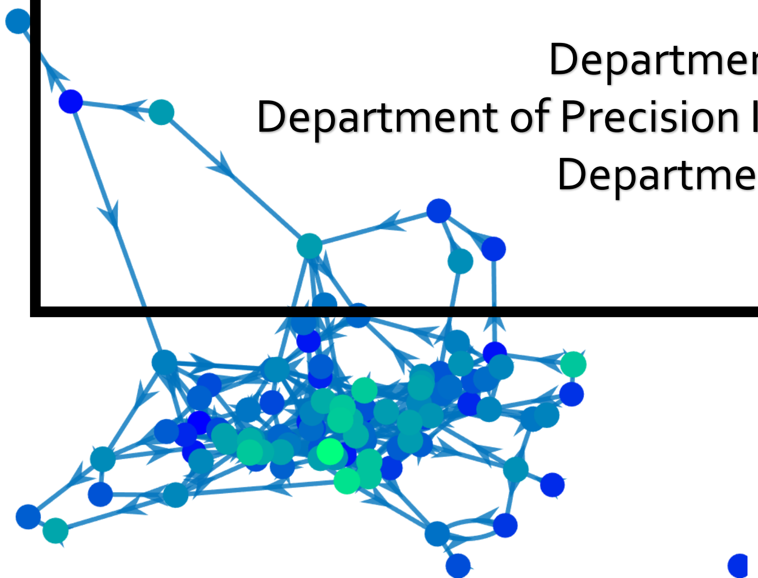
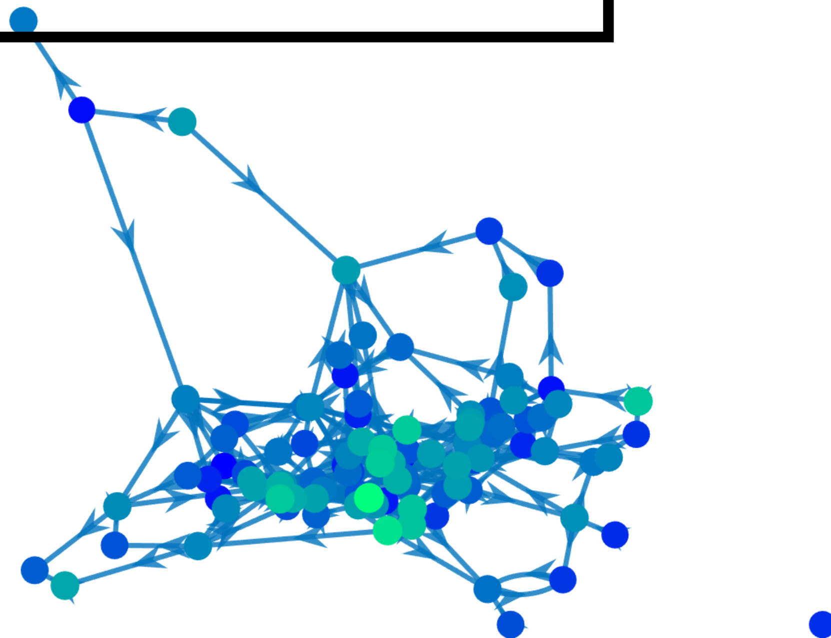
Multi-dimensional Dynamics underlying Neural Information Processing: From Neuroscience Theoretical Foundations to Advanced Artificial Neural Network Applications

Yang Tyan, Guoqi Li, Dan Zhang, Oren Raz, **Pei Sun**

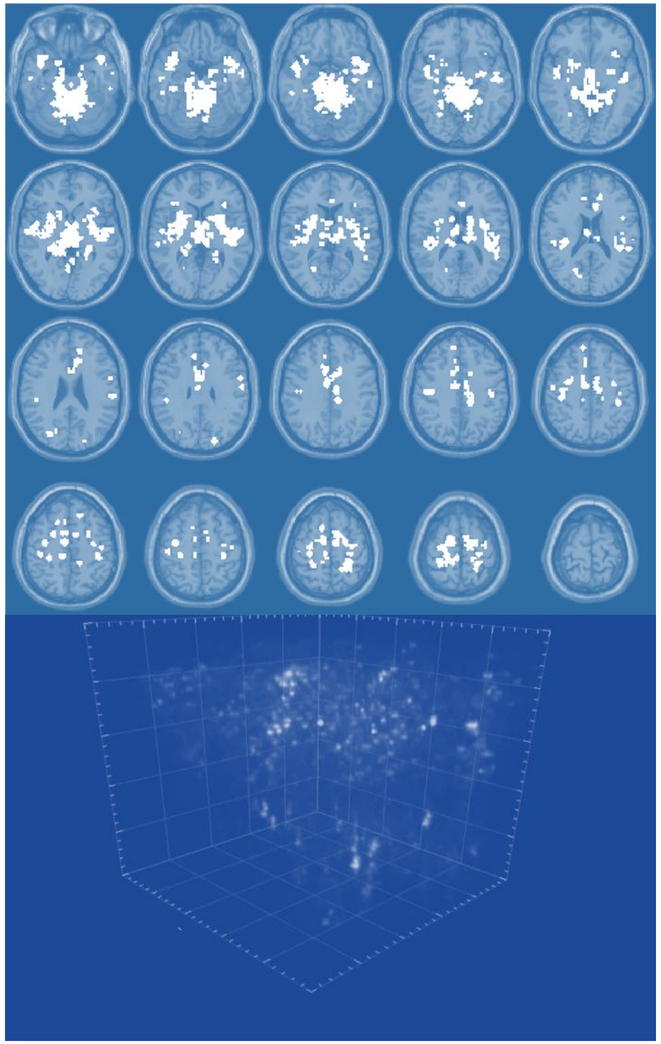
Department of Psychology & Tsinghua Laboratory of Brain and Intelligence, Tsinghua University
Department of Precision Instrumentation & Center for Brain Inspired Computing Research, Tsinghua University
Department of Physics of Complex Systems, Institute of Physics, Weizmann Institute of Science

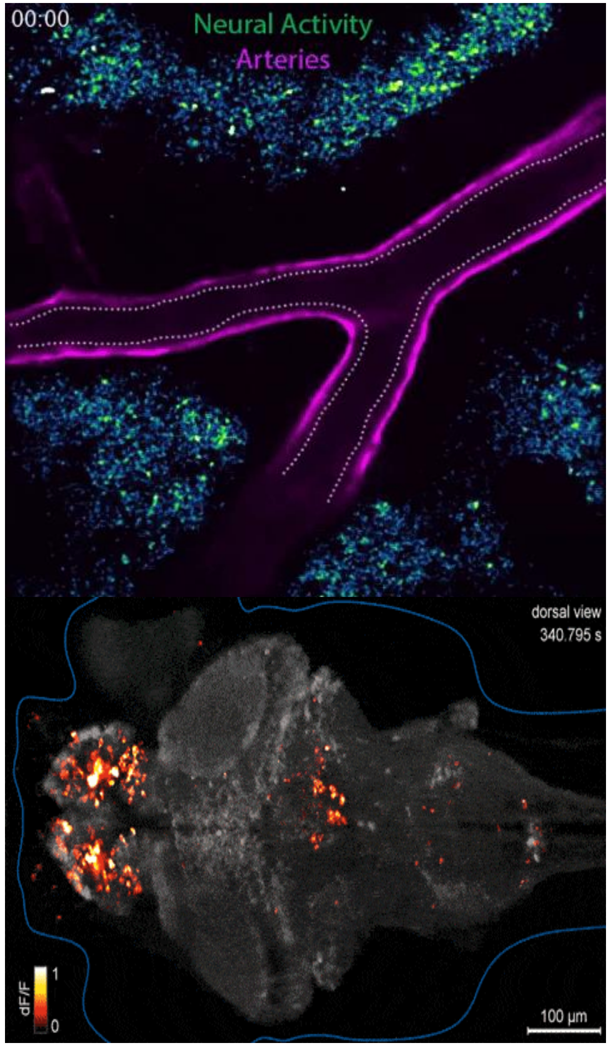


What the background is



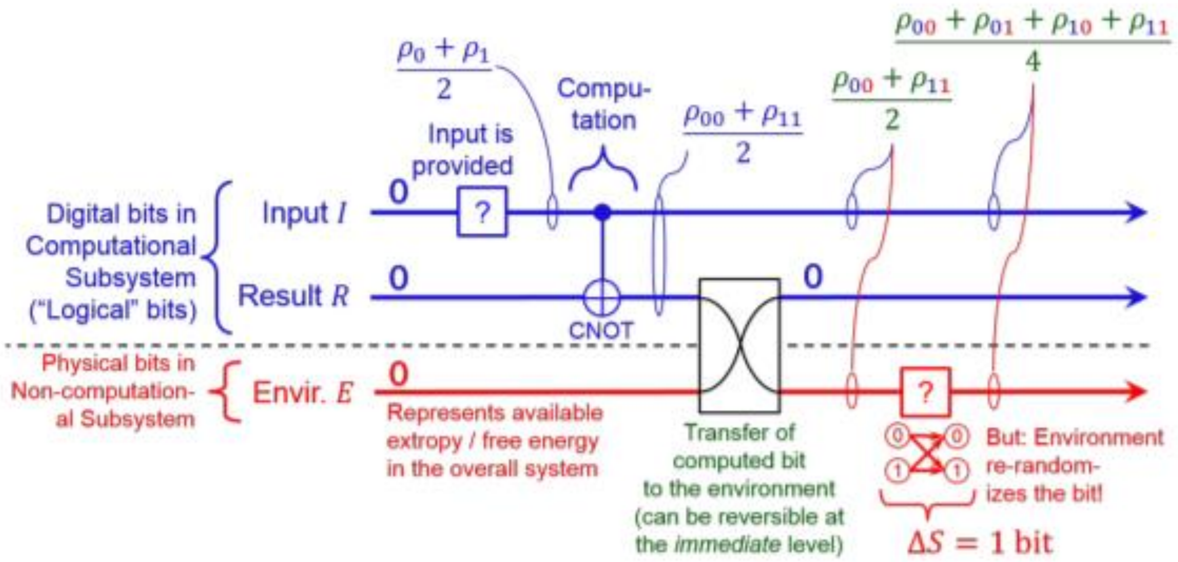
Information - Dynamics

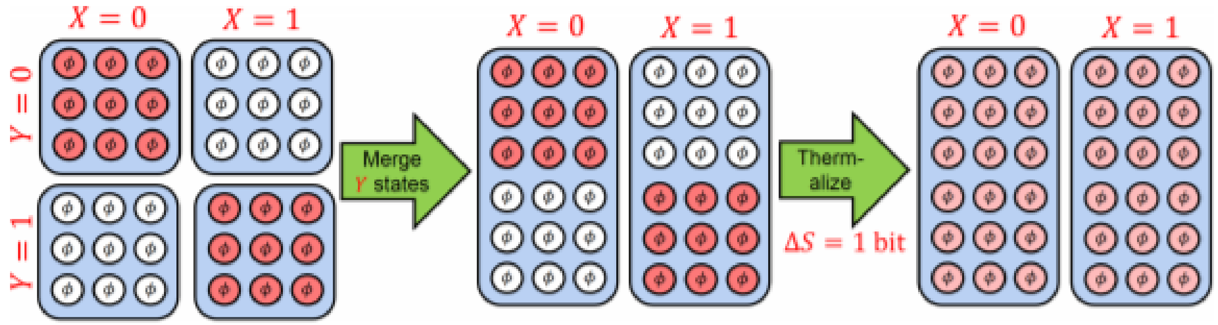


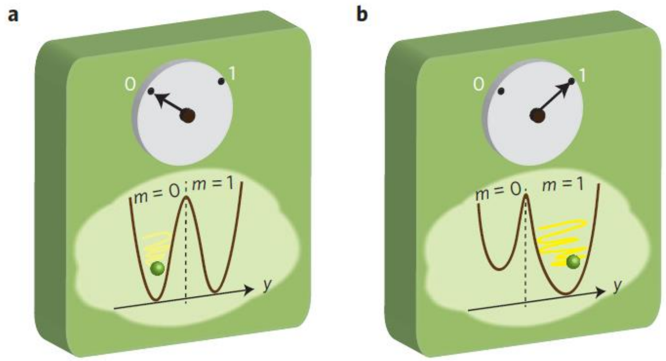


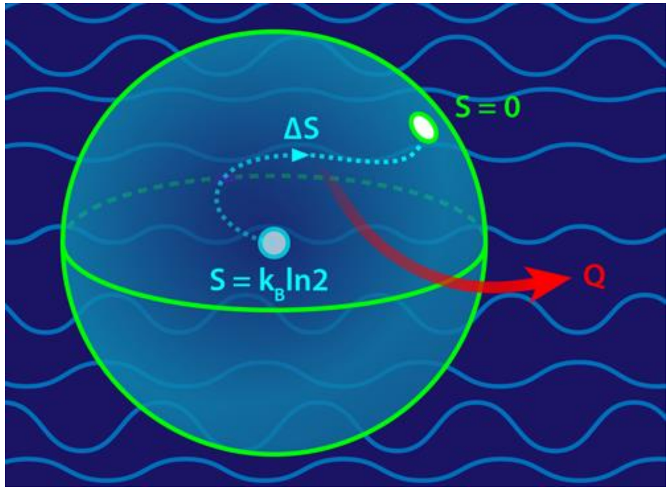
As you are reading this sentence, synergistic neural dynamics emerges in your brain (a non-isolated system of neurons) to encode the text information, changing the thermodynamic state of your brain continuously.

Information - Thermodynamics





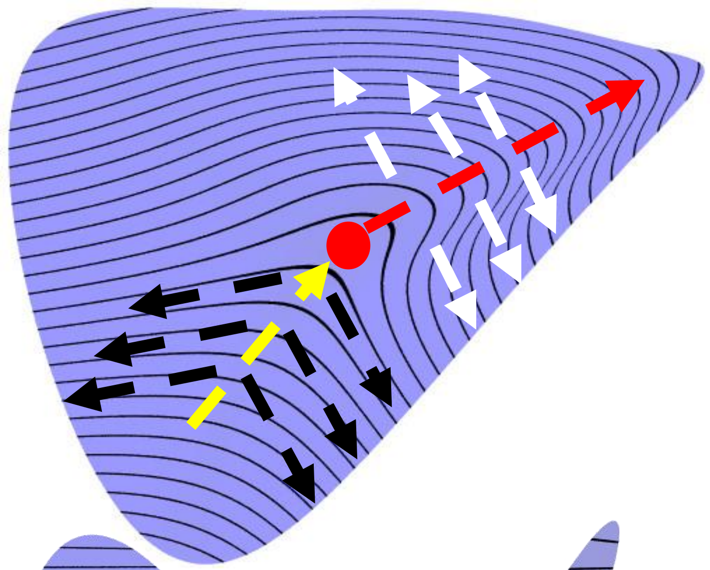


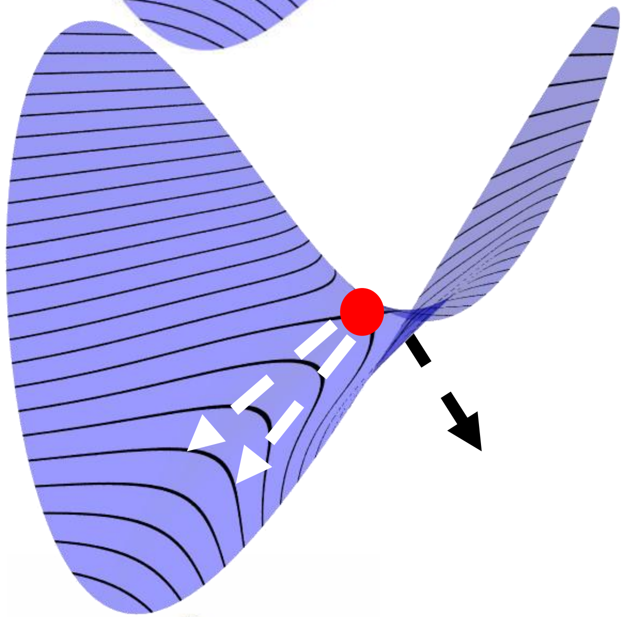


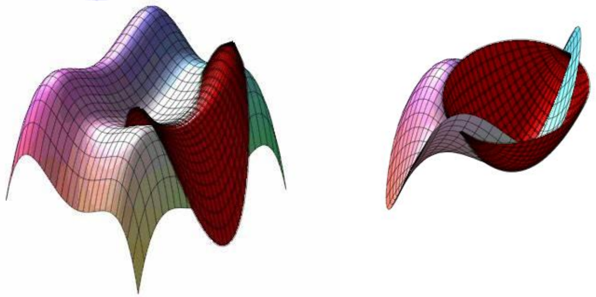
Information is physical.

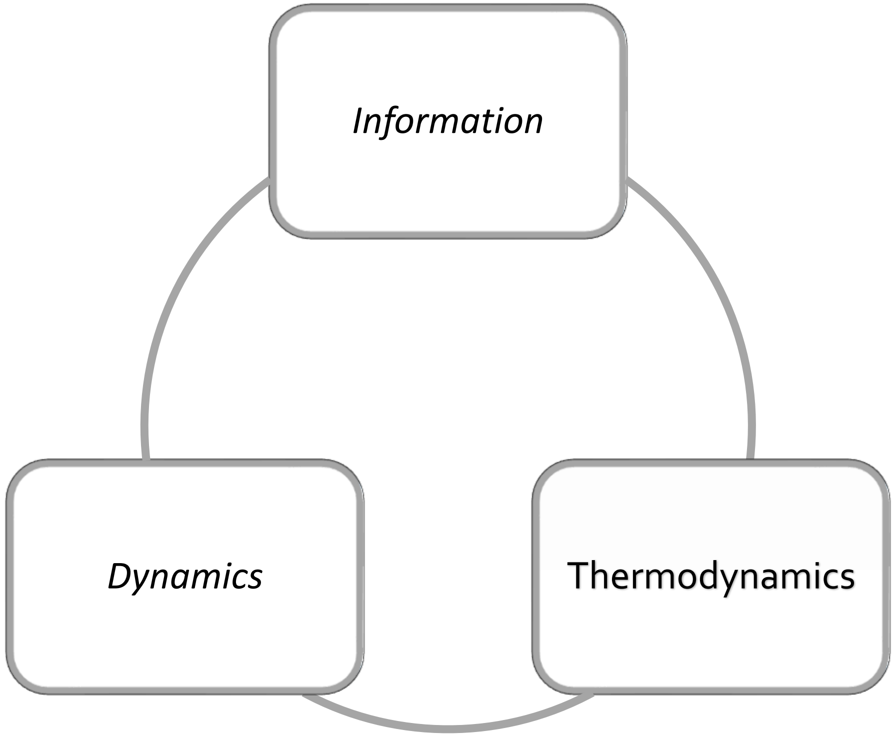
Information corresponds to energy.

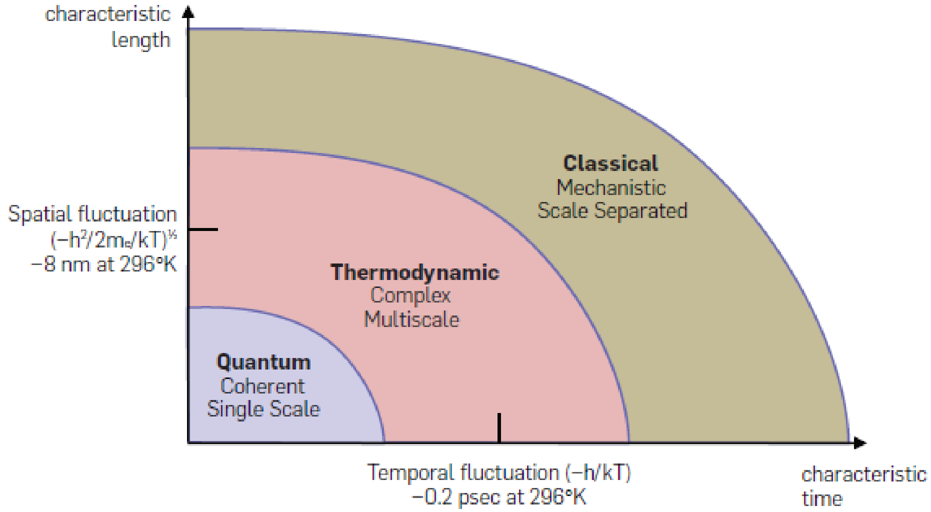
Any information processing is a thermodynamics process as well.

