OR 7240 – Fall 2019

Homework 1

Due: Thursday, 3 October 2019

Instructions:

- This assignment is to be done individually - you must do your own work.

Part I. Modelling Problems (50 Points)

1. (10 Points) Write down the LP formulation of the maximum flow problem and derive its dual problem.
2. (5 Points) Reformulate the problem

$$Minimize\ 3x_2 + |x_1 - x_3| \quad s.t.$$

$$|2x_1 - 1| + |x_2| - |x_3| \leq 4$$

$$x_3^2 - 6x_3 + 8 \leq 0$$

as a linear program in standard form.

3. (5 Points) Solve the above problem using the "linprog" toolbox of MATLAB and attach your code and solution report.

Comment: If you do not have access to MATLAB, feel free to use any open source package. "SolveLP" or "lpSolve" in R are good substitutes.

4. (10 Points) In the lecture you have seen two different formulations for TSP. Another formulation for TSP is provided by Miller-Tucker-Zemlin and is called MTZ formulation. Write down this formulation and prove that it is a valid formulation for TSP. Hint: You need to show that it eliminates all subtours and not any tour.
5. (10 Points) Based on your answer to last problem, prove that the following formulation is also a valid formulation for TSP.

$$Minimize \sum_{(i,j)\in E} w_{ij}x_{ij} \quad s.t.$$

$$\sum_{i:(i,j)\in E} x_{ij} = 1\,, \qquad for\ j \in V$$

$$\sum_{i:(i,j)\in E} x_{ij} = 1\,, \qquad for\ i \in V$$

$$t_i - t_j + nx_{ij} - x_{ij} \leq n - 2, \quad \forall\ 1 \leq i \neq j \leq n$$

$$t_0 = 0$$

$$t_i \in \{0, 1, 2, \cdots\}, \qquad \forall i$$

$$xij \in \{0, 1\}, \qquad \forall (i, j) \in E$$

6. (10 Points) Let f(x): $R_+ \rightarrow R$ be a univariate continuous piecewise linear function.
   (a) Provide a binary ILP formulation for minimization of f(x).
   (b) (5 Bonus Points) Can you provide a different formulation?

Part II. Complexity Theory Problems (50 Points)

7. (5 Points) Prove that $5^{log_2^n} \in O(3^{\sqrt{n}})$.
8. (10 Points) Try to rigorously prove that the independent set problem is polynomially reducible to the vertex cover problem, i.e., Independent set $\leq_p$Vertex cover.

Comment: If you got stuck you can find it in many sources.

9. (10 Points) We know that Problem X_1 is NP-complete and it is polynomially reducable to problem X_2. Problem X_2 is also polynomially reducible to $X_3 \in$ NP. Prove that problem X_3 is NP-complete.

10. (25 Points) List and formally define Karp's initial 21 NP-complete problems.

Hint: *The goal of this problem is to help you to make yourself familiar with these problems. If you need to prove the NP-completeness of a given problem, it really helps to be already familiar with these problems.*