Earthquake Relocation Algorithm

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Contents

1 Introduction

This document contains a description of the Source Specific Station Term (SSST) earthquake relocation algorithm written for hiperseis. This earthquake relocation algorithm fits into the phase picking workflow for performing seismic tomography. The aim is to obtain good hypocentres for earthquakes, and then to match picks on temporary networks (such as the AusArray OA network) to these events so that an inversion of the outputs can be performed, and various tomographic products can be produced.

2 Source Specific Station Term (SSST) relocation algorithm

The algorithm which was developed to perform relocation of events using SSST time corrections is designed to improve the quality of hypocentres by taking into account systematic disagreements between empirical and predicted travel times of seismic waves for event clusters. Consider, for example, the situation depicted in figure 1.

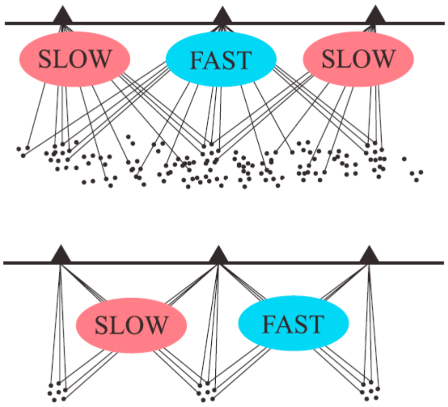


Figure 1: Systematic errors in travel time residuals for picks may be caused by (top) heterogeneity beneath a receiver, in which case all picks will exhibit a similar systematic residual, and, (bottom) heterogeneity along the path between source and receiver, in which case clusters of events will exhibit similar systematic residuals in their picks. Figure adapted from Lin and Shearer (2005).

In figure 1 (top), we see a situation in which all picks recorded by a station will have a similar travel time residual if the events have known hypocentres. In the case where all waves pass through the 'slow' region, for example, every pick will have a positive travel time residual. In such a situation, a Static Station Term (SST) may be applied to all picks recorded by this station. Subtracting the median residual for a phase from the arrival time of each pick will result in a lower L1 error metric for relocation, or subtracting the mean residual will decrease a L2 error metric.

In figure 1 (bottom), we see a situation where picks will have differing travel time residuals based on the path they take, but with events occurring in the same cluster having similar travel time residuals. In this case, a spatially varying travel time correction may be applied to the arrival time of picks. If, for example, we subtract the median (or mean) travel time residual for picks originating in the same cluster from their arrival times, we should see a decrease in the L1 (or L2) error metrics for clustered events when performing relocation. The corrections in this case are known as Source Specific Station Terms (SSST).

2.1 Procedure

A high level overview of the Source Specific Station Term relocation algorithm contains the following steps. Let n be the number of iterations of the algorithm to perform.

1. Compute travel time residuals and redefine phases for picks with respect to current hypocentres.
2. Compute SST travel time corrections for all picks.
3. Relocate events using SST corrections.
4. Find hypocentres which move further than a threshold distance during relocation. Picks corresponding to these events will not have their residuals used to compute the SSST corrections for other picks.
5. For n repetitions, perform the following.
   (a) Compute travel time residuals and redefine phases for picks with respect to current hypocentres.
   (b) Compute SSST travel time corrections for all picks.
   (c) Relocate events using SSST corrections.
   (d) Find hypocentres which move further than a threshold distance during relocation. Add these hypocentres to the list in step (4) above. Picks corresponding to these events will not have their residuals used to compute the SSST corrections for other picks.
6. Compute travel time residuals for all picks with respect to final hypocentres.

During the relocation procedure, only picks from permanent stations are to be used, as events on temporary networks such as the AusArray OA network have not been reviewed by an analyst. Excluding a list of temporary networks in section C.2.2 will result in their picks not being used for relocation purposes, but their phases will still be redefined and residuals computed.

The following sections describe in further detail the methods used during the SSST relocation procedure.

2.2 Phase redefinition and travel time prediction

The phase redefinition and travel time prediction procedure has the following steps. Consider a pick with reported phase 'ph' and travel time t. In the configuration file we specify a list of likely phases that a wave may arrive as (e.g. phase_list = [P, Pg, Pn, Pb, S, Sg, Sn, Sb]).

1. Determine the type of phase (e.g. P or S) of 'ph'.
2. For each phase of the corresponding type in phase_list, compute the predicted travel time, and travel time residuals.
3. Determine the minimum residual and its corresponding phase.
4. If this residual is less than a user defined threshold (e.g. 5 seconds), redefine the pick to be of the corresponding phase. Go to step (6).
5. If this residual is too large (or undefined), compute the predicted travel time for the wave using all phases for which a travel time table is available and repeat steps 3-4. If residuals are all still too large (or undefined), go to step (6).
6. If a match has been found, output the minimum residual and its corresponding phase. Otherwise return residual ∞ and phase 'X'.

Using this method, we avoid redefining P to S (and vice versa). We avoid redefining phases to more exotic types such as core reflections etc. if a close enough match was found in our input phase_list.

Travel time predictions are made by interpolating a travel time value from a pre-computed table of values (see appendix B.1), which is then corrected using an elevation correction (appendix B.2) and ellipticity correction (appendix B.3).

2.3 SST corrections

SST corrections are easy to calculate. The following method is used.

- For all recording stations, perform the following.
    1. Determine the phases of each arrival at the station.
    2. For each unique phase in the phase list for the station (e.g. 'Pg'), perform the following.
        (a) If there are fewer than a user-defined threshold number of picks of the corresponding phase (e.g. 100), return a travel time correction of 0.
        (b) If there are a sufficient number of picks of the corresponding phase, compute the median travel time residual for the picks. Set the time correction for each of these picks to this median travel time residual.

The user defined minimum number of picks is used to ensure corrections are meaningful. If, for example, there are very few picks, then there may not be enough data to deduce that a systematic travel time residual is occurring based on a structure beneath the station.

2.4 SSST corrections

SSST corrections are more difficult to calculate than SST corrections. The following method is used.

- For all recording stations, perform the following.

1. Determine the phases of each arrival at the station.
2. For each unique phase in the phase list for the station (e.g. 'Pg'), perform the following.
   (a) Find all picks of the corresponding phase arriving at the station.
   (b) Compute the matrix of pairwise angular distances between the origins of these picks.
   (c) For each of the picks, perform the following.
      i. Take the row of the distance matrix corresponding to the pick.
      ii. Find all picks where the distance value is below a user defined threshold (e.g. 10km).
      iii. If there are fewer than a user defined number of picks within the threshold distance (e.g. 10 picks), do not compute a time correction.
      iv. If there are a sufficient number of picks within the threshold distance (including the pick under consideration), compute their median travel time residual. Use this value as a time correction to the arrival time of the pick under consideration.