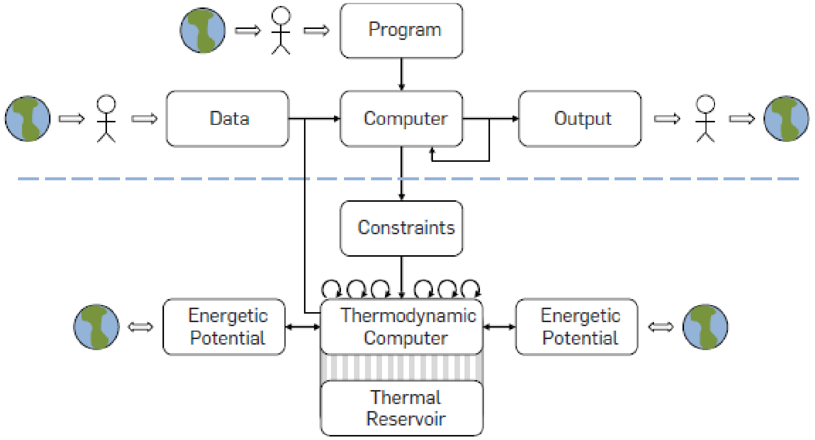




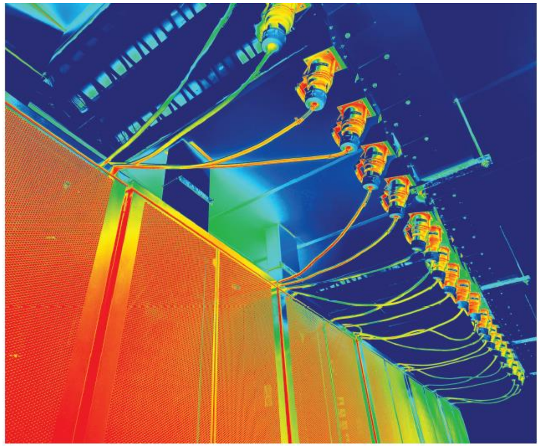




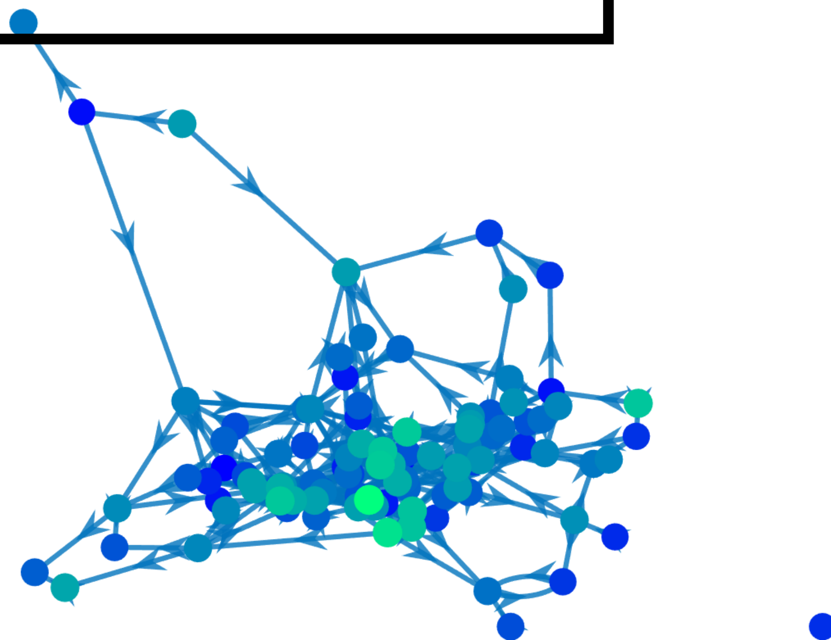


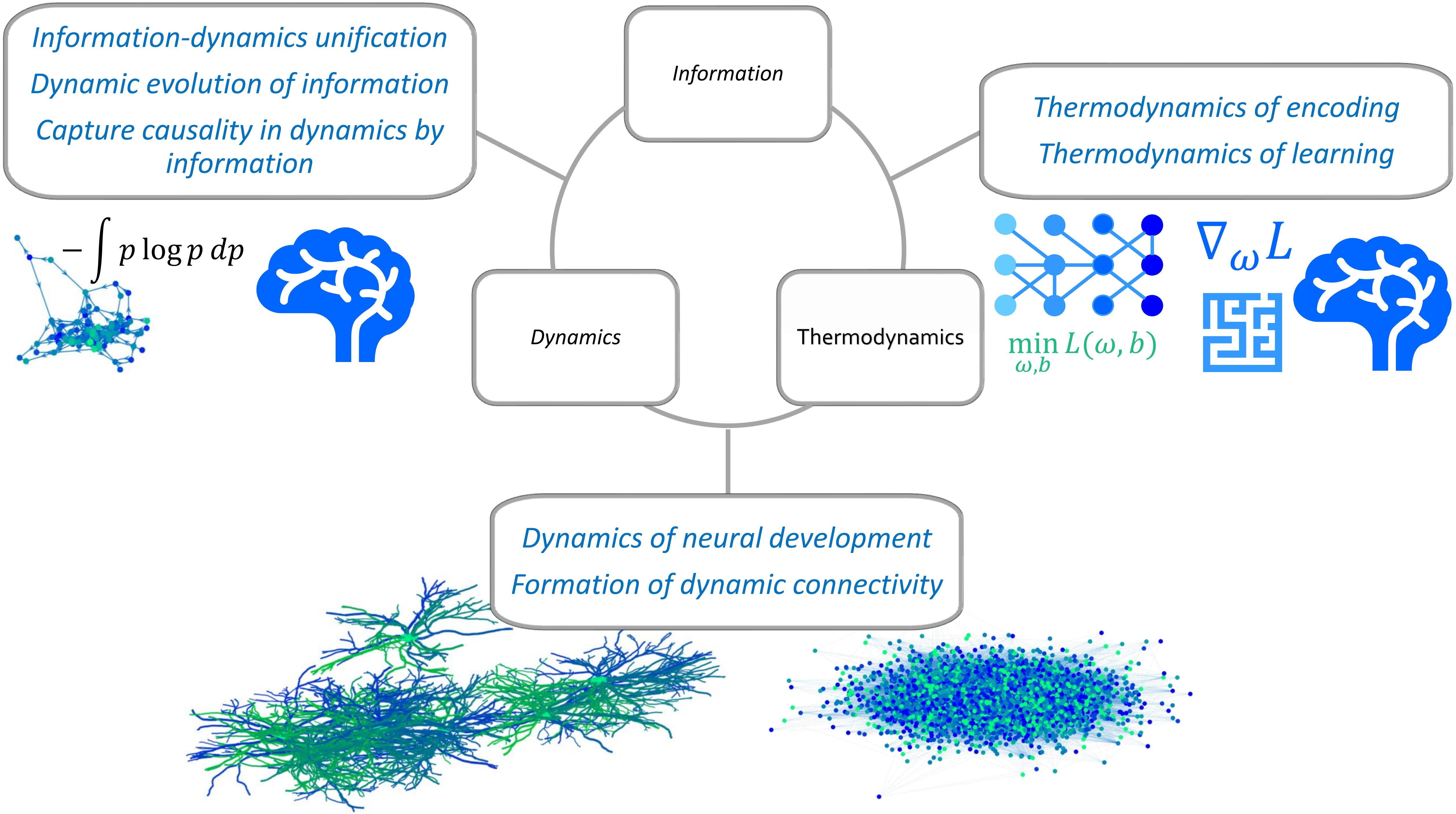


Information, dynamics, and thermodynamics always come together



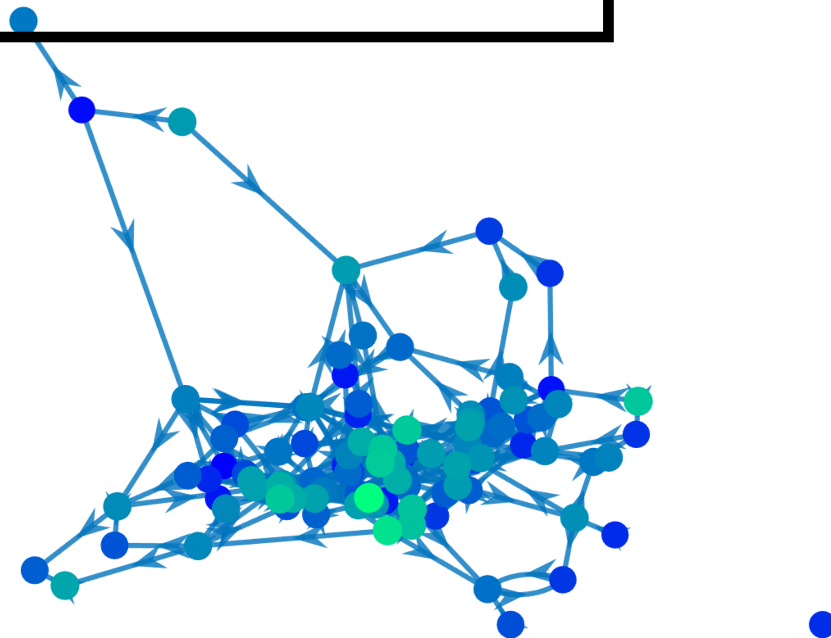
Overview of our research framework





Information-dynamics unification
Dynamic evolution of information
Capture causality in dynamics by information
Information
Thermodynamics of encoding
Thermodynamics of learning
$-\int p \log p \, dp$
$\nabla_\omega L$
Dynamics
Thermodynamics
$\min_{\omega,b} L(\omega, b)$
Dynamics of neural development
Formation of dynamic connectivity

Thermodynamics-Dynamics



Dynamics of neural development

Yang Tian, Aohua Cheng, Yunhui Xu, Hedong Hou, Weihua He, Guoqi Li, Pei Sun,
Neural morphological development for brain modeling and neuromorphic learning

Theoretical contribution

- We present an semi-analytical framework to formalize key biophysical processes underlying single neuron and neural cluster morphological development, such as the coupled reaction-diffusion of tubulin, calcium, and MAP-2 and the molecular guidance of axons.

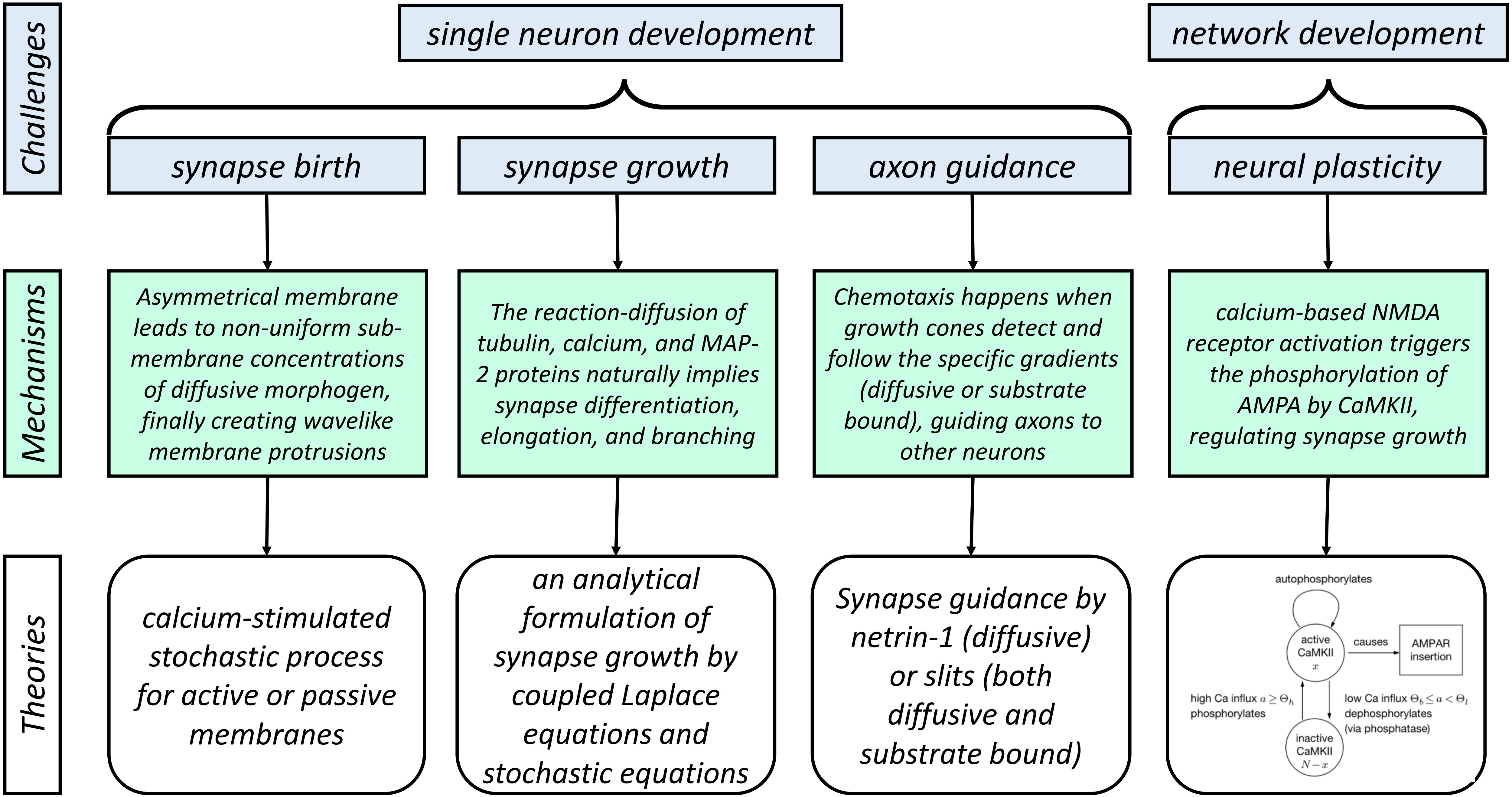
- An efficient computational implementation of this framework, Asuka, is proposed to reproduce critical development steps, such as synapse growing, competing, branching, path finding, and connecting.

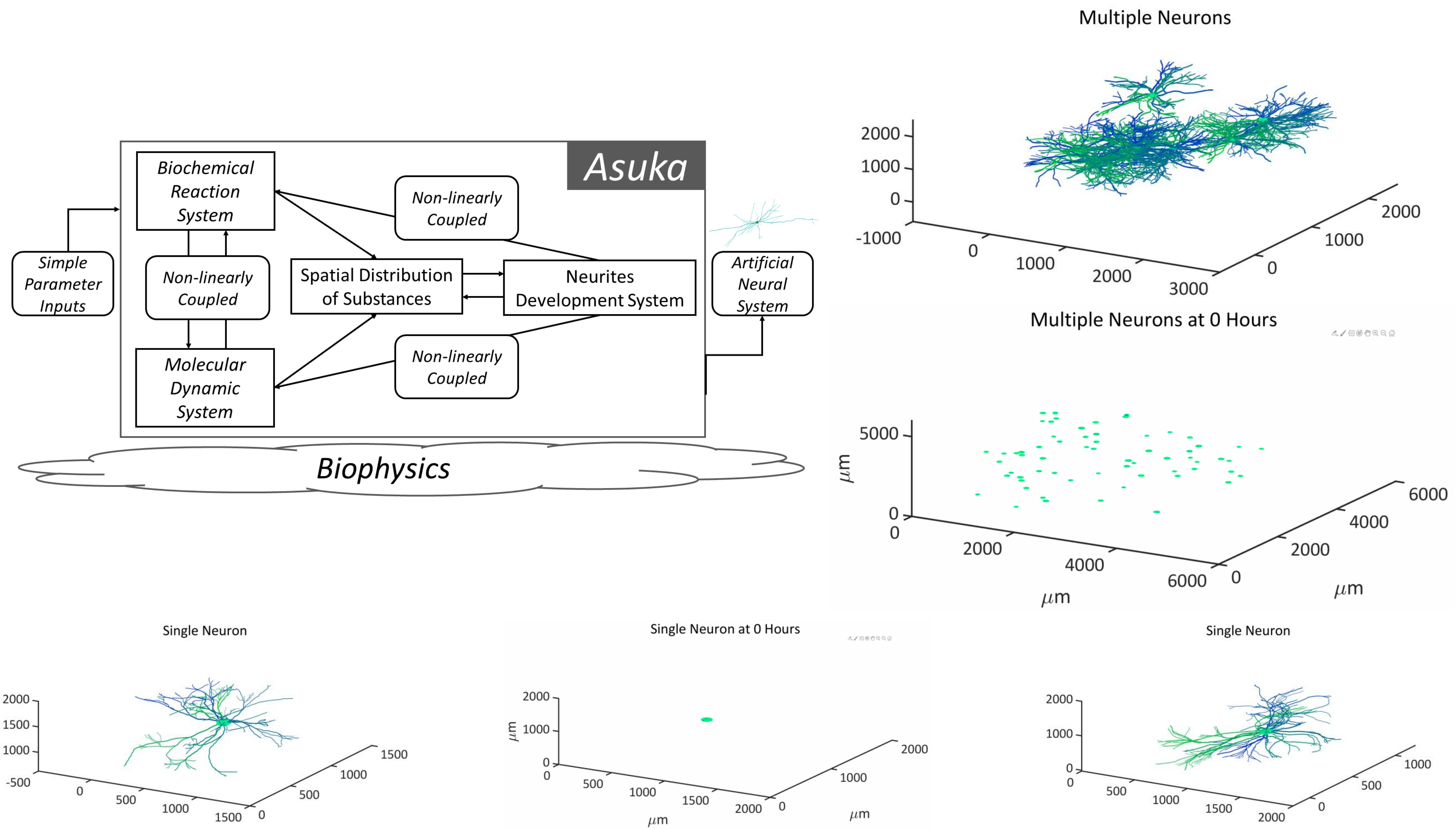
- We attempt to create dynamic coupling between the neural morphodynamics system and the training framework of artificial neural networks

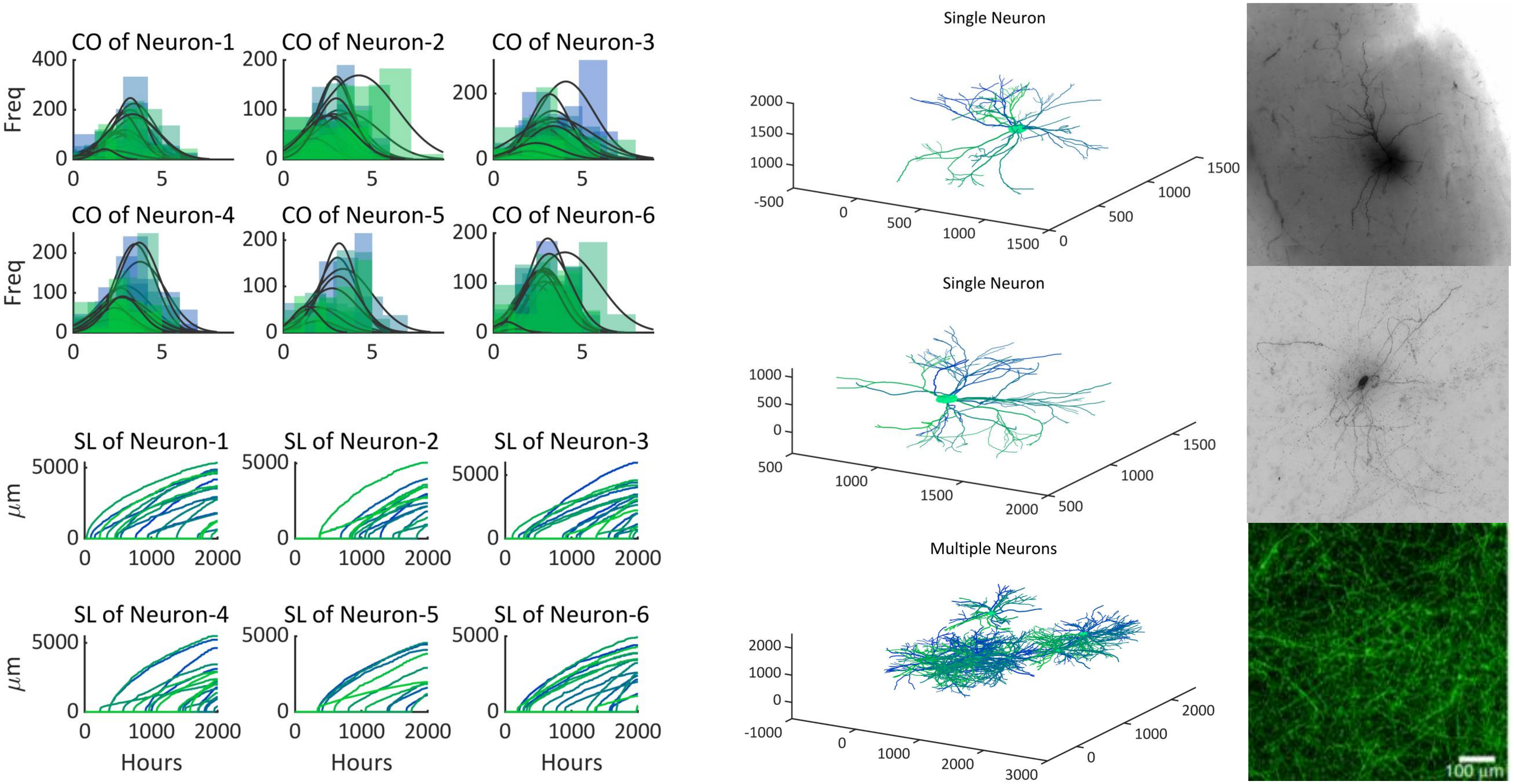
Experimental contribution

- A computational system to analyze the long-term and large scale neural morphodynamics with high spatio-temporal resolution (much higher than the *vivo* experiment).

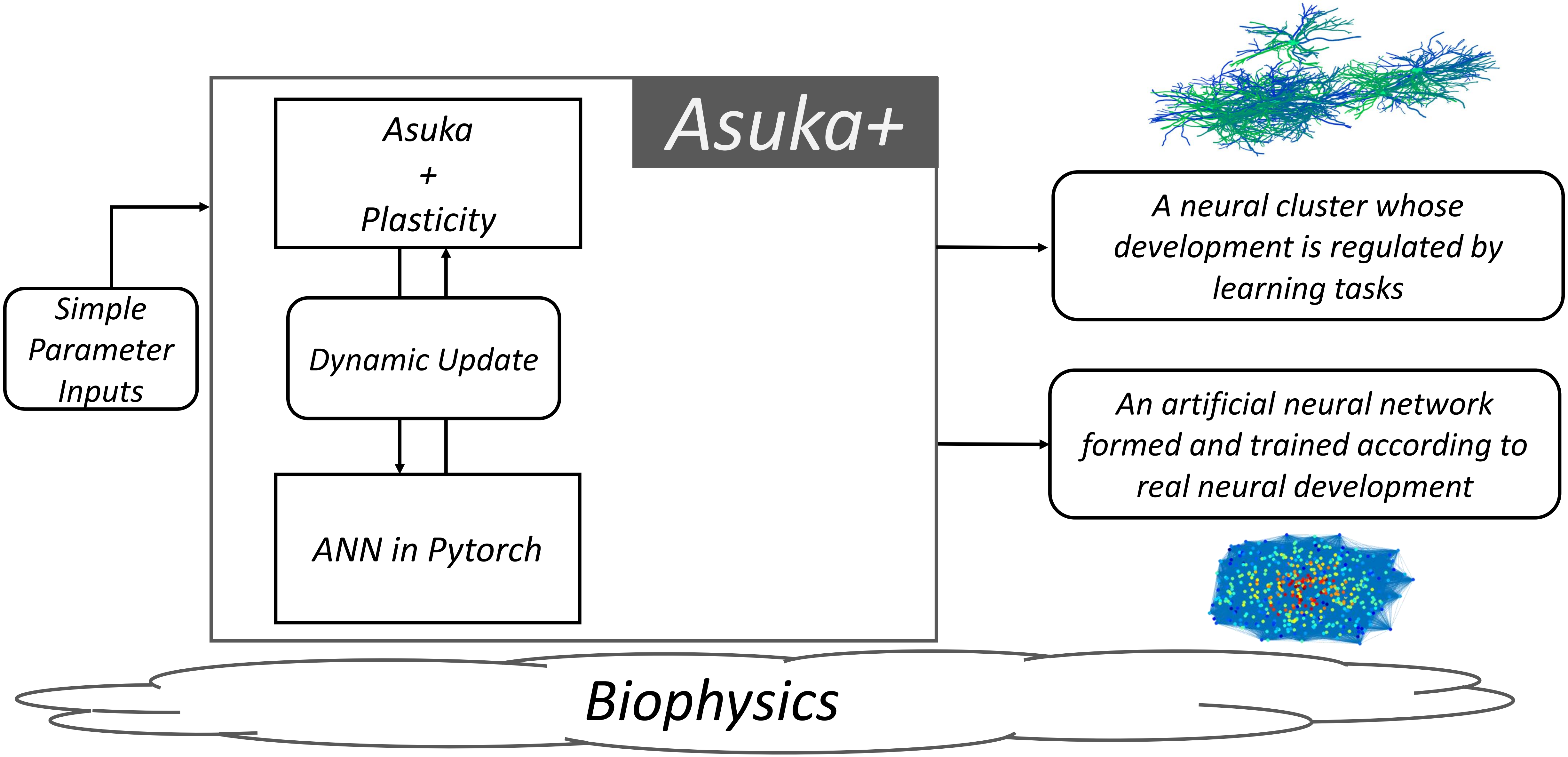
- A possibility to guide the development of neural clusters with the training of artificial neural networks


Challenges
Mechanisms
Theories
single neuron development
network development
synapse birth
synapse growth
axon guidance
neural plasticity
Asymmetrical membrane leads to non-uniform sub-membrane concentrations of diffusive morphogen, finally creating wavelike membrane protrusions
The reaction-diffusion of tubulin, calcium, and MAP-2 proteins naturally implies synapse differentiation, elongation, and branching
Chemotaxis happens when growth cones detect and follow the specific gradients (diffusive or substrate bound), guiding axons to other neurons
calcium-based NMDA receptor activation triggers the phosphorylation of AMPA by CaMKII, regulating synapse growth
calcium-stimulated stochastic process for active or passive membranes
an analytical formulation of synapse growth by coupled Laplace equations and stochastic equations
Synapse guidance by netrin-1 (diffusive) or slits (both diffusive and substrate bound)
autophosphorylates
active CaMKII x
causes
AMPAR insertion
high Ca influx $a \geq \Theta_h$ phosphorylates
low Ca influx $\Theta_b \leq a < \Theta_l$ dephosphorylates (via phosphatase)
inactive CaMKII $N-x$


Asuka
Biochemical Reaction System
Non-linearly Coupled
Simple Parameter Inputs
Non-linearly Coupled
Spatial Distribution of Substances
Neurites Development System
Artificial Neural System
Molecular Dynamic System
Non-linearly Coupled
Biophysics
Multiple Neurons
Multiple Neurons at 0 Hours
μm
Single Neuron
Single Neuron at 0 Hours
Single Neuron



