MATH 135: Extra Practice Set 5
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Question 1. *(a) Use the Extended Euclidean Algorithm to find three integers x, y and $d = gcd(1112, 768)$ such that $1112x + 768y = d$. (b) Determine integers s and t such that $768s - 1112t = gcd(768, -1112)$.*

Question 2. *Prove that for all $a \in \mathbb{Z}$, $gcd(9a + 4, 2a + 1) = 1$.*

Question 3. *Let $gcd(x, y) = d$. Express $gcd(18x + 3y, 3x)$ in terms of d and prove that you are correct.*

Question 4. *Prove that if $gcd(a, b) = 1$, then $gcd(2a + b, a + 2b) \in \{1, 3\}$.*

Question 5. *Prove that for every integer k, $gcd(a, b) \leq gcd(ak, b)$.*

Question 6. *Given a rational number r, prove that there exist coprime integers p and q, with $q \neq 0$, so that $r = \frac{p}{q}$.*

Question 7. *Prove that: if $a \mid c$ and $b \mid c$ and $gcd(a, b) = 1$, then $ab \mid c$.*

Question 8. *Let $a, b, c \in \mathbb{Z}$. Prove that if $gcd(a, b) = 1$ and $c \mid a$, then $gcd(b, c) = 1$.*

Question 9. *Prove that if $gcd(a, b) = 1$, then $gcd(a^m, b^n) = 1$ for all $m, n \in \mathbb{N}$. You may use the result of an example in the notes.*

Question 10. *Suppose a, b and n are integers. Prove that $n \mid gcd(a, n) \cdot gcd(b, n)$ if and only if $n \mid ab$.*