Visualizations of iterative root finding algorithms in the complex plane

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Showcase of a selection of visualizations of iterative root finding algorithms in the complex plane.

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1 Explanation

The images are colored by the number of iterations it takes the algorithm to converge on the root of the equation within some tolerance (for the following images we used a value of 10^{-15}) for a given starting point. If the algorithm exceeds 100 iterations, the pixel is colored white. Less iterations means that the pixel is darker.

2 Algorithms

2.1 Newton's method

$$z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}$$

2.2 Halley's method

$$z_{n+1} = z_n - \frac{2f(z_n)f'(z_n)}{2(f'(z_n))^2 - f(z_n)f''(z_n)}$$

2.3 Stefenssen's method

$$z_{n+1} = z_n - \frac{f(z_n)}{\frac{f(z_n+f(z_n))}{f(z_n)} - 1}$$

3 References

- https://en.wikipedia.org/wiki/Root-finding_algorithms
- https://blbadger.github.io

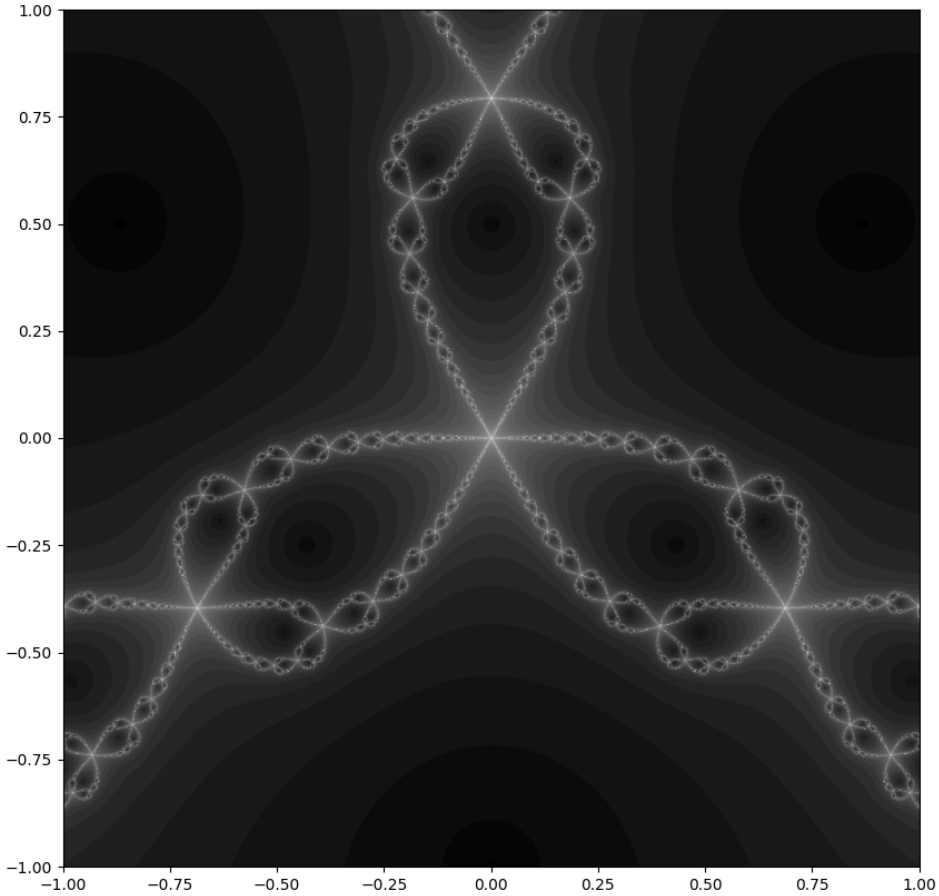

Figure 1: Newton's method for $z^3 - 1$

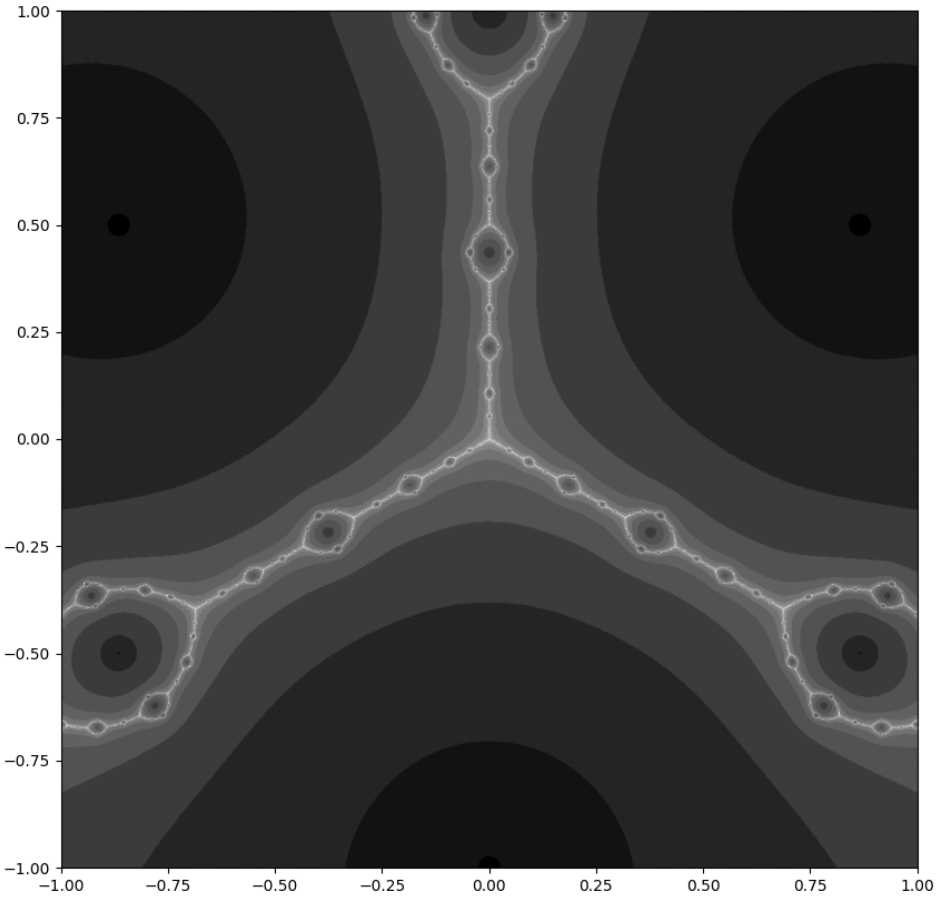

Figure 2: Halley's method for $z^3 - 1$

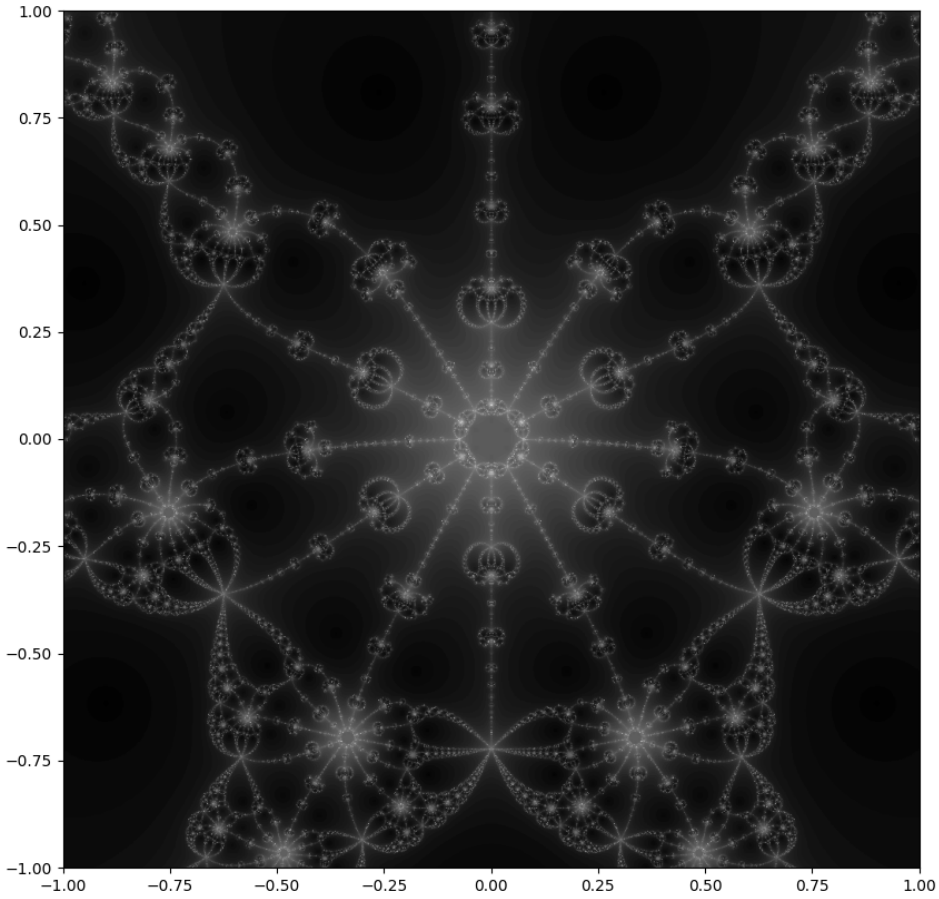

Figure 3: Newton's method for $z^7 - z - 1$

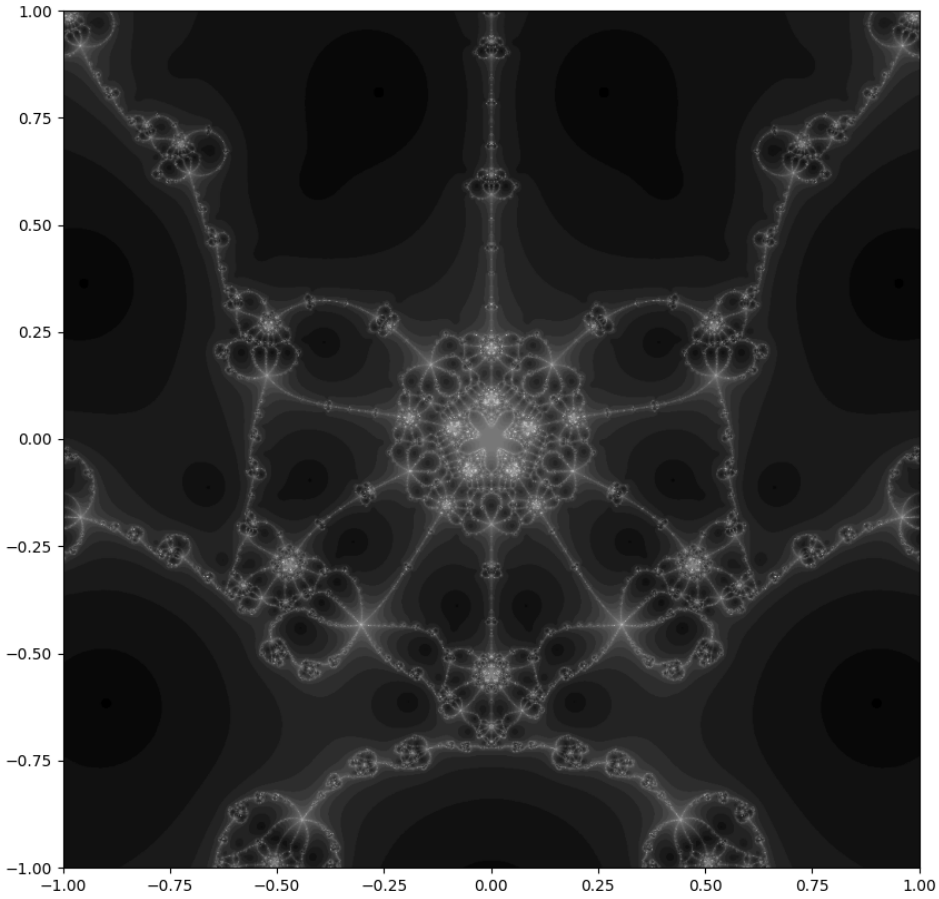