CO of Neuron-1
CO of Neuron-2
CO of Neuron-3
CO of Neuron-4
CO of Neuron-5
CO of Neuron-6
Freq
SL of Neuron-1
SL of Neuron-2
SL of Neuron-3
SL of Neuron-4
SL of Neuron-5
SL of Neuron-6
μm
Hours
Single Neuron
Single Neuron
Multiple Neurons
100 µm


Asuka+
Asuka
+
Plasticity
Simple Parameter Inputs
Dynamic Update
ANN in Pytorch
A neural cluster whose development is regulated by learning tasks
An artificial neural network formed and trained according to real neural development
Biophysics

Formation of dynamic connectivity

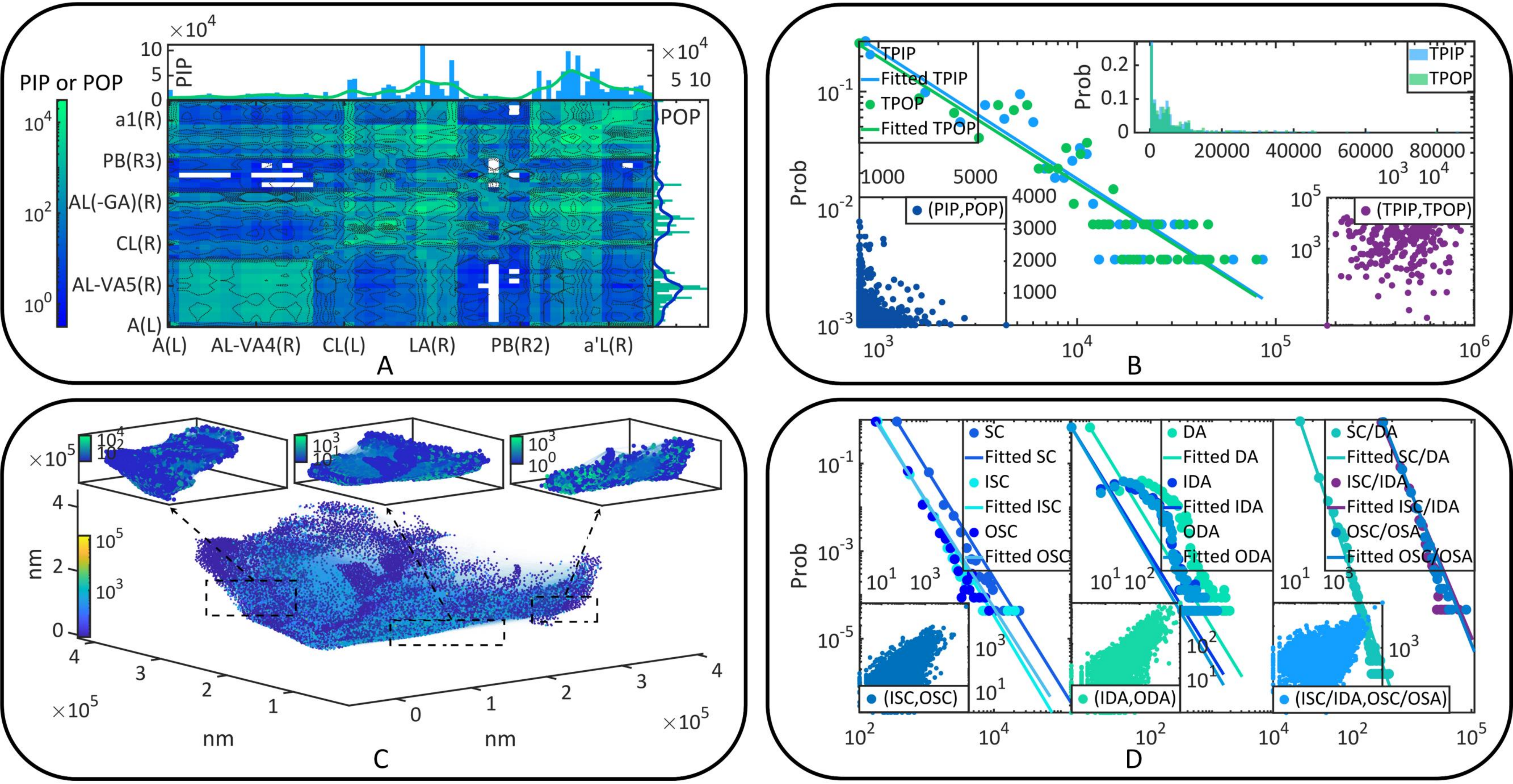
Yang Tian, Pei Sun, ***Percolation may explain efficiency, robustness, and economy of the brain****, under review by Network Neuroscience*

Yang Tian, Pei Sun, ***Percolation in the brain of fruit fly****, under review by Physical Review Research*

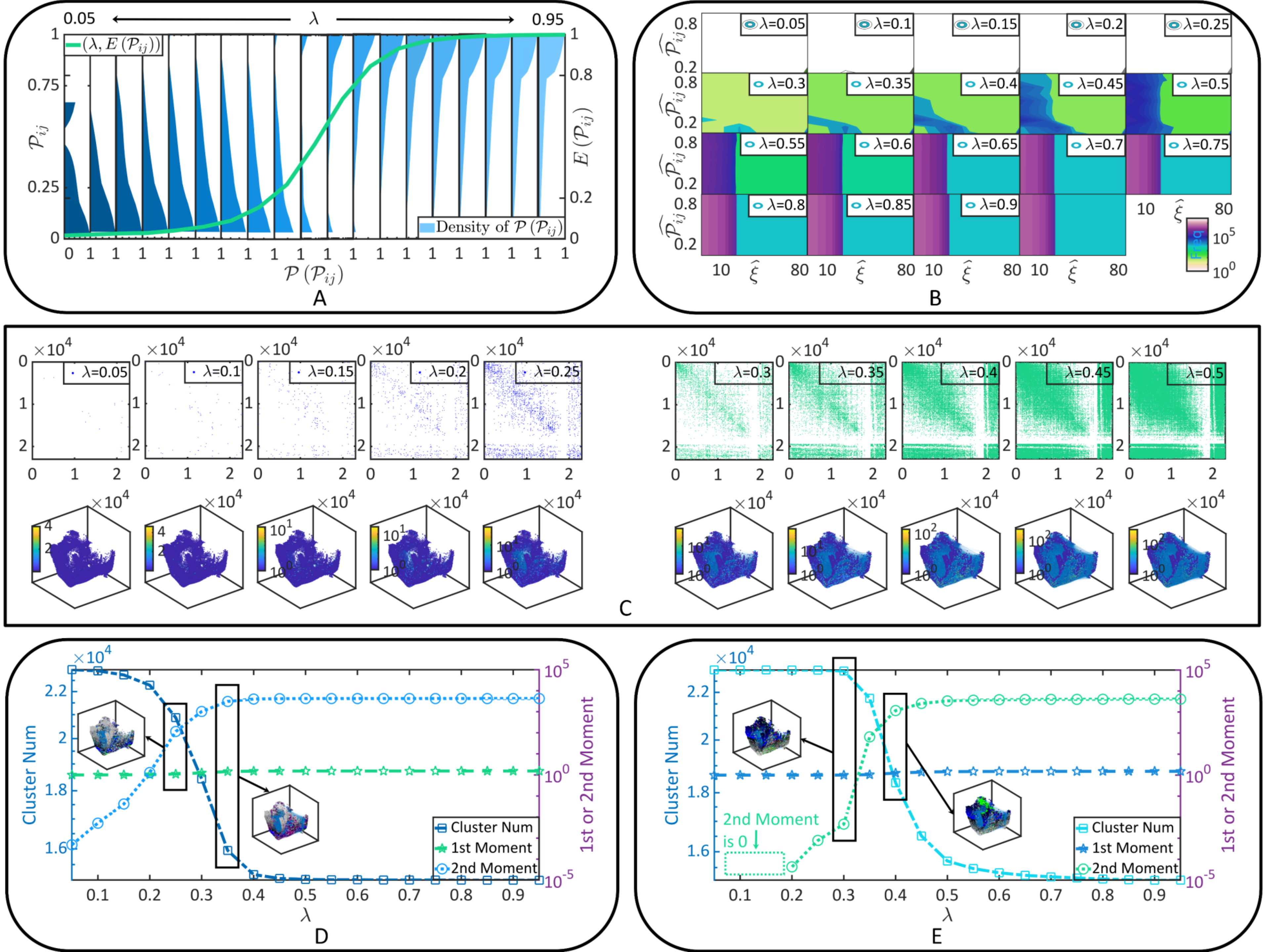
- We study the percolation problem on the largest yet fine-grained brain connectome of the fruit fly (2.5×10^4 neurons and 2×10^7 synapses), whose static connectivity is demonstrated as scale-free. We bridge between massive neural dynamics computation (1.2×10^9 times) and directed percolation analysis, revealing that dynamic connectivity formation in the brain can be explained by a percolation process controlled by synaptic excitation-inhibition (E/I) balance.

- By increasing the E/I balance gradually, we discover the emergence of these properties as bi-products of percolation transition when the E/I balance arrives at 3:7. As the E/I balance keeps increase, an optimal E/I balance 1:1 is unveiled to ensure these three properties simultaneously, consistent with previous in vitro experimental predictions. Once the E/I balance reaches over 3:2, an intrinsic limitation of these properties determined by static brain connectivity can be observed.

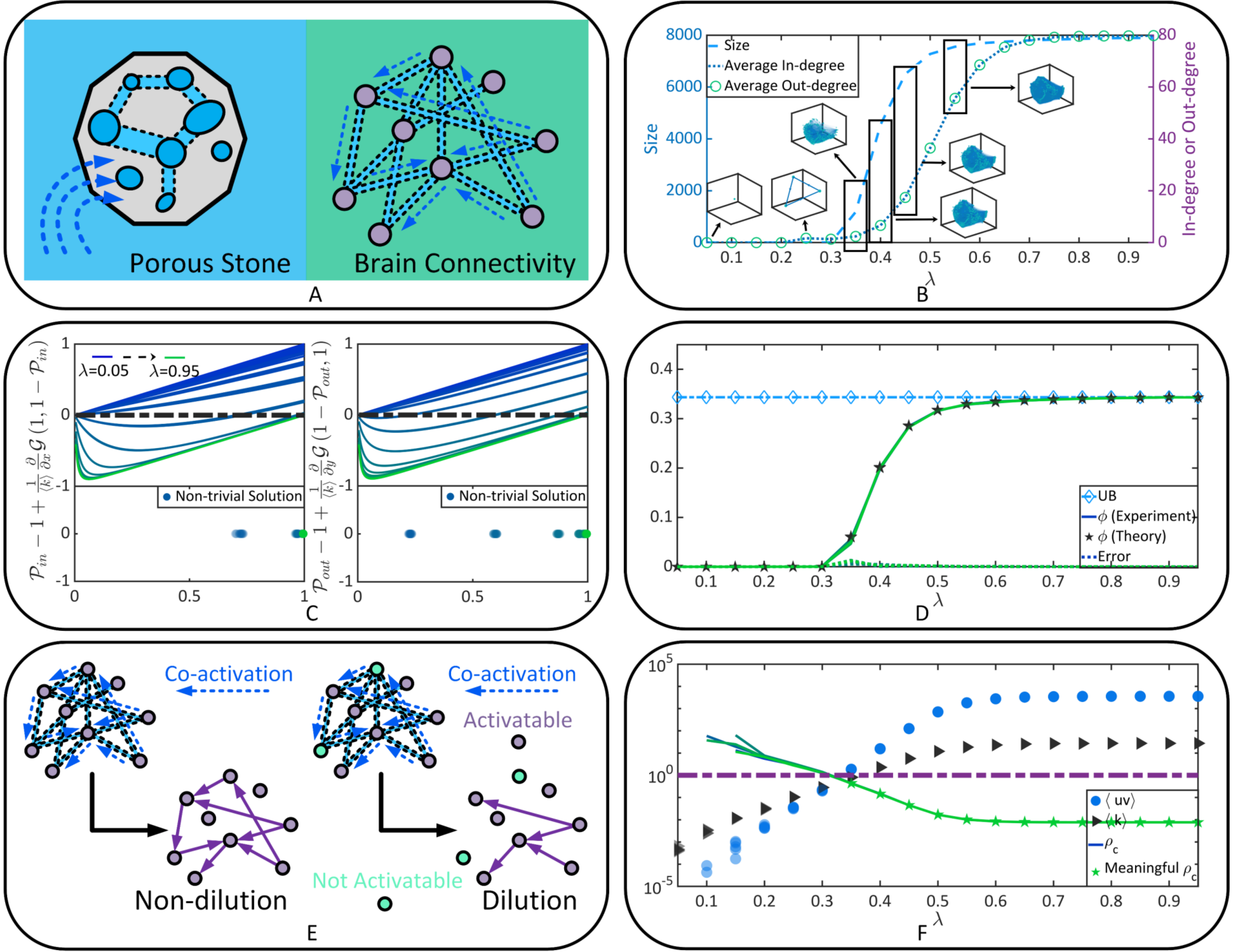
- Our work demonstrates that percolation, a universal characterization of critical phenomena and phase transitions, may serve as a window towards understanding the emergence of various brain properties.



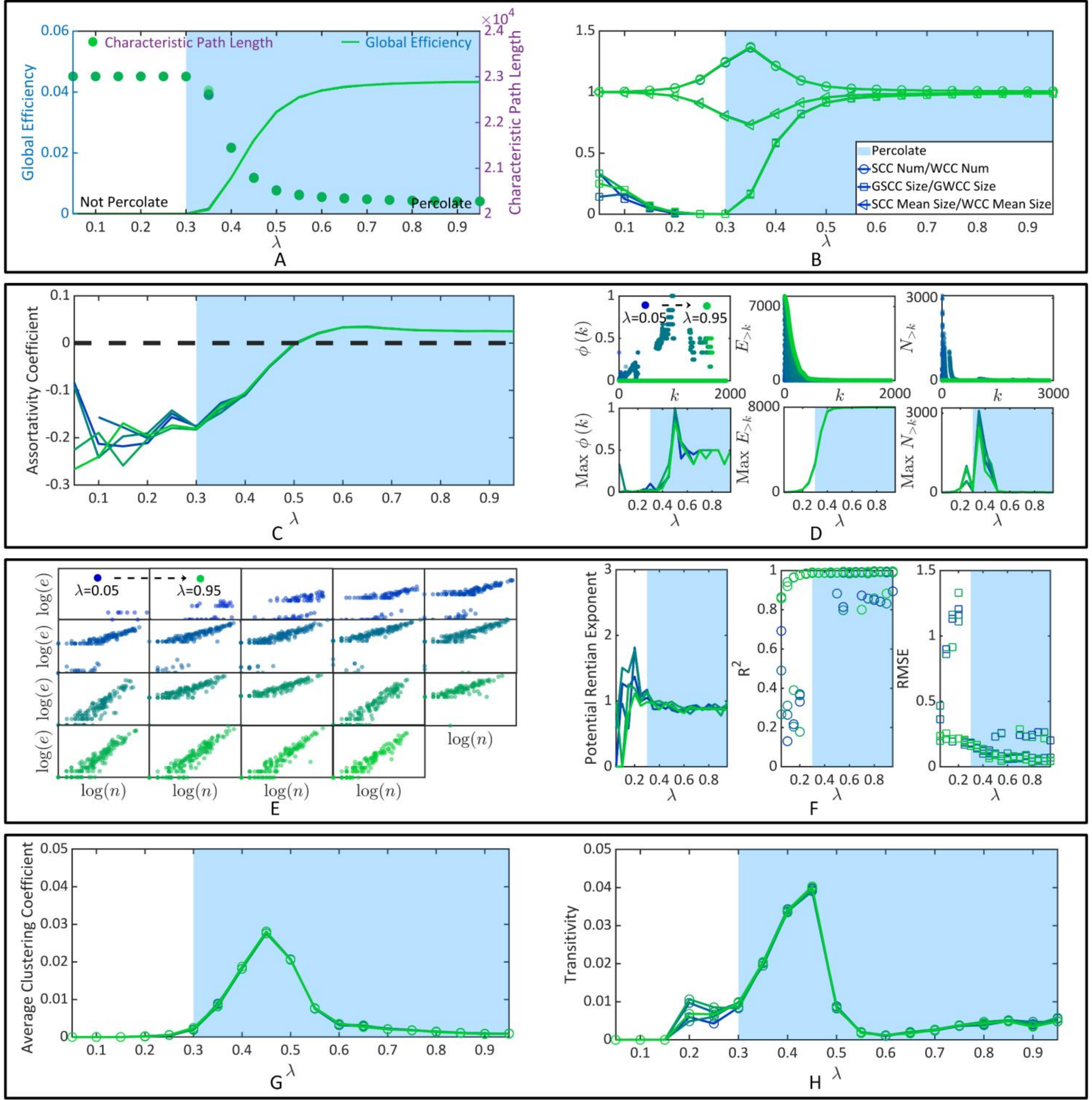
PIP or POP
PIP
POP
a1(R)
PB(R3)
AL(-GA)(R)
CL(R)
AL-VA5(R)
A(L)
A(L)
AL-VA4(R)
CL(L)
LA(R)
PB(R2)
a'L(R)
A
TPIP
Fitted TPIP
TPOP
Fitted TPOP
Prob
(PIP,POP)
(TPIP,TPOP)
B
nm
C
SC
Fitted SC
ISC
Fitted ISC
OSC
Fitted OSC
DA
Fitted DA
IDA
Fitted IDA
ODA
Fitted ODA
SC/DA
Fitted SC/DA
ISC/IDA
Fitted ISC/IDA
OSC/OSA
Fitted OSC/OSA
(ISC,OSC)
(IDA,ODA)
(ISC/IDA,OSC/OSA)
D



A
0.05
λ
0.95
$(\lambda, E(\mathcal{P}_{ij}))$
Density of $\mathcal{P}(\mathcal{P}_{ij})$
$\mathcal{P}_{ij}$
$E(\mathcal{P}_{ij})$
$\mathcal{P}(\mathcal{P}_{ij})$
B
$\widehat{\mathcal{P}_{ij}}$
$\widehat{\xi}$
λ=0.05
λ=0.1
λ=0.15
λ=0.2
λ=0.25
λ=0.3
λ=0.35
λ=0.4
λ=0.45
λ=0.5
λ=0.55
λ=0.6
λ=0.65
λ=0.7
λ=0.75
λ=0.8
λ=0.85
λ=0.9
Freq
C
$\times 10^4$
D
Cluster Num
1st or 2nd Moment
Cluster Num
1st Moment
2nd Moment
λ
E
2nd Moment is 0
Cluster Num
1st Moment
2nd Moment

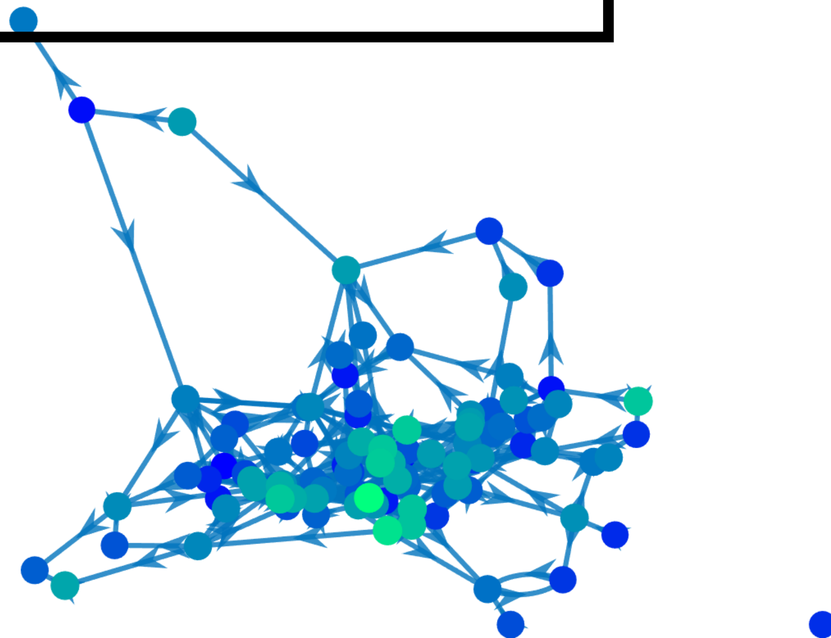


Porous Stone
Brain Connectivity
A
Size
Average In-degree
Average Out-degree
Size
In-degree or Out-degree
λ
B
λ=0.05
λ=0.95
$\mathcal{P}_{in} - 1 + \frac{1}{\langle k \rangle}\frac{\partial}{\partial x}\mathcal{G}(1, 1 - \mathcal{P}_{in})$
$\mathcal{P}_{out} - 1 + \frac{1}{\langle k \rangle}\frac{\partial}{\partial y}\mathcal{G}(1 - \mathcal{P}_{out}, 1)$
Non-trivial Solution
Non-trivial Solution
C
UB
ϕ (Experiment)
ϕ (Theory)
Error
λ
D
Co-activation
Co-activation
Activatable
Not Activatable
Non-dilution
Dilution
E
⟨uv⟩
⟨k⟩
ρ_c
Meaningful ρ_c
λ
F



Characteristic Path Length
Global Efficiency
Not Percolate
Percolate
$\times 10^4$
λ
A
Percolate
SCC Num/WCC Num
GSCC Size/GWCC Size
SCC Mean Size/WCC Mean Size
λ
B
Assortativity Coefficient
λ
C
λ=0.05
λ=0.95
$\phi(k)$
$E_{>k}$
$N_{>k}$
k
Max $\phi(k)$
Max $E_{>k}$
Max $N_{>k}$
λ
D
λ=0.05
λ=0.95
log(e)
log(n)
E
Potential Rentian Exponent
R^2
RMSE
λ
F
Average Clustering Coefficient
G λ
Transitivity
H λ

Information-Dynamics



Information-dynamics unification

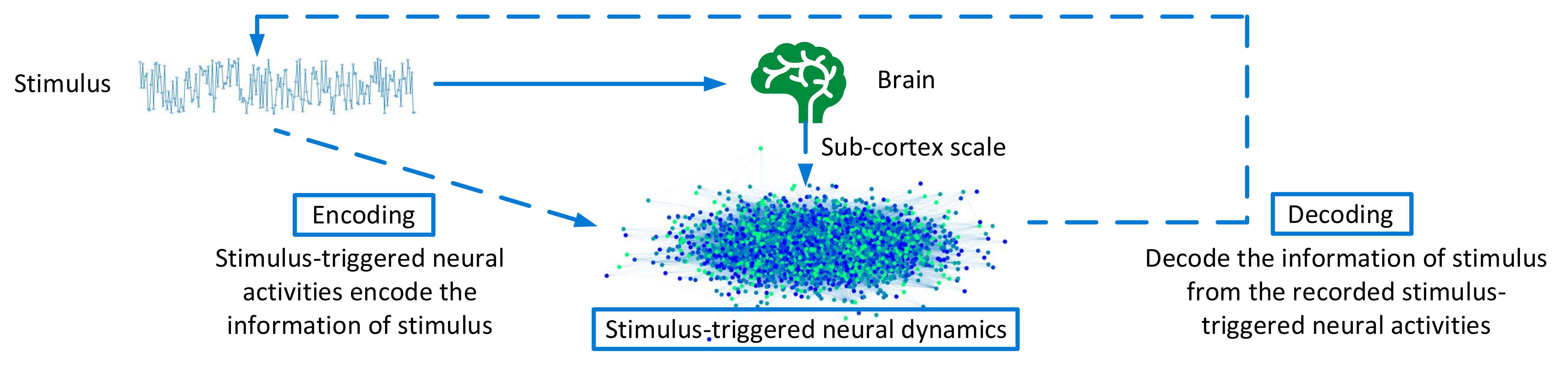
Yang Tian, Guoqi Li, Pei Sun, ***Bridge the Information and Dynamics Attributes of Neural Activities****, accepted by Physical Review Research*