Using this method, assuming the user defined minimum number of picks and maximum threshold distance values are sufficient, the time corrections should reflect the increase or decrease in travel time caused by heterogeneity along a ray path between a cluster of events and a recording station. If the user defined values are sufficient, then events within a cluster should have their pick arrival times corrected only by using residuals for picks originating in the same cluster. Events lying outside of clusters will not have a time correction applied. Additionally, picks for which the hypocentre is unstable are excluded from use when calculating time corrections (but have a time correction calculated for themselves).

2.5 Relocation

The relocation method used is either the single earthquake location method described in appendix A, or the iLoc earthquake location algorithm (Bondar and Storchak, 2011). iLoc uses a SQL database for data input and output, and therefore can not be used on the National Computational Infrastructure (NCI) supercomputer, so a dedicated Amazon Web Service instance (AWS) is used for this purpose. The method in appendix A can be run on the NCI, but is less sophiscitated than iLoc.

If using the method in appendix A, the arrival times for waves are kept separate from the time corrections until computation of the error metric. However, iLoc does not accept time corrections as an input, and so the arrival times listed in the SQL database must have the time corrections subtraced from them before being input into iLoc. The reported arrival time T is kept stored in a separate file so that stability of hypocentres is improved. That is, the arrival time fed into iLoc will be $T + tcor_n$ at each iteration, instead of $T + tcor_0 + tcor_1 + \cdots + tcor_n$.

If using iLoc, we give it the following commands.

- DoNotRenamePhase=1. This command tells iLoc not to redefine phases while performing relocation, as if it does, then the time correction applied becomes meaningless.
- UseRSTTPgLg=0, and UseRSTTPnSn=0. These commands tell iLoc to not use the Regional Seismic Travel Time (RSTT) models during relocation, as for clustered events the effects of regional heterogeneity should be captured by the SSST corrections.

3 Results

Tests of the algorithm were performed on a catalogue of 30,400 events sourced from the International Seismological Centre (ISC) analyst reviewed catalogue, United States Geological Survey (USGS), and Geoscience Australia (GA) analyst reviewed catalogue. GA events were sourced for January 2005 to November 2021, ISC events from January 2005 to November 2019, and USGS events from December 2019 to November 2021. See figure 2 for scatterplot of epicentres for events in the catalogue.

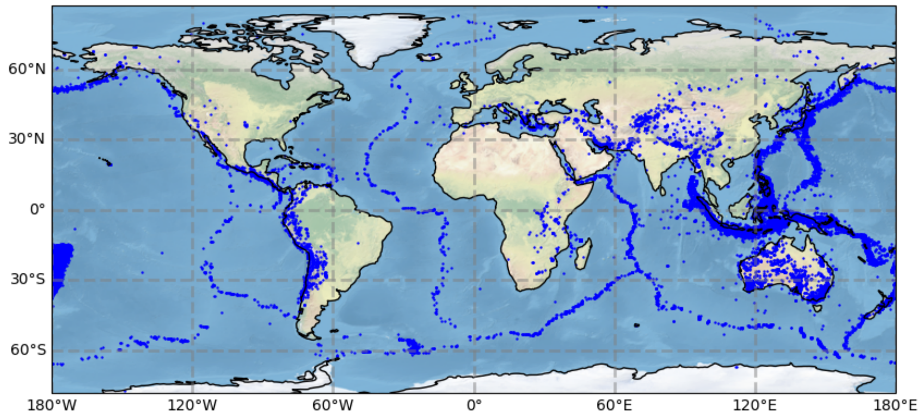


Figure 2: Reported epicentres for the catalogue.

The events were sourced primarily to perform seismic tomography over the Australian continent. To obtain good raypath coverage of this region, the following criteria were used when selecting events from ISC and USGS. The whole GA analyst reviewed catalogue was used.

1. Events across Australia within longitude range $\phi \in [105, 160]$ and latitude range $\theta \in [-40, -13]$ must have an azimuthal gap of less than 180°.
2. Events in the Indian Ocean with $\phi \in [15, 90]$ and $\theta \in [-60, 30]$, or $\phi \in [90, 180]$ and $\theta \in [-60, -45]$, must have a magnitude larger than 5.0.
3. Events elsewhere must have a magnitude above 5.5.

With SST relocation and SSST relocation, we expect to see a decrease in the median absolute value of travel time residuals if using a L1 misfit function, or decrease in the root-mean-square value of the residuals if using a L2 misfit function for relocation. With SSST relocation, provided that the parameters used when computing the time corrections are sufficient, we also expect to see the hypocentres of events 'clustering' along known fault lines.

