Commonalities Analysis for Medical Imaging Applications

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1 Revision History

Date	Version	Notes
Oct 11	1.0	Initial draft

2 Reference Material

This section records information for easy reference.

2.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	unit	SI
N/A		

2.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

symbol	unit	description
a	N/A	dimension of spatial coordinates
b	N/A	dimension of feature values
C_1	N/A	the first class with pixels in $[0, k]$
C_2	N/A	the second class with pixels in $[k+1, L-1]$
f	N/A	function defining an input image
F	N/A	input medical image
g	N/A	function defining an output image
G	N/A	output segmentation image
h	N/A	function defining a mathematical image
H	N/A	2D digital grayscale image
i	N/A	intensity value
k	N/A	threshold value in Otsu' Method
k_1	N/A	threshold value in Otsu' Method with multiple thresholds
k_2	N/A	threshold value in Otsu' Method with multiple thresholds
$k^\star$	N/A	optimal threshold value found by Otsu' Method
$k_1^\star$	N/A	optimal threshold value found by Otsu' Method with multiple thresholds

$k_2^\star$	N/A	optimal threshold value found by Otsu' Method with multiple thresholds
L	N/A	number of the discrete levels of the feature value
m_1	N/A	mean intensity of the pixels in C_1
m_2	N/A	mean intensity of the pixels in C_2
m_3	N/A	mean intensity of the pixels in the third class
m_G	N/A	mean global intensity
p_i	N/A	normalized histogram
P_1	N/A	probability of the first class C_1
P_2	N/A	probability of the second class C_2
P_3	N/A	probability of the third class
$\mathbb{R}$	N/A	set of all real numbers
σ_B	N/A	between-class variance
T_h	N/A	threshold value for segmentation
x	N/A	x-axial coordinate of a image
y	N/A	y-axial coordinate of a image
X	N/A	x-axial pixel length of an image
Y	N/A	y-axial pixel length of an image
$\mathbb{Z}$	N/A	set of all integers

2.3 Abbreviations and Acronyms

symbol	description
2D	Two-Dimensional
3D	Three-Dimensional
A	Assumption
CA	Commonalities Analysis
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
MIA	Medical Image Applications
PS	Physical System Description
R	Requirement
T	Theoretical Model
VTK	the Visualization Toolkit

Contents

3 Introduction

Medical imaging technologies in medical diagnoses have become essential and effective tools for professionals in medicine. Computers are also widely used to generate, process, and visualize medical images. There is a big family of computer software developed to serve these purposes.

The following sections provide an overview of the Commonalities Analysis (CA) for the family of Medical Imaging Applications (MIA). This section explains the purpose of this document, the scope of the family, the characteristics of the intended reader, and the organization of the document.

3.1 Purpose of Document

The major purpose of this document is to describe some commonalities of MIA. Goals, assumptions, theoretical models, definitions, and other model derivation information are specified, allowing the reader to fully understand and verify the purpose and scientific basis of MIA. This CA will remain abstract, describing what problem is being solved, but not how to solve it.

This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the CA fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements (February 1986) point out, the most logical way to present the documentation is still to "fake" a rational design process.

3.2 Scope of the Family

According to Bankman (2000), MIA deal with 6 different basic problems, while Angenent et al. (2006) pointed out that 4 fundamental problems are solved by MIA. While both mentioned Segmentation, Registration and Visualization of medical images, Bankman also included Enhancement, Quantification and a section covering some other functions Bankman (2000). On the other hand, Angenent's team included Simulation Angenent et al. (2006). According to Wikipedia contributors (2019), MIA have major functions in categories such as Segmentation, Registration, Visualization (including the basic display, reformatted views and 3D volume rendering), Statistical Analysis, Image-based Physiological Modelling, etc. As Kim et al. (2011) describe, the general steps of medical image analysis after obtaining digital data include Enhancement, Segmentation, Feature Extraction, Classification and Interpretation.

The major functions of MIA can be divided into several sections and sub sections as shown in Figure 1.