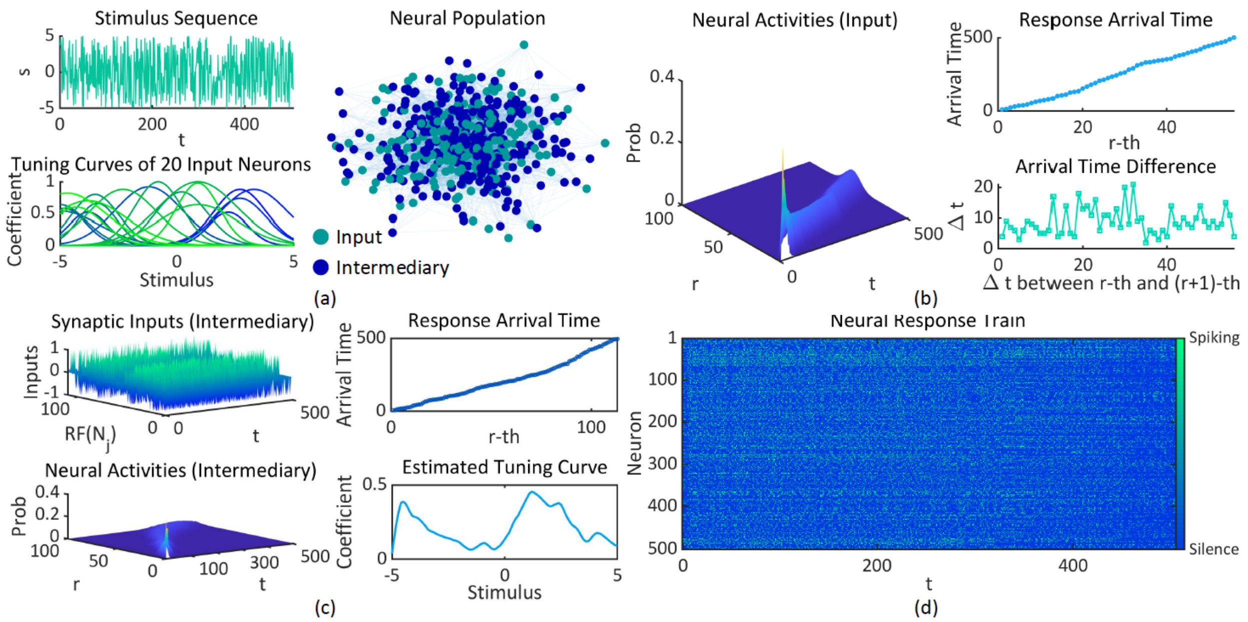
Theoretical contribution

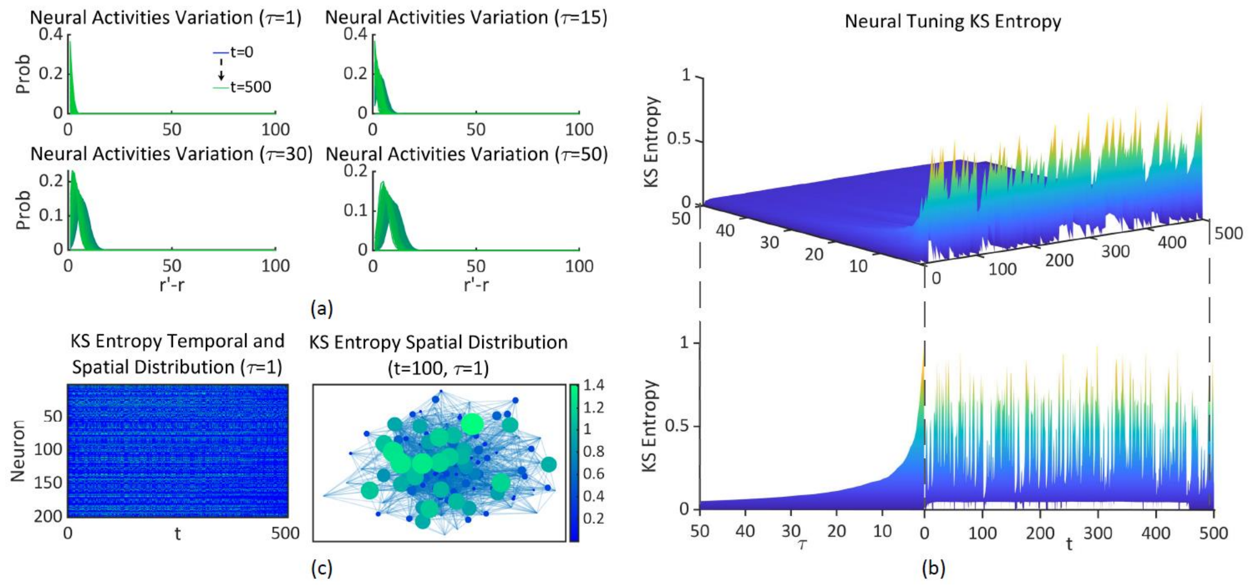
- We bridge dynamics (e.g., Kolmogorov-Sinai entropy) and information (e.g., mutual information and Fisher information) metrics on the stimulus-triggered stochastic dynamics in neural populations.
- Our unified analysis identifies various essential features of the information-processing-related neural dynamics. We discover spatio-temporal differences in the dynamic randomness and chaotic degrees of neural dynamics during neural information processing
- Our framework reveals the fundamental role of neural dynamics in shaping neural information processing. The neural dynamics creates an oppositely directed variation of encoding and decoding properties under specific conditions, and it determines the neural representation of stimulus distribution.

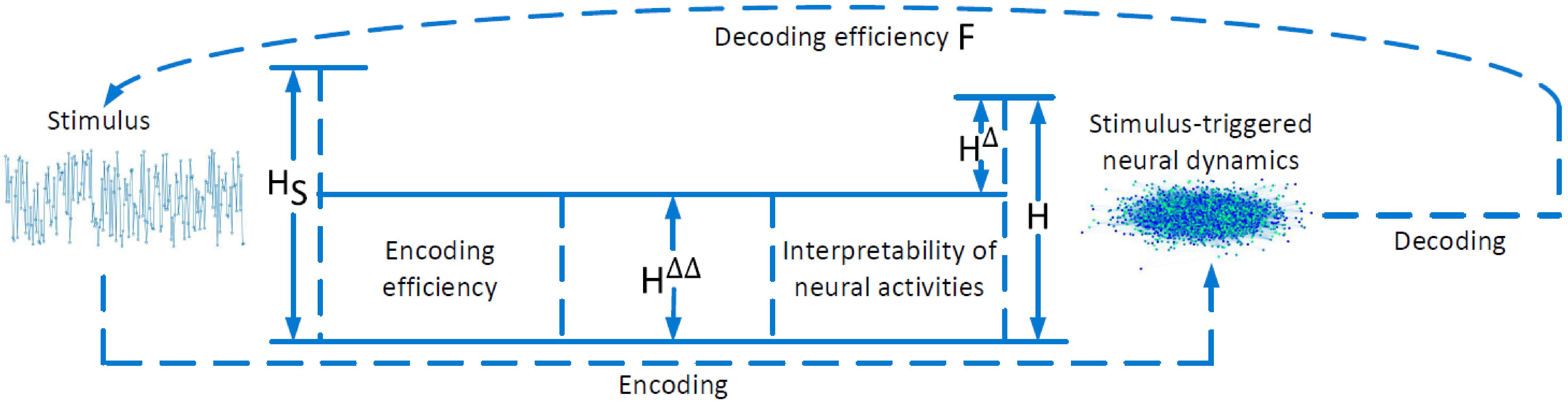
Insights

- Our findings demonstrate a potential direction to explain the emergence of neural information processing from neural dynamics and help glance at the intrinsic connections between the informational and the physical brain.

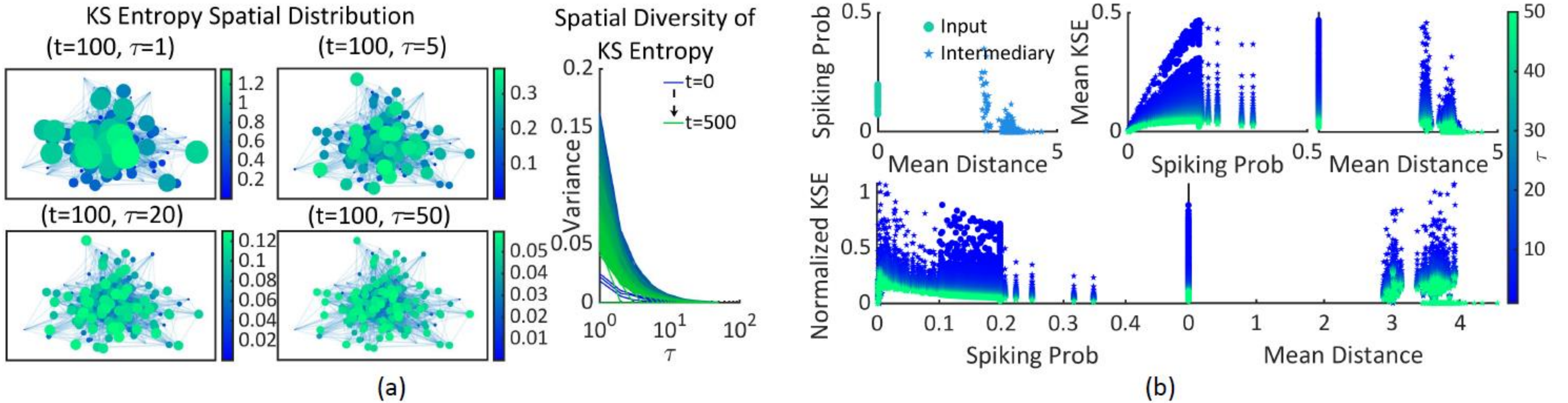

Stimulus
Brain
Sub-cortex scale
Encoding
Stimulus-triggered neural activities encode the information of stimulus
Stimulus-triggered neural dynamics
Decoding
Decode the information of stimulus from the recorded stimulus-triggered neural activities


Stimulus Sequence
Tuning Curves of 20 Input Neurons
Coefficient
Stimulus
Neural Population
Input
Intermediary
(a)
Neural Activities (Input)
Prob
Response Arrival Time
Arrival Time
r-th
Arrival Time Difference
Δ t between r-th and (r+1)-th
(b)
Synaptic Inputs (Intermediary)
Inputs
Neural Activities (Intermediary)
Response Arrival Time
Estimated Tuning Curve
(c)
Neural Response Train
Neuron
Spiking
Silence
(d)

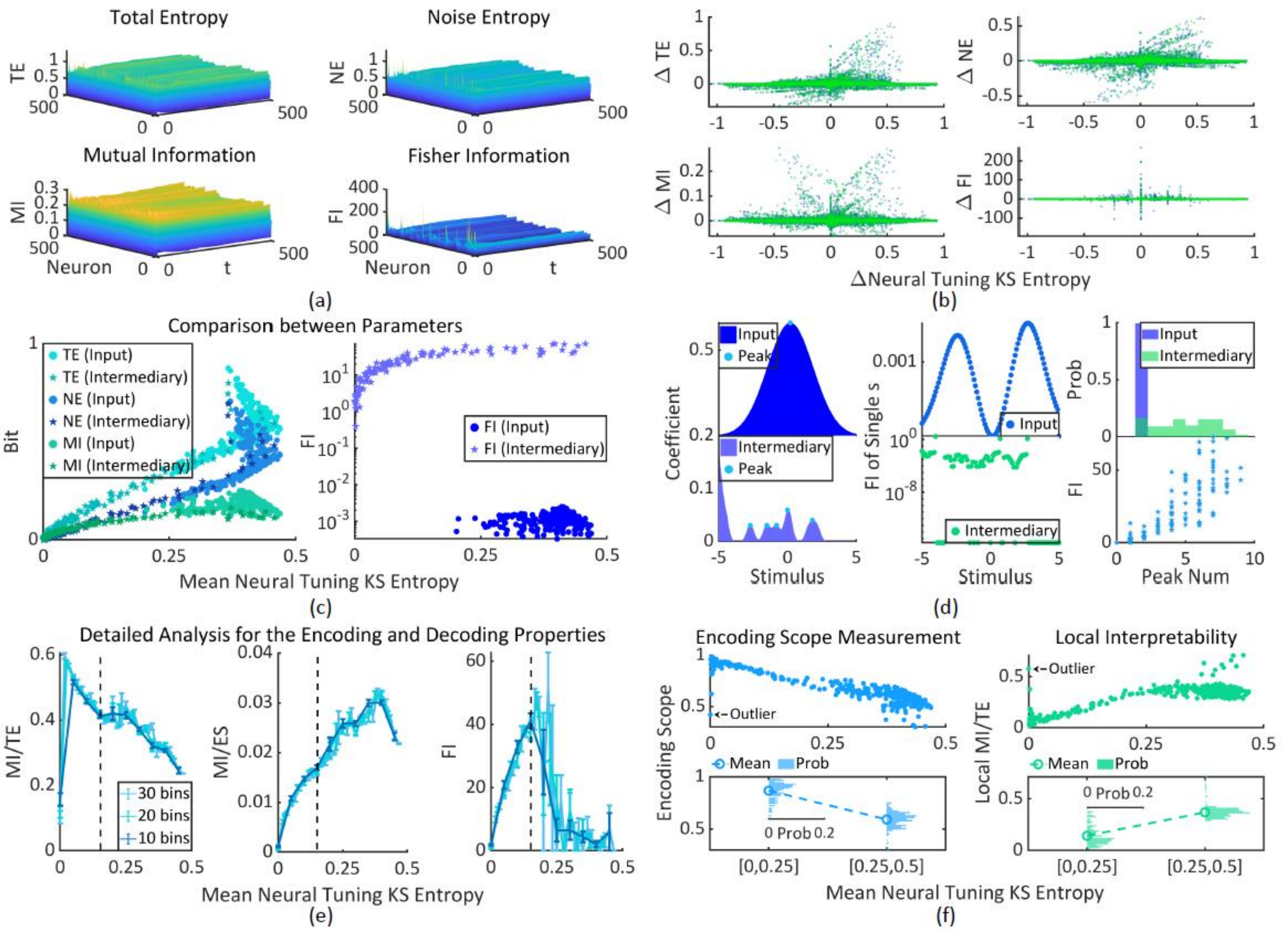

Neural Activities Variation (τ=1)
Neural Activities Variation (τ=15)
Neural Activities Variation (τ=30)
Neural Activities Variation (τ=50)
t=0
t=500
Prob
r'-r
(a)
KS Entropy Temporal and Spatial Distribution (τ=1)
KS Entropy Spatial Distribution (t=100, τ=1)
Neuron
(c)
Neural Tuning KS Entropy
KS Entropy
(b)



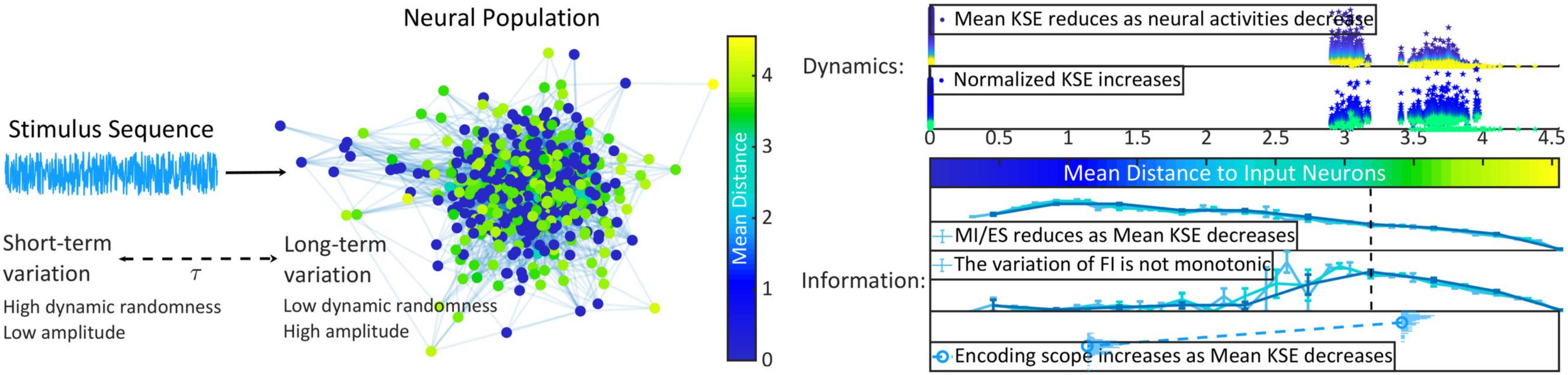
Decoding efficiency F
Stimulus
Stimulus-triggered
neural dynamics
H_S
H^{Δ}
H
$H^{\Delta\Delta}$
Encoding
efficiency
Interpretability of
neural activities
Decoding
Encoding



KS Entropy Spatial Distribution
(t=100, τ=1)
(t=100, τ=5)
(t=100, τ=20)
(t=100, τ=50)
Spatial Diversity of
KS Entropy
Variance
τ
t=0
t=500
(a)
Spiking Prob
Mean Distance
Input
Intermediary
Mean KSE
Normalized KSE
τ
(b)



Total Entropy
Noise Entropy
Mutual Information
Fisher Information
TE
NE
MI
FI
Neuron
t
(a)
Δ TE
Δ NE
Δ MI
Δ FI
ΔNeural Tuning KS Entropy
(b)
Comparison between Parameters
TE (Input)
TE (Intermediary)
NE (Input)
NE (Intermediary)
MI (Input)
MI (Intermediary)
Bit
FI (Input)
FI (Intermediary)
FI
Mean Neural Tuning KS Entropy
(c)
Input
Peak
Intermediary
Peak
Coefficient
Stimulus
FI of Single s
Input
Intermediary
Stimulus
Input
Intermediary
Prob
FI
Peak Num
(d)
Detailed Analysis for the Encoding and Decoding Properties
MI/TE
30 bins
20 bins
10 bins
MI/ES
FI
Mean Neural Tuning KS Entropy
(e)
Encoding Scope Measurement
Outlier
Encoding Scope
Mean
Prob
0 Prob 0.2
[0,0.25]
[0.25,0.5]
Local Interpretability
Outlier
Local MI/TE
Mean
Prob
0 Prob 0.2
[0,0.25]
[0.25,0.5]
Mean Neural Tuning KS Entropy
(f)



Neural Population
Stimulus Sequence
Short-term variation
τ
Long-term variation
High dynamic randomness
Low amplitude
Low dynamic randomness
High amplitude
Mean Distance
0
1
2
3
4
Dynamics:
Mean KSE reduces as neural activities decrease
Normalized KSE increases
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Mean Distance to Input Neurons
Information:
MI/ES reduces as Mean KSE decreases
The variation of FI is not monotonic
Encoding scope increases as Mean KSE decreases

Dynamic evolution of information

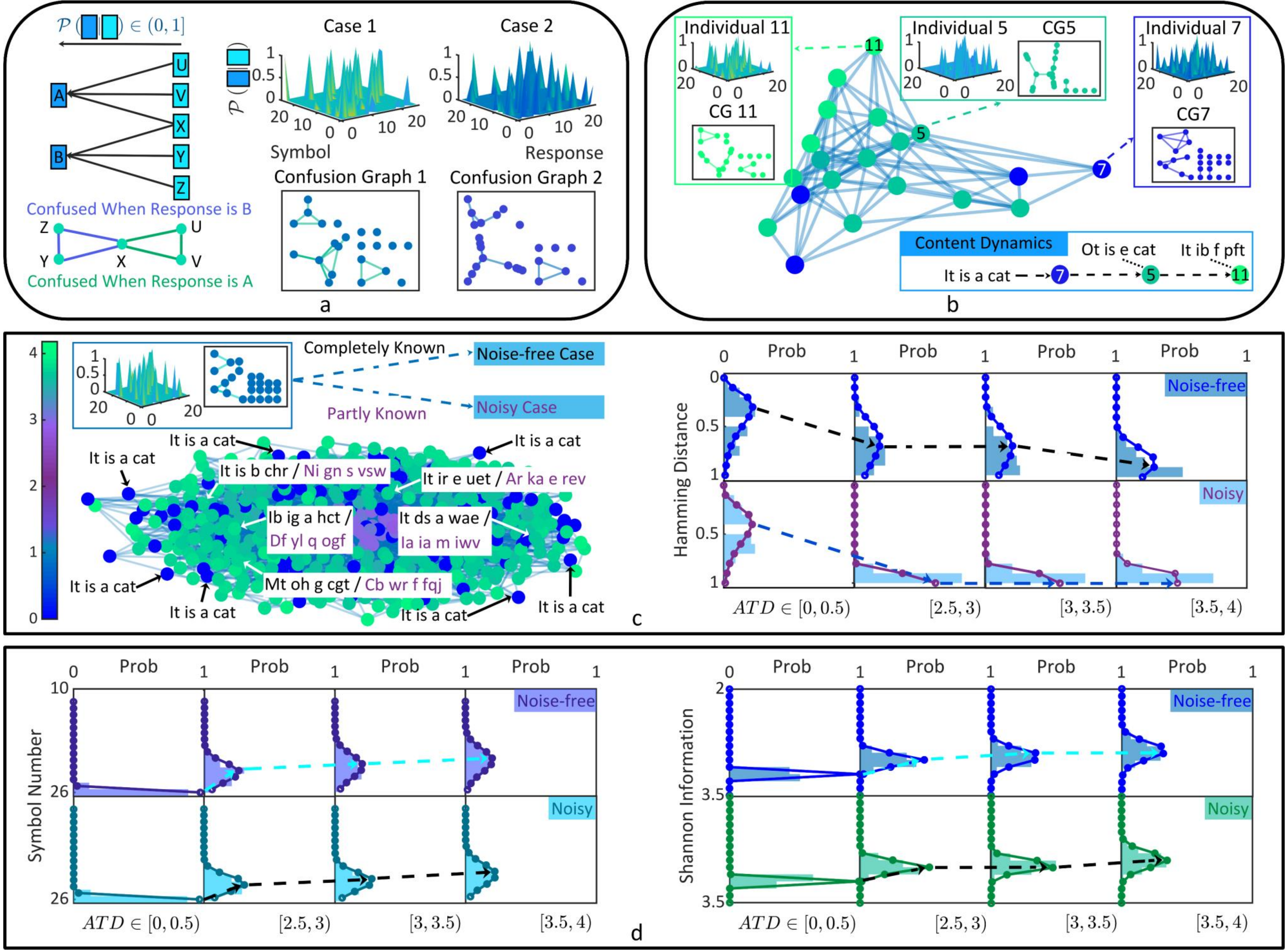
Yang Tian, Justin L. Gardner, Guoqi Li, Pei Sun, ***Information Evolution in Complex Networks****, Submitted to Scipost Physics*

