MATH 135: Extra Practice Set 9
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Question 1. *Find all* $z \in \mathbb{C}$ *which satisfy*

(a) $z^2 + 2z + 1 = 0$,

(b) $z^2 + 2\bar{z} + 1 = 0$,

(c) $z^2 = \dfrac{1+i}{1-i}$.

Question 2. *(a) Find all* $w \in \mathbb{C}$ *satisfying* $w^2 = -15 + 8i$. *(b) Find all* $w \in \mathbb{C}$ *satisfying* $z^2 - (3+2i)z + 5 + i = 0$.

Question 3. *Let* $z, w \in \mathbb{C}$. *Prove that if* $zw = 0$ *then* $z = 0$ *or* $w = 0$.

Question 4. *Let* $a, b, c \in \mathbb{C}$. *Prove: if* $|a| = |b| = |c| = 1$, *then* $\overline{a+b+c} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{c}$.

Question 5. *Find all* $z \in \mathbb{C}$ *satisfying* $z^2 = |z|^2$.

Question 6. *Find all* $z \in \mathbb{C}$ *satisfying* $|z+1|^2 \equiv 3$ *and shade the corresponding region in the complex plane.*

Question 7. *Prove that if* $|z| = 1$ *and* $\bar{z}w \neq 1$, *then* $\left|\dfrac{z-w}{1-\bar{z}w}\right| = 1$.

Question 8. *Show that* $|\operatorname{Re}(z)| + |\operatorname{Im}(z)| \equiv \sqrt{2}|z|$.

Question 9. *Prove that* $\forall z, w \in \mathbb{C}$, $|z-w|^2+|z+w|^2 = 2(|z|^2+|w|^2)$ *(This is the Parallelogram Identity).*

Question 10. *Use De Moivre's Theorem (DMT) to prove that* $sin(4\theta) = 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$.

Question 11. *Let* $n \in \mathbb{N}$ *and* $a, b \in \mathbb{R}$*. Show that* $z = (a+bi)^n + (a-bi)^n$ *is real.*