it can reach into patients' respiratory tract that includes lungs, so we have identified this as an independent part of the risk category. The prevalence of Diabetes (Both Type I and Type II) is 10.5% and the percentage of adults with diagnosed heart disease is 12.1%. Therefore, the total number of patients that have other pre-existing conditions(Heart disease and Diabetes) are  $12.1\% + 10.5\% = 22.6\%$ . According to the statistical report provided by American Lung Association, the percentage of adults with chronic lung disease in Massachusetts State is roughly 11%.
- C. Third is the ages of patients. The risk for severe illness with Covid-19 increases with age, with older adults at highest risk. For example, people in their 50s are at higher risk for severe illness than people in their 40s. Similarly, people in their 60s or 70s are at higher risk for severe illness than people in their 50s. The greatest risk for severe illness from Covid-19 is among those aged 85 or older. So, in this project, we have separated the ages into three groups, age 65 and below, age between 65 and 85, age 85 and older. According to MA demographic statistics, only 1.4% of the population is 85 and older; 9% of the population is from 65 years old to 85 years old. And 89.9% of the population is at the age of 65 and under.

So, in our project, there are 2 types of illness category: mild/severe; 3 types of risk category: With/without pre-existing condition/lung disease; 3 types of ages: 65 and below/ between 65 and 85/ 85 and older. So, there would be  $2 \times 3 \times 3 = 18$  numbers of combinations that

needed to be considered as variables in our project. The following shows one example of how the demand of each patient group is calculated.

*Assume Total Confirmed Cases in Boston Area in a week: 7000 Cases*

*Number of Patient From Group 1 that are 85 and older (1.4%), having chronic lung diseases (11%) and in severe condition(50%)*  
 $= 7000 * 0.014 * 0.11 * 0.5 = 5$

## **Priority rules**

There are priority rules that need to be considered as well in our project. Since the people with severe illness might need to be taken care of first, those patients should have higher priority than others when receiving the correct treatment plans. The treatment methods would also have priority, for example, for a patient who has lung disease/aged 85 and above/severe should have received three types of treatments: intensive therapy ward/ ICU without ventilator and ICU with ventilator. But there should be a priority among those three types of treatment in order to allocate the necessary treatment for the corresponding group because of the limited number of treatment supplies. In this case, ICU with a ventilator has higher priority than ICU without a ventilator than intensive therapy ward. We have indicated the priority of treatment as number 0,1,2,3. 0 means no need to provide, 1 indicates treatment could be provided, 2 means treatments should be provided and 3 means the treatment must be provided to the patients.

## **Model Assumptions and Limitations**

For this project, there were several assumptions we have made. We assume that the number of the supply and demands are combined from three hospitals in the Greater Boston area. The number of supplies was based on the information provided by each hospital on their official website. For example, the number of beds from the “third oldest general hospital in the United States” which is Massachusetts General Hospital is 999. Brigham and Women’s Hospital which is one of the three hospitals we used for this project is a 793 bed teaching hospital of Harvard Medical School. And Beth Israel Deaconess Medical Center has a capacity of 673 beds. So the number of beds available for the patients in Boston would be combined as  $999 + 793 + 673 = 2465$  that we used for the number of beds as supply.

Another assumption we made for this project is that each patient under the corresponding category among the 18 groups should receive exactly one treatment. But the patents belonging to the specific category might receive more than one treatment. For example, in our project, the patients without pre-existing conditions/aged 85 and above/with severe symptoms have received more than one treatment. The reason is that we have assigned priority rules in this project, when the number of supply is smaller than the number of demands, after the primary treatments have been allocated but there are demands left still, the model would assign the secondary treatment to this group of patients in order to fulfill the demand and make it balanced.

One of the limitations of this model is distance and cost were not considered in this project. But distance could be a consideration when we want to allocate the maximum number of treatments to the corresponding patients since we should assign the correct treatment to the critically ill patients first which has a longer distance just because the distance between this patient and hospital should be involved. Same reason as cost, it would change the output when we find the maximum number of treatments in this project. But due to the complexity of the project we did not consider those elements.



## Mathematical Model

### Index Sets

<i>Index</i>		
<i>T</i>	Sets of Treatment Plans	{T = 1,2,3,4,5}; indexed by i
<i>A</i>	Sets of Patient Groups	{A = 1,2,...,17,18}; indexed by j

<i>Index</i>	<i>Treatment Plan</i>
<i>i = 1</i>	Bed
<i>i = 2</i>	Normal Ward
<i>i = 3</i>	Intensive Therapy Ward
<i>i = 4</i>	ICU without Ventilator
<i>i = 5</i>	ICU with Ventilator

<i>Index</i>	<i>Condition/ Pre-existing Condition</i>	<i>Age</i>
<i>j=1</i>	Severe condition with pre-existing lung diseases	85 and older
<i>j=2</i>	Severe condition with pre-existing lung diseases	85 to 65
<i>j=3</i>	Severe condition with pre-existing lung diseases	65 and under
<i>j=4</i>	Severe condition with other pre-existing conditions	85 and older
<i>j=5</i>	Severe condition with other pre-existing conditions	85 to 65
<i>j=6</i>	Severe condition with other pre-existing conditions	65 and under
<i>j=7</i>	Severe condition without other pre-existing conditions	85 and older
<i>j=8</i>	Severe condition without other pre-existing conditions	85 to 65
<i>j=9</i>	Severe condition without other pre-existing conditions	65 and under
<i>j=10</i>	Mild condition with pre-existing lung diseases	85 and older
<i>j=11</i>	Mild condition with pre-existing lung diseases	85 to 65
<i>j=12</i>	Mild condition with pre-existing lung diseases	65 and under
<i>j=13</i>	Mild condition with other pre-existing conditions	85 and older
<i>j=14</i>	Mild condition with other pre-existing conditions	85 to 65
<i>j=15</i>	Mild condition with other pre-existing conditions	65 and under
<i>j=16</i>	Mild condition without other pre-existing conditions	85 and older
<i>j=17</i>	Mild condition without other pre-existing conditions	85 to 65
<i>j=18</i>	Mild condition without other pre-existing conditions	65 and under

## Parameters

$D_j$	Demand of Patient Groups 'j'
$S_i$	Supply of Treatment Plans 'i'
$P_{ij}$	Priority Level of treatment plan 'i' allocate to patient group 'j'. Unitless.

