3/26 Tut

Linear Programming

Ex. Given graph $G=(V,E)$ w/ reqs. $r: E \to \mathbb{R}$

want: a label $\ell: V \to [-10^6, 10^6]$ s.t.
$\forall (u,v) \in E \quad \ell(v) + \ell(u) \geq r(u,v)$

optimize: minimize each $\ell(v)$.

LP formulation:

Variables: $\forall v \in V, \quad x_v$ ← represents $\ell(v)$

Constraints: $\forall v \in V, \quad x_v \leq 10^6$
$\cancel{x_v \geq -10^6} \quad -x_v \leq 10^6$
$\forall (u,v) \in E, \quad \cancel{x_u + x_v \geq r(u,v)}$
$-x_u - x_v \leq -r(u,v)$

Objective: $\cancel{\text{minimize}}\ \sum_{v \in V} x_v$
maximize $\sum_{v \in V} -x_v$

↳ to change to non-neg. standard form, we can replace each x_v w/ $(x_v^+ - x_v^-)$ and add $x_v^+, x_v^- \geq 0$

☆ Every feasible sol^n $x \in \mathbb{R}^n$ to the original LP yields a feasible sol^n $x^+, x^- \in \mathbb{R}^n$ to the new standard LP w/ $x_v = x_v^+ - x_v^-$ (vice versa).

Vertex cover : given $G = (V, E)$,

0-1 ILP : **variables:** $\forall v \in V \; x_v$

NPC, too complex to solve VC

constraints: $\forall v \in V, \; 0 \le x_v \le 1$

$\forall v \in V, \; x_v \in \mathbb{Z}$ (NP-complete)

↳ $x_v = 0 \Rightarrow$ not in cover

$x_v = 1 \Rightarrow$ in cover

$\forall (u,v) \in E, \; x_u + x_v \ge 1$

objective: minimize $\Sigma_{v \in V} x_v$

LP (relaxed) : minimize $\Sigma_{v \in V} x_v$

subject to $\forall (u,v) \in E, \; x_u + x_v \ge 1$

$\forall v \in V, \; 0 \le x_v \le 1$

Note: feasible in original (ILP) $\Rightarrow$ feasible in relaxed

$\therefore$ (min) value of original $\ge$ value of original

A2 Q6 give ex. where $>$.