Figure 4: Halley's method for $z^7 - z - 1$

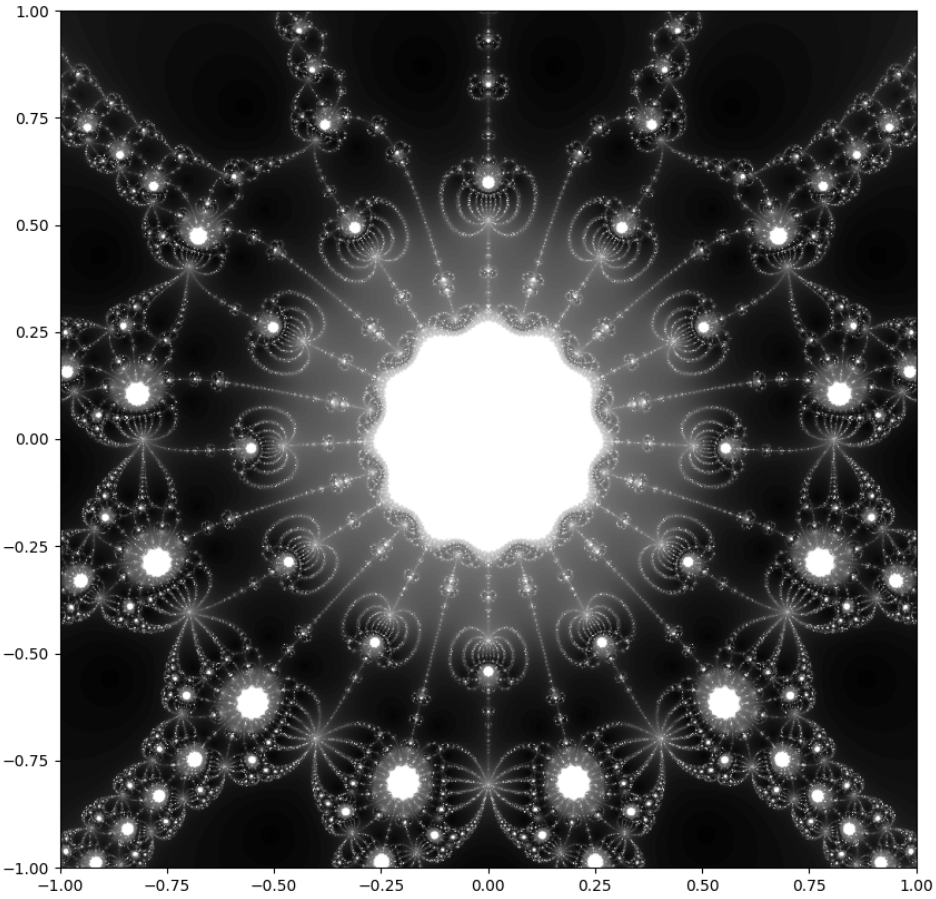

Figure 5: Newton's method for $z^{13} - z - 1$

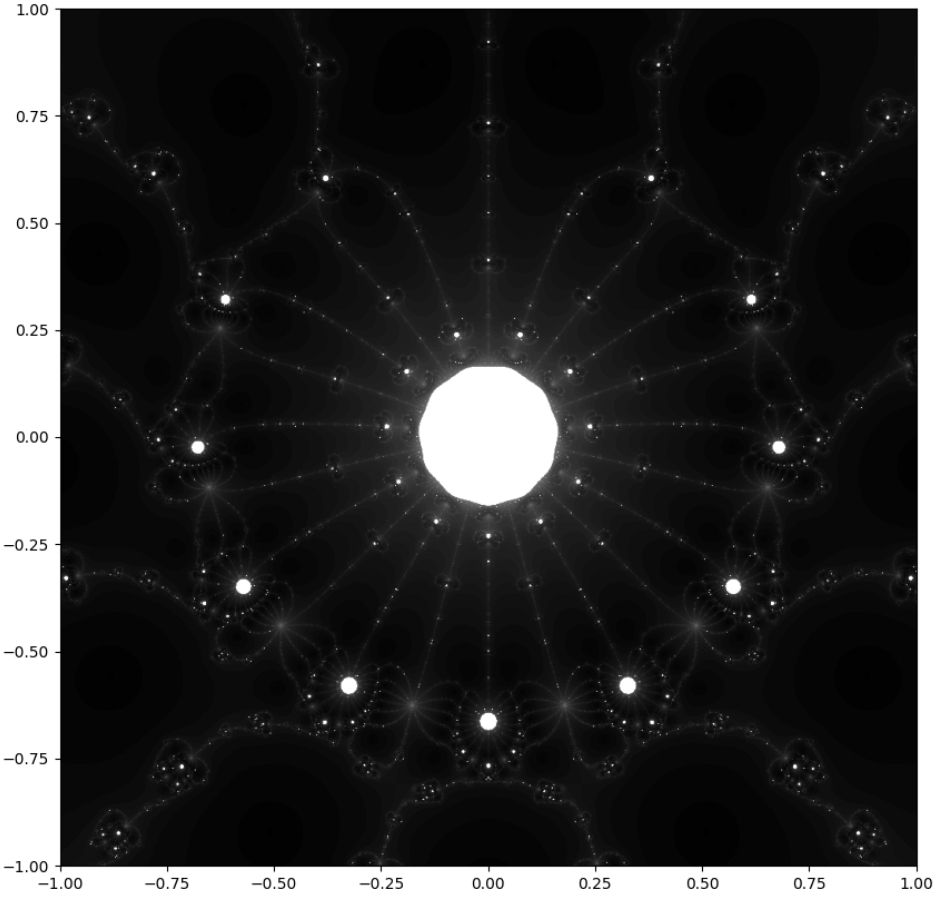

Figure 6: Halley's method for $z^{13} - z - 1$