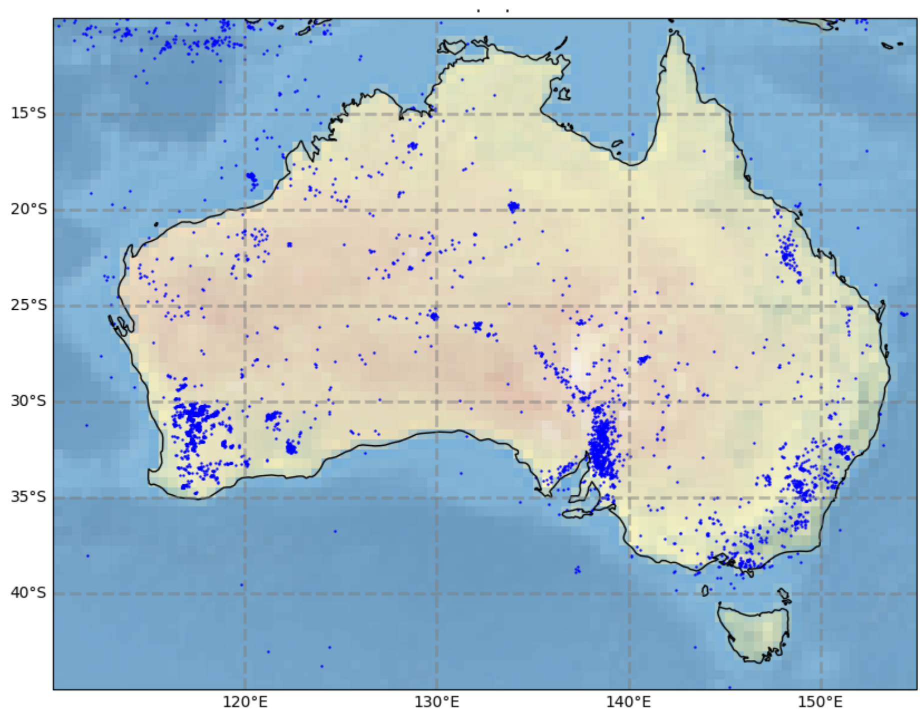


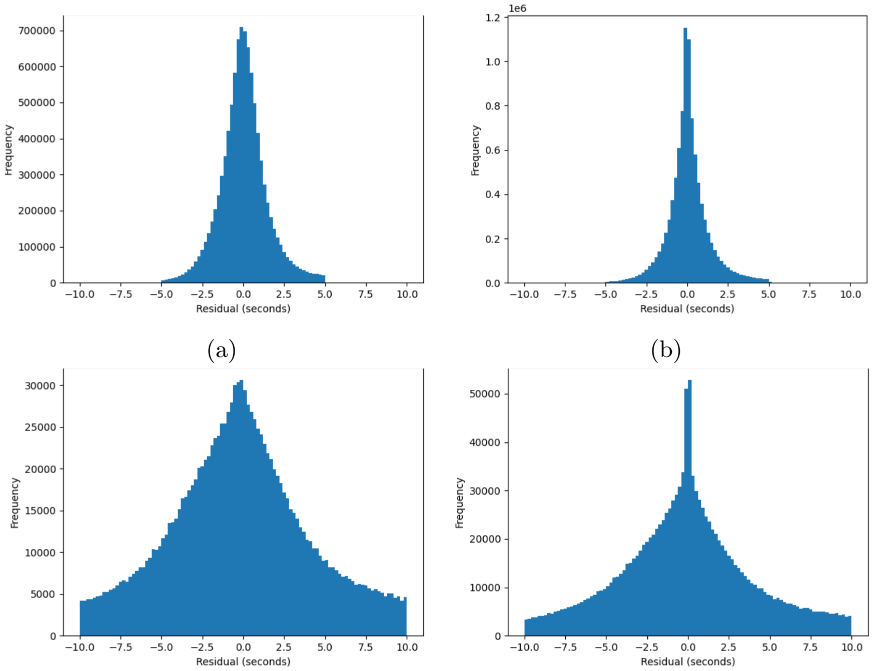
Figure 3: Reported hypocentres on the Australian continent included in the catalogue.

3.1 Analysis of residuals

Using the algorithm described in appendix A, we observe a decrease in both the root-mean-square and mean absolute values of the travel time residuals after relocation of events using SSST corrections. Using iLoc as the relocation algorithm, we don't observe the same decrease in residuals, potentially as a result of clusters of events dispersing during relocation with SSSTs (see section 3.3). Note that in figures 4 and 5, the values shown are $value = ett - ptt - tcor$, for ett = empirical travel time, ptt = predicted travel time, and tcor = time correction.

3.1.1 Residuals using Fortran location algorithm

For picks with phase P, Pg, Pn, and Pb, the mean absolute value of the residuals decreases from 1.173s to 1.125s after 5 iterations of SSST relocation, while the RMS value decreases from 1.544s to 1.501s. For the corresponding S phases, the mean absolute value of the residuals decreases from 3.377s to 3.330s and the RMS value decreases from 4.290s to 4.246s. Figure 4 shows histograms of the P and S wave residuals before and after relocation. Note that for phase redefinition



(a) (b)

(c) (d)

Figure 4: Residuals before and after applying SSST algorithm using single event location method described in appendix A. Top: P wave residuals for (a) picks after relocation without SSST time corrections, and (b) picks after 5 iterations of relocation with SSST corrections. Bottom: S wave residuals for (c) picks after relocation without SSST time corrections, and (d) picks after 5 iterations of relocation with SSST corrections. For (b) and (d), the configuration parameters corr_thr_dist_deg and corr_min_points were set to 0.1 and 10 respectively. The algorithm in appendix A was used for single event location.

a threshold residual of 5 seconds was used for P phases, while 10 seconds was used for S phases, meaning the histograms are truncated at these values on the time axis.

The spikes which appear in the histograms close to 0 seconds demonstrate the effect of the time corrections applied during relocation. Apart from this central peak, the shape of the histogram and hitcount for most bins remain relatively similar after relocation using SSSTs. This occurs because the majority of the picks will not have had a time correction applied to their travel times, due to their distance from other events. The relatively small amount of events which occur in clusters tight enough to have a time correction applied appear to have their travel time residuals significantly reduced. This indicates that the time corrections applied to the picks for these events are relatively close in value to the travel time residuals for the picks, meaning that the misfit between predicted and empirical travel times in these clusters is systematic instead of random.

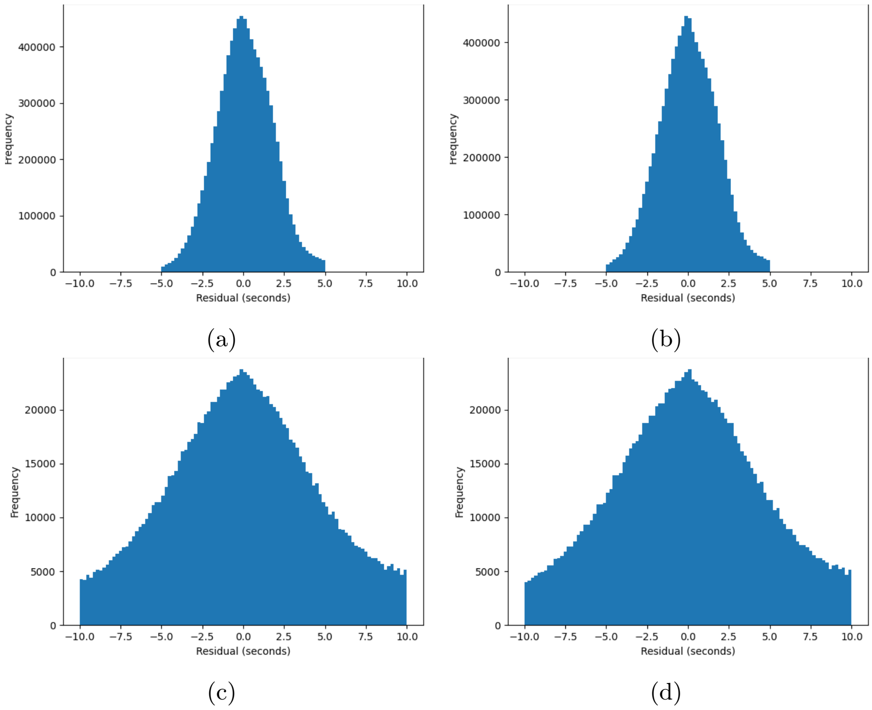


Figure 5: Residuals before and after applying SSST algorithm using iLoc. Top: P wave residuals for (a) picks after relocation without SSST time corrections, and (b) picks after 5 iterations of relocation with SSST corrections. Bottom: S wave residuals for (c) picks after relocation without SSST time corrections, and (d) picks after 5 iterations of relocation with SSST corrections. For (b) and (d), the configuration parameters corr_thr_dist_deg and corr_min_points were set to 0.1 and 10 respectively. The iLoc algorithm was used for single event location.

