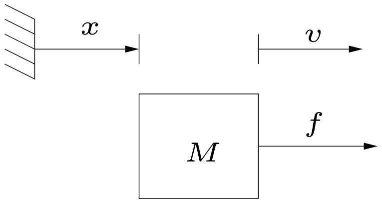
Ideal translatoric elements

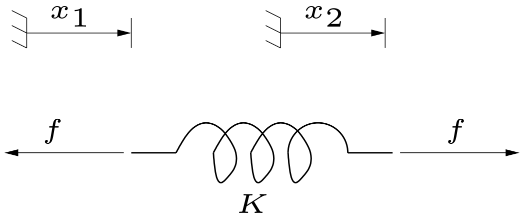
Representation of a mass



Physical law: $M\,\dfrac{dv}{dt} = f$

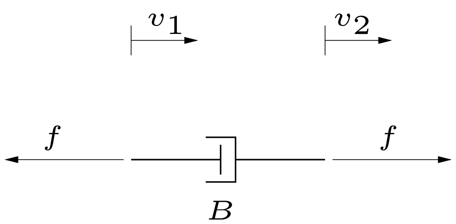
or: $M\,\dfrac{d^2x}{dt^2} = f$

Representation of a spring



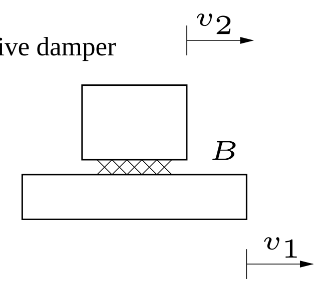
Physical law: $f = K\,(x_2 - x_1)$

Ideal damper



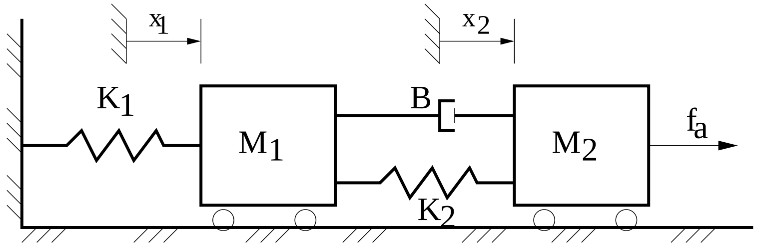
Physical law: $f = B\,(v_2 - v_1)$

Alternative damper

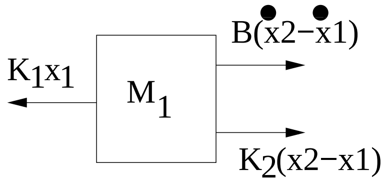


Linear models of translatoric movement can be composed by three idealised elements

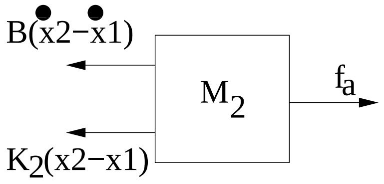
Freebody diagram



FREE BODY FOR M1:



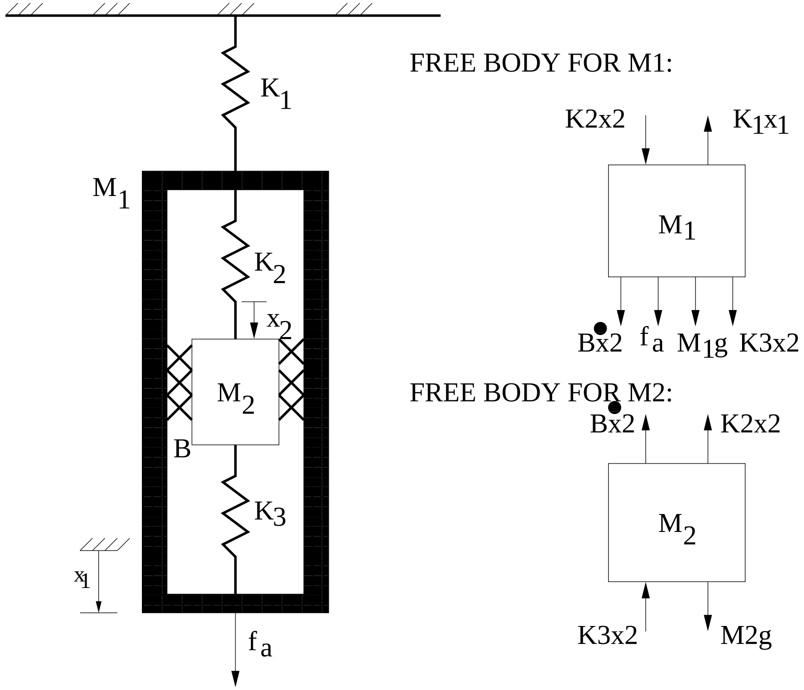
FREE BODY FOR M2:



