

Two dimensional real discrete chaotic attractors

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1 Introduction

Showcase of a selection of two dimensional real discrete chaotic attractors.

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2 Attractors

2.1 Clifford attractor

$$\begin{aligned}x_{n+1} &= \sin(ay_n) + c \cos(ax_n) \\ y_{n+1} &= \sin(by_n) + d \cos(bx_n)\end{aligned}$$

Image parameters:

$$a = 2, b = -2, c = 1, d = -1$$

2.2 Peter de Jong attractor

$$\begin{aligned}x_{n+1} &= \sin(ay_n) - \cos(bx_n) \\ y_{n+1} &= \sin(cx_n) - \cos(dy_n)\end{aligned}$$

Image parameters:

$$a = 0.97, b = -1.899, c = 1.381, d = -1.506$$

2.3 Tinkerbell attractor

$$\begin{aligned}x_{n+1} &= x_n^2 - y_n^2 + ax_n + by_n \\ y_{n+1} &= 2x_ny_n + cx_n + dy_n\end{aligned}$$

Image parameters:

$$a = 0.9, b = -0.6013, c = 2, d = 0.5$$

2.4 Johnny Svensson attractor

$$\begin{aligned}x_{n+1} &= d \sin(ax_n) - \sin(by_n) \\ y_{n+1} &= c \cos(ax_n) + \cos(by_n)\end{aligned}$$

Image parameters:

$$a = 1.4, b = -1.56, c = 1.4, d = -6.56$$

2.5 Gumowski-Mira attractor

$$\begin{aligned}f(x) &= ax + \frac{2(1-a)x^2}{(1+x^2)^2} \\ x_{n+1} &= by_n + f(x_n) \\ y_{n+1} &= f(x_{n+1}) - x_n\end{aligned}$$

Image parameters:

$$a = -0.192, b = 0.982$$

2.6 Fractal Dreams (SSSS) attractor

$$\begin{aligned}x_{n+1} &= \sin(y_n b) + c \sin(x_n b) \\ y_{n+1} &= \sin(x_n a) + d \sin(y_n a)\end{aligned}$$

Image parameters:

$$a = 1.468, b = 2.407, c = 0.194, d = 1.438$$

2.7 Quadratic Strange attractor

$$\begin{aligned}x_{n+1} &= a_0 + a_1 x_n + a_2 x_n^2 + a_3 x_n y_n + a_4 y_n + a_5 y_n^2 \\ y_{n+1} &= a_6 + a_7 x_n + a_8 x_n^2 + a_9 x_n y_n + a_{10} y_n + a_{11} y_n^2\end{aligned}$$

Image parameters:

$$CVQKKGHQTTPHTE$$

2.8 Bogdanov attractor

$$\begin{aligned}x_{n+1} &= x_n + y_{n+1} \\ y_{n+1} &= y_n + \epsilon y_n + k x_n (x_n - 1) + \mu x_n y_n\end{aligned}$$

Image parameters:

$$\epsilon = 0, k = 1.2, \mu = 0$$

2.9 Gingerbread attractor

$$\begin{aligned}x_{n+1} &= 1 - y_n + |x_n| \\ y_{n+1} &= x_n\end{aligned}$$

2.10 Duffing attractor

$$\begin{aligned}x_{n+1} &= y_n \\ y_{n+1} &= -b x_n + a y_n - y_n^3\end{aligned}$$

Image parameters:

$$a = 2.5, b = 1$$

2.11 Hénon attractor

$$\begin{aligned}x_{n+1} &= 1 - ax_n^2 + y_n \\ y_{n+1} &= bx_n\end{aligned}$$

Image parameters:

$$a = 1.2, b = 0.5$$

2.12 Ikeda attractor

$$\begin{aligned}t_n &= 0.4 - \frac{6}{1 + x_n^2 + y_n^2} \\ x_{n+1} &= 1 + u(x_n \cos t_n - y_n \sin t_n) \\ y_{n+1} &= u(x_n \sin t_n + y_n \cos t_n)\end{aligned}$$

Image parameters:

$$u = 0.918$$

2.13 Standard attractor

Values of p and θ were replaced by x and y respectively.