## Decision Variable

$X_{ij}$	Number of treatment plan 'i' allocate to patient group 'j'
----------	--

## Other Variable

$H_j$	Number of patients from group 'j' that are home-isolated
-------	--

## Objective Function

The objective function was to maximize the priority level in order to maximize the benefit of allocating the right treatments to patient groups.

*Total Priority Levbel*  
*= Priority Level of the Patient Group 'j' that needs the Treatment Plan 'i'*  
*× Number of Treatment Plan 'i' allocateto Patient Group 'j'*

$$\text{Maximize } Z = \sum_{i \in T} \sum_{j \in A} P_{ij} \times X_{ij}$$

## Constraints

Supply Constraints: This constraint defined as the availability of the treatment plan resources. The number of treatment plan for each plan that allocate to different patient groups should not exceeds the total supply of that treatment plan.

$$X_{ij} \leq S_i \quad \forall i \in T, \forall j \in A$$

Demand Constraints: This constraint defined as total demand of the hospital resources from each patient group. The number of treatment plan for each plan that allocate to different patient groups should not exceeds the demand of that treatment plan within that group. Therefore, the excess supply can be relocated to other groups.

$$X_{ij} \leq D_j \quad \forall i \in T, \forall j \in A$$

Non-negativity constraint for decision variables

$$X_{ij} \geq 0, \text{integer}$$

### Other variable

Patients that do not receive hospitalization treatment or do not need hospitalization are recommended to isolate themselves at home. This variable defines the number of patients that do not receive treatment due to shortage and the patients that do not need to be hospitalized. This variable can also be defined as a slack variable where the demand is not met. This is the solution to the patients that are not hospitalized. More detail will be described in the result part. Number of patients from each group that are home isolated can be defined as follows:

$$H_j = D_j - \sum_{i \in T} X_{ij}$$

Total number of patients that are home-isolated can be defined as follows:

$$H_{Total} = \sum_{j \in A} \left( D_j - \sum_{i \in T} X_{ij} \right)$$

# Python Model

```
#Import package
import random
import math
import numpy as np
import xlswriter
import pandas as pd

from pulp import *
```

## Preliminary model

```
# Define the model
model = LpProblem(name="Treatments_allocation_Problem", sense=LpMaximize)

# Define the decision variables
x = {i: LpVariable(name=f"x[{i}]", lowBound=0, cat="Integer") for i in range(1, 25)} # x[i] >=0 and integer (i = 1,2,3...24)

#Add constraints
#supply constraints
model += (x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] <= 2000, "bed constrain")
model += (x[9] + x[10] + x[11] + x[12] + x[13] + x[14] + x[15] + x[16] <= 600, "normal ward")
model += (x[17] + x[18] + x[19] + x[20] + x[21] + x[22] + x[23] + x[24] <= 60, "ICU")

# demand constraints
model += (x[1] + x[9] + x[7] <= 8000, "non-con/m/~50")
model += (x[2] + x[10] + x[18] <= 300, "non-con/m/50~")
model += (x[3] + x[11] + x[19] <= 1000, "con/m/~50")
model += (x[4] + x[12] + x[20] <= 50, "con/m/50~")
model += (x[5] + x[13] + x[21] <= 700, "non-con/s/~50")
model += (x[6] + x[14] + x[22] <= 30, "non-con/s/50~")
model += (x[7] + x[15] + x[23] <= 500, "con/s/~50")
model += (x[8] + x[16] + x[24] <= 20, "con/s/50~")

#Set the objective
model += x[1] + 3* x[3] + 2* x[7] + x[9] + 2*x[11] + 2*x[12] + x[13] + 2*x[14] + x[15] + 2*x[16] + 3* x[17] + 3*x[18] + 3*x[20]
#Solve the optimization problem
status = model.solve()

#Get the results
print(f"status: {model.status}, {LpStatus[model.status]}")
print(f"objective: {model.objective.value()}")

for var in x.values():
    print(f"{var.name}: {var.value()}")

for name, constraint in model.constraints.items():
    print(f"{name}: {constraint.value()}")

# Sensitive analysis
sr = [{'Constraint Names':cname, 'Slack Values':cinfo.slack, 'Shadow Price':cinfo.pi} for cname, cinfo in model.constraints.items()] # slack values: how many supply or demand left at th
print(pd.DataFrame(sr))
```