On the freebody diagram note

- Direction of force arrows define sign convention
- Free bodies represent masses. Additional points can represent massless junctions
- Coordinates represent position of centres of mass. Origin of coordinate axis are best chosen to be an equilibrium point. Constant forces like gravity disappears

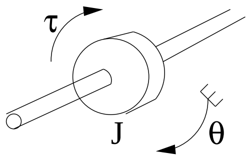
Choice of coordinates axis



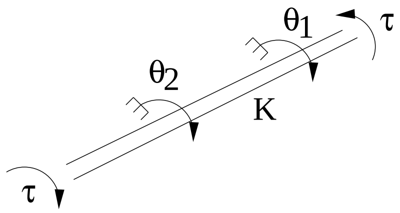
- First choose origin of axis such that $x_1 = x_2 = 0$ when springs are relaxed
- Next determine equilibrium point $\bar{x}_1,\ \bar{x}_2$ (blackboard)
- If origin is changed to equilibrium point ($x_1^{\Delta} = x_1 - \bar{x_1},\ x_2^{\Delta} = x_2 - \bar{x_2}$), constant forces like gravity disappear from equation (blackboard)

Ideal elements, rotation

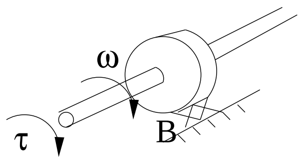
Det ideelle inertimoment:



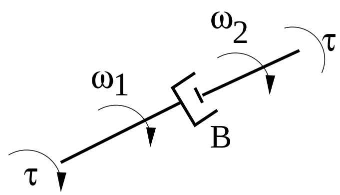
Den ideelle rotationsfjeder:



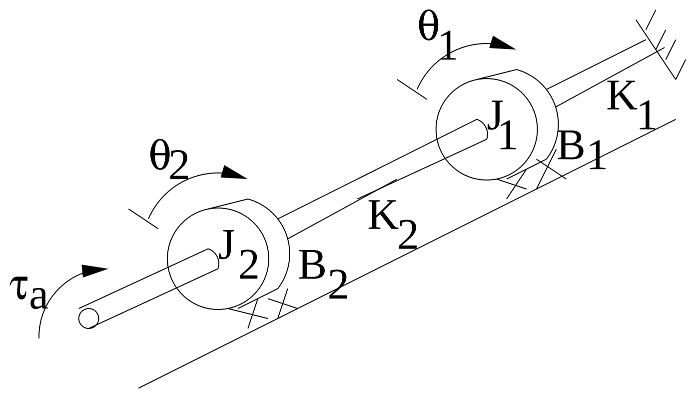
Den ideelle rotationsdæmper:



