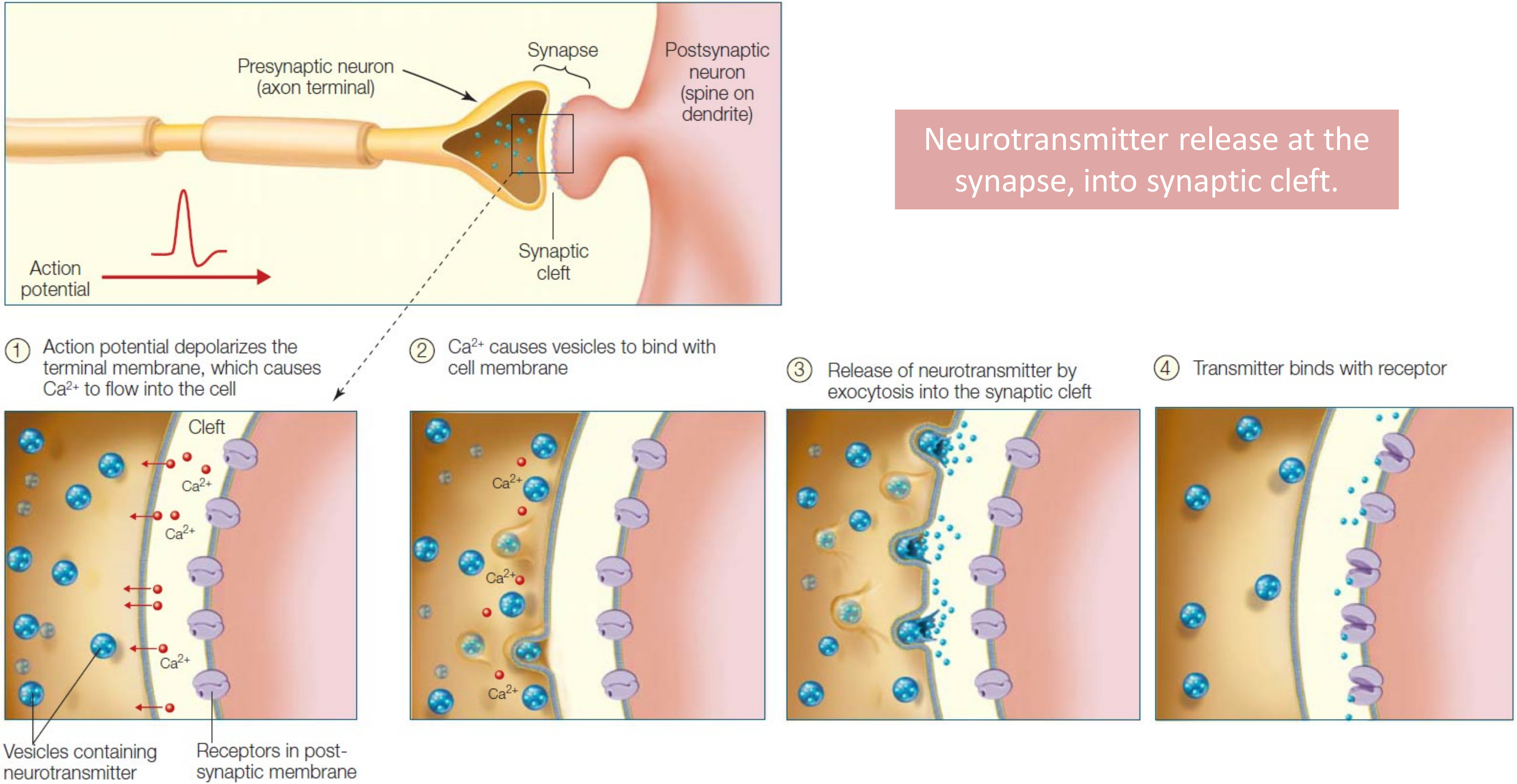
Synaptic Model (1):