## Load file

```
# Load the dataset
df = pd.read_excel('supply_demand.xlsx')
# set the index equal to the 's/d'
df.index = df['s/d']
df = df.drop('s/d',axis =1)
df = df.fillna(1)
# get the sum of all demand(use it Later)
col = list(df)
col.remove('Supply')

sum_demand = df[col].sum(axis=1)[5]
```

```

# Define the model
model = LpProblem(name="Treatments_allocation_Problem", sense=LpMaximize)

# Define the decision variables
x = {i: LpVariable(name=f"x[{i}]", lowBound=0, cat="Integer") for i in range(1, 91)} # x[i] >=0 and integer (i = 1,2,3...90)

#Add constraints
#Supply constraints
model += (x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] + x[9] + x[10] + x[11] + x[12] + x[13] + x[14] + x[15] + x[16] + x[17] + x[18] <= df['Supply'][0], "bed constrain")
model += (x[19] + x[20] + x[21] + x[22] + x[23] + x[24] + x[25] + x[26] + x[27] + x[28] + x[29] + x[30] + x[31] + x[32] + x[33] + x[34] + x[35] + x[36] <= df['Supply'][1], "normal ward")
model += (x[37] + x[38] + x[39] + x[40] + x[41] + x[42] + x[43] + x[44] + x[45] + x[46] + x[47] + x[48] + x[49] + x[50] + x[51] + x[52] + x[53] + x[54] <= df['Supply'][2], "intensive th")
model += (x[55] + x[56] + x[57] + x[58] + x[59] + x[60] + x[61] + x[62] + x[63] + x[64] + x[65] + x[66] + x[67] + x[68] + x[69] + x[70] + x[71] + x[72] <= df['Supply'][3], "ICU without")
model += (x[73] + x[74] + x[75] + x[76] + x[77] + x[78] + x[79] + x[80] + x[81] + x[82] + x[83] + x[84] + x[85] + x[86] + x[87] + x[88] + x[89] + x[90] <= df['Supply'][4], "ICU with ven")

# demand constraints
model += (x[1] + x[19] + x[37] + x[55] + x[73] <= df.loc['Demand',][17], "non-con/m/~65")
model += (x[2] + x[20] + x[38] + x[56] + x[74] <= df.loc['Demand',][16], "non-con/m/65~85")
model += (x[3] + x[21] + x[39] + x[57] + x[75] <= df.loc['Demand',][15], "non-con/m/85~")
model += (x[4] + x[22] + x[40] + x[58] + x[76] <= df.loc['Demand',][14], "con/m/~65")
model += (x[5] + x[23] + x[41] + x[59] + x[77] <= df.loc['Demand',][13], "con/m/65~85")
model += (x[6] + x[24] + x[42] + x[60] + x[78] <= df.loc['Demand',][12], "con/m/85~")
model += (x[7] + x[25] + x[43] + x[61] + x[79] <= df.loc['Demand',][11], "lung/m/~65")
model += (x[8] + x[26] + x[44] + x[62] + x[80] <= df.loc['Demand',][10], "lung/m/65~85")
model += (x[9] + x[27] + x[45] + x[63] + x[81] <= df.loc['Demand',][9], "lung/m/85~")
model += (x[10] + x[28] + x[46] + x[64] + x[82] <= df.loc['Demand',][8], "non-con/s/~65")
model += (x[11] + x[29] + x[47] + x[65] + x[83] <= df.loc['Demand',][7], "non-con/s/65~85")
model += (x[12] + x[30] + x[48] + x[66] + x[84] <= df.loc['Demand',][6], "non-con/s/85~")
model += (x[13] + x[31] + x[49] + x[67] + x[85] <= df.loc['Demand',][5], "con/s/~65")
model += (x[14] + x[32] + x[50] + x[68] + x[86] <= df.loc['Demand',][4], "con/s/65~85")
model += (x[15] + x[33] + x[51] + x[69] + x[87] <= df.loc['Demand',][3], "con/s/85~")
model += (x[16] + x[34] + x[52] + x[70] + x[88] <= df.loc['Demand',][2], "lung/s/~65")
model += (x[17] + x[35] + x[53] + x[71] + x[89] <= df.loc['Demand',][1], "lung/s/65~85")
model += (x[18] + x[36] + x[54] + x[72] + x[90] <= df.loc['Demand',][0], "lung/s/85~")

#Set the objective
model += 3*x[11] + 2*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40] +

#Solve the optimization problem
status = model.solve()

#Get the results
print(f"status: {model.status}, {lpStatus[model.status]}")
print(f"objective: {model.objective.value()}")
home = 0
for var in x.values():
    home += var.value()
    print(f"{var.name}: {var.value()}")
print(f"Stay at home: {format(sum_demand - home)}")
for name, constraint in model.constraints.items():
    print(f"{name}: {constraint.value()}")

```

## Sensitivity Analysis

Sensitivity analysis is a systematic student of how sensitive solutions are to the changes in the model. In this study, sensitivities of objective function, resources data and constraints are investigated.

### Objective function

Four new objective function were built to test the sensitivity of the final result.

The four different objective functions are shown below:

```
#Set the objective
model1 += 0.3*x[11] + 2*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
#model2 += 300*x[11] + 2*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
#model3 += 3*x[11] + 200*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
#model4 += 3*x[11] + 2*x[12] + 300*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
```

### Result:

Model 1: 

```
status: 1, Optimal
objective: 5327.7
```

Model 2: 

```
status: 1, Optimal
objective: 14019.0
```

Model 3: 

```
status: 1, Optimal
objective: 95100.0
```

Model4: 

```
status: 1, Optimal
objective: 8673.0
```

It can be concluded that in our model if the objective function changes, the optimal solution will change at the same time.

### Resource changes

All that information about shadow price and slack value has been stored in `model.constraints.items()`. Therefore, these Information can be extracted from it using a for loop function. The result is show below.

	Constraint Names	Slack Values	Shadow Price
0	bed_constrain	435.0	-0.0
1	normal_ward_constrain	-0.0	-0.0
2	intensive_therapy_ward_constrain	-0.0	-0.0
3	ICU_without_ventilator_constrain	-0.0	-0.0
4	ICU_with_ventilator_constrain	51.0	-0.0
5	non_con/m/~65	-0.0	-0.0
6	non_con/m/65~85	-0.0	-0.0
7	non_con/m/85~	-0.0	-0.0
8	con/m/~65	-0.0	-0.0
9	con/m/65~85	-0.0	-0.0
10	con/m/85~	-0.0	-0.0
11	lung/m/~65	-0.0	-0.0
12	lung/m/65~85	-0.0	-0.0
13	lung/m/85~	1159.0	-0.0
14	non_con/s/~65	-0.0	-0.0
15	non_con/s/65~85	-0.0	-0.0
16	non_con/s/85~	-0.0	-0.0
17	con/s/~65	-0.0	-0.0
18	con/s/65~85	-0.0	-0.0
19	con/s/85~	-0.0	-0.0
20	lung/s/~65	-0.0	-0.0
21	lung/s/65~85	-0.0	-0.0
22	lung/s/85~	7840.0	-0.0

It can be concluded that there are still 435 bed and 51 ICU with ventilator supply left and 8999(1159+7840) patients with lung disease, severe or mild illness category and at least 85 years old have not been assigned. Additionally, our shadow price all equal to zero means if we have one more supply or demand available and give it to the constrain, our model benefit will increase zero. It means our constrains are nonbinding.

### Additional Constraints

A new constraint is added to limit the up bound of patients with severe illness category and patients that 85 and older. The reason for this constraint is that the elderly will be more susceptible to infection and the condition will be more serious, which is more common in the elderly.

