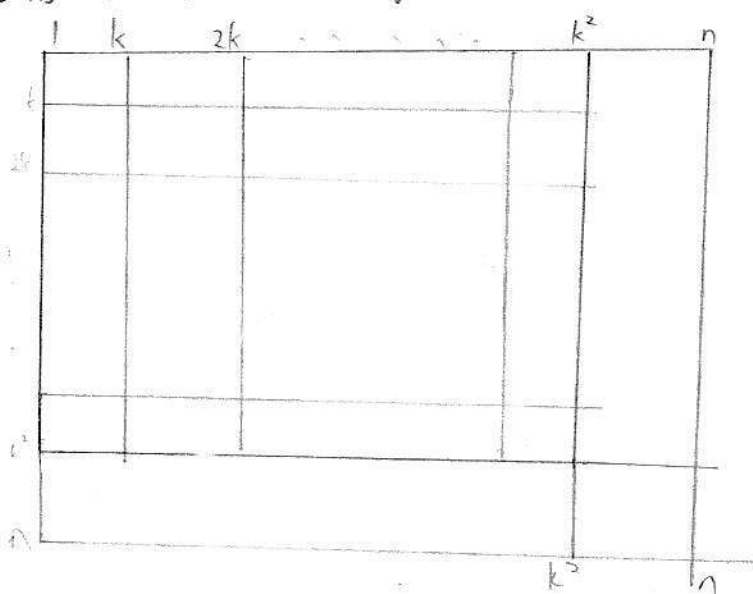


Answer: $\lceil \sqrt{n} \rceil - 1$

First, we show that $k = \lceil \sqrt{n} \rceil - 1$ fits. Suppose that there exists such peaceful configuration, where this k fails. Knowing that $n > k^2$, consider the following:



Now, consider the k^2 squares with ~~diagonal~~ ~~vertices~~ diagonals

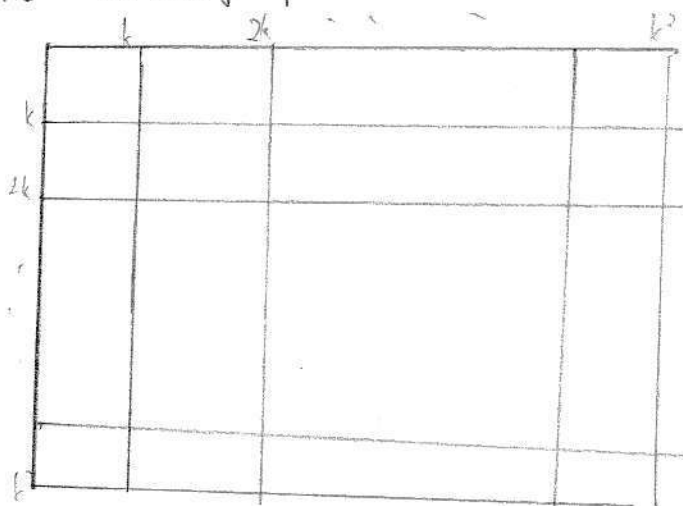
$[(a-1)k+1, (b-1)k+1]$ and (ak, bk) , $\forall 1 \leq a, b \leq k$. Since every such square has at least a rook, we know that this $k^2 \times k^2$ square contains at least k^2 rooks. These k^2 rooks all have distinct coordinates, and in the $k^2 \times k^2$ chessboard, hence for each column $1, 2, \dots, k^2$ and each row $1, 2, \dots, k^2$ there is exactly a rook. This means, the remaining $n^2 - k^2$ rooks have ~~row~~ row and column number greater ^{vertices} than k^2 , and there are $n - k^2$ rooks in the square with diagonals at $(k^2+1, k^2+1), (n, n)$.

However, ~~#~~ by ~~rotating~~ rotation, and consider k^2 squares with diagonals $[(a-1)k+1+(n-k^2), (b-1)k+1]$ and $(ak+(n-k^2), bk)$ for $k \leq a, b \leq k$ we know that the square with diagonals $(1, k^2+1), (n-k^2, n)$ has $n-k^2$ rooks too. For $k \geq 2$ (i.e. $n \geq 5$), we know that $n \leq 2k^2$ as $n \leq (k+1)^2 - 1 = k^2 + 2k$, so the ^{two} squares, ~~each~~ with diagonals $(k^2+1, k^2+1), (n, n)$, and $(1, k^2+1), (n-k^2, n)$ are disjoint, and we get that there are ~~two~~ $2(n-k^2)$ rooks in row k^2+1, k^2+2, \dots, n , which is impossible. (For $k=1$, it is obvious that some square doesn't contain a rook).

This implies that any k greater than this fails.

Now, we show that $k = \lfloor \sqrt{n} \rfloor$ fails. It suffices to find one such configuration. Let us start with $n = k^2$.

Consider the following partition:



such coordinate:

$(x-1)k + a, (y-1)k + b$

$xk + a, (y-1)k + b$

(x_k, y_k)

$(x-1)k + a, yk + b$

$xk + a, yk + b$

boundary

boundary

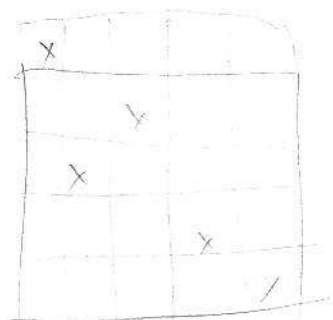
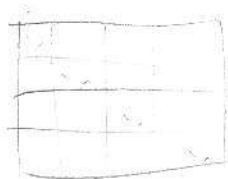
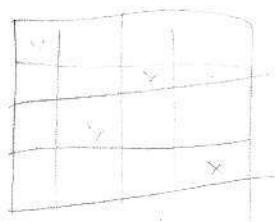
$(xk+a, yk+b)$

Since each ~~at~~ of $(\overset{(n-1)k}{\cancel{x(k-1)}}+y, \overset{(y-1)k}{\cancel{y(k-1)}}+x), (\overset{xk}{\cancel{(k-1)(k-1)}}+y, \overset{(y-1)k}{\cancel{y(k-1)}}+x+1),$
 $((n-1)k+y+1, yk+x), (xk+y+1, yk+x+1)$ have a rook, ~~if~~ ^{we have:} ~~all~~ of
~~if~~ (1) \rightarrow if ~~$a+1 \leq y$~~ $a \leq y, b \leq x$, then $((n-1)k+y, (y-1)k+x)$ is in
the square.

- (2) if $a \leq y, b \leq x$, then $(x+y, (y-1)k+x+1)$ is in the square.
 (3) if $a \leq y, b \geq x$, then $((x-1)k+y+1, y+k+x)$ is in the square.
 (4) if $a \geq y, b \leq x$, then $(xk+y+1, yk+x+1)$ is in the square.

Since any combination of $[a, b]$ falls into one of the four cases above, any $k \times k$ square contains a rook.

Finally, for $n < k^2$, consider the $n \times n$ subgrid with diagonals $(1,1)$ described above and delete all squares and rooks not in the $n \times n$ board, and (n,n) in the $k^2 \times k^2$ board and ~~remove~~ the ~~rest~~ this grid has all its rooks in different columns and rows. Moreover, ~~each~~ ^{each} $k \times k$ square ~~is~~ contains a rook. If there is any rook "missing" we can assign it arbitrarily and the condition still holds.
 QED



2-1
3-1
4-1
5-2?