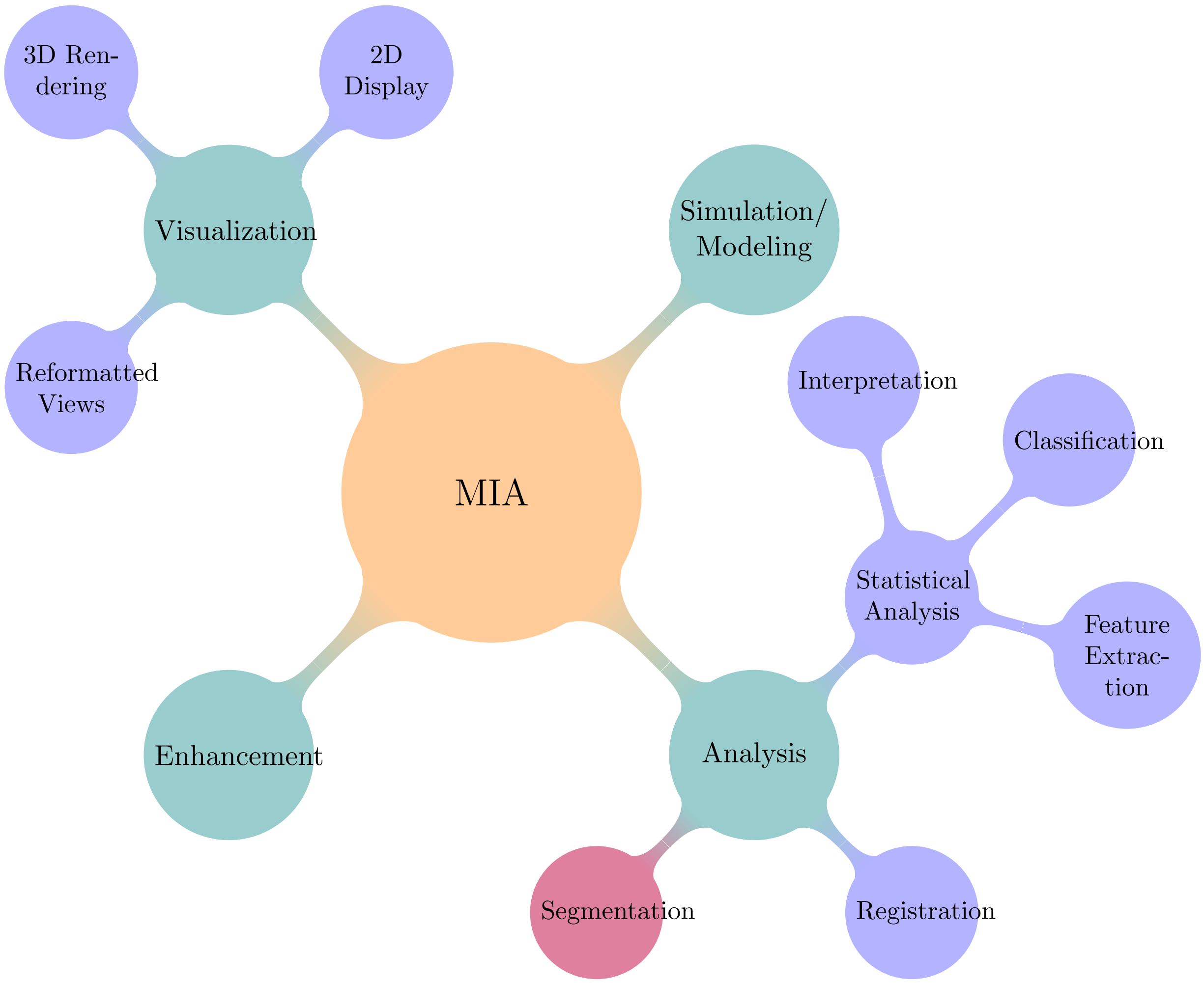

Figure 1: Major functions of MIA

In this project, the scope of the family is limited to the software with the Segmentation functions.