3.1.2 Residuals using iLoc location algorithm

For picks with phase P, Pg, Pn, and Pb, the mean absolute value of the residuals increases slightly from 1.414s to 1.458s after 5 iterations of SSST relocation, while the RMS value also increases slightly from 1.765s to 1.813s. For the corresponding S phases, the mean absolute value of the residuals increases slightly from 3.566s to 3.572s and the RMS value remains steady at 4.430s. Figure 4 shows histograms of the P and S wave residuals before and after relocation.

There are many potential reasons why the residuals appear to increase after a few iterations of this procedure. For example:

1. iLoc uses bounce point corrections for phases which reflect off an interface at least once. This is not currently considered when applying time corrections.

2. iLoc drops some arrivals when it computes a preferred hypocentre if their residual is too large to be used for relocation. If this occurs over several iterations, the fewer picks may lead to instability of the hypocentre. In contrast, the grid search algorithm in appendix A retains all arrivals for each iteration.

3.2 Epicentres outside Australia

SSST relocation has been used successfully in the past to improve the quality of hypocentres for events in regions with high seismicity (see, for example, Richards-Dinger and Shearer (2000) or Lin and Shearer (2005)). However, in these cases, the number of events available to use for relocation purposes is many orders of magnitude higher than for the Australian region. In addition, the density of seismic monitoring stations in regions such as California or New Zealand where SSST relocation has been performed in the past is much higher than the coverage for Australia.

3.2.1 New Zealand

In New Zealand, where there is a larger number of earthquakes with high magnitude, the SSST relocation algorithm appears to perform as expected. In areas where there are several events, specifically Christchurch, Bienheim, and off the south-west tip of South Island, clusters of events are grouped together tighter after 10 iterations of the SSST algorithm compared to relocation without SSSTs. Figure 6 provides scatter plots showing epicentres in this region.

3.2.2 Chile and western Argentina

Similarly, we observe many clusters of events grouped together more tightly after relocation using SSSTs for Chile and western Argentina (figure 7). A larger cluster near the city of Salta in Argentina, and smaller clusters near Catamarca (Argentina), Santiago (Chile), and Concepcion (Chile) all are grouped together tighter after relocation. In addition, we see an entire cluster near Iquique in Chile shift south-east by about a quarter of a degree.

We also notice, however, that many clusters occurring beneath the sea floor just off the coast of Chile are more spread apart after relocation using SSSTs. The likely cause of this is a larger azimuthal gap for these events, as the configuration was set such that only [P, Pg, Pn, Pb, S, Sg, Sn, Sb] phases were used by the locator described in appendix A. Waves traversing the Pacific Ocean are likely to have more exotic phases, as they may be reflected off the surface may times, or may interact with the mantle or core before being detected.

3.3 Epicentres inside Australia

For the catalogue used in this analysis, there are only a few regions in Australia where SSST relocation may make a difference to the hypocentres. Figure 3 contains a scatter plot of event epicentres occurring on the Australian continent. In south-western Western Australia, south-eastern South Australia, and eastern New South Wales and Victoria, there are regions with a significant amount of seismic activity where clusters of events are reported along known fault lines. We would not expect epicentres of events in this catalogue to change significantly if the velocity model used for travel time prediction is sufficient, given that the majority of the events come from the GA and ISC analyst reviewed catalogues. However, hypocentres from the USGS catalogue reported more recently have not been reviewed, and when relocated they may exhibit the expected clustering behaviour.

3.3.1 SSST relocation vs single event location - Fortran algorithm

When comparing hypocentres in New South Wales and Victoria relocated with SSST time corrections against the same events with hypocentres relocated using single event location only, we see that there are some groups of events which are clustered slightly closer together. However, at the same time, we see some clusters of events have dispersed (see figure 8). For example, consider the three clusters of events in a line between Orange (149E, 33.3S) and Canberra (149E, 35.3S). The cluster closest to Orange appears to be more tightly grouped together after relocation, while the cluster closest to Canberra appears more spread out. Similarly, two small clusters near Wilson's Promontory (146.4E, 39.1S) appear to spread apart after relocation, while a cluster slightly further north (146.5E, 37.5S) is grouped together tighter after relocation.

3.3.2 SSST relocation vs single event location - iLoc

Using the iLoc earthquake location algorithm, we see that using SSST corrections does not appear to improve the quality of the hypocentres. In figure 9 we see that for the New South Wales and Victoria region, the epicentres output by using iLoc for single event location are comparable with those computed using the single event location algorithm described in appendix A. However, we do not observe the expected clustering behaviour when using SSST corrections with iLoc as the relocation algorithm. Only events in the cluster close to (146.5E, 37.5S) remains tightly grouped together, while most others begin to spread apart after a few iterations of the algorithm. It should be noted that many events in the region with longitude range $\phi \in [150.5E, 151.5E]$ and latitude range $\theta \in [32S, 33S]$ are potentially mine blasts detected from coal mines in the Hunter Valley.

3.4 Summary of results

As shown, SSST time corrections have the potential to significantly reduce the misfit function computed during relocation for events occurring in clusters. The elimination of systematic differences between empirical and predicted travel times during relocation means that the resulting residuals should better demonstrate the effects of heterogeneity between event clusters and the recording stations. For the application of this method in hiperseis, this means that tomographic products inverted from this data may be better able to show the features within the Earth responsible for the systematic differences between empirical and predicted travel times.

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A Single event location algorithm

The algorithm which was developed to perform single earthquake location is based on the 'SingleLoc' procedure included in the XCorLoc relocation package (Lin, 2018). This algorithm is contained within relocation_helper.f.

The following inputs are required by the location algorithm.

- Event longitude, colatitude, and depth (ϕ_e, θ_e, z_e).
- Recording station longitude, colatitude, and elevation (ϕ_s, θ_s, z_s).
- Pick phases, arrival times t_s, and time corrections $tcor_s$ (if available).
- Predicted travel time tables.
- Ellipticity correction coefficient tables.
- $\delta\phi, \delta\theta, \delta z$ values to use for grid search.
- Number of iterations n_{iter}, and fraction to shrink grid search by per iteration f.
- Norm to use for minimisation, i.e. $L1$ (Manhattan) or $L2$ (Euclidian) norm.

The procedure used is as follows. Assume n_{pick} is the number of picks.