$$\begin{aligned}p_{n+1} &= [p_n + K \sin(\theta_n)] \mod 2\pi \\ \theta_{n+1} &= [\theta_n + p_{n+1}] \mod 2\pi\end{aligned}$$

Image parameters:

$$u = 0.918$$

2.14 Zaslavskii attractor

$$\begin{aligned}\mu &= \frac{1 - e^{-r}}{r} \\ x_{n+1} &= [x_n + \nu(1 + \mu y_n) + \epsilon \nu \mu \cos(2\pi x_n)] \mod 1 \\ y_{n+1} &= e^{-r}(y_n + \epsilon \cos(2\pi x_n))\end{aligned}$$

Image parameters:

$$\epsilon = 1.5, \nu = 0.8, r = 1.2$$

3 References

- <https://seuelaencollection.home.blog/2d-chaotic-attractors/>
- <https://blbadger.github.io/>
- https://en.wikipedia.org/wiki/List_of_chaotic_maps

Clifford Attractor

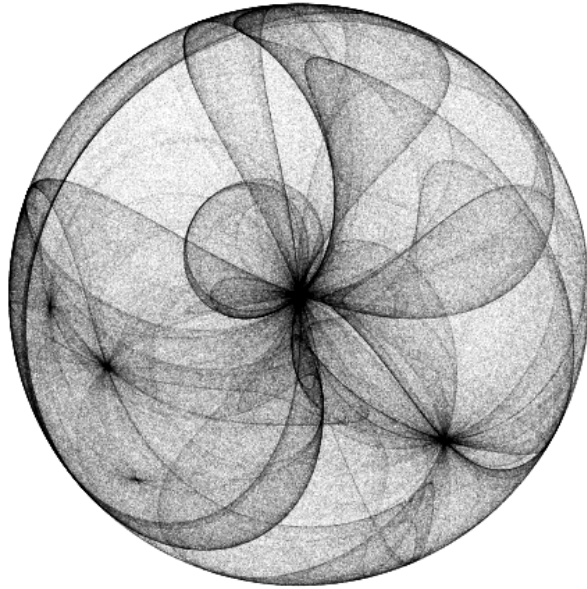


Figure 1: Clifford attractor

Peter de Jong Attractor

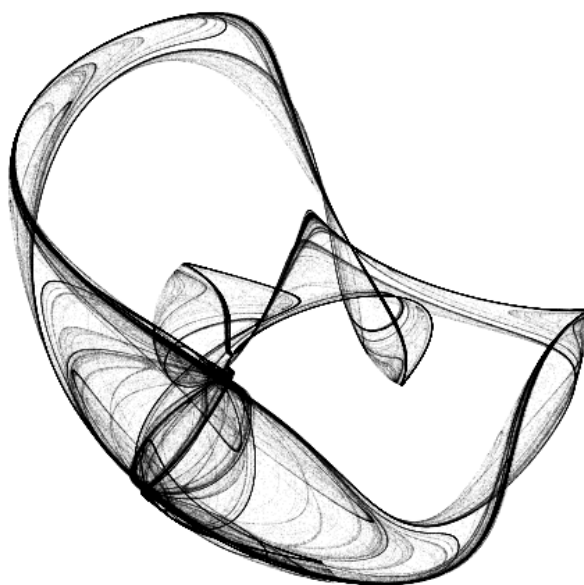