3.3 Characteristics of Intended Reader

Reviewers of this documentation should have an understanding of heat transfer theory from level 3 or 4 mechanical engineering and differential equations from level 1 and 2 calculus. The users of MIA can have a lower level of expertise, as explained in Section 4.2.

3.4 Organization of Document

The organization of this document follows the template for an CA for scientific computing software proposed by Parnas (1972) and Parnas and Clements (February 1986). The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section 5.6 and trace back to find any additional information they require. The goal statements (Section 5.4) are refined to the theoretical models and the theoretical models (Section 5.5) to the instance models (Section 5.6). The instance models to be solved are referred to as IM1, IM2, IM3, IM4.

4 General System Description

This section identifies the interfaces between the system and its environment, describes the potential user characteristics and lists the potential system constraints.

4.1 Potential System Contexts

Figure 2 shows the system context. A circle represents an external entity outside the software, the user in this case. A rectangle represents the software system itself (MIA). Arrows are used to show the data flow between the system and its environment. MIA are mostly self-contained. The only external interaction is through the user interface. The responsibilities of the user and the system are as follows:

- User Responsibilities:
  - Provide the input data to the system, ensuring no errors in the data entry
  - Take care that consistent units are used for input variables
- MIA Responsibilities:
  - Detect data type mismatch, such as a text file instead of a image file
  - Determine if the inputs satisfy the required mathematical and software constraints
  - Calculate the required outputs

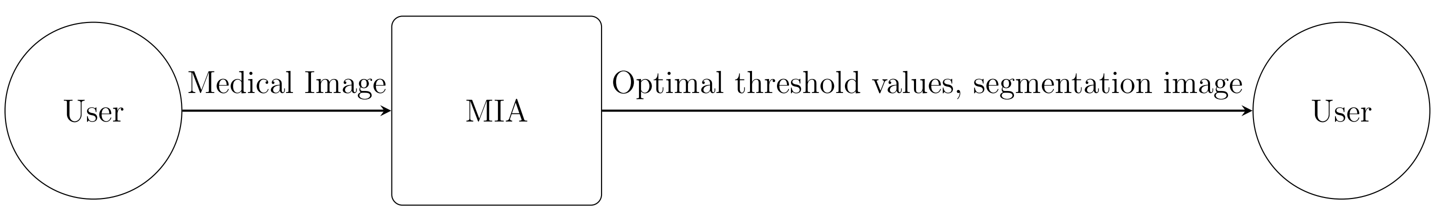

Figure 2: System Context

4.2 Potential User Characteristics

The end user of MIA should have an understanding of undergraduate Level 1 Calculus and Physics.

4.3 Potential System Constraints

There are no system constraints.

5 Commonalities

This section first presents the background overview, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

5.1 Background Overview

Segmentation, separation of structures of interest from the background and from each other Bankman (2000). Image segmentation is the process of partitioning an image into different meaningful segments. In medical imaging, these segments often correspond to different tissue classes, organs, pathologies, or other biologically relevant structures Forouzanfar et al. (2010). Image segmentation is one of the most interesting and challenging problems in computer vision generally and medical imaging applications specifically Elnakib et al. (2011).

Regarding the different medical image segmentation methods, Withey and Koles (2007) suggested that they can be divided into 3 generations from low-level to high-level technologies as shown in table 1.

This document is focusing on the Intensity Threshold method.

Generation	**Category**		
	Region-based	**Boundary Following**	**Pixel Classification**
1^{st}	• Region growing	• Edge tracing (heuristic)	• Intensity threshold
2^{nd}	• Deformable models • Graph search	• Minimal path • Target tracking • Graph search • Neural networks • Multiresolution	• Statistical pattern recognition • C-means clustering • Neural networks • Multiresolution
3^{rd}	• Shape models • Appearance models • Rule-based • Coupled surfaces		• Atlas-based • Rule-based

Table 1: Segmentation Methods Withey and Koles (2007)