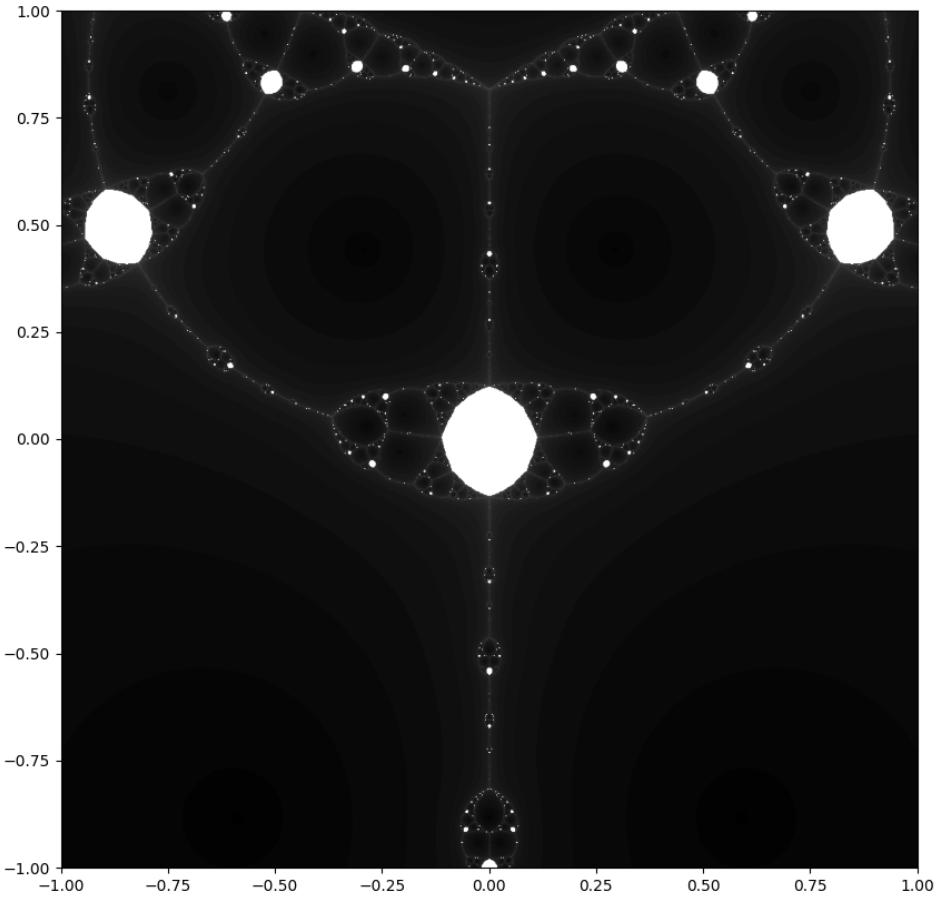

Figure 7: Newton's method for $z^3 - 2z + 2$

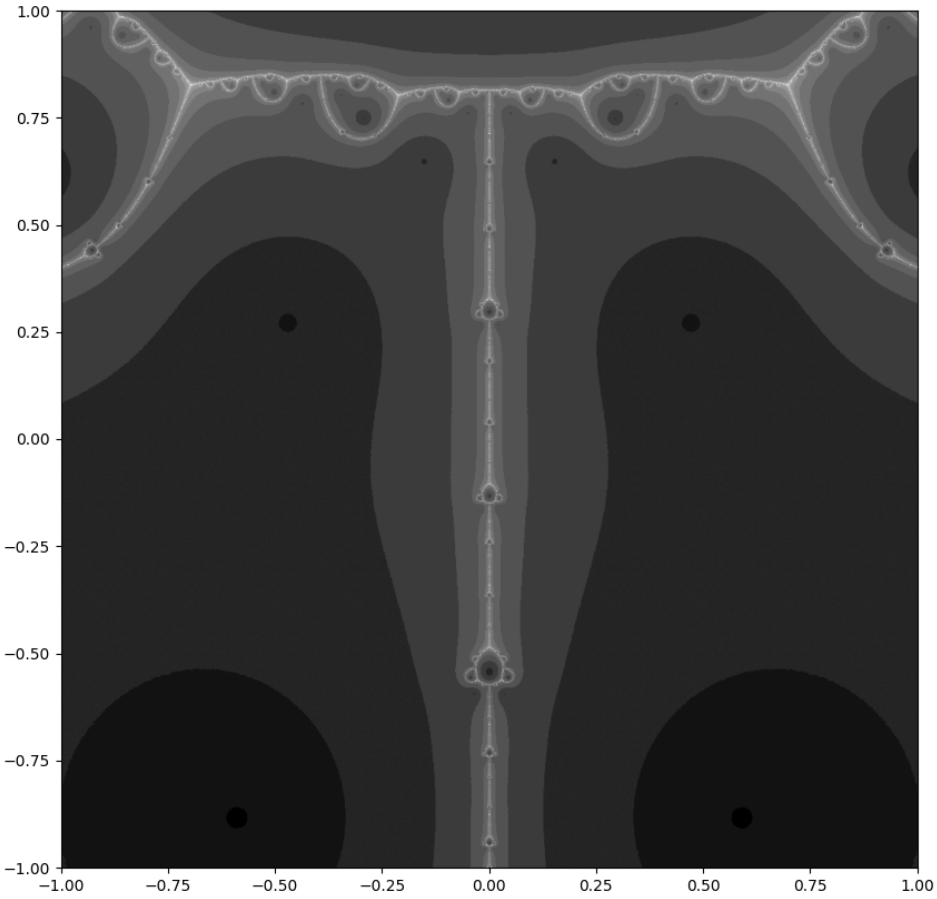

Figure 8: Halley's method for $z^3 - 2z + 2$

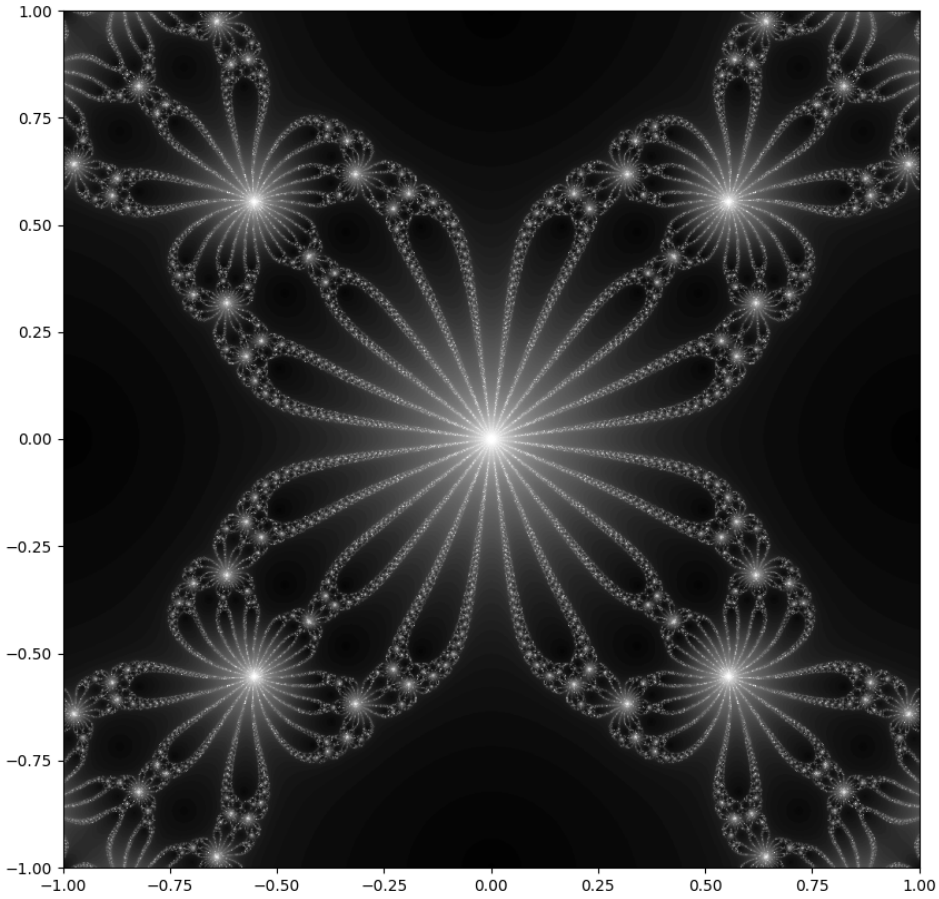

Figure 9: Newton's method for $z^8 + 15z^4 - 16$