1. Start with initial location (ϕ_e, θ_e, z_e), set $(\phi_{best}, \theta best, z_{best})$ equal to the initial coordinates, set $t_e = t_{best} = 0$, and set error metric $E_{best} = \infty$.
2. For $i \in \{1, n_{iter}\}$, perform the following.
   (a) For $\phi_0 \in \{\phi_e - \delta\phi, \phi_e, \phi_e + \delta\phi\}$, $\theta_0 \in \{\theta_e - \delta\theta, \theta_e, \theta_e + \delta\theta\}$, and $z_0 \in \{z_e - \delta z, z_e, z_e + \delta z\}$, perform the following.
      i. For each pick (denoting the station as s):
         A. Compute the azimuth α_s and angular distance β_s from (ϕ_0, θ_0) to (ϕ_s, θ_s).
         B. Compute the predicted travel time ptt_s for a wave of the corresponding phase with angular distance β_s and event depth z_0.
         C. Compute a time correction to account for elevation of the station z_s based on P (or S) wave surface velocity.
         D. Compute an elevation correction based on epicentral colatitude θ_0, azimuth α_s, angular distance β_s, and event depth z_0.
         E. Correct the predicted travel time using these two time corrections.
         F. Compute the residual $ttr_s = t_s - tcor_s - ptt_s$.
      ii. If using $L1$ norm, compute the median residual $\widetilde{ttr}$. Otherwise, if using $L2$ norm, compute the mean residual $\overline{ttr}$. Set this as the origin time t_0.

   iii. If using the L1 norm, compute the error metric $E_0 = \frac{1}{n_{pick}} \sum_s |ttr_s - \widetilde{ttr}|$. Otherwise, if using the L2 norm, compute the error metric $E_0 = \sqrt{\frac{1}{n_{pick}} \sum_s \left(ttr_s - \overline{ttr}\right)^2}$.
   iv. If $E_0 < E_{best}$, set $(\phi_{best}, \theta_{best}, z_{best}, t_{best}) = (\phi_0, \theta_0, z_0, t_0)$ and set $E_{best} = E_0$.

   (b) Set $(\phi_e, \theta_e, z_e, t_e) = (\phi_{best}, \theta_{best}, z_{best}, t_{best})$.
   (c) Set $(\delta\phi, \delta\theta, \delta z) = f \cdot (\delta\phi, \delta\theta, \delta z)$.

3. Return best fitting hypocentre $(\phi_e, \theta_e, z_e, t_e)$.

This algorithm performs a grid search about the reported hypocentre, to find a new best fitting hypocentre. Consider n iterations of the algorithm. If given, for example, $\delta\theta = d$ and 'shrinking fraction' f, then the best fitting hypocentre within a colatitude range of $\theta = \theta_0 \pm \sum_{i=1}^{n} d \cdot f^{i-1}$ is found, with an uncertainty of $\pm\delta\theta \cdot f^{n-1}$.

The methods used for calculating predicted travel times, ellipticity corrections, and elevation corrections are included in appendix B.

B Travel time, distance, and azimuth calculation

B.1 Travel time prediction

In the Fortran single event location subroutine, travel time prediction for a wave with phase ph, angular distance β, and event depth z is computed using a bicubic spline interpolating function of a travel time table provided for the phase. In the SSST algorithm, travel times are interpolated using a Clough-Tocher interpolating function (Alfeld, 1984). The differences between the output of the two routines is negligible.

To compute the spline for the Fortran subroutines, consider $\beta_m \leq \beta < \beta_{m+1}$ and $z_n \leq z \leq z_{n+1}$. The 16 data points surrounding (β, z) with indices $i \in \{m-1, \ldots, m+2\}$ and $j \in \{n-1, \ldots, n+2\}$ are used, with the natural condition applied. If there is an insufficient number of points to perform bicubic spline interpolation of the table, e.g. if (β, z) is close to the edges, bilinear interpolation is performed instead using the values at $i \in \{m, m+1\}$ and $j \in \{n, n+1\}$. If (β, z) lies outside the angular distance and depth range covered by the table, an invalid value is returned and the wave is not used for locating the event.

B.2 Elevation correction

Elevation corrections are computed in the following method. Consider a wave with phase ph, surface velocity v, epicentral distance (in km) D, station elevation z_s and event depth z_e. We compute $\frac{\partial t}{\partial D}$ by interpolating the value of the first horizontal derivative of the travel time table using the same interpolation method described in section B.1. Then, the time correction is computed using the formula

$$tcor = \frac{z_s}{v} \sqrt{1 - min\left(\left(v\frac{\partial t}{\partial D}\right)^2, \frac{1}{\left(v\frac{\partial t}{\partial D}\right)^2}\right)}.$$

This formula is the same as that used in the iLoc earthquake relocation algorithm.

B.3 Ellipticity correction

The ellipticity correction is designed to account for differences in travel times due to Earth's eccentricity. The ellipticity correction algorithm follows the same procedure as Brian Kennett's 'ellipcorr' calculator (Kennett and Gudmundsson, 1996), with modifications made to improve computation time and different interpolation methods used.

Let θ be the colatitude of an event, α be the azimuth from the event to a receiver, D be distance (in degrees) from the event to the receiver, and z be the event depth. The time correction is then calculated to be, for a particular phase,

$$\begin{aligned}\delta t = &\frac{1}{4}\left(1 + 3\cos(2\theta)\right)\tau_0(D,z)\\ &+ \frac{\sqrt{3}}{2}\sin(2\theta)\cos(\alpha)\tau_1(D,z)\\ &+ \frac{\sqrt{3}}{2}\sin^2(\theta)\cos(2\alpha)\tau_2(D,z).\end{aligned}$$

The coefficients τ_0, τ_1, and τ_2 are interpolated using bicubic spline interpolation of a coefficient table produced by Kennett and Gudmundsson for the AK135 model. The coefficients vary by phase.

B.4 Azimuth and distance calculation

To compute azimuth and angular distance between points, Earth is approximated as a sphere. It is assumed that the uncertainty introduced into travel time calculations by this approximation is insignificant compared to the uncertainty introduced by interpolation of the travel time values from a pre-computed table, or changes in travel time due to Earth's heterogeneous structure. Let (ϕ_1, θ_1) and (ϕ_2, θ_2) be the longitude and colatitude of points 1 and 2 respectively.

The azimuth from point 1 to 2 is then

$$\alpha = \arctan\left(\frac{\sin(\phi_2 - \phi_1)}{\sin(\theta_1)\cot(\theta_2) - \cos(\theta_1)\cos(\phi_2 - \phi_1)}\right).$$

The angular distance between point 1 and 2 is then

$$D = 2\arcsin\left(\sqrt{\sin^2\left(\frac{\theta_2 - \theta_1}{2}\right) + \sin(\theta_1)\sin(\theta_2)\sin^2\left(\frac{\phi_2 - \phi_1}{2}\right)}\right).$$

C Structure of repository

On hiperseis, the code for this algorithm is stored in two directories. The repository is stored at https://github.com/GeoscienceAustralia/hiperseis.

