We will prove a stronger result. For any positive integer n there exists a partition of finite collection of coins with value at most $n-\frac{1}{2}$ into at most n groups, with each group having value at most 1. The problem ~~condition~~ asks for $n=100$.

The base case $n=1$ is clear as a group of coins has value at most $\frac{1}{2}$.

Inductive step. Suppose for some $n \geq 2$, the above claim is true for $n-1$ ~~piles~~ groups of total value at most $n-\frac{3}{2}$. Now, we ~~will~~ have a collection of coins of value at most $n-\frac{1}{2}$, and we are to partition it into at most n piles.

We can safely assume that no combination of coins add up to exactly 1. Indeed, if this is the case, put ~~it~~ these coins into a single pile, and we have another collection of coins of value no more than $n-\frac{3}{2}$. By induction hypothesis this forms $n-1$ groups of coins with value at most 1, and we are done. With this assumption, ~~let us look into this lemma:~~ we split into two cases:

~~Lemma: It is safe to assume that the~~

Case 1: ~~There is at most one coin in the form $\frac{1}{k}$, s.t. $k \geq 2n$.~~
The coins of value at most $\frac{1}{2n+1}$ made up of ~~at most half of~~ value at most $\frac{1}{2}$.

Let us do ~~severe~~ "regroup" of coins, one step at a time:
"For a integer k, ~~let~~ p be prime divisor of k, ~~then~~ and a_k the ~~p and~~ ~~di~~ number of coins of value ~~a_k~~ $\frac{1}{k}$ with $a_k \geq p$. Then we 'coagulate' p coins into one coin of value $\frac{1}{\left(\frac{k}{p}\right)}$."

This process can be done only finitely many times, since at each step the number of coin strictly decreases. ~~After~~ Observe that the number of coins with value at most $\frac{1}{2n+1}$ cannot increase either, so this ~~assu~~ assumption holds true after the process. (The total value of coins, each of at most $\frac{1}{2n+1}$, cannot increase).

At the end of the process where it cannot be done anymore, if a_m represents ~~m~~ the number of coins with ~~at most~~ value $\frac{1}{m}$ then a_m is strictly less than ~~p~~ any divisor of m, so $a_m < m$. Moreover, for m even, $a_m \leq 1$, and we already assumed that $a_1 = 0$. (otherwise there is a pile of coins with value exactly one).

Thus, since ~~each~~ the coin of at most $\frac{1}{2n+1}$ made up of value at most $\frac{1}{2}$, we can group it into group 1 consisting of them and a_2 coins of $\frac{1}{2}$ (so $a_2 \cdot \frac{1}{2} \leq \frac{1}{2}$ as $a_2 \leq 1$). Also, for group m ($\cancel{2} \leq m \leq n$) we ~~to~~ group a_{2m-1} coins of $\frac{1}{2m-1}$ and a_{2m} coins of $\frac{1}{2m}$, so $\frac{a_{2m-1}}{2m-1} + \frac{a_{2m}}{2m}$
$\leq \frac{2m-2}{2m-1} + \frac{1}{2m} = 1 - \frac{1}{2m-1} + \frac{1}{2m} < 1$. This gives legal partition of n groups of coins.

Case 2: The ~~a~~ ~~t~~ coins of value at most $\frac{1}{2n+1}$ made up of value more than $\frac{1}{2}$ ~~of~~ ~~the~~ ~~the~~

Now, using suitable algorithm, Our goal is to create a pile of coins of value more than 1, s.t. ~~tota~~ it contains at least a coin of value of at most $\frac{1}{2n+1}$, ~~Denote this~~ but removal of this coin yields a pile of value ~~of~~ less than 1 (by assumption, equality doesn't hold in both situations).

If ~~this pile~~ the pile of coins of at most $\frac{1}{2n+1}$ (denote the set of coins as S) made up of value at least 1, we take a coin from S, one by one, until the first time the value exceeds 1. Denote the last coin ~~as~~ with value $\frac{1}{c}$, $c \geq 2n+1$.

(denote it as T)

Otherwise, the pile of coins with value at least $\frac{1}{2n}$ has value at least $n-\frac{3}{2}$, moreover no coin has value 1 (by induction assumption), so ~~the value~~ each coin has value at most $\frac{1}{2}$. Let us take ~~the~~ coins from T, one by one, until the first time the sum of value of coin exceeds $\frac{1}{2}$. Since, before this last step, ~~the previously~~ we have total coin ~~value~~ value of at most $\frac{1}{2}$, and ~~now~~ the last coin has value at most $\frac{1}{2}$, the resulting pile has value at most 1.

Finally, with this pile of value more than $\frac{1}{2}$ and at most 1, and knowing at S has value more than $\frac{1}{2}$, we can add coin(s) from S one by one, and stop immediately after the total value exceeds 1. Also denote the last coin with value $\frac{1}{c}$.

Contestant MAS 1

Problem 5

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Now, the pile P has value more than 1, but value of $P \setminus \{\text{coin with value } \frac{1}{c}\}$ is less than 1, so the other one has at most $n-\frac{3}{2}$. By induction hypothesis we can split it into $n-1$ groups with each ~~value~~ group of value at most 1. Since the average value is at most $\frac{n-\frac{3}{2}}{n-1} = \frac{2n-3}{2n-2}$, there is a group with total value at most $\frac{2n-3}{2n-2}$, and we can insert the coin of ~~wet~~ value $\frac{1}{c}$ from P into it since $\frac{2n-3}{2n-2} + \frac{1}{c} \leq 1 - \frac{1}{2n-2} + \frac{1}{2n-1} < 1$. These $n-1$ groups are still legal.

Finally, after bring $\frac{1}{c}$ into the other group, by definition, now pile P has value less than 1. We are done, since this P, with the $n-1$ groups described above, all have value at most 1.

$2 \ \ldots \ n+1$

$2n$

$\frac{3}{4} + \frac{2}{3} = \frac{17}{12}$

$\frac{1}{2}$?

$\frac{1}{4}$