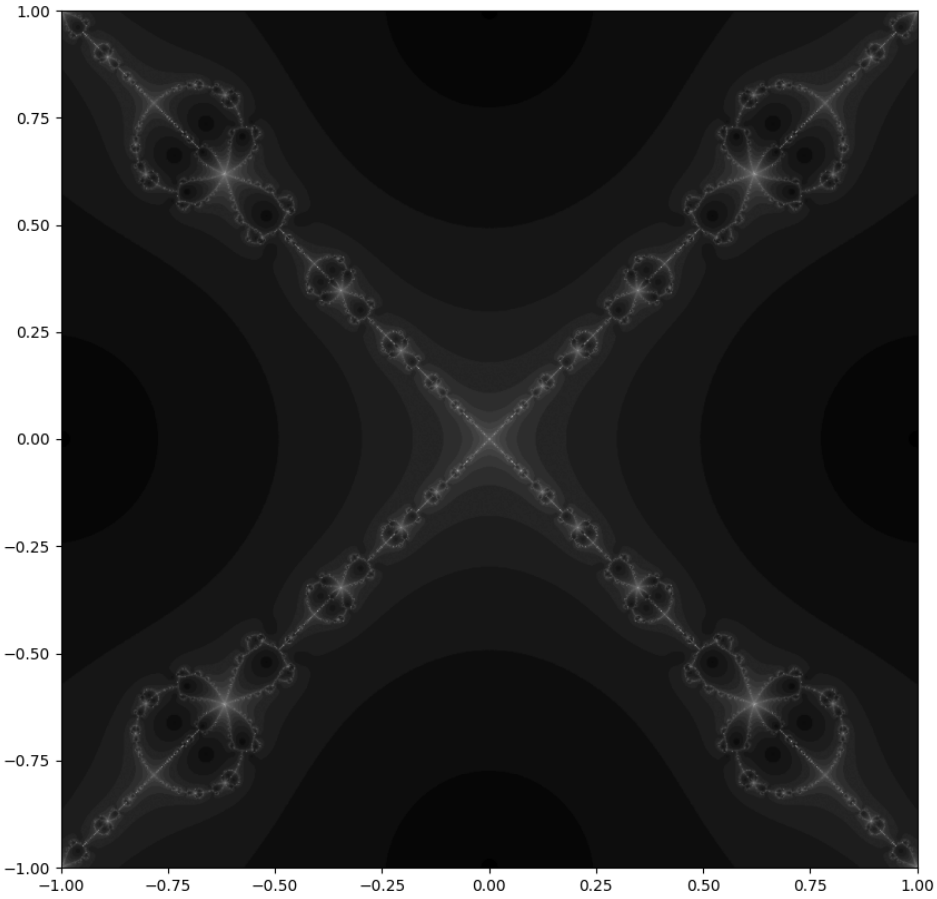

Figure 10: Halley's method for $z^8 + 15z^4 - 16$

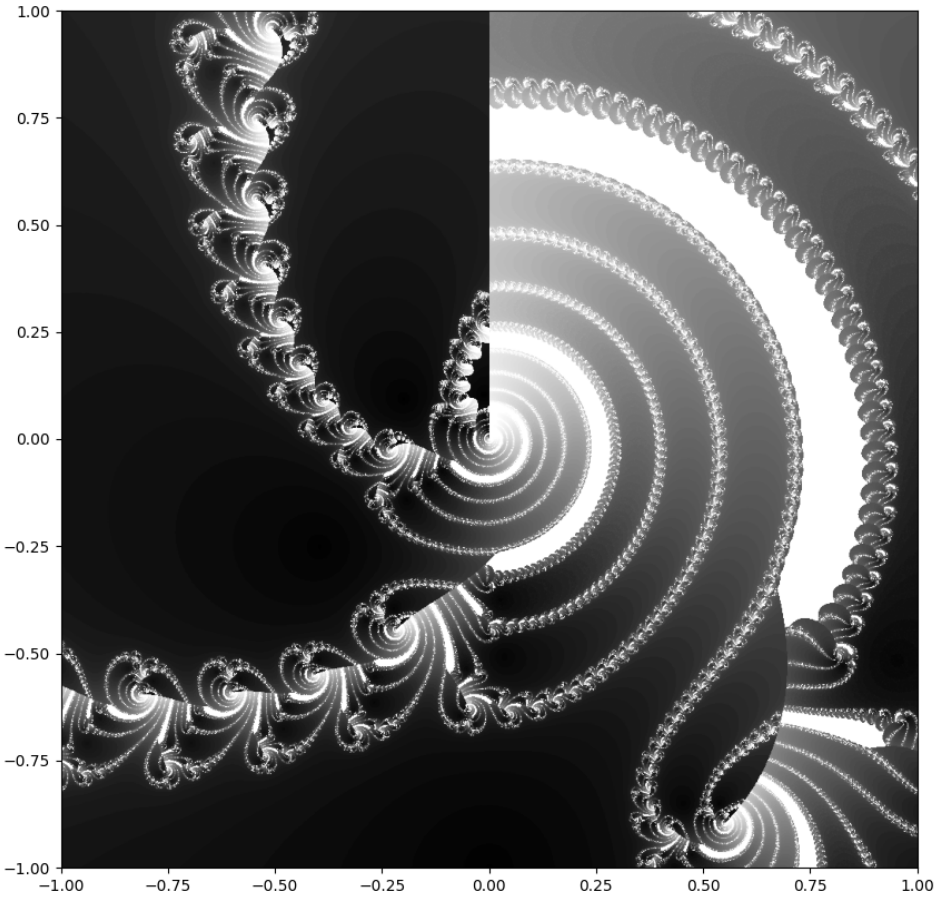

Figure 11: Newton's method for $z^{4+3i} - 1$

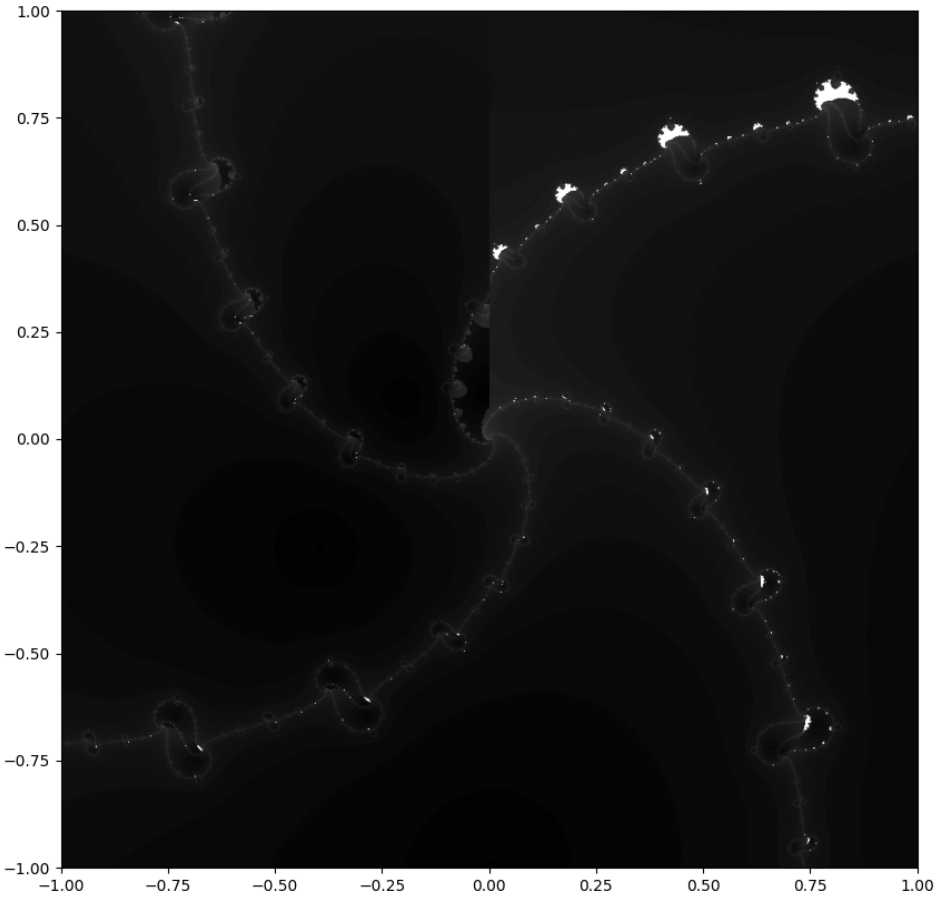

Figure 12: Halley's method for $z^{4+3i} - 1$