C.1 Data conversion

The folder seismic/ssst_relocation/data_conversion/ contains three scripts:

- convert_for_relocation.py - convert data to a format usable by the relocation algorithm.
- convert_array_to_catalogue.py - create XML files for use by iLoc.
- convert_after_relocation.py - convert output data into a form usable by inversion software.

C.2 Relocation

The folder seismic/ssst_relocation/relocation/ contains several files. Some of these (Relocation.py, Travel_Times.py, Station_Corrections.py) are modules containing functions to be used in main.py. Executable scripts are described in the following subsections. In addition, there is a Fortran script relocation_helper.f which must be converted into a python module before use.

C.2.1 tt_table_calculator.py

The script tt_table_calculator.py and its corresponding configuration file are used to produce pre-computed travel time tables which are to be used by the module Travel_Times.py. These may be created using the TauP toolkit (Crotwell et al., 1999). Alternatively, if the iLoc relocation algorithm is to be used (see section the iLoc travel time tables can be used.

If the iLoc travel time tables are to be used, they can be converted into the correct format using the the following script.

```
python3 tt_table_calculator.py --iloc_path <path_to_iloc_tables>
↪ --output_path <output_path> --from_iloc_tables True
```

If the travel time tables are to be computed using the TauP toolkit, the same script may be used, however a configuration file is needed to tell the algorithm the phases, epicentral distance coordinates, and depth coordinates to use for the tables. The script may be run using the following command.

```
python3 tt_table_calculator.py --model <model_name> --config_file
↪ <config_file_name> --output_path <output_path>
```

The configuration file should be a .txt file in the format below. Rows are (1) phase, (2) epicentral distance in degrees, (3) depth in kilometres.

```
----------------------------------------------------
[P]
dists = 0.000 1.000 2.000 3.000 4.000 5.000
depths = 0.000 1.000 2.500 5.000 7.500 10.000 15.000

[Pg p]
dists = 0.000 0.025 0.050 0.075 1.000
```

```
depths = 0.000 1.000 2.500 5.000 7.500 10.000

[S]
dists = 0.000 1.000 2.000 3.000 4.000 5.000
depths = 0.000 1.000 2.500 5.000 7.500 10.000 15.000
----------------------------------------------------
```

This example configuration file tells the algorithm to produce 3 tables. The first table will be a 6×7 grid of travel times for 'P' waves. The second will be a 5×6 grid of travel times for the first arriving 'Pg' or 'p' wave. The third will be a 6×7 grid of travel times for 'S' waves.

The output of tt_table_calculator.py for each specified phase is as follows.

- phase.tt - predicted travel times.
- phase.dtdd - first horizontal derivative of phase.tt.
- phase.dtdh - first vertical derivative of phase.tt.

The first row contains a length m array of epicentral distance values, and the second row contains a length n array of depth values. The rest of the rows form a $m \times n$ matrix of the computed values.

C.2.2 main.py

The script main.py is the script which runs the full relocation algorithm. It takes the following arguments.

- config_file: Name of configuration file to use.
- tt_table_path: Directory containing pre-computed travel time tables.
- input_path: Directory containing pick array save file and, if iteration > 0, list of events with unstable hypocentres.
- elcordir: Directory containing ellipticity correction coefficient tables.
- iteration: Iteration number.
- output_path: Directory to output updated pick array and list of events with unstable hypocentres
- relocation_algorithm: If relocation_algorithm is 'iloc' then the iLoc relocation algorithm is used, or if not, the relocation algorithm in relocation_helper.f is used.

The script main.py is accompanied by a configuration file relocation_config.py. This file contains the following configuration parameters.

- hypo_thr_dist_deg: float, representing the allowed distance a hypocentre may move during relocation before being classed as unstable.
- hypo_thr_time_sec: float, representing the allowed number of seconds an event origin time may change by during relocation before being classed as unstable.
- corr_thr_dist_deg: float, radius of sphere (in degrees) used for computing SSST corrections.
- thr_p: float, time threshold for P wave phase redefinition.
- thr_s: float, time threshold for S wave phase redefinition.
- no_relocation: boolean. Set to True if only phase redefinition is desired.
- correction_method: string. Set to 'SSST' if SSST corrections are desired, 'SST' if static station term corrections are desired, or 'None' for no time corrections.

- phases: list, describing which phases to try to match arrivals to in the first pass of the phase redefinition algorithm. Additionally, if the default relocation algorithm is used, only these phases are used for relocation. Example: phases = P, Pg, Pn, Pb, S, Sg, Sn, Sb.

If the iLoc relocation algorithm is to be used, also define the following variables.

- iloc_redefine_phases: boolean. If True, iLoc will be allowed to redefine phases during its relocation procedure.
- iloc_use_rstt: boolean. If True, iLoc will be allowed to use the RSTT model for travel time calculation for Pg, Sg, Pn, and Sn phases.

If the default relocation algorithm is to be used, also define the following variables.

- reloc_dlat: float, describing the initial spacing between grid points for the grid search algorithm. This decreases by 'reloc_sfrac' amount each iteration.
- reloc_ddep: float, decribing the initial depth spacing between grid points for the grid search algorithm. This also decreases by 'reloc_sfrac' amount each iteration.
- reloc_nit: integer, number of iterations to use for default relocation algorithm.
- reloc_norm: integer. Set to 1 if L1 norm is used for minimisation routine, or 2 if L2 norm (RMS) is to be used.
- reloc_sfrac: float, describing the proportion that reloc_dlat and reloc_ddep will shrink by each iteration. E.g. if reloc_sfrac = 0.5, then on each iteration, reloc_dlat and reloc_ddep are halved.
- temp_networks: list, describing temporary seismic station deployments which are not to be used for relocation. Picks on these networks still have their phases redefined and residuals calculated. For example: temp_networks = 7B, 7D, 7E, 7F, 7G, 7J, 7Q, 7T, 7U, 7W, 7X, OA, W1, AQ.

Run multiple iterations of this algorithm using the command below, e.g. by executing "for i in ‘seq 1 $number_of_iterations‘; do <command>; done". On the first iteration, the algorithm only finds hypocentres which are unstable, using no time corrections. On all subsequent iterations, the configuration specified is followed.

```
mpirun -np $number_of_processors main.py --config_file <configuration
↪  file> --tt_table_path .../tt_table_path/ --input_path .../input_path/
↪  --elcordir .../elcordir/ --iteration $i --output_path
↪  .../output_path/
```

C.2.3 Plots.py

Plots.py is a script used to generate diagnostic plots after relocation has been performed. It currently produces a histogram of travel time residuals, a scatter plot of earthquake epicentres, and a plots of the ray path coverage for a region specified by a user defined bounding box.

Arguments for Plots.py are as follows.

- files (list)
- output_path (string)
- show (boolean) - if True, plots will be displayed while script is executing.
- plot_epicentres, plot_residuals, plot_raypaths - all boolean, describing which plots to create.
- tcor_available (boolean) - specifies whether or not output files have a column for time corrections used.