The new constraint is shown below:

```
model += (x[12]+x[30]+x[48]+x[66]+x[84]+x[15]+x[33]+x[51]+x[69] +x[87]+x[18]+x[36]+x[54]+x[72]+x[90] <= 40) # add a constrain
```

### Result:

```
status: 1, Optimal
objective: 3476.0
- - -
```

The result indicates that the new constraints worked and will change the optimal solution.

# Sensitivity Analysis Python Code

Sensitive analysis: resource(supply or demand) available change.

Left at the end. Shadow price: if we have one more supply or demand available and give it to the constrain, our model benefit will increase the value of shadow price (in our model, all is

	Constraint Names	Slack Values	Shadow Price
0	bed_constrain	435.0	-0.0
1	normal_ward_constrain	-0.0	-0.0
2	intensive_therapy_ward_constrain	-0.0	-0.0
3	ICU_without_ventilator_constrain	-0.0	-0.0
4	ICU_with_ventilator_constrain	51.0	-0.0
5	non_con/m/~65	-0.0	-0.0
6	non_con/m/65~85	-0.0	-0.0
7	non_con/m/85~	-0.0	-0.0
8	con/m/~65	-0.0	-0.0
9	con/m/65~85	-0.0	-0.0
10	con/m/85~	-0.0	-0.0
11	lung/m/~65	-0.0	-0.0
12	lung/m/65~85	-0.0	-0.0
13	lung/m/85~	1159.0	-0.0
14	non_con/s/~65	-0.0	-0.0
15	non_con/s/65~85	-0.0	-0.0
16	non_con/s/85~	-0.0	-0.0
17	con/s/~65	-0.0	-0.0
18	con/s/65~85	-0.0	-0.0
19	con/s/85~	-0.0	-0.0
20	lung/s/~65	-0.0	-0.0
21	lung/s/65~85	-0.0	-0.0
22	lung/s/85~	7840.0	-0.0

Sensitive analysis: objective function changed

solution : optimal solution change.

```
[23]: # Define the model
model = LpProblem(name="Treatments_allocation_Problem", sense=LpMaximize)

# Define the decision variables
x = [{}: LpVariable(name=f"x[{i}]", lowBound=0, cat="Integer") for i in range(1, 91)] # x[i] >= 0 and integer (i = 1,2,3...90)

#Add constraints
#supply constraints
model += (x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] + x[9] + x[10] + x[11] + x[12] + x[13] + x[14] + x[15] + x[16] + x[17] + x[18] <= df['Supply'][0], "bed constrain")
model += (x[19] + x[20] + x[21] + x[22] + x[23] + x[24] + x[25] + x[26] + x[27] + x[28] + x[29] + x[30] + x[31] + x[32] + x[33] + x[34] + x[35] + x[36] <= df['Supply'][1], "normal ward")
model += (x[37] + x[38] + x[39] + x[40] + x[41] + x[42] + x[43] + x[44] + x[45] + x[46] + x[47] + x[48] + x[49] + x[50] + x[51] + x[52] + x[53] + x[54] <= df['Supply'][2], "intensive th")
model += (x[55] + x[56] + x[57] + x[58] + x[59] + x[60] + x[61] + x[62] + x[63] + x[64] + x[65] + x[66] + x[67] + x[68] + x[69] + x[70] + x[71] + x[72] <= df['Supply'][3], "ICU without ven")
model += (x[73] + x[74] + x[75] + x[76] + x[77] + x[78] + x[79] + x[80] + x[81] + x[82] + x[83] + x[84] + x[85] + x[86] + x[87] + x[88] + x[89] + x[90] <= df['Supply'][4], "ICU with ven")

# demand constraints
model += (x[1] + x[19] + x[37] + x[55] + x[73] <= df.loc['Demand',][17], "non-con/m/~65")
model += (x[2] + x[20] + x[38] + x[56] + x[74] <= df.loc['Demand',][16], "non-con/m/65~85")
model += (x[3] + x[21] + x[39] + x[57] + x[75] <= df.loc['Demand',][15], "non-con/m/85~")
model += (x[4] + x[22] + x[40] + x[58] + x[76] <= df.loc['Demand',][14], "con/m/~65")
model += (x[5] + x[23] + x[41] + x[59] + x[77] <= df.loc['Demand',][13], "con/m/65~85")
model += (x[6] + x[24] + x[42] + x[60] + x[78] <= df.loc['Demand',][12], "con/m/85~")
model += (x[7] + x[25] + x[43] + x[61] + x[79] <= df.loc['Demand',][11], "lung/m/~65")
model += (x[8] + x[26] + x[44] + x[62] + x[80] <= df.loc['Demand',][10], "lung/m/65~85")
model += (x[9] + x[27] + x[45] + x[63] + x[81] <= df.loc['Demand',][9], "lung/m/85~")
model += (x[10] + x[28] + x[46] + x[64] + x[82] <= df.loc['Demand',][8], "non-con/s/~65")
model += (x[11] + x[29] + x[47] + x[65] + x[83] <= df.loc['Demand',][7], "non-con/s/65~85")
model += (x[12] + x[30] + x[48] + x[66] + x[84] <= df.loc['Demand',][6], "non-con/s/85~")
model += (x[13] + x[31] + x[49] + x[67] + x[85] <= df.loc['Demand',][5], "con/s/~65")
model += (x[14] + x[32] + x[50] + x[68] + x[86] <= df.loc['Demand',][4], "con/s/65~85")
model += (x[15] + x[33] + x[51] + x[69] + x[87] <= df.loc['Demand',][3], "con/s/85~")
model += (x[16] + x[34] + x[52] + x[70] + x[88] <= df.loc['Demand',][2], "lung/s/~65")
model += (x[17] + x[35] + x[53] + x[71] + x[89] <= df.loc['Demand',][1], "lung/s/65~85")
model += (x[18] + x[36] + x[54] + x[72] + x[90] <= df.loc['Demand',][0], "lung/s/85~")

#Set the objective
#model += 0.3*x[11] + 2*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
#model += 300*x[11] + 2*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
#model += 3*x[11] + 200*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]
model += 3*x[11] + 2*x[12] + 300*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40]

#Solve the optimization problem
status = model.solve()
```

```
#Get the results
print(f"status: {model.status}, {LpStatus[model.status]}")
print(f"objective: {model.objective.value()}")
home = 0
for var in x.values():
    home += var.value()
    print(f"{var.name}: {var.value()}")
print(f"Stay at home: {home}")
for name, constraint in model.constraints.items():
    print(f"{name}: {constraint.value()}")

# Sensitive analysis
sr = [{}: {'Constraint Names':cname, 'Slack Values':cinfo.slack, 'Shadow Price':cinfo.pi} for cname, cinfo in model.constraints.items()] # slack values: how many supply or demand left at th
print(pd.DataFrame(sr))
```