Figure 2: Peter de Jong attractor

Tinkerbell Attractor

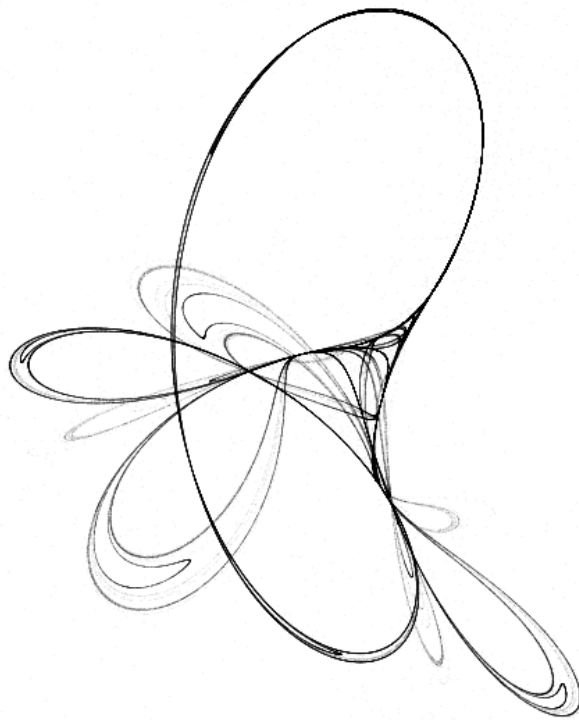


Figure 3: Tinkerbell attractor

Johnny Svensson Attractor

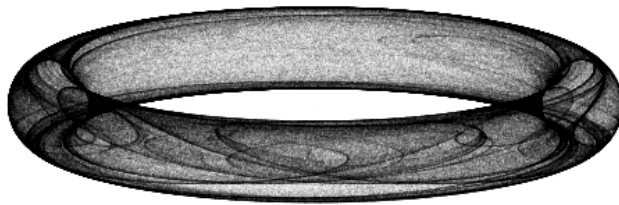


Figure 4: Johnny Svensson attractor

Gumowski-Mira Attractor

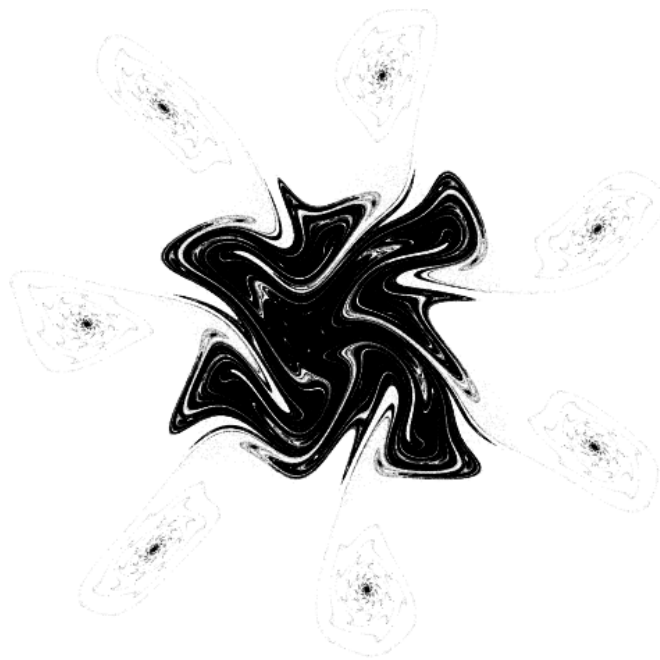


Figure 5: Gumowski-Mira attractor

Fractal Dreams (SSSS) Attractor

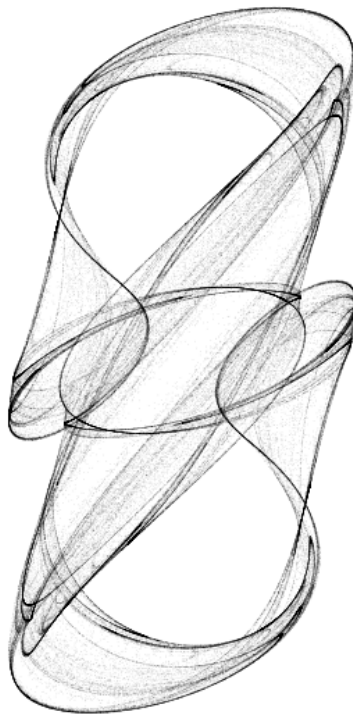