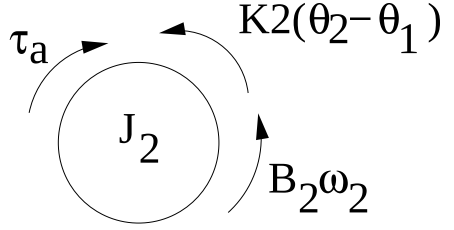
ELLER



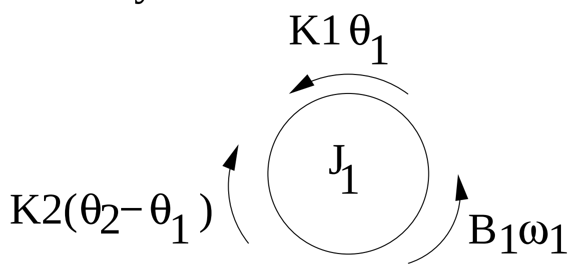
Freebody, rotation



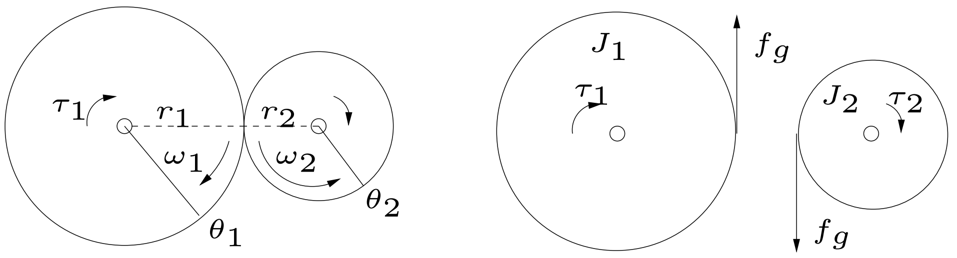
Free body for J2:



Free body for J1:



Gear



Assume no sliding such that we have the geometric relations between angles, angular velocities and angular accelerations

$$\theta_1 r_1 = \theta_2 r_2 \;\Rightarrow\; N = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

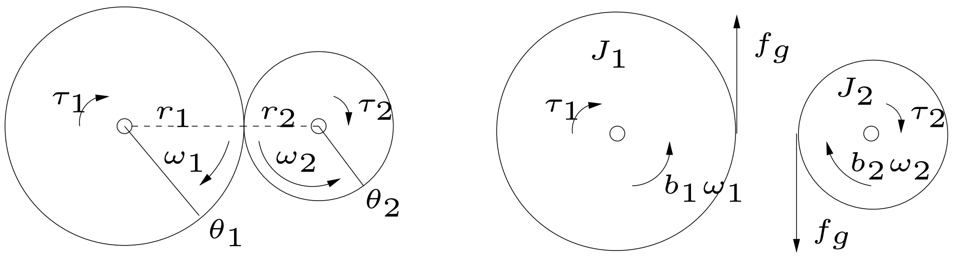
As a start consider steady state and the find conditions for equilibrium

$$\tau_1 - r_1 f_g = 0 \;\wedge\; \tau_2 - r_2 f_g = 0$$

Giving the steady state relation between torques

$$\frac{\tau_2}{\tau_1} = \frac{r_2}{r_1} = \frac{1}{N}$$

Gear



Now we will write up dynamic equations. We have added friction on both wheels

$$J_1\dot{\omega_1} = \tau_1 - r_1 f_g - b_1\omega_1$$

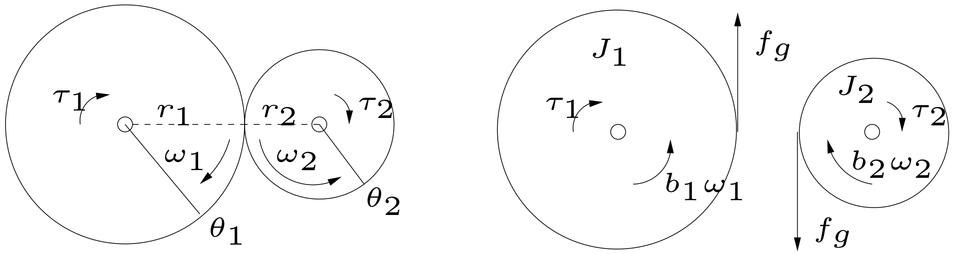
$$J_2\dot{\omega_2} = -\tau_2 + r_2 f_g - b_2\omega_2$$

We can now isolate f_g from the second equation and insert in the first

$$f_g = \frac{1}{r_2}J_2\dot{\omega_2} + \frac{1}{r_2}\tau_2 + \frac{1}{r_2}b_2\omega_2 = \frac{r_1}{r_2^2}J_2\dot{\omega_1} + \frac{1}{r_2}\tau_2 + \frac{r_1}{r_2^2}b_2\omega_1$$

$$J_1\dot{\omega_1} = \tau_1 - r_1\left(\frac{r_1}{r_2^2}J_2\dot{\omega_1} + \frac{1}{r_2}\tau_2 + \frac{r_1}{r_2^2}b_2\omega_1\right) - b_1\omega_1$$