## Sensitive analysis: add a new constrain

constrain: The number of patients with a sever condition and larger than 85 years old are less than 40.

solution: Optimal solution changed. x12 changed from 453 to 40.

```
15]: # Define the model
model = LpProblem(name="Treatments_allocation_Problem", sense=LpMaximize)

# Define the decision variables
x = {i: LpVariable(name=f"x{i}", lowBound=0, cat="Integer") for i in range(1, 91)} # x[i] >=0 and integer (i = 1,2,3...90)

#Add constraints
#supply constraints
model += (x[1] + x[2] + x[3] + x[4] + x[5] + x[6] + x[7] + x[8] + x[9] + x[10] + x[11] + x[12] + x[13] + x[14] + x[15] + x[16] + x[17] + x[18] <= df['Supply'][0], "bed constrain")
model += (x[19] + x[20] + x[21] + x[22] + x[23] + x[24] + x[25] + x[26] + x[27] + x[28] + x[29] + x[30] + x[31] + x[32] + x[33] + x[34] + x[35] + x[36] <= df['Supply'][1], "normal ward")
model += (x[37] + x[38] + x[39] + x[40] + x[41] + x[42] + x[43] + x[44] + x[45] + x[46] + x[47] + x[48] + x[49] + x[50] + x[51] + x[52] + x[53] + x[54] <= df['Supply'][2], "intensive th")
model += (x[55] + x[56] + x[57] + x[58] + x[59] + x[60] + x[61] + x[62] + x[63] + x[64] + x[65] + x[66] + x[67] + x[68] + x[69] + x[70] + x[71] + x[72] <= df['Supply'][3], "ICU without")
model += (x[73] + x[74] + x[75] + x[76] + x[77] + x[78] + x[79] + x[80] + x[81] + x[82] + x[83] + x[84] + x[85] + x[86] + x[87] + x[88] + x[89] + x[90] <= df['Supply'][4], "ICU with ven")
model += (x[12] + x[30] + x[48] + x[66] + x[84] + x[15] + x[33] + x[51] + x[69] + x[87] + x[18] + x[36] + x[54] + x[72] + x[90] <= 40) # add a constrain

# demand constraints
model += (x[1] + x[19] + x[37] + x[55] + x[73] <= df.loc['Demand',][17], "non-con/m/~65")
model += (x[2] + x[20] + x[38] + x[56] + x[74] <= df.loc['Demand',][16], "non-con/m/65~85")
model += (x[3] + x[21] + x[39] + x[57] + x[75] <= df.loc['Demand',][15], "non-con/m/85~")
model += (x[4] + x[22] + x[40] + x[58] + x[76] <= df.loc['Demand',][14], "con/m/~65")
model += (x[5] + x[23] + x[41] + x[59] + x[77] <= df.loc['Demand',][13], "con/m/65~85")
model += (x[6] + x[24] + x[42] + x[60] + x[78] <= df.loc['Demand',][12], "con/m/85~")
model += (x[7] + x[25] + x[43] + x[61] + x[79] <= df.loc['Demand',][11], "lung/m/~65")
model += (x[8] + x[26] + x[44] + x[62] + x[80] <= df.loc['Demand',][10], "lung/m/65~85")
model += (x[9] + x[27] + x[45] + x[63] + x[81] <= df.loc['Demand',][9], "lung/m/85~")
model += (x[10] + x[28] + x[46] + x[64] + x[82] <= df.loc['Demand',][8], "non-con/s/~65")
model += (x[11] + x[29] + x[47] + x[65] + x[83] <= df.loc['Demand',][7], "non-con/s/65~85")
model += (x[12] + x[30] + x[48] + x[66] + x[84] <= df.loc['Demand',][6], "non-con/s/85~")
model += (x[13] + x[31] + x[49] + x[67] + x[85] <= df.loc['Demand',][5], "con/s/~65")
model += (x[14] + x[32] + x[50] + x[68] + x[86] <= df.loc['Demand',][4], "con/s/65~85")
model += (x[15] + x[33] + x[51] + x[69] + x[87] <= df.loc['Demand',][3], "con/s/85~")
model += (x[16] + x[34] + x[52] + x[70] + x[88] <= df.loc['Demand',][2], "lung/s/~65")
model += (x[17] + x[35] + x[53] + x[71] + x[89] <= df.loc['Demand',][1], "lung/s/65~85")
model += (x[18] + x[36] + x[54] + x[72] + x[90] <= df.loc['Demand',][0], "lung/s/85~")

#Set the objective
model += 3*x[11] + 2*x[12] + 3*x[13] + 2*x[14] + x[15] + 2*x[16] + x[17] + 3*x[24] + 3*x[26] + 3*x[27] + 3*x[28] + 2*x[29] + 2*x[31] + x[32] + x[34] + x[37] + x[38] + 3*x[39] + x[40] +

#Solve the optimization problem
status = model.solve()
```

```
#Get the results
print(f"status: {model.status}, {LpStatus[model.status]}")
print(f"objective: {model.objective.value()}")
home = 0
for var in x.values():
    home += var.value()
    print(f"{var.name}: {var.value()}")
print(f"Stay at home:{home}")
for name, constraint in model.constraints.items():
    print(f"{name}: {constraint.value()}")

# Sensitive analysis
sr = [{"Constraint Names":cname, 'Slack Values':cinfo.slack, 'Shadow Price':cinfo.pi} for cname, cinfo in model.constraints.items()] # slack values: how many supply or demand Left at th
print(pd.DataFrame(sr))
```