Theoretical contribution

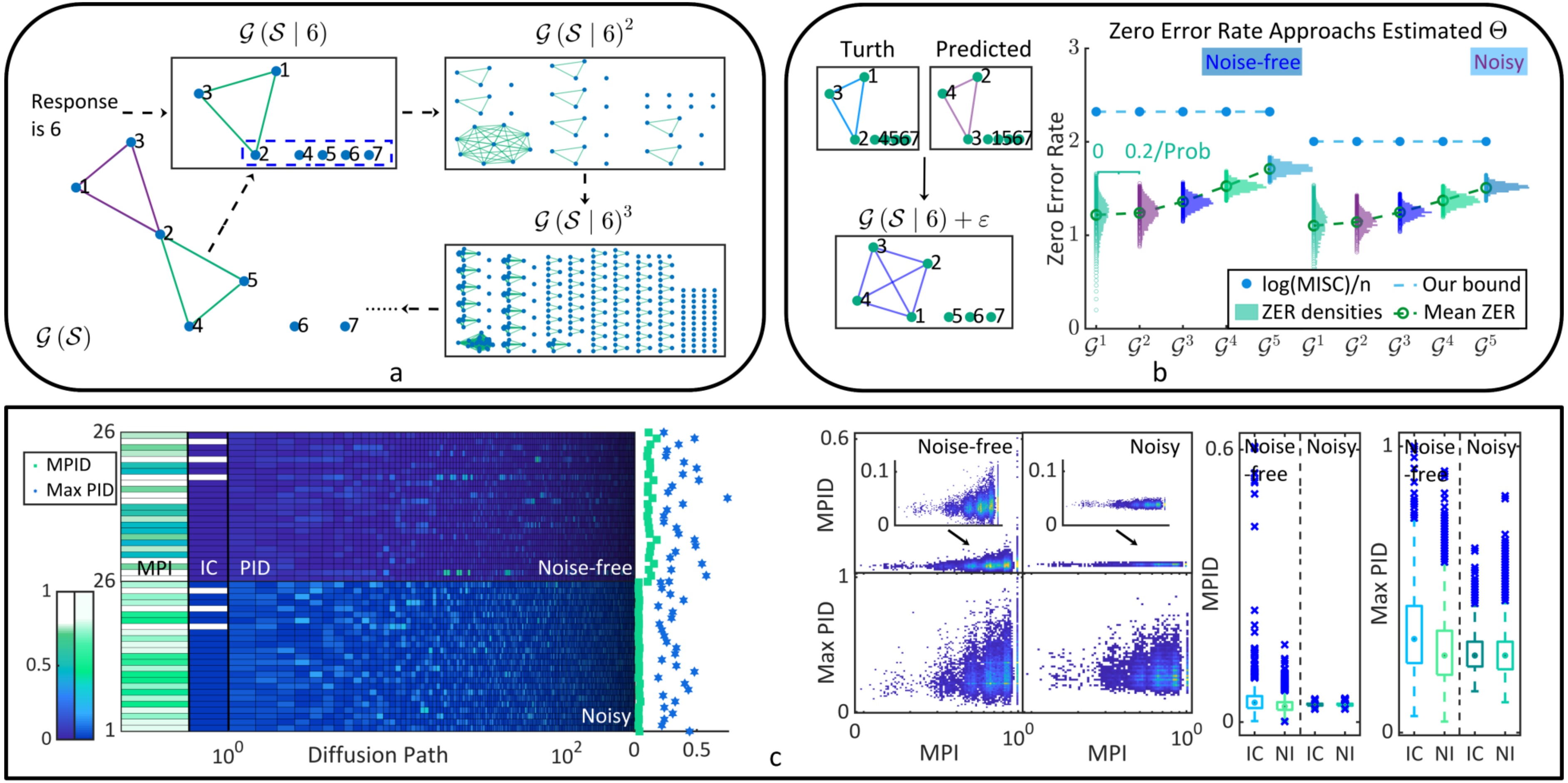
- We pursue to glance at the laws governing the evolution of information during its diffusion in complex networks
- We reveal that the maximum information invariants during diffusion are limited by specific information selectivity characteristics in complex networks. Information contents out of this bound will inevitably experience distortion and dissipation, whose speeds are shaped by the diversity of information selectivity in networks. The existence of this process is irrespective of noise, and the noise only accounts for accelerating it

Insights

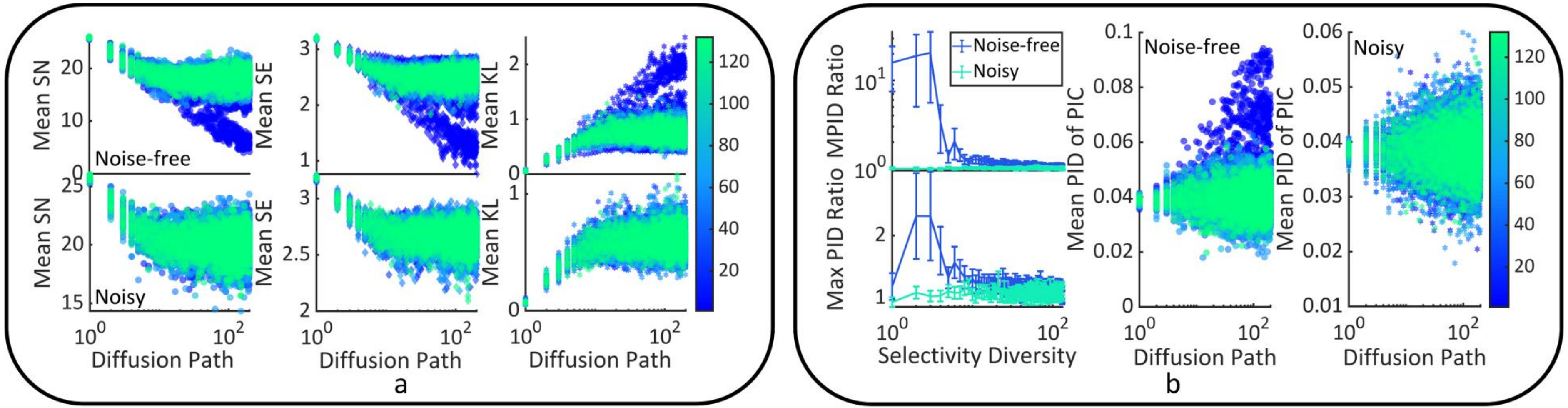
- Applying the discovered laws, we offer possible explanations for the emergence of neural tuning properties in the V1 and MT cortices of the human brain, and the emergence of extreme opinions in social networks.
- These findings demonstrate the profound effects of information evolution in shaping the characteristics of complex systems on multiple scales.



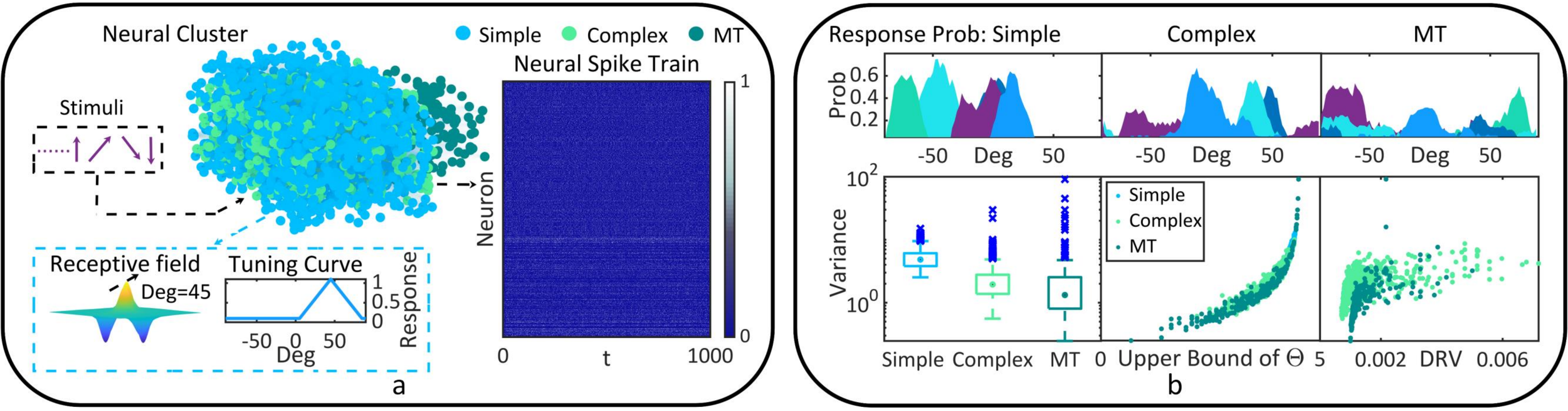
Case 1
Case 2
Symbol
Response
Confusion Graph 1
Confusion Graph 2
Confused When Response is B
Confused When Response is A
a
Individual 11
CG 11
Individual 5
CG5
Individual 7
CG7
Content Dynamics
It is a cat
Ot is e cat
It ib f pft
b
Completely Known
Noise-free Case
Noisy Case
Partly Known
It is a cat
It is b chr / Ni gn s vsw
It ir e uet / Ar ka e rev
Ib ig a hct / Df yl q ogf
It ds a wae / Ia ia m iwv
Mt oh g cgt / Cb wr f fqj
Prob
Hamming Distance
Noise-free
Noisy
ATD ∈ [0, 0.5)
[2.5, 3)
[3, 3.5)
[3.5, 4)
c
Symbol Number
Shannon Information
d



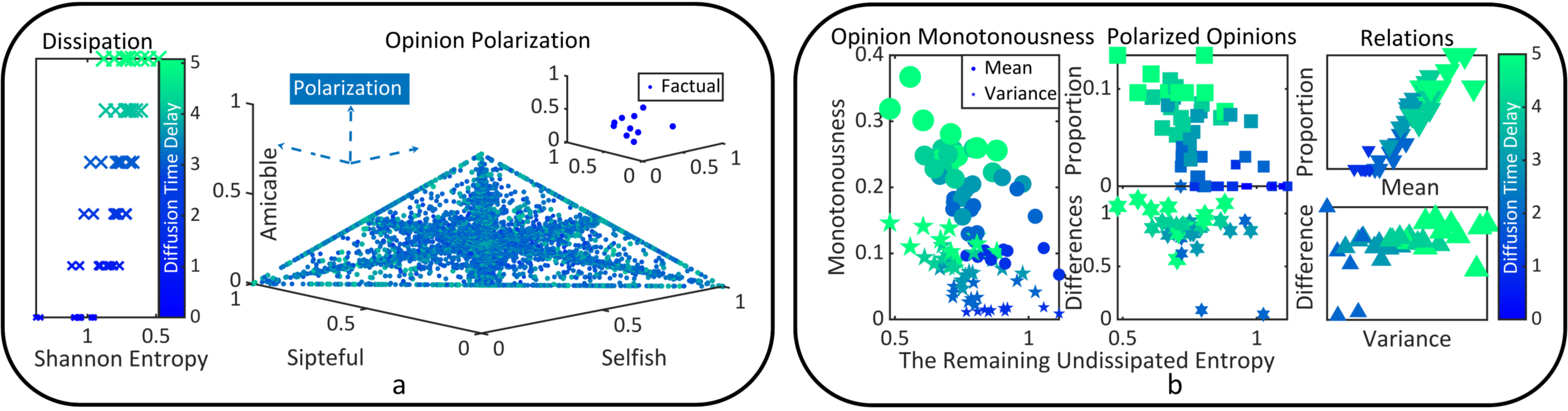
$\mathcal{G}(\mathcal{S} \mid 6)$
$\mathcal{G}(\mathcal{S} \mid 6)^2$
Response is 6
$\mathcal{G}(\mathcal{S} \mid 6)^3$
$\mathcal{G}(\mathcal{S})$
a
Turth
Predicted
Zero Error Rate Approachs Estimated Θ
Noise-free
Noisy
$\mathcal{G}(\mathcal{S} \mid 6) + \varepsilon$
0 0.2/Prob
Zero Error Rate
log(MISC)/n
Our bound
ZER densities
Mean ZER
b
MPID
Max PID
MPI
IC
PID
Noise-free
Noisy
Diffusion Path
MPID
Max PID
MPI
Noise-free
Noisy
IC
NI
c



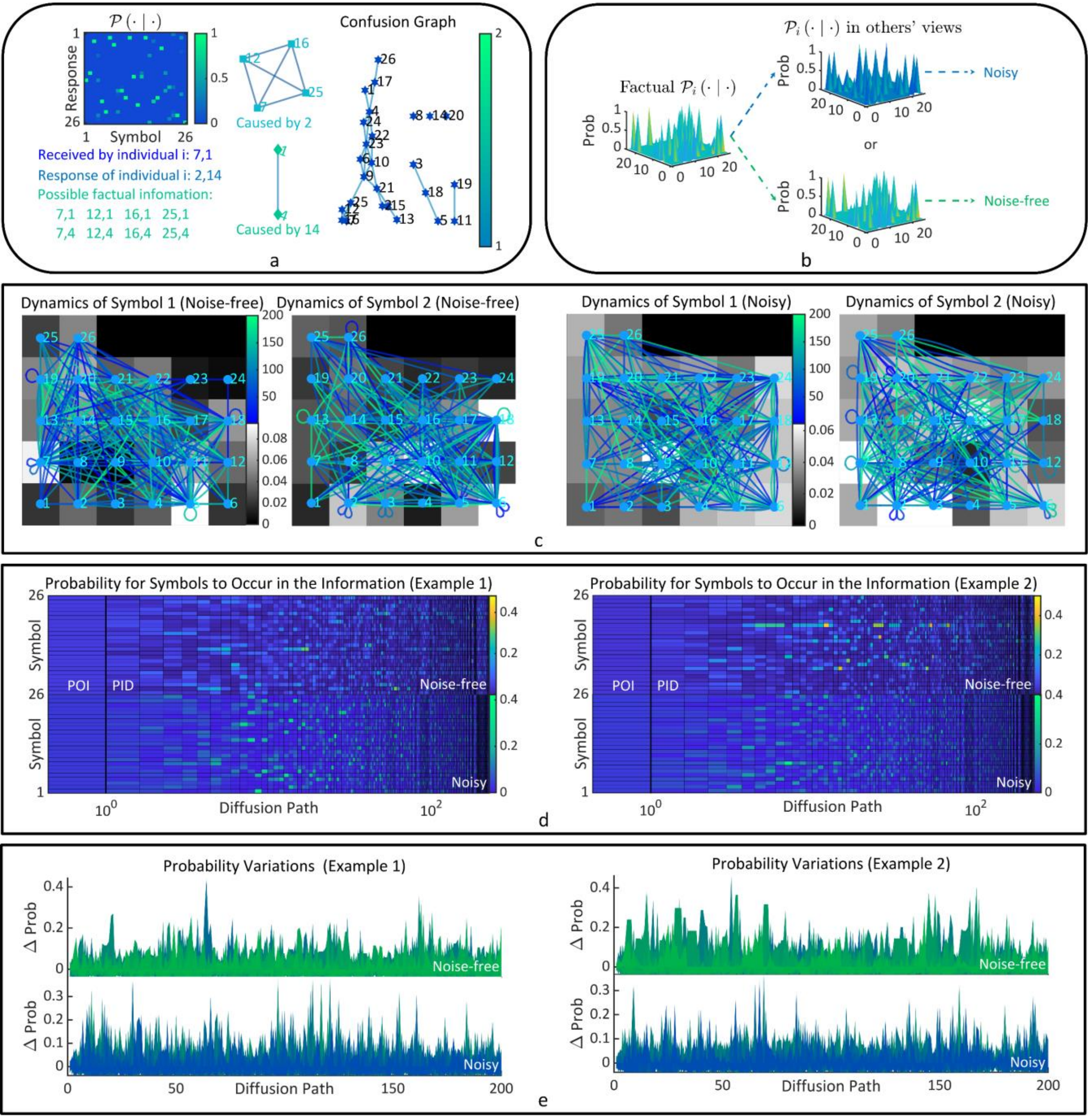
Mean SN
Mean SE
Mean KL
Noise-free
Noisy
Diffusion Path
a
Max PID Ratio MPID Ratio
Noise-free
Noisy
Selectivity Diversity
Mean PID of PIC
Noise-free
Noisy
Diffusion Path
b



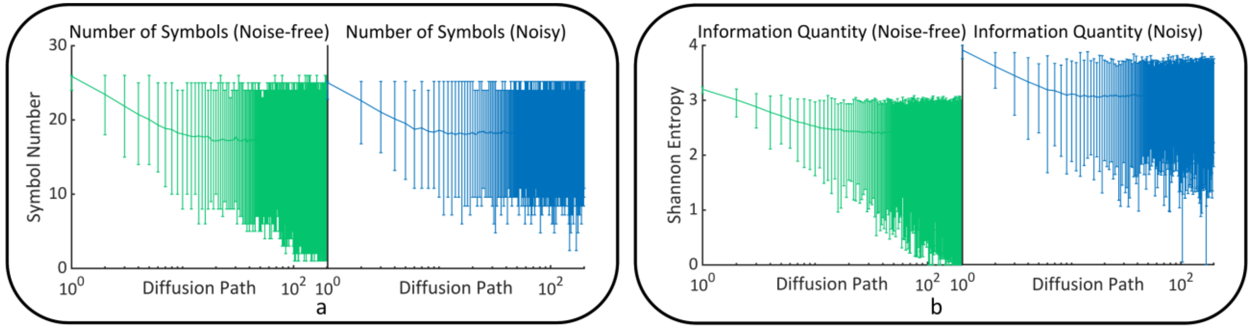
Neural Cluster
Simple
Complex
MT
Stimuli
Neural Spike Train
Neuron
t
0
1000
1
0
Receptive field
Deg=45
Tuning Curve
Response
Deg
-50
0
50
1
0.5
a
Response Prob: Simple
Complex
MT
Prob
0.6
0.4
0.2
-50
Deg
50
Variance
10^2
10^0
Simple
Complex
MT
Upper Bound of Θ
0
5
0.002
DRV
0.006
b



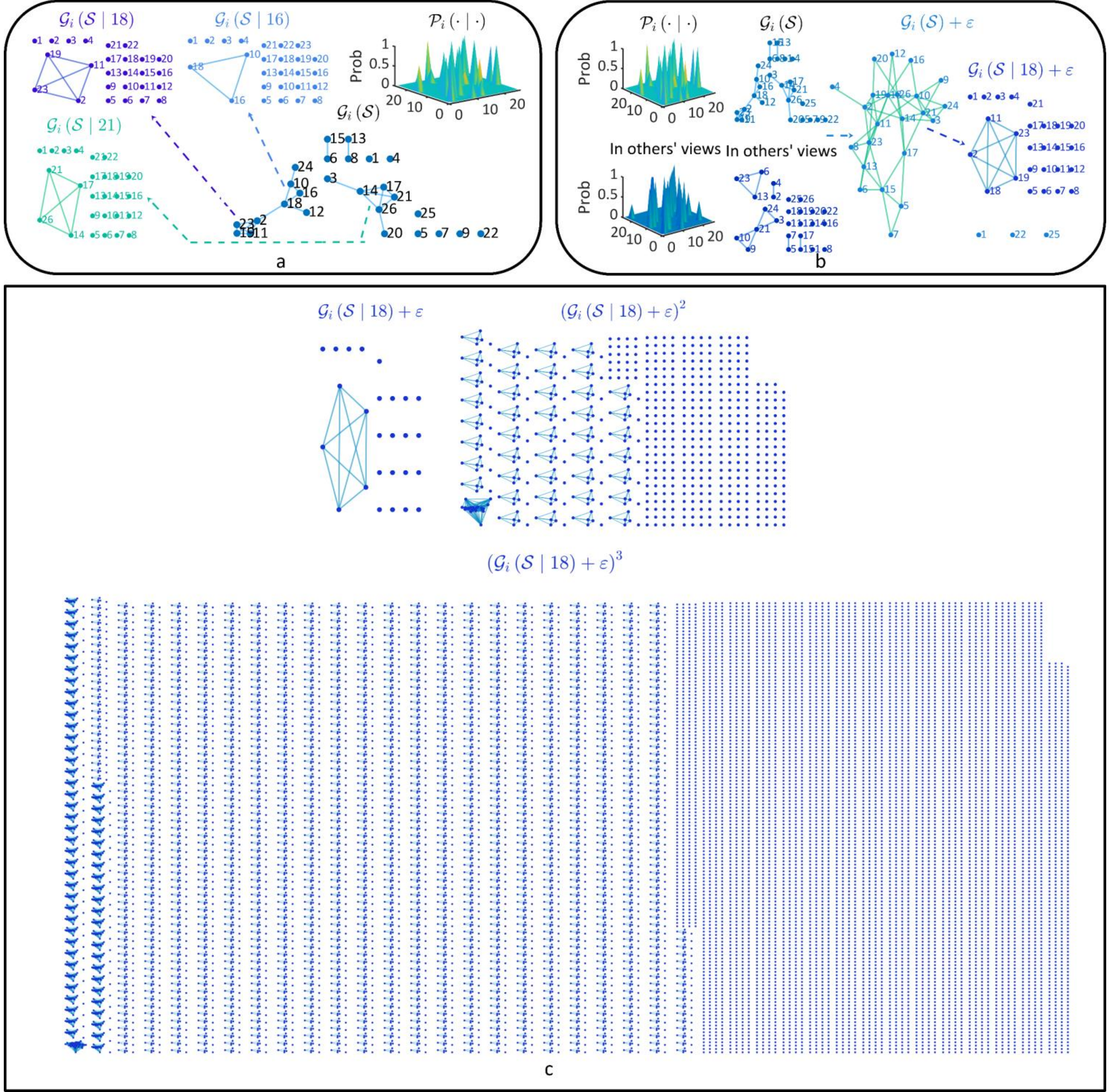
Dissipation
Diffusion Time Delay
Shannon Entropy
1
0.5
Opinion Polarization
Polarization
Amicable
Sipteful
Selfish
Factual
a
Opinion Monotonousness
Mean
Variance
Monotonousness
The Remaining Undissipated Entropy
Polarized Opinions
Proportion
Differences
Relations
Proportion
Mean
Difference
Variance
Diffusion Time Delay
b



$\mathcal{P}(\cdot\mid\cdot)$
Response
Symbol
Confusion Graph
Caused by 2
Caused by 14
Received by individual i: 7,1
Response of individual i: 2,14
Possible factual infomation:
7,1 12,1 16,1 25,1
7,4 12,4 16,4 25,4
a
$\mathcal{P}_i(\cdot\mid\cdot)$ in others' views
Factual $\mathcal{P}_i(\cdot\mid\cdot)$
Prob
Noisy
or
Noise-free
b
Dynamics of Symbol 1 (Noise-free)
Dynamics of Symbol 2 (Noise-free)
Dynamics of Symbol 1 (Noisy)
Dynamics of Symbol 2 (Noisy)
c
Probability for Symbols to Occur in the Information (Example 1)
Probability for Symbols to Occur in the Information (Example 2)
Symbol
POI
PID
Noise-free
Noisy
Diffusion Path
d
Probability Variations (Example 1)
Probability Variations (Example 2)
Δ Prob
Noise-free
Noisy
Diffusion Path
e



Number of Symbols (Noise-free)
Number of Symbols (Noisy)
Symbol Number
Diffusion Path
a
Information Quantity (Noise-free)
Information Quantity (Noisy)
Shannon Entropy
Diffusion Path
b



$\mathcal{G}_i(\mathcal{S} \mid 18)$
$\mathcal{G}_i(\mathcal{S} \mid 16)$
$\mathcal{P}_i(\cdot \mid \cdot)$
Prob
$\mathcal{G}_i(\mathcal{S})$
$\mathcal{G}_i(\mathcal{S} \mid 21)$
a
$\mathcal{P}_i(\cdot \mid \cdot)$
$\mathcal{G}_i(\mathcal{S})$
$\mathcal{G}_i(\mathcal{S}) + \varepsilon$
$\mathcal{G}_i(\mathcal{S} \mid 18) + \varepsilon$
In others' views
In others' views
Prob
b
$\mathcal{G}_i(\mathcal{S} \mid 18) + \varepsilon$
$(\mathcal{G}_i(\mathcal{S} \mid 18) + \varepsilon)^2$
$(\mathcal{G}_i(\mathcal{S} \mid 18) + \varepsilon)^3$
c

Capture causality in dynamics by information

Yang Tian, Yaoyuan Wang, Ziyang Zhang, Pei Sun, ***Fourier-domain transfer entropy spectrum****, Submitted to Physical Review Letter*

Challenges to overcome

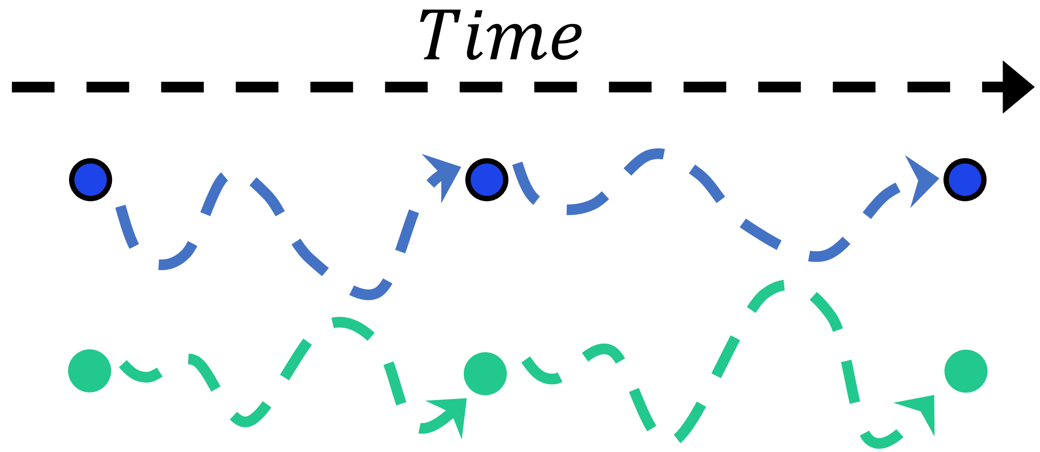
(I) Causality should not be treated as a entirety; (II) Causality is time-varying. (III) Probability density estimation for non-stationary and high-dimensional systems

Theoretical contribution

- We propose the Fourier-domain transfer entropy spectrum, a novel generalization of transfer entropy, as a model-free metric of causality. For arbitrary systems, this approach systematically quantifies the causality among their different system components rather than merely analyze systems as entireties. The generated spectrum offers a rich-information causality representation, tackling problems (I-III).