Gear



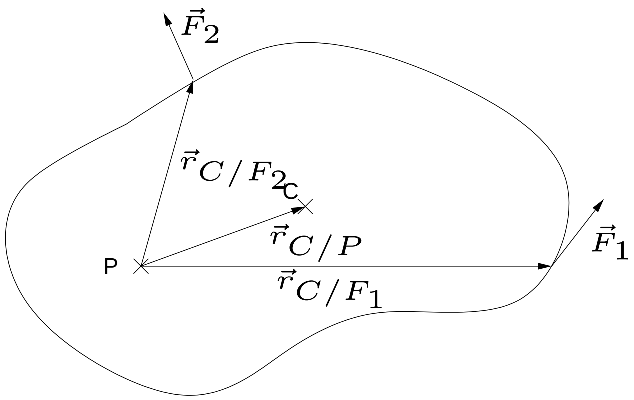
$$J_1\dot{\omega_1} = \tau_1 - r_1(\frac{r_1}{r_2^2}J_2\dot{\omega_1} + \frac{1}{r_2}\tau_2 + \frac{r_1}{r_2^2}b_2\omega_1) - b_1\omega_1$$

Collecting the terms gives the following relation, showing how the gear transfers torques, moments of inertia and damping factors

$$(J_1 + \frac{r_1^2}{r_2^2}J_2)\dot{\omega_1} = \tau_1 - \frac{r_1}{r_2}\tau_2 - (b_1 + \frac{r_1^2}{r_2^2}b_2)\omega_1$$

$$(J_1 + N^2 J_2)\dot{\omega_1} = \tau_1 - N\tau_2 - (b_1 + N^2 b_2)\omega_1$$

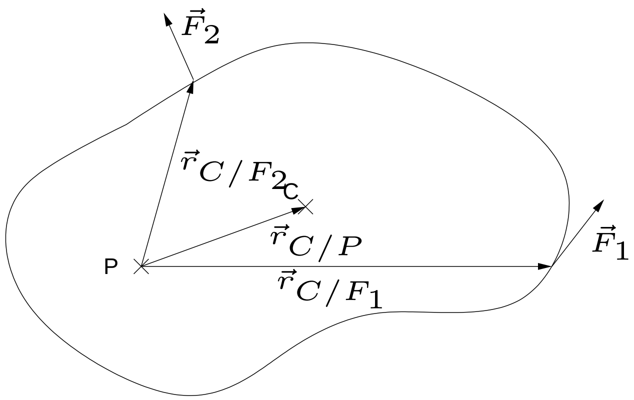
General Momentum



The equation for momentum we used til now was valid for a rigid body which can rotate about one axis. A more general equation can be formulated for arbitrary motion for moment around a point, P, which may be accelerated.