5.2 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Image: in Mathematics, an image is defined as a function $h : \mathbb{R}^a \rightarrow \mathbb{R}^b$.
- Digital image: When the spatial coordinates and the function value are finite and discrete, the image is called digital, shown as $h : \mathbb{Z}^a \rightarrow \mathbb{Z}^b$.
- Grayscale image: In digital photography, computer-generated imagery, and colorimetry, a grayscale or greyscale image is one in which the value of each pixel is a single sample representing only an amount of light, that is, it carries only intensity information.
- 2D Digital Image: the computer-based generation of digital images - mostly from two-dimensional models (such as 2D geometric models, text, and digital images) and by techniques specific to them.
- Medical Image: visual representations of the interior of a body for clinical analysis and medical intervention, as well as visual representation of the function of some organs or tissues (physiology).
- Grayscale Intensity: represents gray levels, where the intensity 0 usually represents black and the intensity 255 usually represents full intensity, or white.
- Shades of Gray: variations of gray or grey include achromatic grayscale shades, which lie exactly between white and black, and nearby colors with low colorfulness.
- Coordinates: in geometry, a coordinate system is a system that uses one or more numbers, or coordinates, to uniquely determine the position of the points or other geometric elements on a manifold such as Euclidean space.
- Histogram: a histogram is an accurate representation of the distribution of numerical data. It is an estimate of the probability distribution of a continuous variable.
- Pixel: in digital imaging, a pixel, pel, or picture element is a physical point in a raster image, or the smallest addressable element in an all points addressable display device; so it is the smallest controllable element of a picture represented on the screen.
- VTK: the Visualization Toolkit is an open-source, freely available software system for 3D computer graphics, modeling, image processing, volume rendering, scientific visualization, and 2D plotting.

5.3 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

Number	DD1
Label	**Digital Image**
Symbol	N/A
SI Units	N/A
Equation	$h : \mathbb{Z}^a \to \mathbb{Z}^b$
Description	In Mathematics, an image is defined as a function $h : \mathbb{R}^a \to \mathbb{R}^b$. Usually, $a = 2$ and, in the simplest case, $b = 1$. When the spatial coordinates and the function value are finite and discrete, the image is called digital, shown as $h : \mathbb{Z}^a \to \mathbb{Z}^b$.
Sources	Ferrari (2018a)
Ref. By	DD2

Number	DD2
Label	**2D Digital Grayscale Image**
Symbol	H
SI Units	N/A
Equation	$H_{X \times Y} = [h(x, y)]_{X \times Y}$
Description	In this project, we only consider 2D grayscale images, as stated in A1. The images can be defined as $h : \mathbb{Z}^2 \to \mathbb{Z}$, or the above equation. $X \times Y$ is the size of the image. (x, y) denotes the 2D spatial coordinates, where $x \in [0, X - 1]$ and $y \in [0, Y - 1]$
Sources	Pal and Pal (1993)
Ref. By	DD3

Number	DD3
Label	**Input and Output Medical Image**
Symbol	F and G
SI Units	N/A
Equation	$F_{X \times Y} = [f(x, y)]_{X \times Y}$ and $G_{X \times Y} = [g(x, y)]_{X \times Y}$
Description	F and G denote the input and output medical image respectively, which are both 2D Digital Grayscale. $X \times Y$ is the size of the image. (x, y) denotes the 2D spatial coordinates, where $x \in [0, X - 1]$ and $y \in [0, Y - 1]$
Sources	Pal and Pal (1993)
Ref. By	T2 T3 IM1

Number	DD4
Label	**Input and Output Image Feature Value**
Symbol	f and g
SI Units	N/A
Equation	$f(x, y), g(x, y) \in \{0, 1, ..., L - 1\}$
Description	(x, y) denotes the 2D spatial coordinates and $f(x, y)$, $g(x, y)$ the feature values at (x, y) of the input and output image respectively. Depending on the type of image, the feature value could be light intensity, depth, intensity of radio wave or temperature. $\{0, 1, ..., L - 1\}$ is the set of discrete levels of the feature value, and L is the number of the levels. In this project, we refer to $f(x, y)$ and $g(x, y)$ as gray intensity value (or intensity) at (x, y).
Sources	Pal and Pal (1993)
Ref. By	DD3 DD7 T2 T3 IM1