Figure 6: Fractal Dreams (SSSS) attractor

Quadratic Strange Attractor

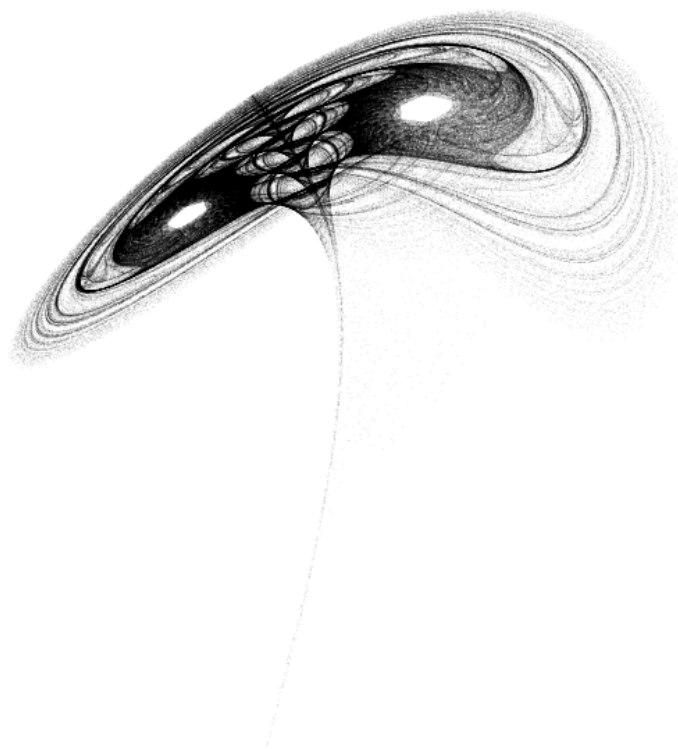


Figure 7: Quadratic Strange attractor

Bogdanov Attractor

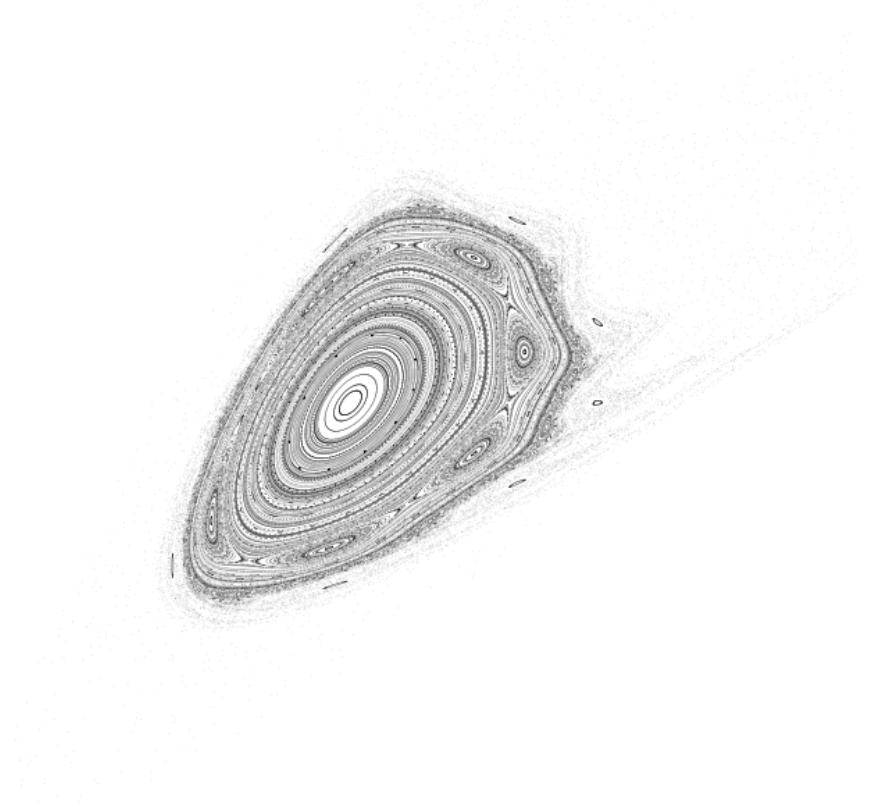


Figure 8: Bogdanov attractor

Gingerbreadman Attractor

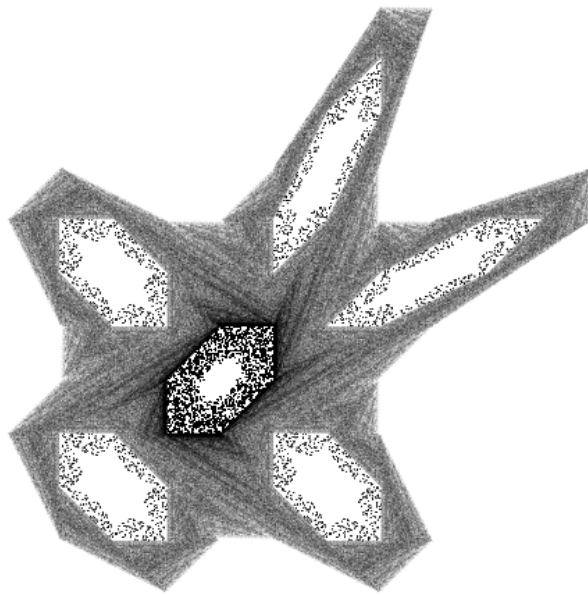


Figure 9: Gingerbreadman attractor

Duffing Attractor