$$\Sigma M_p = \dot{H}_p + m\vec{r}_{C/P} \times \vec{a}_p$$

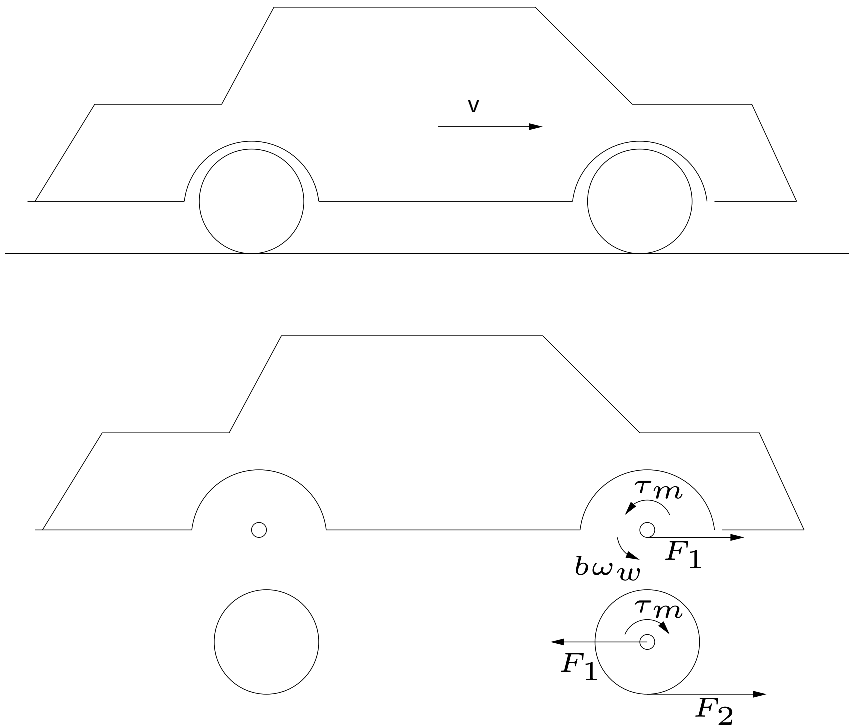
General Momentum



$$\Sigma M_p = \dot{H}_p + m\vec{r}_{C/P} \times \vec{a}_p$$

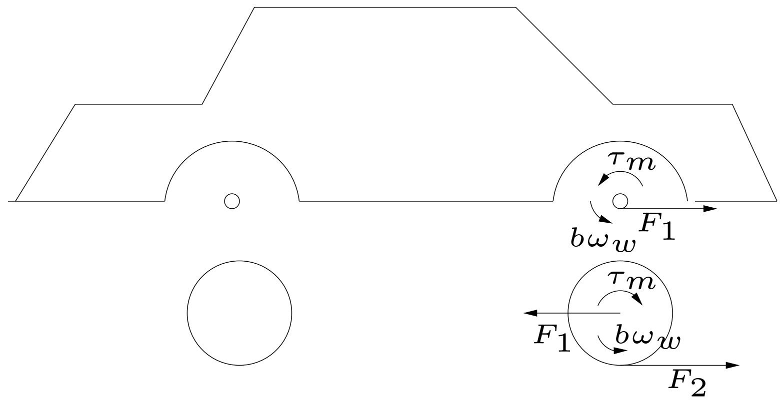
- first term : $\Sigma M_p = \Sigma_i \vec{r}_{P/F_i} \times \vec{F}_i$, the sum of three axis moments of forces around the point P.
- second term: $\dot{H}_p$, derivative of the three axis angular momentum around P: $H_P = \Sigma_i \vec{r}_{Pi} \times (m_i \vec{v}_i)$ or $H_P = \int \vec{r} \times \vec{v} dm$.
- third term: $m\vec{r}_{C/P} \times \vec{a}_p = \vec{r}_{C/P} \times (m\vec{a}_p)$ the moment of an artificial force $m\vec{a}_p$ due to the acceleration of the point P.

Car



For the car we want to control velocity. Therefore we want to formulate sufficient equations to give a model which connect torque from the motor to translational velocity of the car.

Car freebody



The wheel has translation and rotation, so we formulate two equations for the wheel and one translational equation for the body of the car

Car translation:

$$M_c \dot{v} = F_1$$

Wheel translation

$$M_w \dot{v} = F_2 - F_1$$

Wheel rotation. Angular momentum around center of mass

$$J_w \dot{\omega_w} = \tau_m - r_w F_2$$

Car freebody

$$M_c \dot{v} = F_1$$

$$M_w \dot{v} = F_2 - F_1$$

$$J_w \dot{\omega_w} = \tau_m - r_w F_2 - b\omega_w$$

We insert $\omega_w = \frac{v}{r_w}$ and divide the last equation with r_w

$$\frac{J_w}{r_w^2}\dot{v} = \frac{1}{r_w}\tau_m - F_2 - \frac{b}{r_w^2}v$$

Now the three equations can be added, thereby eliminating the unknown forces F_1 and F_2

$$(M_c + M_w + \frac{J_w}{r_w^2})\dot{v} = -\frac{b}{r_w^2}v + \frac{1}{r_w}\tau_m$$

which also give us the transfer function from motor torque to velocity

$$\frac{V(s)}{\tau_m(s)} = \frac{\frac{1}{r_w}}{(M_c + M_w + \frac{J_w}{r_w^2})s + \frac{b}{r_w^2}}$$