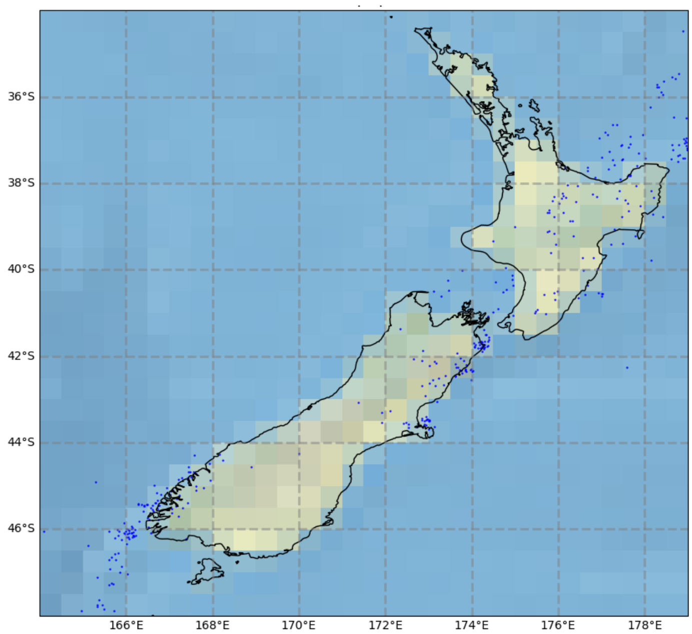
- ray_interval (integer) - to avoid over-crowding the plot, a user may for example choose to display one out of every ten rays.
- ray_heatplot_bin_width (float) - specifies the size of a box for the ray path heat plot.
- lonmin, lonmax, latmin, latmax, timemin, timemax, depthmin, depthmax - all floats, giving boundaries for filtering epicentres to be plotted.

C.2.4 relocation_helper.f

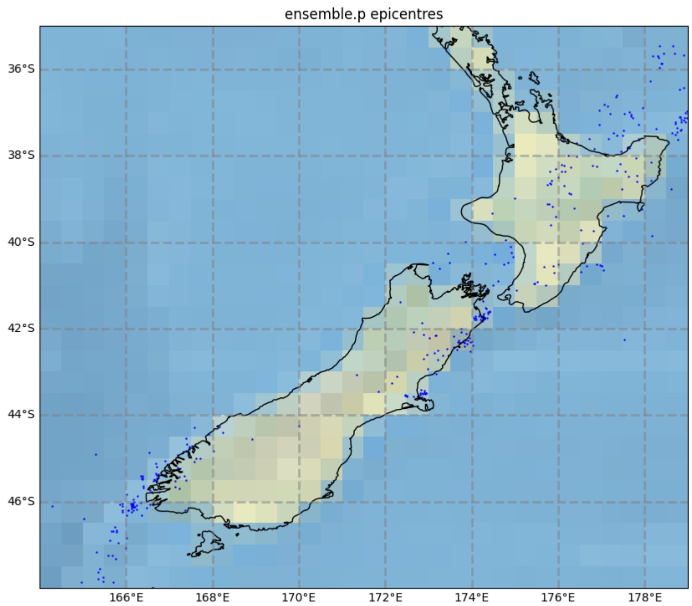
The file relocation_helper.f contains several subroutines which may be compiled using the python numpy.f2py compiler, to produce python modules. The module Relocation.py depends on some of the functions in relocation_helper.f. Before running the relocation algorithm, produce the relocation_helper module using the following command.

```
python -m numpy.f2py -c relocation_helper.f -m relocation_helper
```

On Linux, this will produce a module relocation_helper.so.

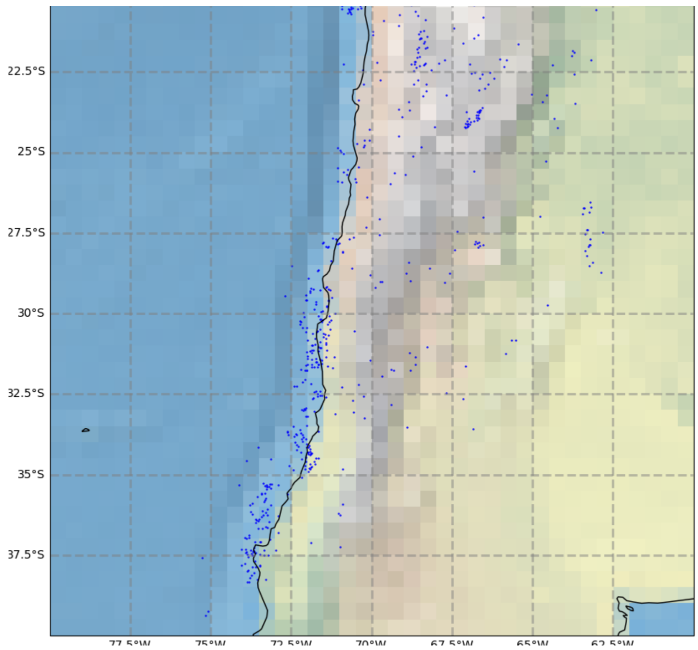


(a)

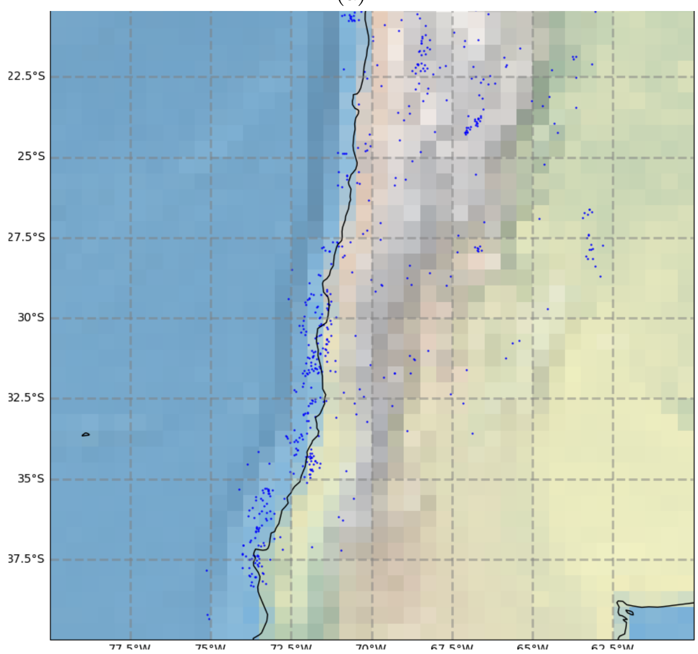


(b)

Figure 6: Events occurring in New Zealand. Epicentres after relocation without SSST time corrections are shown in (a). Epicentres after 5 iterations of relocation with SSST corrections are shown in (b). For (b), the configuration parameters corr_thr_dist_deg and corr_min_points were set to 0.1 and 10 respectively.