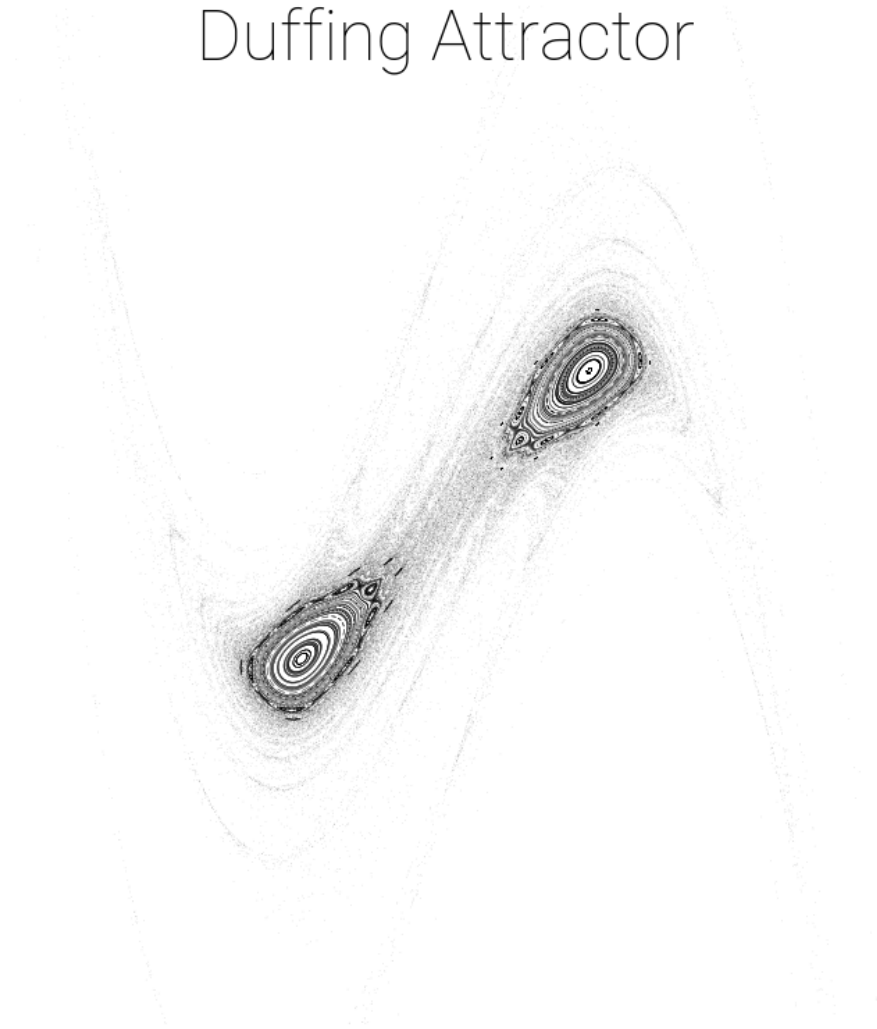


Figure 10: Duffing attractor

Hénon Attractor

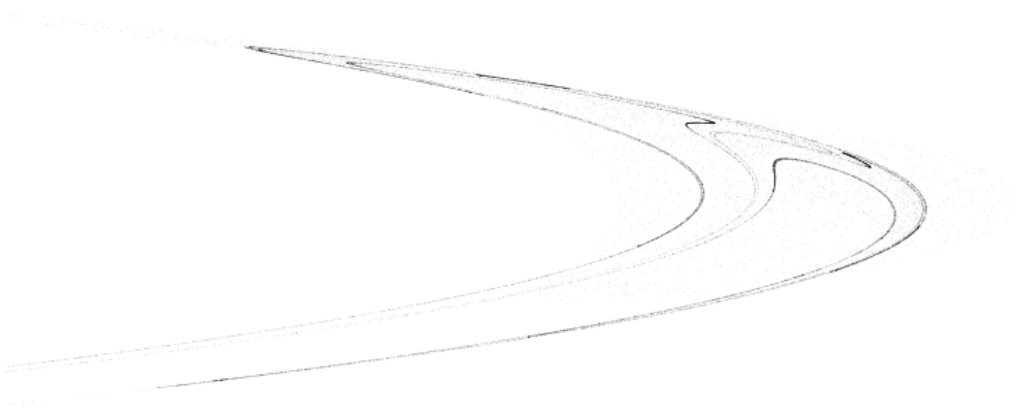


Figure 11: Hénon attractor

Ikeda Attractor

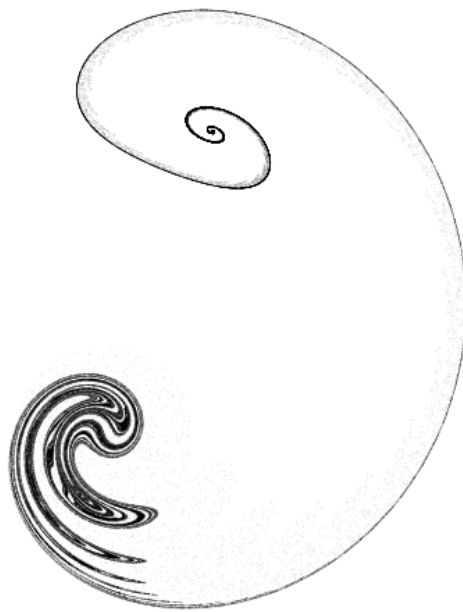


Figure 12: Ikeda attractor

Standard Attractor

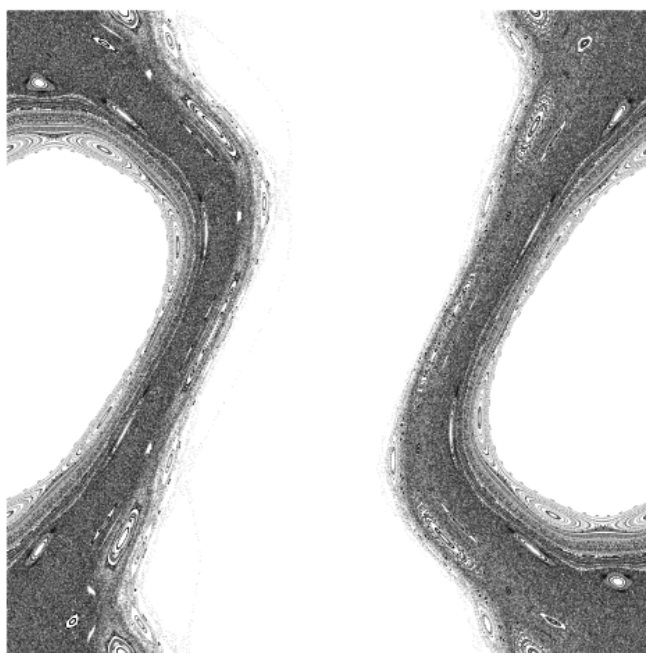


Figure 13: Standard attractor

Zaslavskii Attractor



Figure 14: Zaslavskii attractor