- Our metric can quantify time-varying latent causal relations and efficiently deal with non-stationary processes and high-dimensional conditions. Therefore, it can be generally applied to arbitrary systems to solve causality quantification problems in different scientific disciplines.

- We systematically demonstrate the validity of our approach in the aspects of parameter dependence, statistic significance test, and sensibility. A series of validation experiments are implemented on neuroscience data sets and diffusively coupled logistic oscillators.



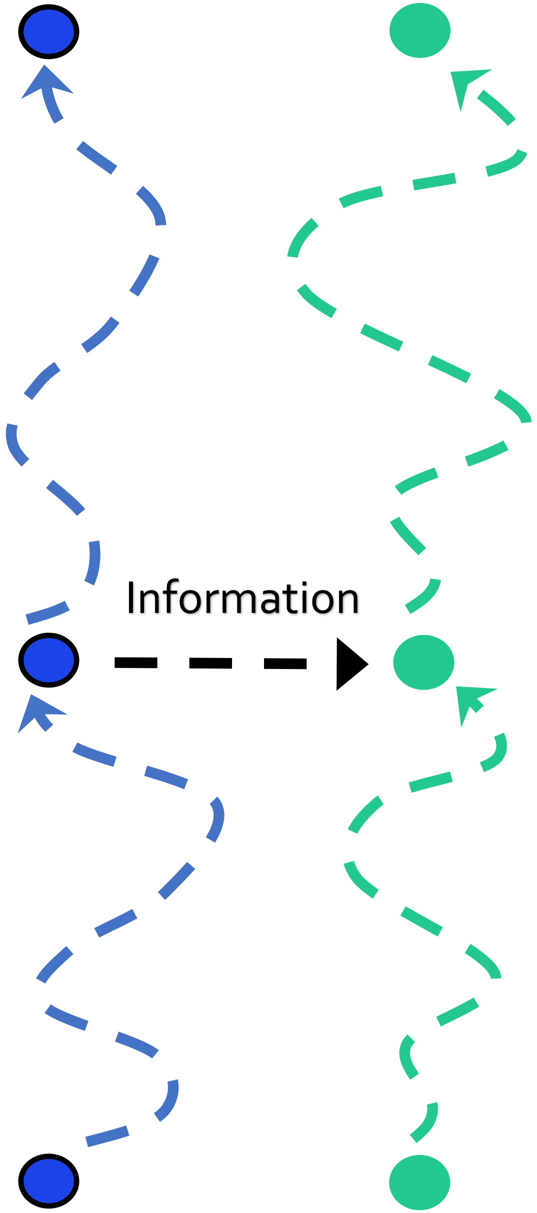
Causality lies in the **synchronization** between systems.

Following their ideas, the existence of causality between systems X and Y (e.g., X is the cause of Y) can be interpreted as

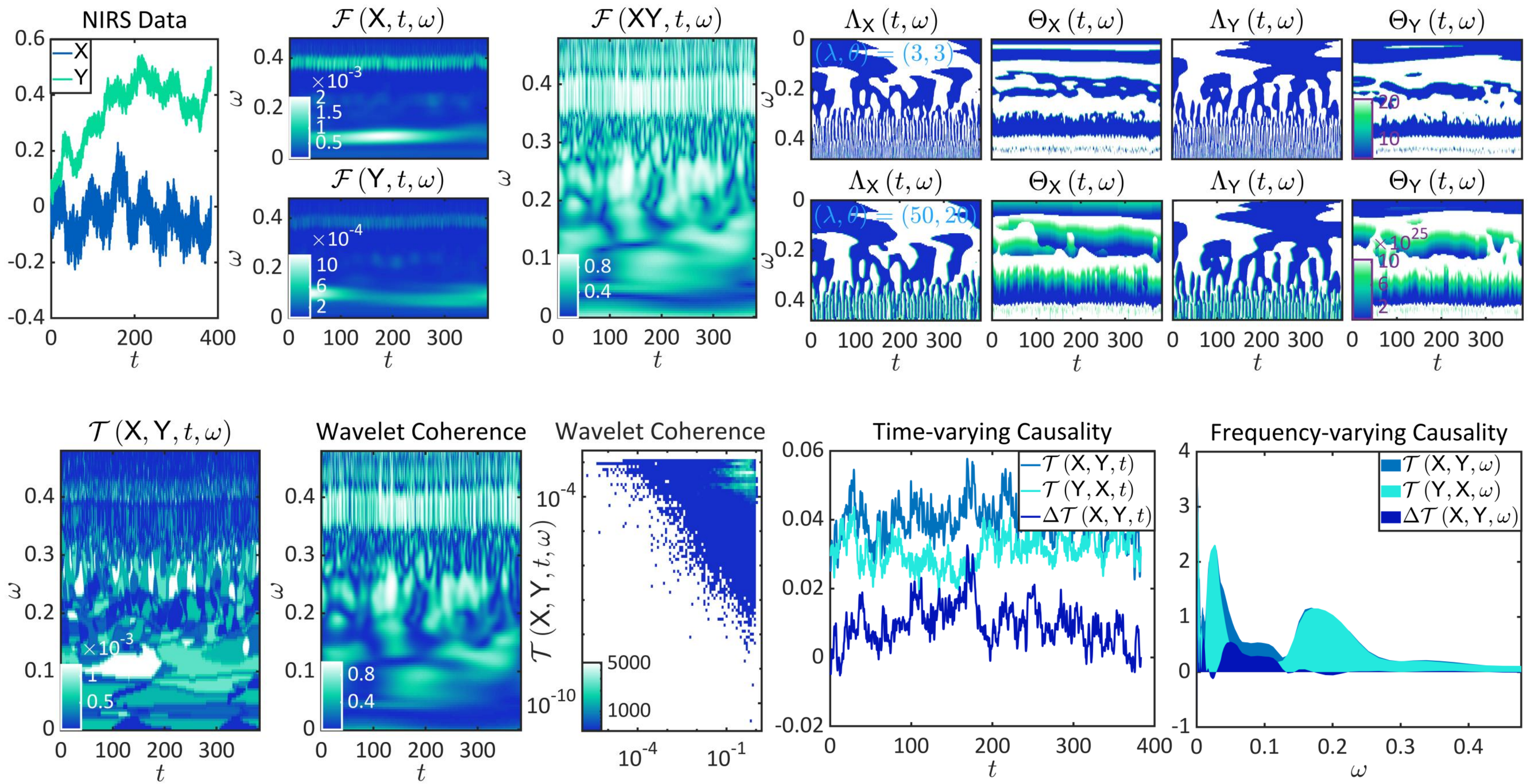
$$\mathcal{P}\left(\mathsf{Y}(t), \mathsf{X}_{[t,\beta,\Delta t]} \mid \mathsf{Y}_{[t,\beta,\Delta t]}\right) \neq \mathcal{P}\left(\mathsf{Y}(t) \mid \mathsf{Y}_{[t,\beta,\Delta t]}\right) \mathcal{P}\left(\mathsf{X}_{[t,\beta,\Delta t]} \mid \mathsf{Y}_{[t,\beta,\Delta t]}\right), \tag{1}$$

where $\mathcal{P}(\cdot)$ denotes the probability. We mark $\mathsf{X}_{[t,\beta,\Delta t]} = (\mathsf{X}(t-\beta\Delta t), \ldots, \mathsf{X}(t-\Delta t))$ as the history of X with time lag unit Δt and maximum lag number β, representing the information of system X during a time interval $[t-\beta\Delta t, t-\Delta t]$. In general, inequality (1) means that the uncertainty of Y at moment t given its own historical information (term $\mathsf{Y}_{[t,\beta,\Delta t]}$) will be regulated if the historical information of X (term $\mathsf{X}_{[t,\beta,\Delta t]}$) is added.

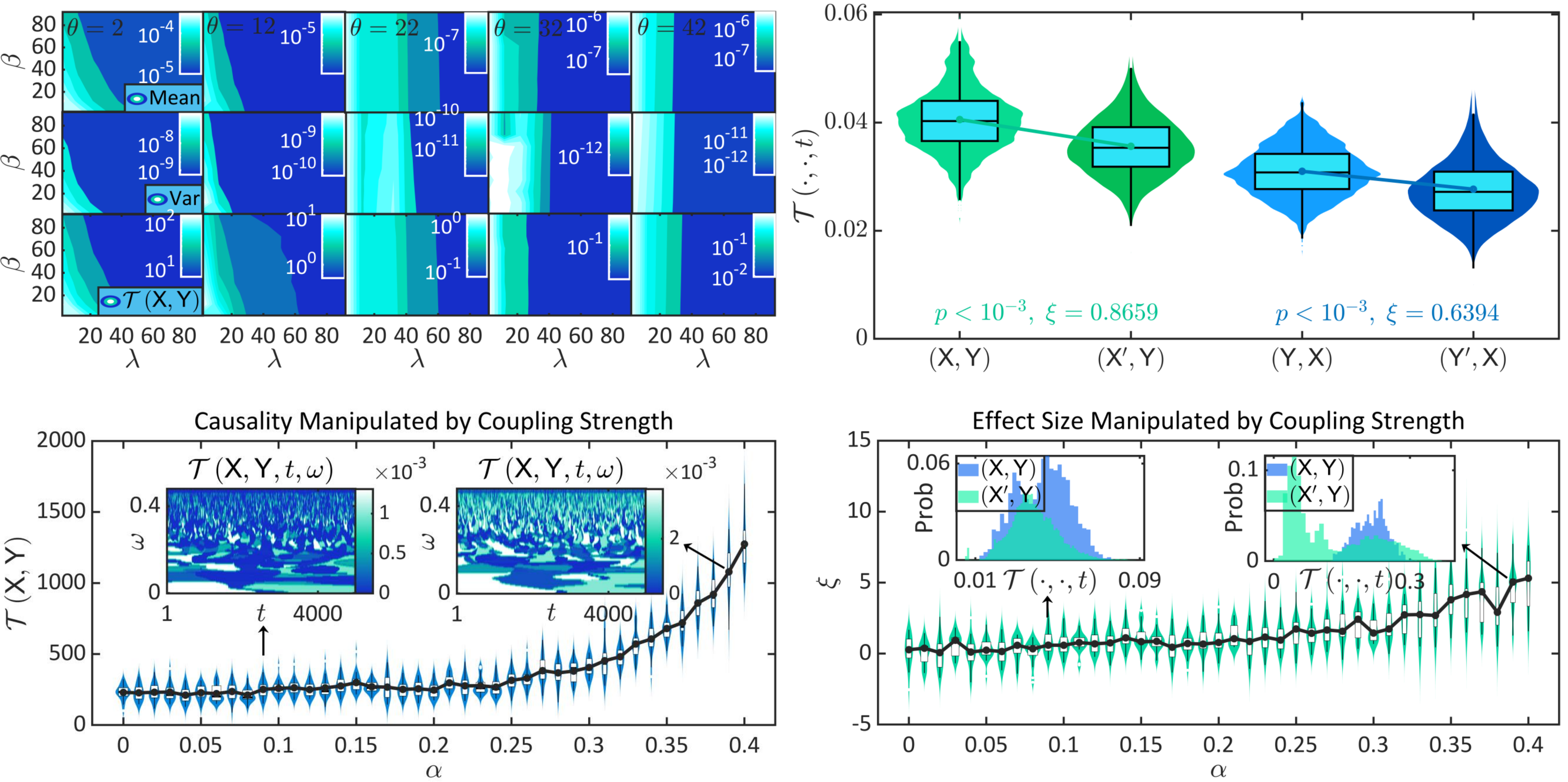
Causality lies in the **modification of uncertainty**.



Causality lies in the **information flow**.

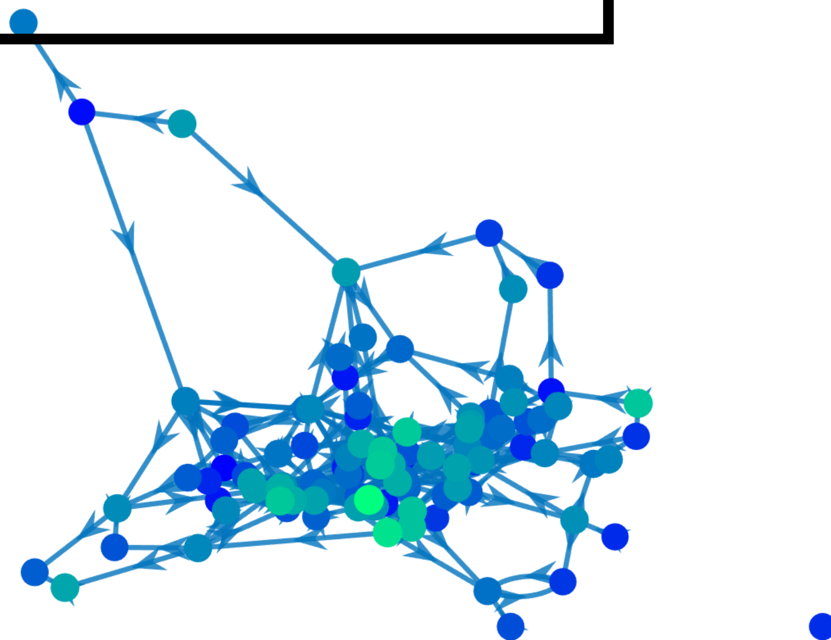


NIRS Data
X
Y
$\mathcal{F}(\mathsf{X}, t, \omega)$
$\mathcal{F}(\mathsf{Y}, t, \omega)$
$\mathcal{F}(\mathsf{XY}, t, \omega)$
$\Lambda_{\mathsf{X}}(t, \omega)$
$\Theta_{\mathsf{X}}(t, \omega)$
$\Lambda_{\mathsf{Y}}(t, \omega)$
$\Theta_{\mathsf{Y}}(t, \omega)$
$(\lambda, \theta) = (3, 3)$
$(\lambda, \theta) = (50, 20)$
$\mathcal{T}(\mathsf{X}, \mathsf{Y}, t, \omega)$
Wavelet Coherence
Wavelet Coherence
Time-varying Causality
$\mathcal{T}(\mathsf{X}, \mathsf{Y}, t)$
$\mathcal{T}(\mathsf{Y}, \mathsf{X}, t)$
$\Delta\mathcal{T}(\mathsf{X}, \mathsf{Y}, t)$
Frequency-varying Causality
$\mathcal{T}(\mathsf{X}, \mathsf{Y}, \omega)$
$\mathcal{T}(\mathsf{Y}, \mathsf{X}, \omega)$
$\Delta\mathcal{T}(\mathsf{X}, \mathsf{Y}, \omega)$
t
ω



$\theta = 2$
$\theta = 12$
$\theta = 22$
$\theta = 32$
$\theta = 42$
Mean
Var
$\mathcal{T}$ (X, Y)
β
λ
$\mathcal{T}(\cdot,\cdot,t)$
$p < 10^{-3},\ \xi = 0.8659$
$p < 10^{-3},\ \xi = 0.6394$
(X, Y)
(X′, Y)
(Y, X)
(Y′, X)
Causality Manipulated by Coupling Strength
$\mathcal{T}$ (X, Y, t, ω)
$\times 10^{-3}$
ω
t
$\mathcal{T}$ (X, Y)
α
Effect Size Manipulated by Coupling Strength
Prob
ξ

Information-Thermodynamics



Thermodynamics of encoding

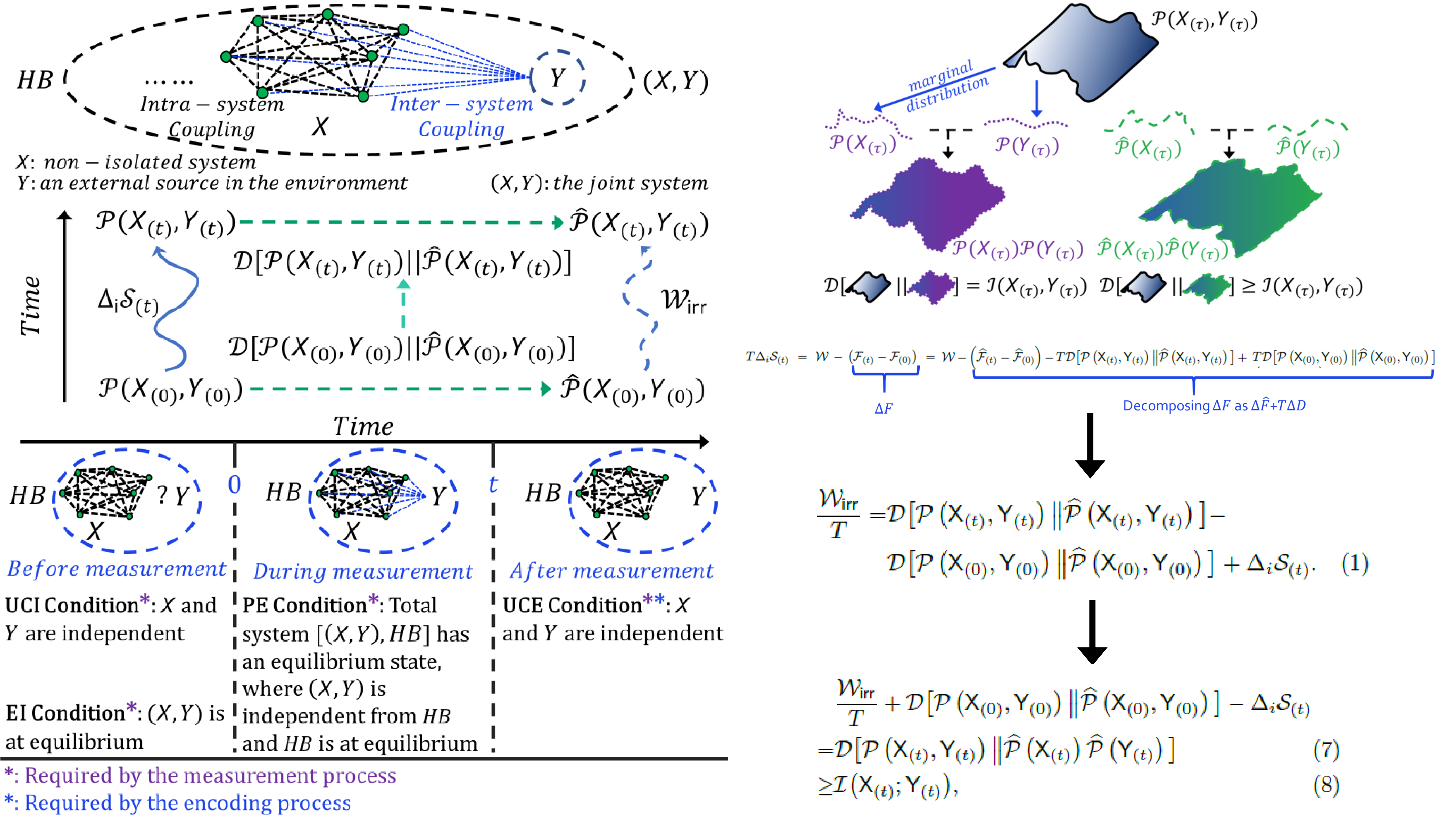
Theoretical contribution

- We derive the thermodynamics of encoding from the nonequilibrium second law of thermodynamics.
- We define information thermodynamics perceptron, a kind of generalized computer
- We prove that during encoding the information of an external source Y, a non-isolated system X with internal correlation might have specific sub-systems that can exceed the information thermodynamics bound on (X,Y) and encode more information than X itself.