Number	DD5
Label	**Number of the shades of gray**
Symbol	L
SI Units	N/A
Equation	$L = 256$
Description	L is the number of the shades of gray, also referred as number of intensity levels, where $L = 2^8 = 256$, as stated in A2.
Sources	https://homepages.inf.ed.ac.uk/rbf/HIPR2/value.htm
Ref. By	DD4 DD6 DD7 T2 T3 T4 IM1

Number	DD6
Label	**Threshold Value**
Symbol	T_h
SI Units	N/A
Equation	$T_h \in \{1, ..., L - 2\}$
Description	In this project, we refer to $f(x, y)$ and $g(x, y)$ as gray intensity value (or intensity) at (x, y). Then, $T_h \in [1, 254]$.
Sources	Ferrari (2018b)
Ref. By	DD7 T2 T3 IM1

Number	DD7
Label	**Between-class Variance**
Symbol	σ_B^2
SI Units	N/A
Equation	$\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2$
Description	n_i is the number of pixels of intensity i, where $i \in \{0, 1, ..., L-1\}$. p_i is the normalized histogram, where $p_i = \frac{n_i}{\sum_{i=1}^{L-1} n_i}$. Using $k, 0 < k < L-1$, as threshold, there are two classes: C_1 (pixels in $[0, k]$) and C_2 (pixels in $[k+1, L-1]$). P_1 is the probability of the class C_1, where $P_1 = P(C_1) = \sum_{i=1}^{k} p_i$. P_2 is the probability of the class C_2, where $P_2 = P(C_2) = \sum_{i=k+1}^{L-1} p_i = 1 - P_1$. m_1 is the mean intensity of the pixels in C_1, where $m_1 = \sum_{i=1}^{k} i \cdot P(i\vert C_1) = \sum_{i=1}^{k} i \cdot \frac{P(C_1\vert i)P(i)}{P(C_1)} = \frac{1}{P_1}\sum_{i=1}^{k} i \cdot p_i$, since $P(C_1\vert i) = 1, P(i) = p_i$, and $P(C_1) = P_1$. m_2 is the mean intensity of the pixels in C_2, similarly $m_2 = \frac{1}{P_2}\sum_{i=k}^{L-1} i \cdot p_i$. m_G is the mean global intensity, where $m_G = \sum_{i=1}^{L-1} i \cdot p_i$.
Sources	Ferrari (2018b)
Ref. By	T4 IM1 IM3

5.4 Goal Statements

Given the medical images as inputs, the goal statements are:

GS1: Calculate the optimal threshold value $k^\star$ with Otsu's Method.

GS2: Output the processed images representing the segmentation results with one threshold.

GS3: Calculate multiple optimal thresholds value $k_1^\star$ and $k_2^\star$ with Otsu's Method.

GS4: Output the processed images representing the segmentation results with multiple thresholds.

5.5 Theoretical Models

This section focuses on the general equations and laws that MIA is based on.

Number	T1
Label	**Mathematical Image**
Equation	$h : \mathbb{R}^a \rightarrow \mathbb{R}^b$
Description	In Mathematics, an image is defined as a function $h : \mathbb{R}^a \rightarrow \mathbb{R}^b$, where $\mathbb{R}^a$ denotes the set of a-dimensional spatial coordinates and $\mathbb{R}^b$ the set of b-dimensional feature values.
Source	Ferrari (2018a)
Ref. By	DD1

Number	T2
Label	**Global Threshold Method**
Equation	$g(x, y) = \begin{cases} L-1, & if\ f(x,y) > T_h \\ 0, & if\ f(x,y) \leq T_h \end{cases}$
Description	$f(x, y)$ and $g(x, y)$ are the feature values at (x, y) of the input and output image respectively. L is the number of the discrete levels of the feature value. T_h is the threshold value.
Source	Ferrari (2018b)
Ref. By	DD7 T3 IM2