## Assignment Problem

An imbalanced assignment problem has been constructed based on hospital resources and number of different types of patients. Each block (e.g.  $X_{11}, X_{12}$ , etc.) of the **table 1** shown below represents the number of each treatment (e.g. Bed, Normal Ward, etc.) available to be assigned to each type of patient (e.g. A1-A18). Index Definition has shown in the **table 2**

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	Supply
Bed	$X_{11}$	$X_{12}$	$X_{13}$	$X_{14}$	$X_{15}$	$X_{16}$	$X_{17}$	$X_{18}$	$X_{19}$	$X_{110}$	$X_{111}$	$X_{112}$	$X_{113}$	$X_{114}$	$X_{115}$	$X_{116}$	$X_{117}$	$X_{118}$	2465
Normal Ward	$X_{21}$	$X_{22}$	$X_{23}$	$X_{24}$	$X_{25}$	$X_{26}$	$X_{27}$	$X_{28}$	$X_{29}$	$X_{210}$	$X_{211}$	$X_{212}$	$X_{213}$	$X_{214}$	$X_{215}$	$X_{216}$	$X_{217}$	$X_{218}$	616
Intensive Ward	$X_{31}$	$X_{32}$	$X_{33}$	$X_{34}$	$X_{35}$	$X_{36}$	$X_{37}$	$X_{38}$	$X_{39}$	$X_{310}$	$X_{311}$	$X_{312}$	$X_{313}$	$X_{314}$	$X_{315}$	$X_{316}$	$X_{317}$	$X_{318}$	205
ICU (No Ventilator)	$X_{41}$	$X_{42}$	$X_{43}$	$X_{44}$	$X_{45}$	$X_{46}$	$X_{47}$	$X_{48}$	$X_{49}$	$X_{410}$	$X_{411}$	$X_{412}$	$X_{413}$	$X_{414}$	$X_{415}$	$X_{416}$	$X_{417}$	$X_{418}$	130
ICU (Ventilator)	$X_{51}$	$X_{52}$	$X_{53}$	$X_{54}$	$X_{55}$	$X_{56}$	$X_{57}$	$X_{58}$	$X_{59}$	$X_{510}$	$X_{511}$	$X_{512}$	$X_{513}$	$X_{514}$	$X_{515}$	$X_{516}$	$X_{517}$	$X_{518}$	67
Demand	5	29	113	11	69	276	49	315	1254	5	29	453	11	69	1104	49	315	7840	

Table 1

INDEX	CONDITION/ PRE-EXISTING CONDITION	AGE
A1	Severe condition with pre-existing lung diseases	85 and older
A2		85 to 65
A3		65 and under
A4	Severe condition with other pre-existing conditions	85 and older
A5		85 to 65
A6		65 and under
A7	Severe condition without other pre-existing conditions	85 and older
A8		85 to 65
A9		65 and under
A10	Mild condition with pre-existing lung diseases	85 and older
A11		85 to 65
A12		65 and under
A13	Mild condition with other pre-existing conditions	85 and older
A14		85 to 65
A15		65 and under
A16	Mild condition without other pre-existing conditions	85 and older
A17		85 to 65
A18		65 and under

Table 2

The treatment plan model would fulfill the demand from the group of patients who has the highest priority level (3) to the lowest priority level (0). The algorithm of the model tends to fulfill the demand of the group with highest priority level and then fulfill the next level within each treatment type. For example, in the first treatment plan, the model seeks to allocate the available beds to the group of patients with priority level 3 until the demand has meet. If there are more supply available, the model would then allocate the rest of the beds to the group with next highest priority level until the demand has been fulfilled or the supply has been used up. The model would utilize the available supply to fulfill the demand from each group of patients. The priority table is shown below

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	Supply
<b>Bed</b>	0	0	0	0	0	0	0	0	0	0	3	2	3	2	1	2	1	0	2465
<b>Normal Ward</b>	0	0	0	0	0	3	0	3	3	3	2	0	2	1	0	1	0	0	616
<b>Intensive Ward</b>	1	1	3	1	3	2	3	2	1	2	0	0	1	0	0	0	0	0	205
<b>ICU (No Ventilator)</b>	2	3	2	2	2	0	2	1	0	0	0	0	0	0	0	0	0	0	130
<b>ICU (Ventilator)</b>	3	2	1	3	1	0	1	0	0	0	0	0	0	0	0	0	0	0	67
<b>Demand</b>	5	29	113	11	69	276	49	315	1254	5	29	453	11	69	1104	49	315	7840	

Table 3

## Result and discussion

Under given supply and demand, the optimal priority level of 5406 has been obtained. Every group of patients has been assigned to at least 1 treatment plan. Hospital supply such as NORMAL WARD, INTENSIVE WARD, ICU(NO VENTILATOR) has been fully utilized. Demand for each type of patients has been met with exception of group A9(Severe condition without other pre-existing conditions). Only 95 normal ward has been assigned to group A9 and excess demand is 1159 patients. The complete result table is shown in table 4. And result from python can be found in figure 1.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	Supply
Bed											29	453	11	69	1104	49	315		2465
Normal Ward						276		245	95										616
Intensive Ward			113				17	70		5									205
ICU (No Ventilator)		29			69		32												130
ICU (Ventilator)	5			11															67
Home Isolation									1159									7840	
Demand	5	29	113	11	69	276	49	315	1254	5	29	453	11	69	1104	49	315	7840	