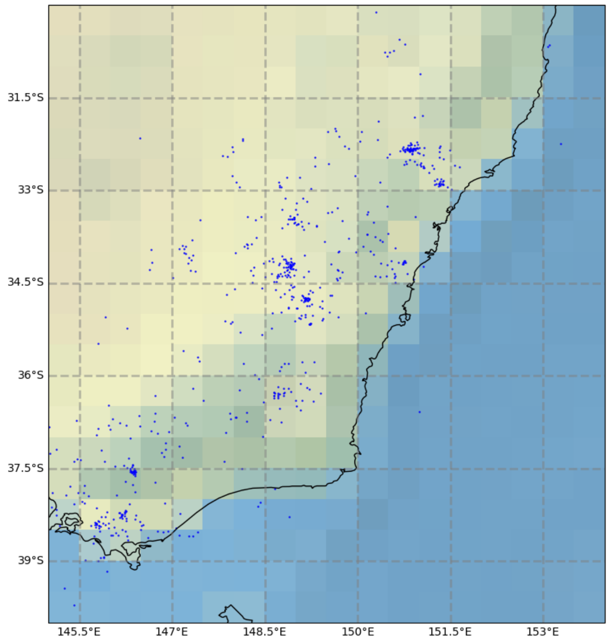


(a)

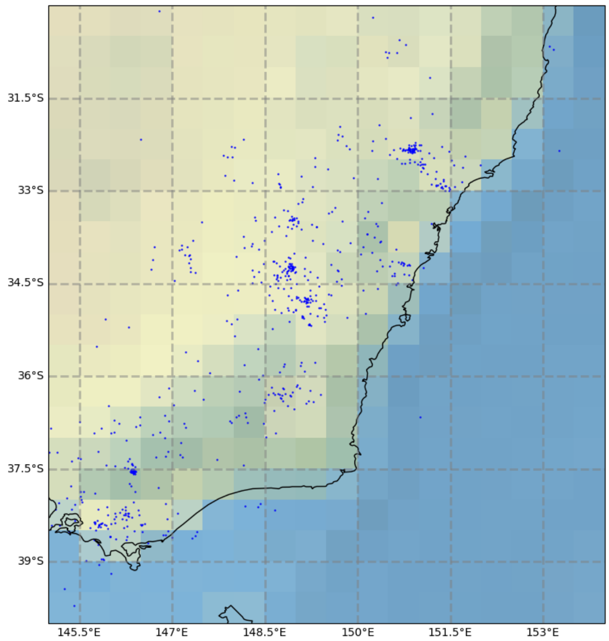


(b)

Figure 7: Events occurring along part of Chile and western Argentina. Epicentres after relocation without SSST time corrections are shown in (a). Epicentres after 10 iterations of relocation with SSST corrections are shown in (b). For (b), the configuration parameters corr_thr_dist_deg and corr_min_points were set to 0.1 and 10 respectively.

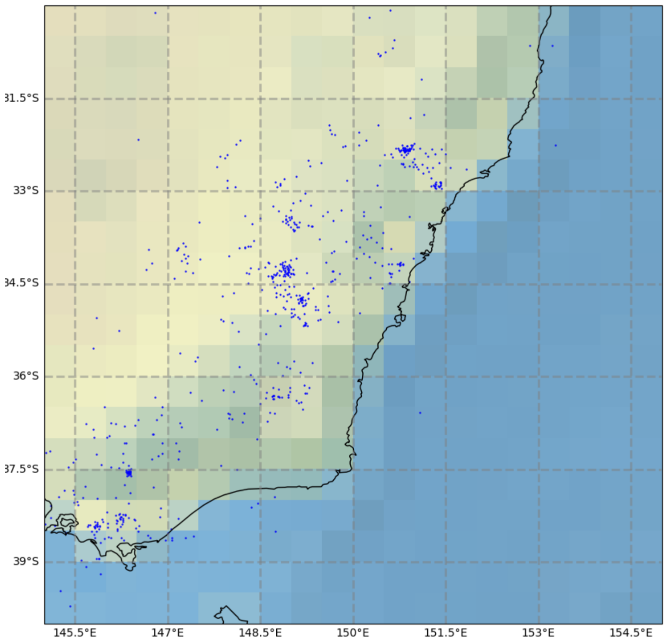


(a)

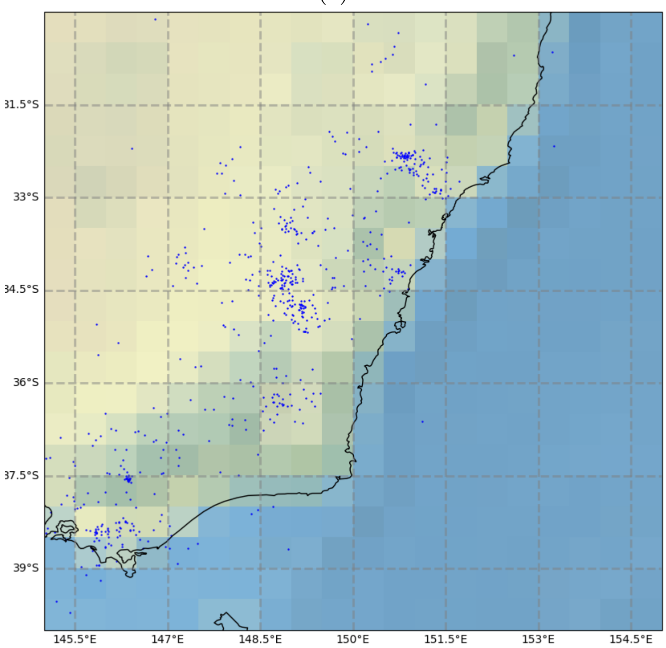


(b)

Figure 8: Epicentres for events occurring in eastern New South Wales and Victoria before and after relocation with SSST corrections, using single event location algorithm described in appendix A. Epicentres after relocation without SSST time corrections are shown in (a). Epicentres after 5 iterations of relocation with SSST corrections are shown in (b). For (b), the configuration parameters corr_thr_dist_deg and corr_min_points were set to 0.1 and 10 respectively.



(a)



(b)

Figure 9: Epicentres for events occurring in eastern New South Wales and Victoria before and after relocation with SSST corrections, using iLoc for single event location. Epicentres after relocation without SSST time corrections are shown in (a). Epicentres after 5 iterations of relocation using SSST corrections are shown in (b). For (b), the configuration parameters corr_thr_dist_deg and corr_min_points were set to 0.1 and 10 respectively.