Number	T3
Label	**Multiple Threshold Method**
Equation	$g(x,y) = \begin{cases} L-1, & if\ f(x,y) > T_{h2} \\ L/2, & if\ T_{h1} < f(x,y) \leq T_{h2} \\ 0, & if\ f(x,y) \leq T_{h1} \end{cases}$
Description	$f(x,y)$ and $g(x,y)$ are the feature values at (x,y) of the input and output image respectively. L is the number of the discrete levels of the feature value. T_{h1} and T_{h2} are the threshold values.
Source	Ferrari (2018b)
Ref. By	IM3

Number	T4
Label	**Otsu's Method**
Equation	$\sigma_B^2(k^\star) = \underset{0<k<L-1}{max}\ \sigma_B^2(k)$
Description	Otsu's method is aimed in finding the optimal value for the global threshold T. The optimal threshold value, $k^\star$, satisfies the above equation, where using k as threshold to divide all pixels of the image into two classes: C_1 (pixels in $[0,k]$) and C_2 (pixels in $[k+1, L-1]$). σ_B is the between-class variance.
Source	Ferrari (2018b)
Ref. By	IM1 IM3

5.6 Instance Models

This section transforms the problem defined in Section 5.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 5.3 to replace the abstract symbols in the models identified in Sections 5.5.

Medical image segmentation by Threshold Method can be solved by T2, T4, DD3, DD4, DD6

Number	IM1
Label	**Otsu's Method to find the optimal threshold value $k^\star$**
Input	$F_{X\times Y}$ from DD3, $f(x,y)$ from DD4, L from DD5, σ_B, k, P_1, P_2, m_1, m_2, m_G from DD7
Output	$k^\star$, such that $\sigma_B^2(k^\star) = \underset{0<k<L-1}{max} \sigma_B^2(k)$
Description	Since $P_1m_1 + P_2m_2 = m_G$ and $P_1 + P_2 = 1$, $\sigma_B^2 = P_1(m_1 - m_G)^2 + P_2(m_2 - m_G)^2 = P_1P_2(m_1 - m_2)^2$ Hence, for each value of k, σ_B can be computed, where $\sigma_B^2(k) = P_1(k)P_2(k)(m_1(k) - m_2(k))^2$
Sources	Ferrari (2018b)
Ref. By	IM2

Number	IM2
Label	**Use Otsu's Method to output segmentation image**
Input	$F_{X\times Y}$ from DD3, $f(x,y)$ from DD4, L from DD5, $k^\star$ from IM1
Output	$G_{X\times Y}$, such that for pixel at each (x,y), $g(x,y) = \begin{cases} 255, & if\ f(x,y) > k^\star \\ 0, & if\ f(x,y) \leq k^\star \end{cases}$
Description	The output image $G_{X\times Y}$ has the same size and format as the input image $F_{X\times Y}$. As a piece of 8-bit grayscale image, all of its pixels are with intensity of either 255 or 0, so it only shows the same information as a binary image.
Sources	Ferrari (2018b)
Ref. By	

Number	IM3
Label	**Otsu's Method to find multiple optimal threshold values $k_1^\star$ and $k_2^\star$**
Input	$F_{X\times Y}$ from DD3, $f(x,y)$ from DD4, L from DD5, σ_B, k, P_1, P_2, m_1, m_2, m_G from DD7
Output	$k_1^\star$ and $k_2^\star$, such that $\sigma_B^2(k_1^\star,k_2^\star) = \underset{0<k_1<k_2<L-1}{max} \sigma_B^2(k_1,k_2)$
Description	Similarly as the normal Otsu's Method, $\sigma_B^2(k_1,k_2) = P_1(k_1)(m_1(k_1)-m_G)^2+P_2(k_2)(m_2(k_2)-m_G)^2+P_3(k_3)(m_3(k_3)-m_G)^2$
Sources	Ferrari (2018b)
Ref. By	IM2

Number	IM4
Label	**Use multiple thresholds to output segmentation image**
Input	$F_{X\times Y}$ from DD3, $f(x,y)$ from DD4, L from DD5, $k_1^\star$ and $k_2^\star$ from IM3
Output	$G_{X\times Y}$, such that for pixel at each (x,y), $g(x,y) = \begin{cases} 255, & if\ f(x,y) > k_2^\star \\ 128, & if\ k_1^\star < f(x,y) \leq k_2^\star \\ 0, & if\ f(x,y) \leq k_1^\star \end{cases}$
Description	The output image $G_{X\times Y}$ has the same size and format as the input image $F_{X\times Y}$. As a piece of 8-bit grayscale image, all of its pixels are with intensity of 255, 188 or 0, so it shows more information than a binary image.
Sources	Ferrari (2018b)
Ref. By	

6 Variabilities

6.1 Assumptions

A1: The images are 2D and grayscale.