Experimental contribution

- First time to analytically calculate information thermodynamics in non-isolated systems
- First time to experimentally verify information thermodynamics in the human brain

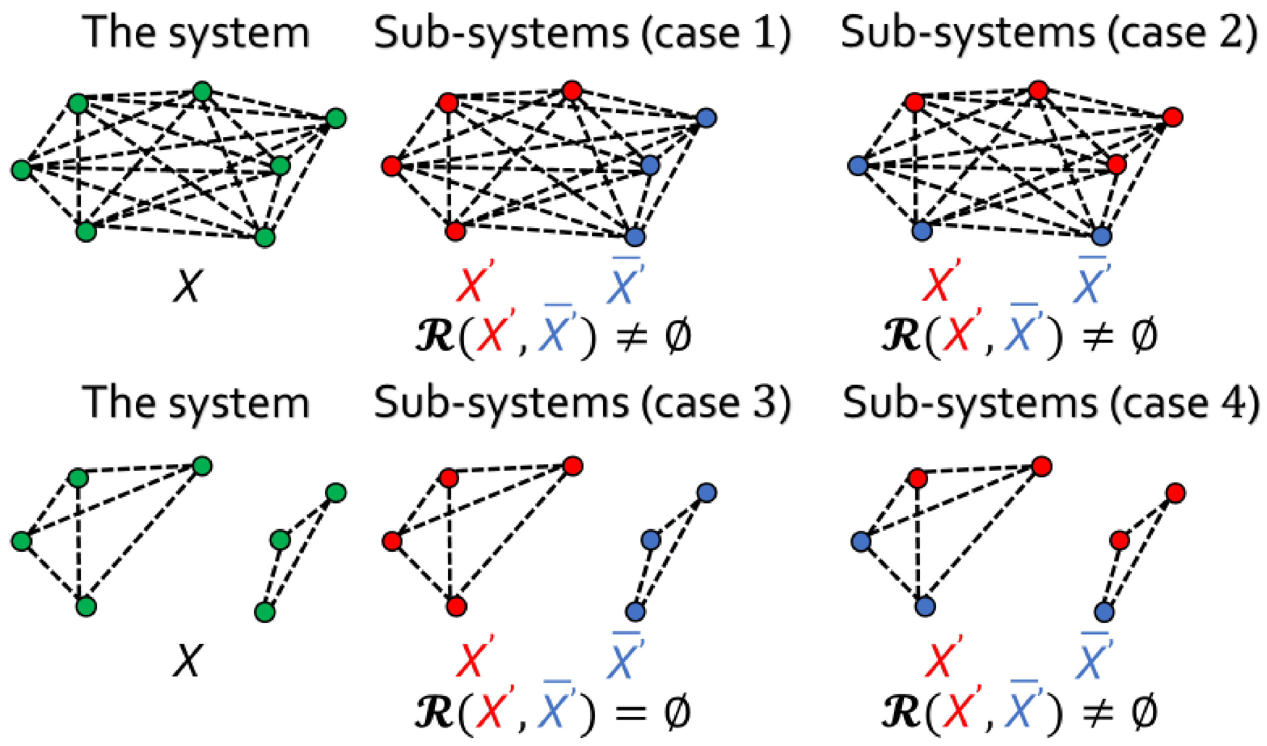
Yang Tian, Pei Sun, ***Thermodynamics of Encoding and Encoders****, under review by Chaos*



HB
Intra − system Coupling
X
Inter − system Coupling
Y
(X, Y)
X: non − isolated system
Y: an external source in the environment
(X, Y): the joint system
Time
$\mathcal{P}(X_{(t)},Y_{(t)})$
$\hat{\mathcal{P}}(X_{(t)},Y_{(t)})$
$\mathcal{D}[\mathcal{P}(X_{(t)},Y_{(t)})||\hat{\mathcal{P}}(X_{(t)},Y_{(t)})]$
$\Delta_i\mathcal{S}_{(t)}$
$\mathcal{W}_{\mathrm{irr}}$
$\mathcal{D}[\mathcal{P}(X_{(0)},Y_{(0)})||\hat{\mathcal{P}}(X_{(0)},Y_{(0)})]$
$\mathcal{P}(X_{(0)},Y_{(0)})$
$\hat{\mathcal{P}}(X_{(0)},Y_{(0)})$
Time
0
t
HB ? Y
X
Before measurement
During measurement
After measurement
UCI Condition*: X and Y are independent
EI Condition*: (X, Y) is at equilibrium
PE Condition*: Total system [(X, Y), HB] has an equilibrium state, where (X, Y) is independent from HB and HB is at equilibrium
UCE Condition**: X and Y are independent
*: Required by the measurement process
*: Required by the encoding process
$\mathcal{P}(X_{(\tau)},Y_{(\tau)})$
marginal distribution
$\mathcal{P}(X_{(\tau)})$
$\mathcal{P}(Y_{(\tau)})$
$\hat{\mathcal{P}}(X_{(\tau)})$
$\hat{\mathcal{P}}(Y_{(\tau)})$
$\mathcal{P}(X_{(\tau)})\mathcal{P}(Y_{(\tau)})$
$\hat{\mathcal{P}}(X_{(\tau)})\hat{\mathcal{P}}(Y_{(\tau)})$
$\mathcal{D}[\ ||\ ] = \mathcal{I}(X_{(\tau)},Y_{(\tau)})$
$\mathcal{D}[\ ||\ ] \geq \mathcal{I}(X_{(\tau)},Y_{(\tau)})$
$T\Delta_i\mathcal{S}_{(t)} = \mathcal{W} - \underbrace{(\mathcal{F}_{(t)} - \mathcal{F}_{(0)})}_{\Delta F} = \mathcal{W} - \underbrace{(\hat{\mathcal{F}}_{(t)} - \hat{\mathcal{F}}_{(0)}) - T\mathcal{D}[\mathcal{P}(X_{(t)},Y_{(t)})||\hat{\mathcal{P}}(X_{(t)},Y_{(t)})] + T\mathcal{D}[\mathcal{P}(X_{(0)},Y_{(0)})||\hat{\mathcal{P}}(X_{(0)},Y_{(0)})]}_{\text{Decomposing } \Delta F \text{ as } \Delta\hat{F}+T\Delta D}$
$\frac{\mathcal{W}_{\mathrm{irr}}}{T} = \mathcal{D}[\mathcal{P}(X_{(t)},Y_{(t)})||\hat{\mathcal{P}}(X_{(t)},Y_{(t)})] - \mathcal{D}[\mathcal{P}(X_{(0)},Y_{(0)})||\hat{\mathcal{P}}(X_{(0)},Y_{(0)})] + \Delta_i\mathcal{S}_{(t)}. \quad (1)$
$\frac{\mathcal{W}_{\mathrm{irr}}}{T} + \mathcal{D}[\mathcal{P}(X_{(0)},Y_{(0)})||\hat{\mathcal{P}}(X_{(0)},Y_{(0)})] - \Delta_i\mathcal{S}_{(t)}$
$= \mathcal{D}[\mathcal{P}(X_{(t)},Y_{(t)})||\hat{\mathcal{P}}(X_{(t)})\hat{\mathcal{P}}(Y_{(t)})] \quad (7)$
$\geq \mathcal{I}(X_{(t)};Y_{(t)}), \quad (8)$

My research question:

Let us consider an arbitrary sub-system X' of system X, is the maximum information of Y that can be encoded in X' is bound by (no more than) the irreversible work $\mathcal{W}_{\text{irr}}$ from the joint system (X, Y) as well?



$$\mathcal{I}(X'; \overline{X'}; Y) = \mathcal{I}(X'; Y) + \mathcal{I}(\overline{X'}; Y) - \mathcal{I}((X', \overline{X'}); Y)$$

3-order mutual information

The information of Y that is encoded in X'

The information of Y that is encoded in $\overline{X'}$

The information of Y that is encoded in X

Possibility 1

$$\mathcal{I}(X'; \overline{X'}; Y) \leq 0 \longrightarrow \mathcal{I}((X', \overline{X'}); Y) \geq \mathcal{I}(X'; Y) + \mathcal{I}(\overline{X'}; Y) \longrightarrow \mathcal{I}((X', \overline{X'}); Y) \geq \mathcal{I}(X'; Y)$$

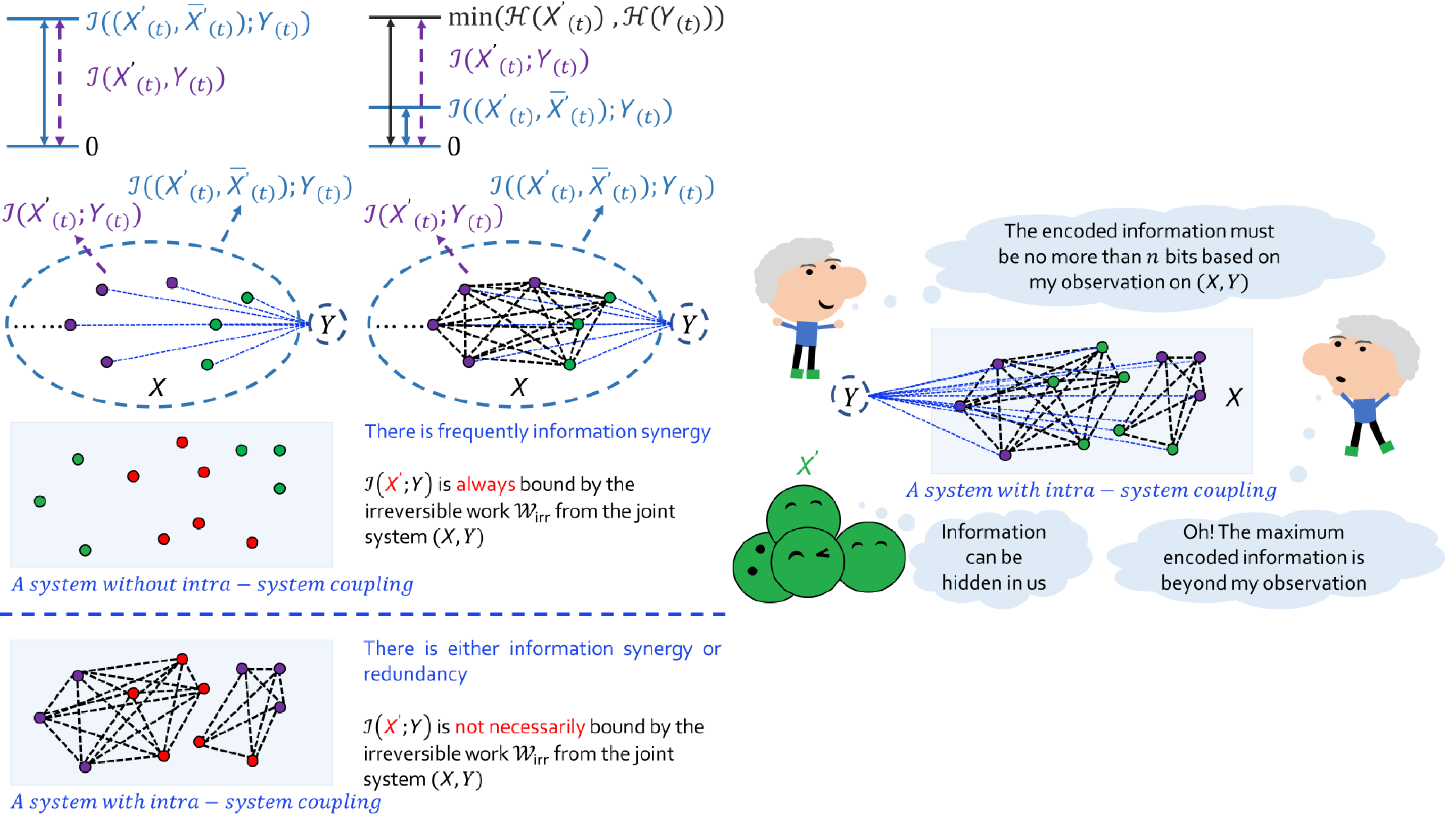
Possibility 2

$$\mathcal{I}(X'; \overline{X'}; Y) > 0 \longrightarrow \mathcal{I}((X', \overline{X'}); Y) < \mathcal{I}(X'; Y) + \mathcal{I}(\overline{X'}; Y) \longrightarrow$$

$\mathcal{I}((X', \overline{X'}); Y) \geq \mathcal{I}(X'; Y)$

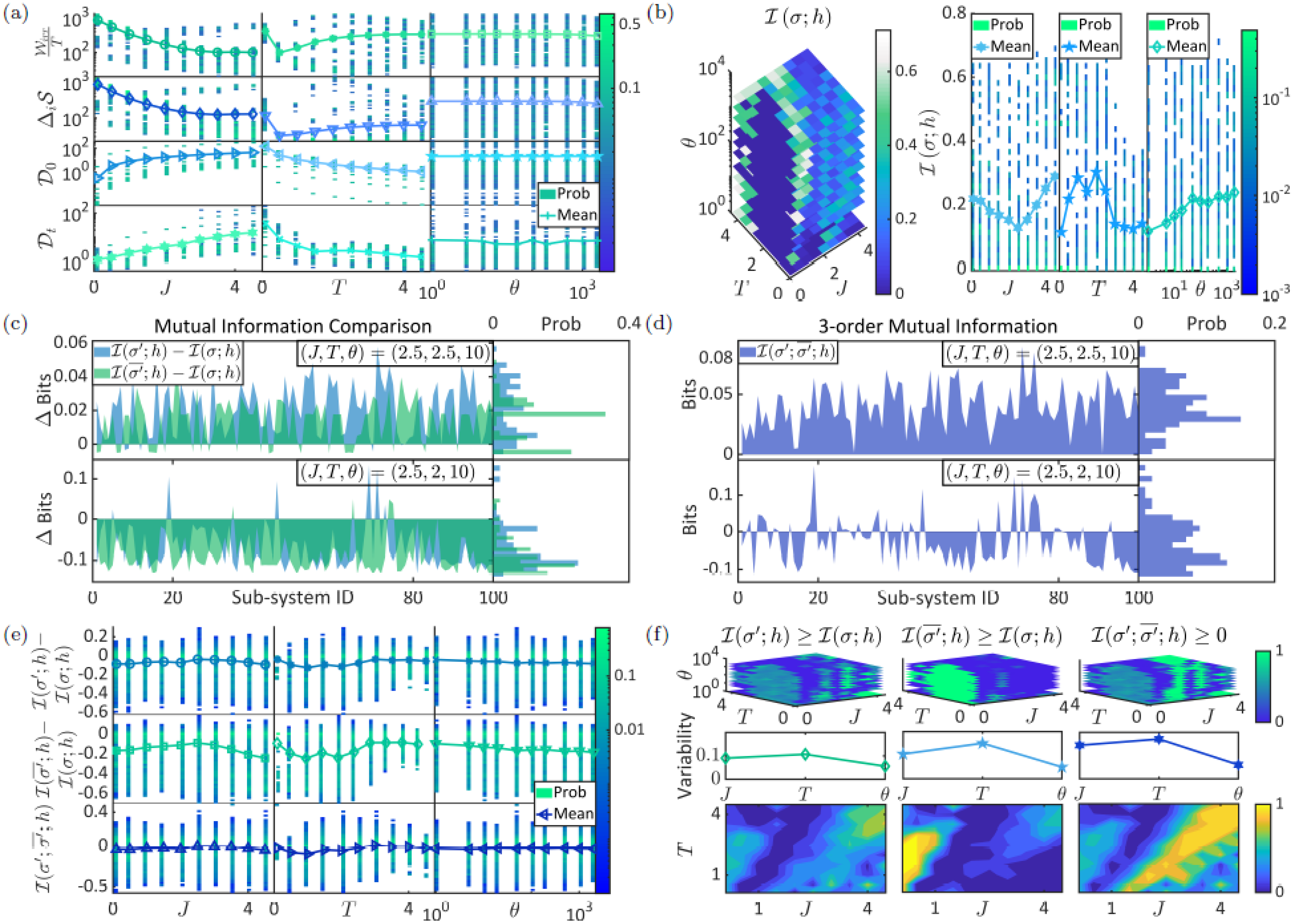
or

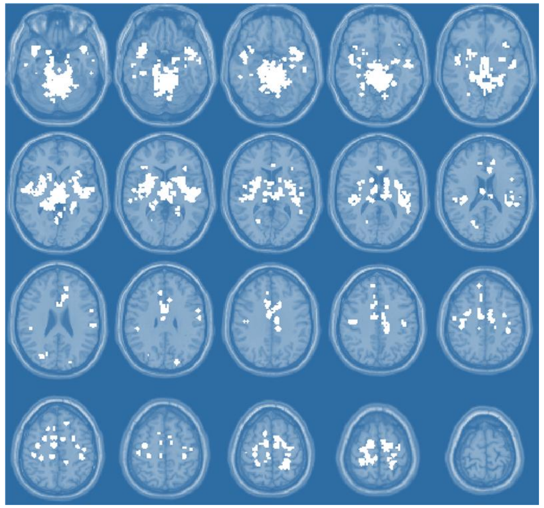
$\mathcal{I}((X', \overline{X'}); Y) < \mathcal{I}(X'; Y)$

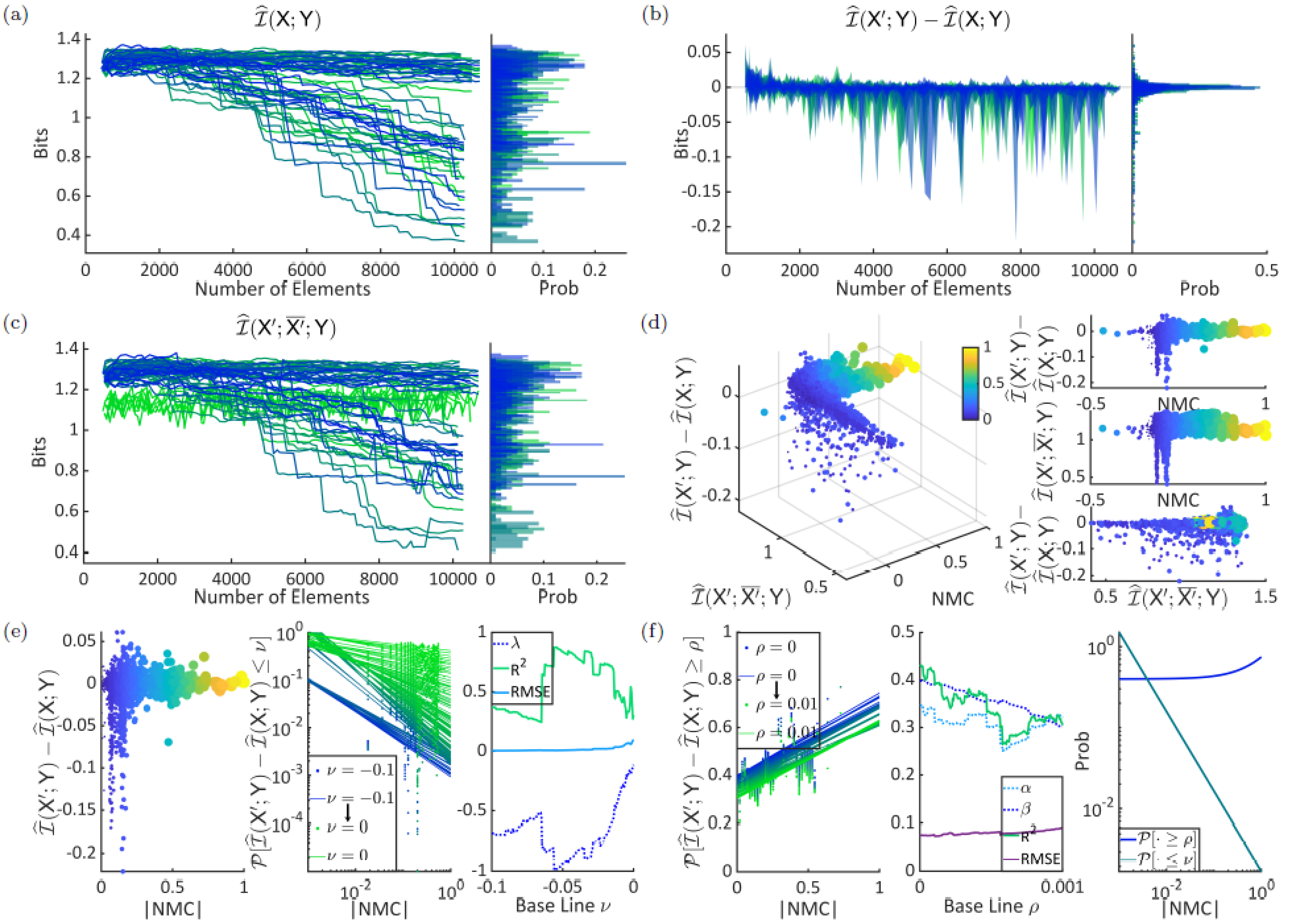


$\mathcal{I}((X'_{(t)},\bar{X}'_{(t)});Y_{(t)})$
$\mathcal{I}(X'_{(t)},Y_{(t)})$
0
$\min(\mathcal{H}(X'_{(t)}),\mathcal{H}(Y_{(t)}))$
$\mathcal{I}(X'_{(t)};Y_{(t)})$
$\mathcal{I}((X'_{(t)},\bar{X}'_{(t)});Y_{(t)})$
0
$\mathcal{I}((X'_{(t)},\bar{X}'_{(t)});Y_{(t)})$
$\mathcal{I}(X'_{(t)};Y_{(t)})$
Y
X
$\mathcal{I}((X'_{(t)},\bar{X}'_{(t)});Y_{(t)})$
$\mathcal{I}(X'_{(t)};Y_{(t)})$
Y
X
There is frequently information synergy
$\mathcal{I}(X';Y)$ is always bound by the irreversible work $\mathcal{W}_{\text{irr}}$ from the joint system (X,Y)
$A\ system\ without\ intra-system\ coupling$
There is either information synergy or redundancy
$\mathcal{I}(X';Y)$ is not necessarily bound by the irreversible work $\mathcal{W}_{\text{irr}}$ from the joint system (X,Y)
$A\ system\ with\ intra-system\ coupling$
The encoded information must be no more than n bits based on my observation on (X,Y)
Y
X
X'
$A\ system\ with\ intra-system\ coupling$
Information can be hidden in us
Oh! The maximum encoded information is beyond my observation

First time to analytically calculate information thermodynamics in non-isolated systems

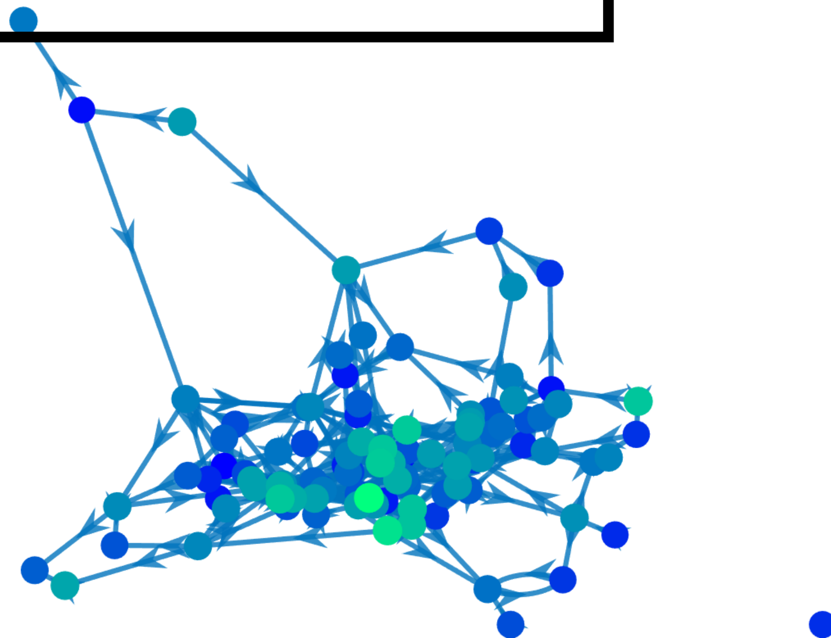


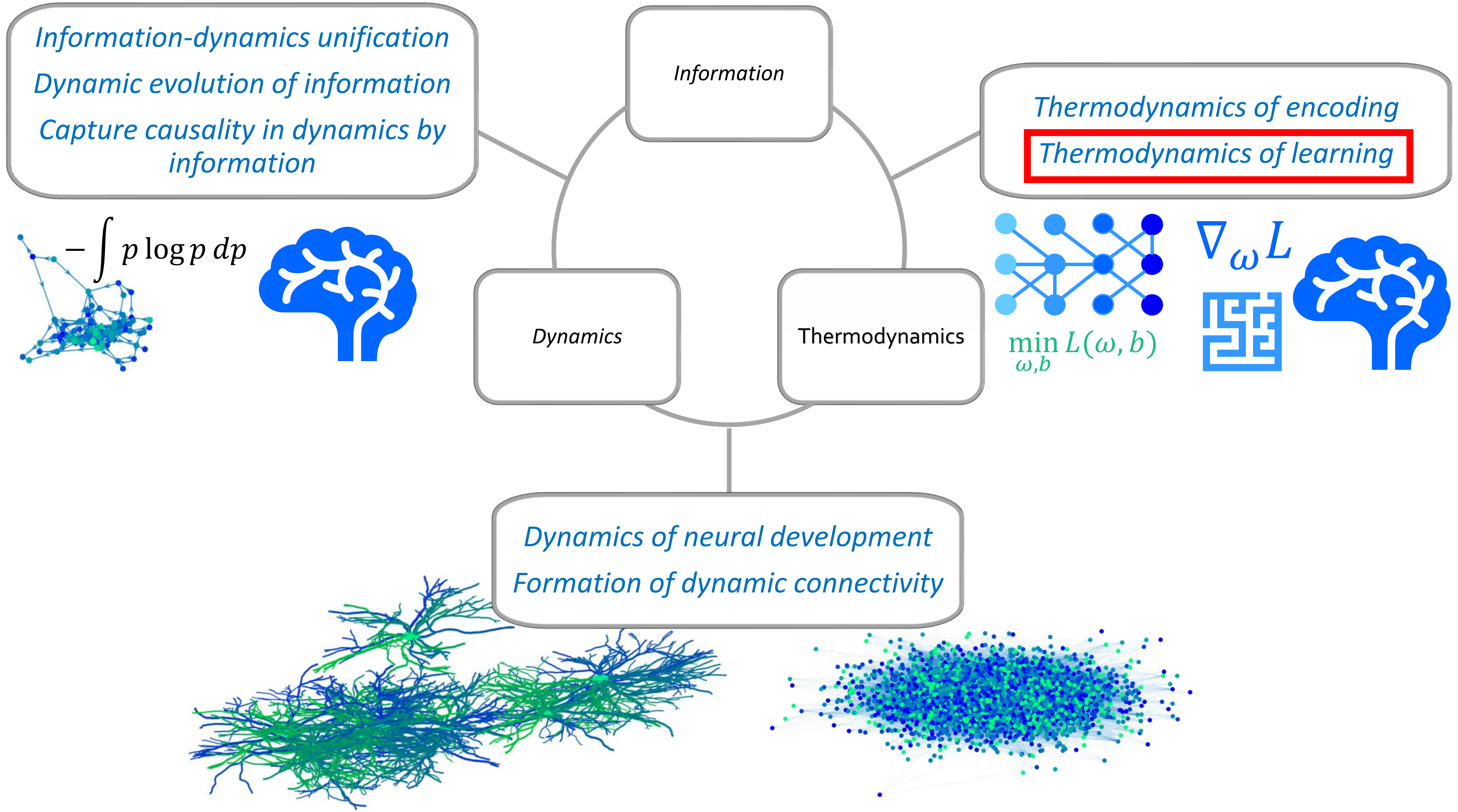




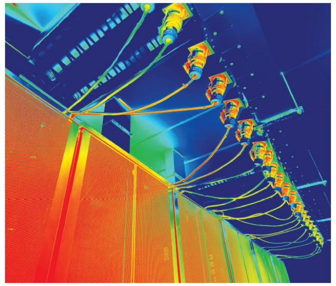
First time to experimentally verify information thermodynamics in the human brain