Table 4

---

```

status: 1, Optimal
objective: 8673.0
x1: 0.0
x2: 0.0
x3: 0.0
x4: 0.0
x5: 0.0
x6: 0.0
x7: 0.0
x8: 0.0
x9: 0.0
x10: 0.0
x11: 29.0
x12: 453.0
x13: 11.0
x14: 69.0
x15: 1104.0
x16: 49.0
x17: 315.0
x18: 0.0
x19: 0.0
x20: 0.0
x21: 0.0
x22: 0.0
x23: 0.0
x24: 276.0
x25: 0.0
x26: 245.0
x27: 95.0
x28: 0.0
x29: 0.0
x30: 0.0
x31: 0.0
x32: 0.0
x33: 0.0
x34: 0.0
x35: 0.0
x36: 0.0
x37: 0.0
x38: 0.0
x39: 113.0
x40: 0.0
x41: 0.0
x42: 0.0
x43: 17.0
x44: 70.0
x45: 0.0
x46: 5.0
x47: 0.0
x48: 0.0
x49: 0.0
x50: 0.0
x51: 0.0
x52: 0.0
x53: 0.0
x54: 0.0
x55: 0.0
x56: 29.0
x57: 0.0
x58: 0.0
x59: 69.0
x60: 0.0
x61: 32.0
x62: 0.0
x63: 0.0
x64: 0.0
x65: 0.0
x66: 0.0
x67: 0.0
x68: 0.0
x69: 0.0
x70: 0.0
x71: 0.0
x72: 0.0
x73: 5.0
x74: 0.0
x75: 0.0
x76: 11.0
x77: 0.0
x78: 0.0
x79: 0.0
x80: 0.0
x81: 0.0
x82: 0.0
x83: 0.0
x84: 0.0
x85: 0.0
x86: 0.0
x87: 0.0
x88: 0.0
x89: 0.0
x90: 0.0
Stay at home:8999.0
bed_constrain: -435.0
normal_ward_constrain: 0.0
intensive_therapy_ward_constrain: 0.0
ICU_without_ventilator_constrain: 0.0

```

---

Figure 1

## Bed Allocation

Patient groups with priority level of 0 indicate these group does not need the corresponding treatment. Patient groups with priority levels greater than 0 ( $P_{1j}=1,2,3$ ) have all been assigned to bed treatment. It is due to the sufficient supply of the hospital beds. The demand of group of patient that must receive a bed in the hospital has been met. However, there are still 435 beds available for allocation. Calculation is shown blow:

$$\text{Total Bed Supply} = 2465$$

$$\begin{aligned} &\text{Total beds that allocated to patient groups} \\ &= 29 + 453 + 11 + 69 + 1104 + 49 + 315 = 2030 \end{aligned}$$

$$\text{Total bed supply left} = 2465 - 2030 = 435$$

## Normal Ward

Normal wards haven been allocated to groups that have higher risk. Total supply of 616 normal wards have been distributed to group A6, A8 and A9 until the supply has been depleted. There are total of four patient groups have a priority level of 3 (A6, A8,A9,A10), two patient groups have a priority level of 2 (A11, A13)and two patient groups have a priority level of 1 (A14, A16). However, the demand of only 3 of the four groups that has the highest priority level has been fulfilled. Group A10 does not receive any NORMAL WARD supply due to shortage. The supply of 616 normal wards has prioritized the demand of A6, A8 and A9 and is depleted after allocation to these three groups. As a result, 276 normal wards are assigned to A6. 245 normal wards are assigned to A8 and 95 normal wards are assigned to A9.

$$\text{Total Normal Ward Supply} = 616$$

$$\text{Total Normal Wards that allocated to patient groups} = 276 + 245 + 95 = 616$$

$$\text{Total Normal Wards supply left} = 0$$

## Intensive Ward

Intensive ward is designed to take care of patients that are in severe condition. Total supply of 205 units have been distribution to group A3, A7 A8 and A10 until the supply has been depleted. There are total of three patient group have a priority level of 3, but only two of them were assigned. Intensive ward supply fulfilled the demand of group A3, A7 but skip A5 which has the same priority level as A3 and A7. A8 and A10 are the groups with priority level 2 that have been fulfilled their demands. No supply left after distributing the rest of the unit to A8 and A10. 113 units are assigned to A3. 17 units are assigned to A7. 70 units are assigned to A8 and 5 units are assigned to A10.

$$\text{Total Intensive Ward Supply} = 205$$

$$\text{Total Intensive Wards that allocated to patient groups} = 113 + 17 + 70 + 5 = 205$$

$$\text{Total Intensive Wards supply left} = 0$$

### **ICU without Ventilator Unit**

ICU is designed to take care of patients with severe or life-threatening illnesses of Covid-19. ICU is divided into two categories due to the limitation of ventilator units. There are total of 130 units of ICU that are not equipped with ventilators. These units would be distributed to type of patients that need life supporting equipment but not necessarily need ventilators. ICU that are not equipped with ventilators are allocated to A2, A5 and A7. The supply is used up after distributing to the groups with higher priority levels (3 and 2). 29 units are assigned to group A2. 69 units are assigned to group A5 and the rest 32 units are assigned to A7.

$$\text{Total ICU (No Ventilator) Supply} = 130$$

$$\text{Total ICU (No Ventilator) that allocated to patient groups} = 29 + 69 + 32 = 130$$

$$\text{Total ICU (No Ventilator) supply left} = 0$$

### **ICU with Ventilator Unit**

There are total of 67 units of ICU that are equipped with ventilators. These units would be distributed to type of patients that need life supporting equipment, as well as intubation through ventilators to help them breath. ICU that are equipped with ventilators are allocated to A1 and A4. The supply is not used up after distribution. Only patient groups with priority level 3 have received and reserved for the treatment with ICU and ventilators. Other groups with lower priority level would not receive ICU and ventilator treatment is because the demand has already fulfilled. In other word, different treatment plans have been assigned to these groups. ICUs that are equipped with ventilator are only allocated to patients that are in life-threatening condition, which matches the patient group condition A1 and A4. Both groups consist of severe patients that are 85 and older and have dangerous pre-existing conditions.

$$\text{Total ICU (Ventilator) Supply} = 67$$

$$\text{Total ICU (Ventilator) that allocated to patient groups} = 5 + 11 = 16$$

$$\text{Total ICU (Ventilator) supply left} = 67 - 16 = 51$$

The excess supply of ICU ventilator can be reserved for patients who may have medical emergencies or for patients whose illness conditions are aggravated.