A2: The pixel format of images is the byte image, where the feature value is the gray intensity value stored as an 8-bit integer giving a range of possible values from 0 to

255.

6.2 Calculation

Variabilities	**Parameter of Variation**
Allowed input	set of {digital image, binary data representing the image}
Dimension of spatial coordinates (a)	$\mathbb{N}$
Dimension of feature values (b)	$\mathbb{N}$
Number of the discrete levels of the feature value (L)	$\mathbb{N}$
Calculate methods	set of {Global threshold method, multiple threshold method}
Number of threshold values in Otsu' Method with multiple thresholds (number of $k_1, k_2, ...$)	set of $\{1, 2, 3, ..., L-2\}$

Table 2: Calculation Variabilities

6.3 Output

Variabilities	**Parameter of Variation**
Output	set of {digital image, binary data representing the image}
Optimal threshold value ($k^\star$)	set of $\{1, 2, 3, ..., L-2\}$
Optimal threshold values ($k_1^\star$ and $k_2^\star$)	set of $[1, k_2^\star - 2]$ and $[k_1^\star + 2, L-2]$
Number of optimal threshold values in Otsu' Method (number of $k_1^\star, k_2^\star, ...$)	set of $\{1, 2, 3, ..., L-2\}$

Table 3: Output Variabilities

7 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit.

7.1 Functional Requirements

R1: The user shall make sure that the input image must be 2D 8-bit grayscale medical image, the pixel format must be the byte image, where the feature value must be the gray intensity value stored as an 8-bit integer giving a range of possible values from 0 to 255. The user shall also decide which method to use, global threshold or multiple threshold.

R2: MIAshall guarantee that the output file is the same format and size as the input file.

R3: The user shall provide correct calculate according to Instance Models.

R4: MIAshall verify that the input data are valid and meet R1.

R5: MIAshall output the correct optimal threshold value $k^\star$ or $k_1^\star$ and $k_2^\star$ accordingly. MIAshall also output segmentation image $G_{X\times Y}$.

7.2 Nonfunctional Requirements

- Correctness: The output image will be generally similar to the output from VTK.
- Maintainability: MIA shall be documented with an SRS, VnV, MG, and MIS.
- Performance: MIA shall be able to calculate the output not much slower then VTK.
- Reliability: The mean time between failures (MTBF) will be longer than the average MTBF of a sample of the OTS solutions.
- Robustness: The system will not crash when a user provides invalid input.
- Usability: MIA should be easy and satisfying for users to learn and use.

8 Likely Changes

LC1: This document only describes multi-threshold method with 2 thresholds. It could list more Instance Models with more thresholds in the future.

LC2: This document only specifies one segmentation method. It could list specifications for all the methods in Table 1 in the future.

LC3: This document only describes segmentation. It could list specifications for all the sections in analysis in Figure 1 in the future.

9 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 4 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other.

	T1	T2	T3	T4	DD1	DD2	DD3	DD4	DD5	DD6	DD7	IM1	IM2	IM3	IM4
T1															
T2							X	X	X	X					
T3		X					X	X	X	X					
T4									X		X				
DD1	X														
DD2					X										
DD3						X		X				X	X	X	X
DD4									X			X	X	X	X
DD5												X	X	X	X
DD6									X						
DD7		X						X	X	X		X		X	
IM1				X			X	X	X	X	X		X		
IM2		X										X		X	
IM3			X	X							X				X
IM4															

Table 4: Traceability Matrix Showing the Connections Between Items of Different Sections

References

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