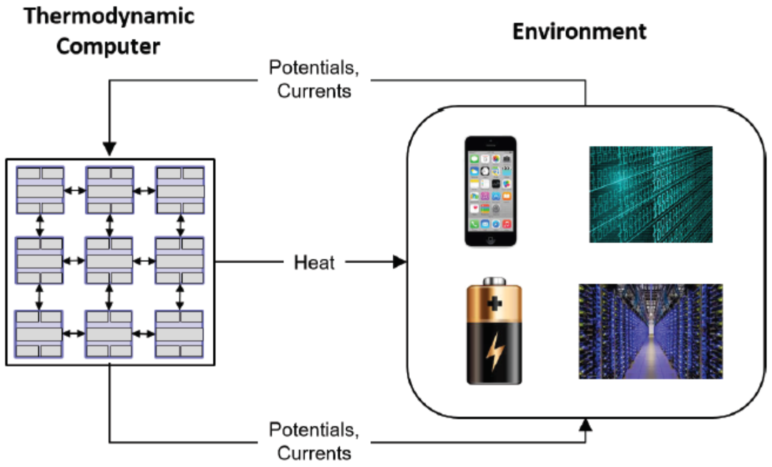
Summary and outlook of our research

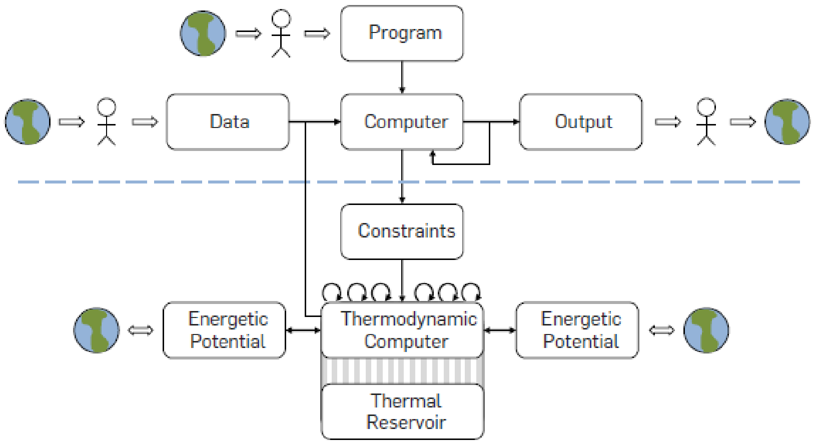


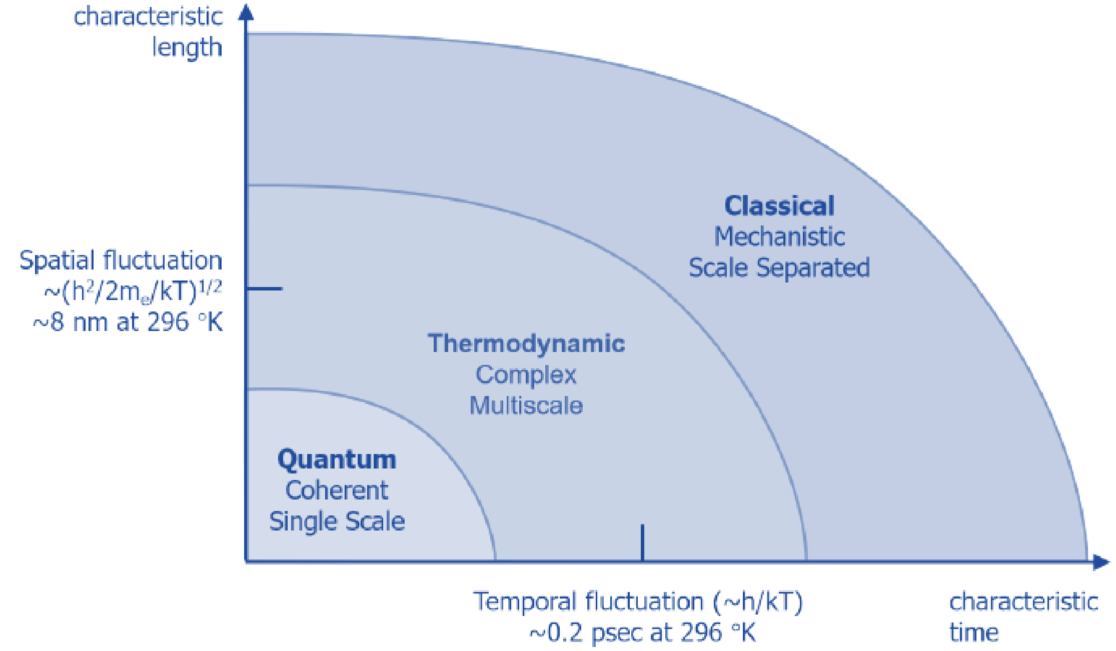

Information-dynamics unification
Dynamic evolution of information
Capture causality in dynamics by information
Information
Thermodynamics of encoding
Thermodynamics of learning
$-\int p \log p \, dp$
$\nabla_\omega L$
Dynamics
Thermodynamics
$\min_{\omega,b} L(\omega, b)$
Dynamics of neural development
Formation of dynamic connectivity

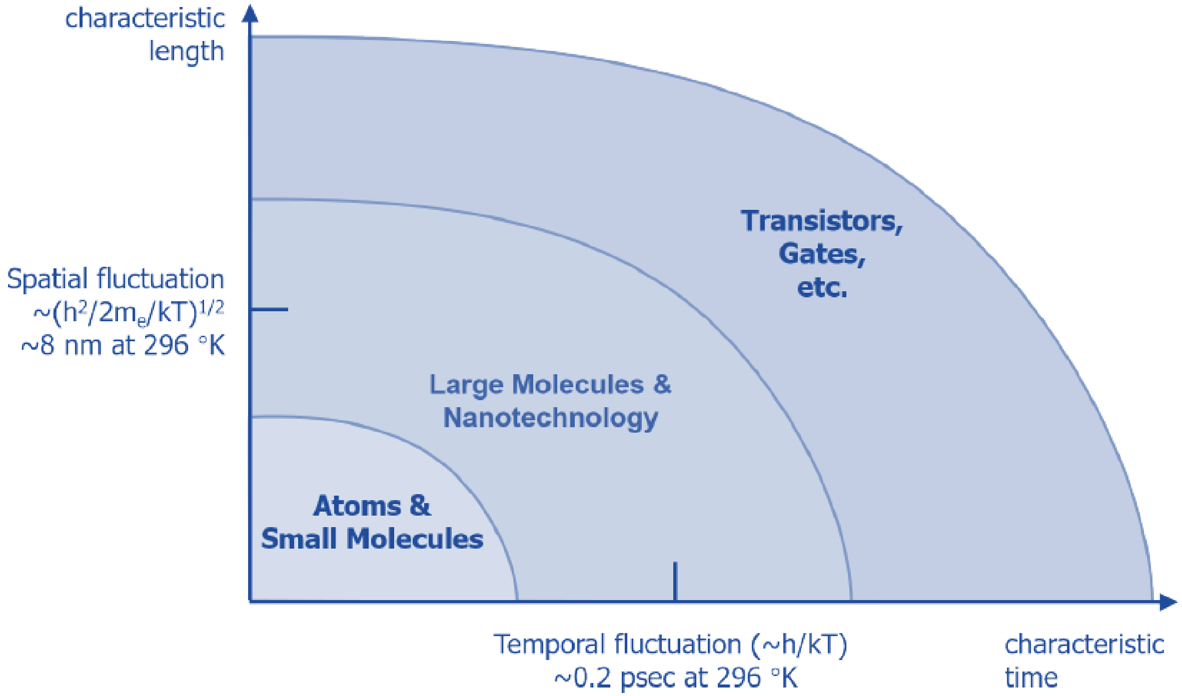
Thermodynamics of learning & computation





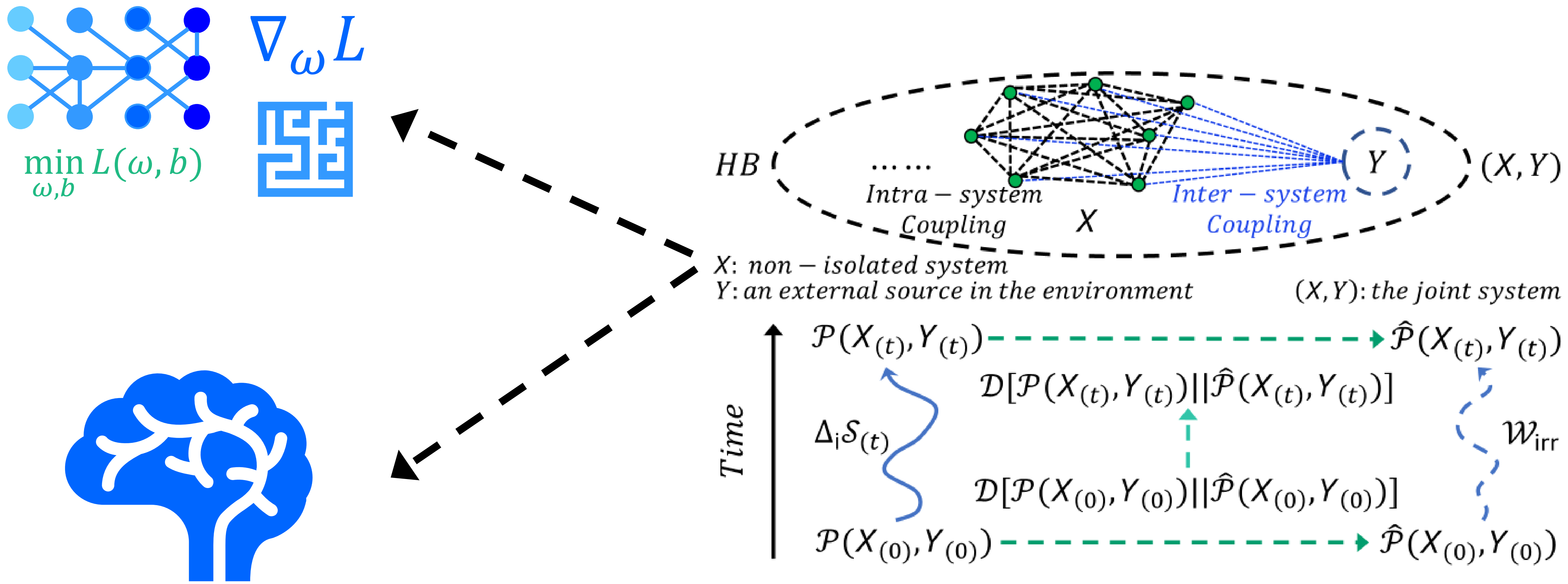






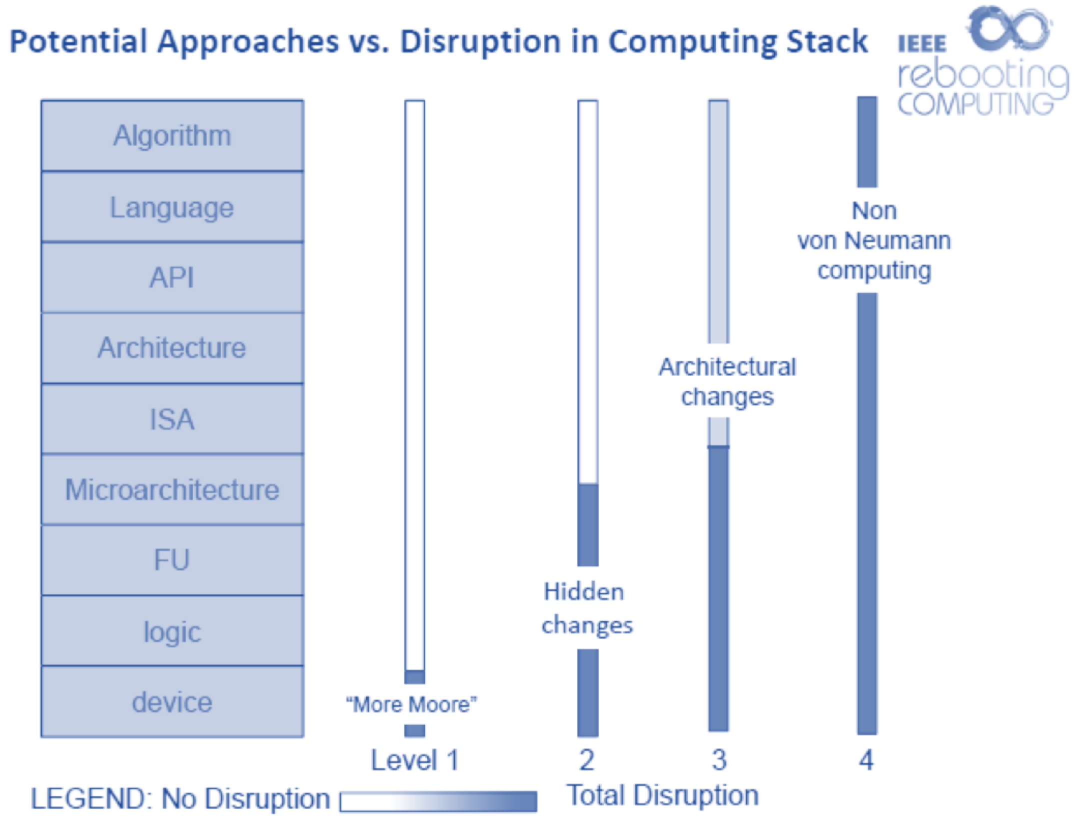
Principle	Meaning
Information Destruction $\langle W \rangle \geq k_B T \ln(2)$	Logically irreversible operations dissipate energy (Landauer, 1961)
Reciprocity $\langle W_{min}^{t-sym} \rangle = k_B T \langle \Psi \rangle - \langle W \rangle$	Logically nonreciprocal operations dissipate energy
Information Creation $\dot{Q} \geq k_B T \ln(2)\left(h_\mu - h'_\mu / \hat{L}\right)$	Creating information dissipates heat (Aghamohammdi & Crutchfield, 2017)
Information Process Second Law $\langle W \rangle \geq k_B T \left(h'_\mu - h_\mu\right)$	Work to drive (or energy dissipated) during computation (Boyd, Mandal, & Crutchfield, 2016a)
Requisite Complexity $\langle W \rangle \leq k_B T \ln(2) \min\{\Delta H_1, \Delta h_\mu\}$	Advantage maximized when controller matches environment (Boyd, Mandal, & Crutchfield, 2016b)
Synchronization & Error Correction $\langle Q^{tran} \rangle_{min} \geq k_B T \ln(2)\, \mathrm{I}[X_0 : \tilde{Y}'] - \boldsymbol{E}'$	Work to correct errors or synchronize to environment (Boyd, Mandal, & Crutchfield, 2017)
Modularity $\langle \Sigma_{t\to t+\tau}^{mod} \rangle_{min} = k_B T \ln(2)\, \Delta I_{t\to t+\tau}$	Controller modularity is thermodynamically expensive (Boyd, Mandal, & Crutchfield, 2018)
Information Dynamics $\lambda > 0$	Maxwellian demons are chaotic dynamical systems (Boyd & Crutchfield, 2016)
Steady-State Transitions $\Pr(W_{ex}, \Psi)/\Pr(-W_{ex}, -\Psi) = e^{\Psi} e^{\beta W_{ex}}$	Work to drive transitions between information storage states (Riechers & Crutchfield, 2017)
Functional Fluctuations $Iu=β−1−1hμPβ−β−1log2λ\ I(u) = (\beta^{-1} - 1) h_\mu(P_\beta) - \beta^{-1} \log_2(\hat{\lambda}_\beta)$	Engine functionality fluctuates in small systems, short times (Crutchfield & Aghamohammdi, 2016)
Control tradeoffs $\dot{\Sigma} = f(1/\tau, L^2)$	Counterdiabatic control dissipation design (Campbell & De, 2017) (Boyd, Patra, Jarzynski, & Crutchfield, 2018)
Reliability $\dot{\Sigma} = f(-\ln(\epsilon))$	Dissipation costs of high-reliability information processing
Trajectory-class fluctuations $\langle e^{-W/k_B T} \rangle_C = R(C^{\mathrm{R}})/P(C)\ e^{-\Delta F / k_B T}$	Success and failure have thermodynamic signatures (Wimsatt, et al., 2019)

There are thermodynamics costs of computation, optimization, memory, renewal, and encoding

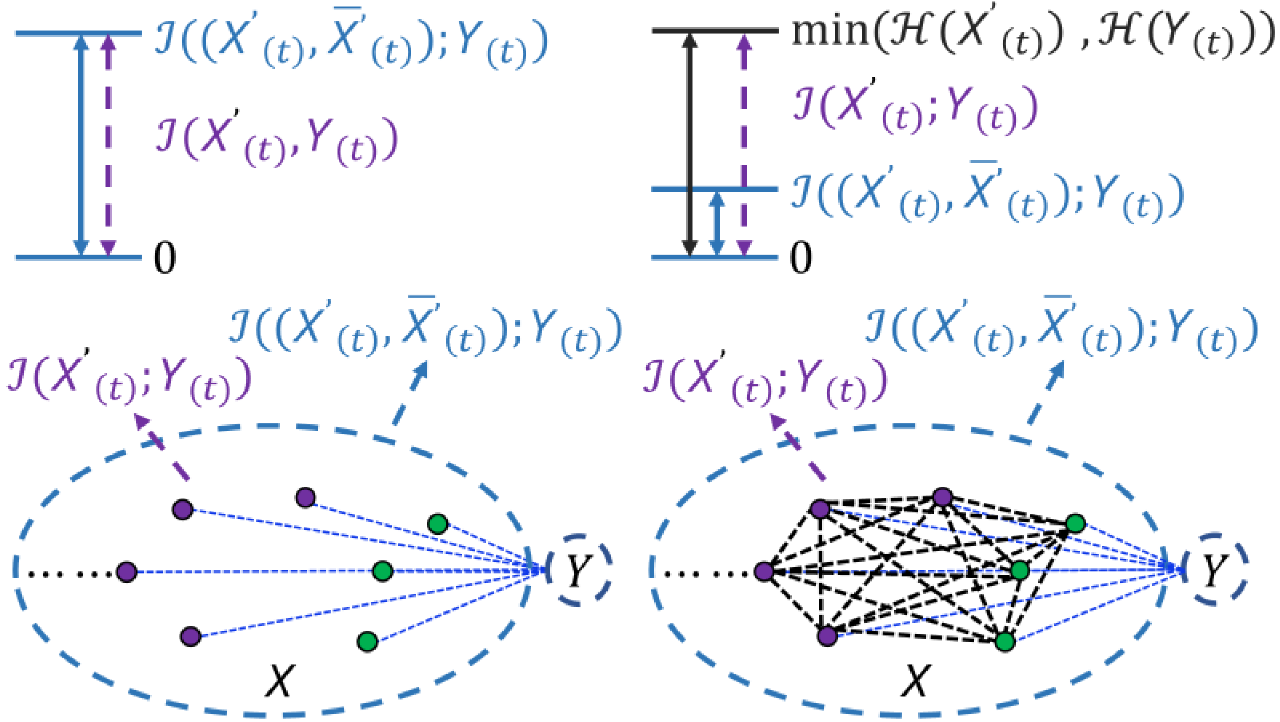


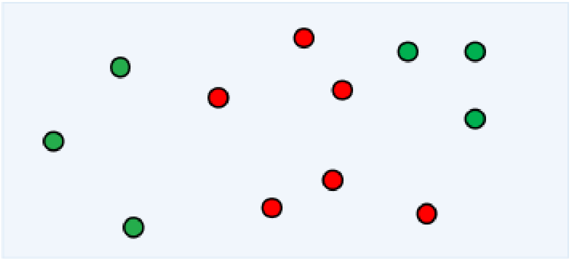
If there exists the thermodynamics of learning?

Learning can be treated as a kind of encoding where the information thermodynamics perceptron
Attempts to maximize the correctly encoded information.



Can we optimize the internal orders (or structures) to create more possibility for a sub-system to encode more information given the thermodynamics cost?

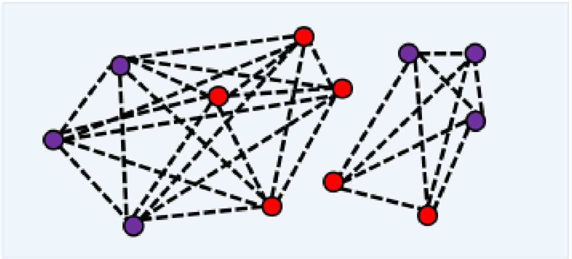




A system without intra − system coupling

There is frequently information synergy

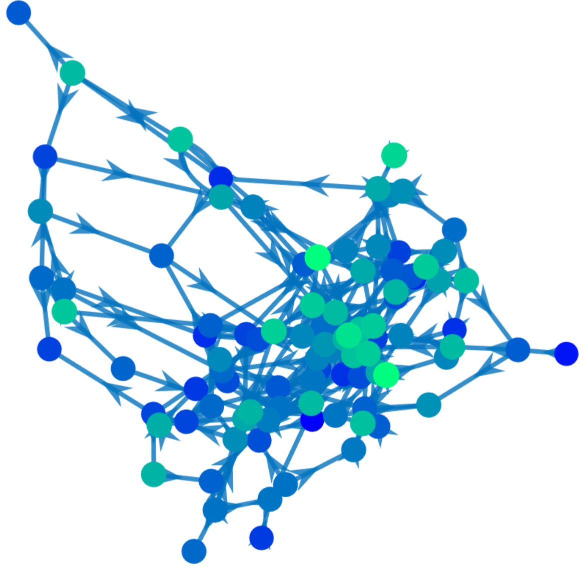
$\mathcal{I}(X';Y)$ is always bound by the irreversible work $\mathcal{W}_{irr}$ from the joint system (X, Y)



A system with intra − system coupling

There is either information synergy or redundancy

$\mathcal{I}(X';Y)$ is not necessarily bound by the irreversible work $\mathcal{W}_{irr}$ from the joint system (X, Y)



Thanks and any question?