### **Home Isolation**

Patients from group A18 are considered the type that does not need to utilize the hospital resources. Patients from this groups are under age of 65 , without any pre-existing condition and only have mild symptom. They are likely to heal by themselves and are not necessarily need to be hospitalized. Therefore, patients from this group are recommended to isolate at home by themselves for 14 days. In group A9, only 95 patients received normal ward treatment. The rest 1159 are allocated for home isolation due to supply shortage. These 1159 patients are in severe condition, which means normal wards are needed for treatment due to the highly contagious nature

of Covid-19. However, normal wards supply has been depleted by other groups. These patients need to wait until further notice from the hospital based on capacity availability.

### **Special Cases: A5 and A10**

A5 and A10 are two special cases according to the result generated. These two patient groups did not receive the corresponding treatments that were labeled as priority level of 3. Instead, they received the treatments that only has a priority level of 2. In group A5, the highest priority is to receive intensive wards for the 69 patients in this group. However, these 69 patients are allocated with an even higher level of treatment: ICU (No ventilator). Similarly, 5 patients in group A10 received one treatment level higher than demanded. This exception happens because the model seeks to find the maximum value of priority level. Therefore, this combination yields the greatest outcomes.

### **Limitation**

The resources of the hospital has not been fully utilized. Based on this priority rule, in the category of A9, only 95 of 1254 patients have received hospitalization treatment. The rest 1159 patients were sent home for isolation. However, some facilities have not been depleted. Based on the result, there are still bed and ICU with ventilator available that can be allocated to patients within this group.

As mention in previous section, the model built was designed to find the maximum sum of the priority level while satisfying the demand and supply constraints. In other hand, in order to solve this limitation, a better relationship between demand and number of treatments should be constructed. However, due to the algorithm's limitation of Python Linear Programming Library used, it is unlikely achievable with the use of in-built library.



## Conclusion

There is a constant need for COVID-19 infected patients to be provided with the necessary treatment for the patients to fully recover. Fundamentals of Operations Research were applied to build a treatment plan model to optimize in terms of maximizing the number of patients that can be given the required treatment plan without violating the constraints by allocating essential hospital equipment to them. The team has conducted a study to determine the veracity of such a phenomenon.

It is observed from the assignment model that the priority level which has been obtained from calculations is 5406. The model worked out well as we were able to assign every single group of COVID-19 infected patients to a minimum, one treatment plan. Analyzing the demands for each group of patients, the only group where the demand was not met was group A9, which had an additional demand of 1159 patients that could not receive necessary treatment.

Furthermore, results indicate that every single hospital equipment supply was utilized, however, only the ICU with a ventilator unit and the beds were in excess supply, which means that every other medical equipment came out to be either completely used up or in shortage. The priority level of a patient group was taken into account when determining the quantity of medical equipment to be assigned to a patient.

Programming languages, AMPL and Python were used to aid this project, with finding the mathematical values when optimizing the medical equipment allocated to the COVID-19 infected patients in Boston.

## Future Scope and Improvement

With the constant rise of the COVID-19 infections, the pandemic is exponentially worsening. This trend suggests a strong need for medical facilities to give to the people. In this study, we solely focus on the people of Boston.

The findings relating to the medical equipment supply concluded that there is a strong need for more medical equipment to treat the COVID-19 infected patients in Boston. This result was deduced by analyzing the number of equipment that was in excess supply, which turned out to be only 2 groups of equipment, the remaining was all used or in shortage. To successfully treat more patients in the area of Boston, the hospitals would need to accommodate more medical equipment such as normal wards, intensive wards, and ICU with ventilator units.

The limitation of the study was the unfair allocation of medical equipment to the patients in Boston. Because a priority rule was implemented that categorized each patient group with a priority level, many patients had to be sent to their homes for self-isolation. Another issue was that the ICU with ventilators and bed equipment were in excess supply, along with the patients being sent home proved to be a major limitation. Because many patients could be allocated to the excess equipment and could have received the necessary treatment. However, due to the limitations of the model, it was not possible.

Due to these findings, there are several recommendations for improvements in the future extension of this study. For one, expanding the priority rule may yield better and more accurate results by dynamically calculating the threshold of supply for each group of people accurately. The priority rule currently stands at a range from 0 to 3. If the number of bins here increases to more than 5 priorities, the allocation can work in a much more accurate manner.

The study can also be extended by creating more categories in terms of age, the specific pre-existing medical condition, and the location of the patients. Because the actual treatment of COVID-19 requires much more complex equipment, these may also be expanded by naming more specific equipment and forming more treatment plans. The current study focuses on the people of Boston, the same study can be extended to account for people from all over the state, Massachusetts.

The implementation of this model on a similar scale may result in some interesting findings. A major challenge one might face when building this model is the efficient allocation of medical equipment to all groups, findings suggested that a group was left out of medical equipment allocation due to the shortage in equipment, this problem arises due to the priority rule defined in the model. The careful understanding and extension of this priority rule are essential.

The dataset chosen to work on this model must be following the demand and supply for medical equipment and patients weekly, hence, similar scaling must be applied to the dataset. Otherwise, incorrect results may be given.

## Reference

“Boston, MA Demographic Statistics.” Infoplease.

*[www.infoplease.com/us/census/massachusetts/boston/demographic-statistics](http://www.infoplease.com/us/census/massachusetts/boston/demographic-statistics).*

“Brigham and Women's Hospital”

*<https://www.brighamandwomens.org/>*

“Beth Israel Deaconess Medical Center”

*<https://www.bidmc.org/>*

“COVID-19 Response Reporting”

*<https://www.mass.gov/info-details/covid-19-response-reporting>*

“Estimated Prevalence and Incidence of Lung Disease” lung.org

*<https://www.lung.org/research/trends-in-lung-disease/prevalence-incidence-lung-disease>*

“Heart Disease”, CDC.org

*<https://www.cdc.gov/nchs/fastats/heart-disease.htm>*

“Massachusetts General Hospital”

*[https://en.wikipedia.org/wiki/Massachusetts\\_General\\_Hospital](https://en.wikipedia.org/wiki/Massachusetts_General_Hospital)*

“National Diabetes Statistics Report, 2020”, CDC.org

*<https://www.cdc.gov/diabetes/data/statistics-report/index.html>*