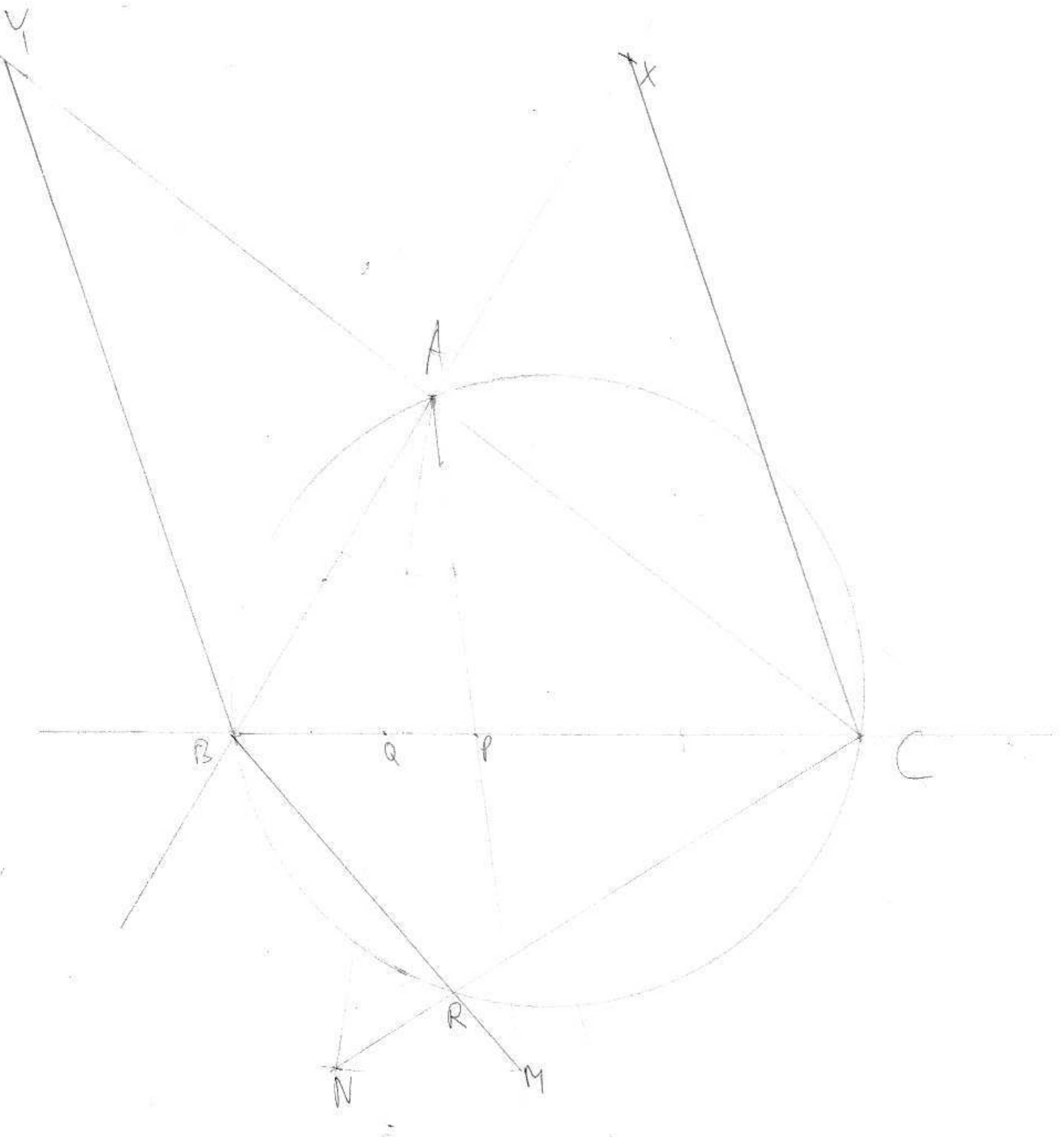
Let BM and CN to intersect at R. We show that $\angle ABR + \angle ACR = 180°$.

~~From~~ Define also X and Y as: X = reflection of point B in point A, Y = reflection of point C in point A.

We claim that $\angle BCX = \angle ACN$. As $\angle CAQ = \angle ABC$, $\angle ACQ = \angle ACB$, we have $\angle AQC = \angle BAC$. ~~Moreover~~ Hence, $\triangle ACQ \sim \triangle BCA$, and $\frac{QC}{QA} = \frac{\cancel{AQ}\,AC}{AB}$. However, we have $NQ = QA$, and $AX = AB$, and $\angle CQN = \angle CAX = 180° - \angle AQC = 180° - \angle BAC$. Therefore, $\frac{QC}{QA} = \frac{QC}{NQ} = \frac{\cancel{AQ}\,AC}{AB} = \frac{\cancel{AQ}\,AC}{AX}$. $\frac{QC}{NQ} = \frac{AC}{AX}$ and $\angle CQN = \angle CAX$ implies $\triangle CAX \sim \triangle CQN$, hence $\angle ACX = \angle QCN$, ~~with~~ so $\angle BCX = \angle BCA + \angle ACX = \angle BCA + \angle QCN = \angle NCA$. Similarly, $\angle ABM = \angle CBY$.

Now, as A is midpoint of BX and CY, XYBC is a parallelogram, and $BY \parallel CX$, so $\angle ABR + \angle ACR = \angle ABM + \angle ACN = \angle CBY + \angle BCX = 180°$.

QED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