Phenomenological Models

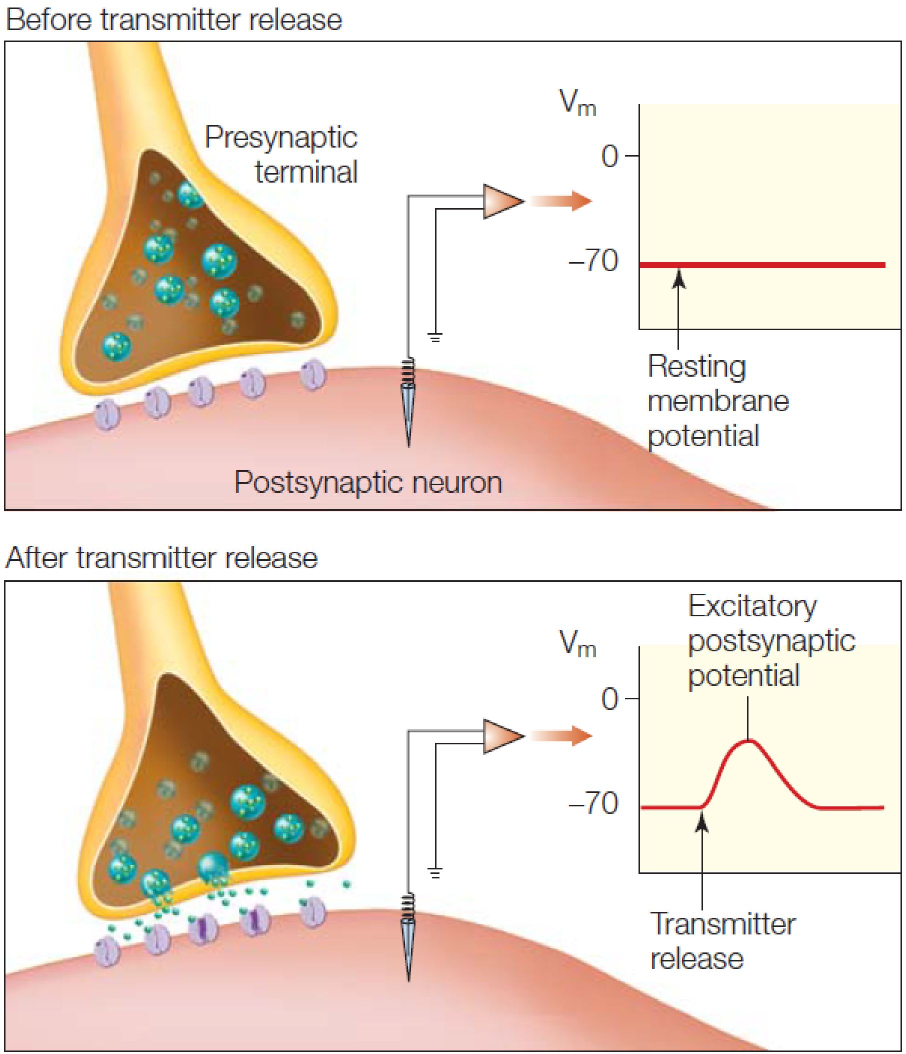
Neuron models	HH neuron model
	LIF neuron model
	Exponential IF model
Synapse models	Exponential/Alpha synapse
	AMPA/GABA/NMDA synapse
Network Models	E/I balance network
	Continuous attractor network
	Working memory model
	Decision making model

Synaptic Transmission

突触：大脑内信息交流的基本媒介



Neurotransmitter release at the synapse, into synaptic cleft.



Neurotransmitter leading to postsynaptic potential

What makes a molecule a neurotransmitter?

1. It is synthesized by and localized within the presynaptic neuron, and stored in the presynaptic terminal before release.
2. It is released by the presynaptic neuron when action potentials depolarize the terminal (mediated primarily by Ca2+).
3. Th e postsynaptic neuron contains receptors specific for the neurotransmitter.
4. When artificially applied to a postsynaptic cell, the neurotransmitter elicits the same response that stimulating the presynaptic neuron would.

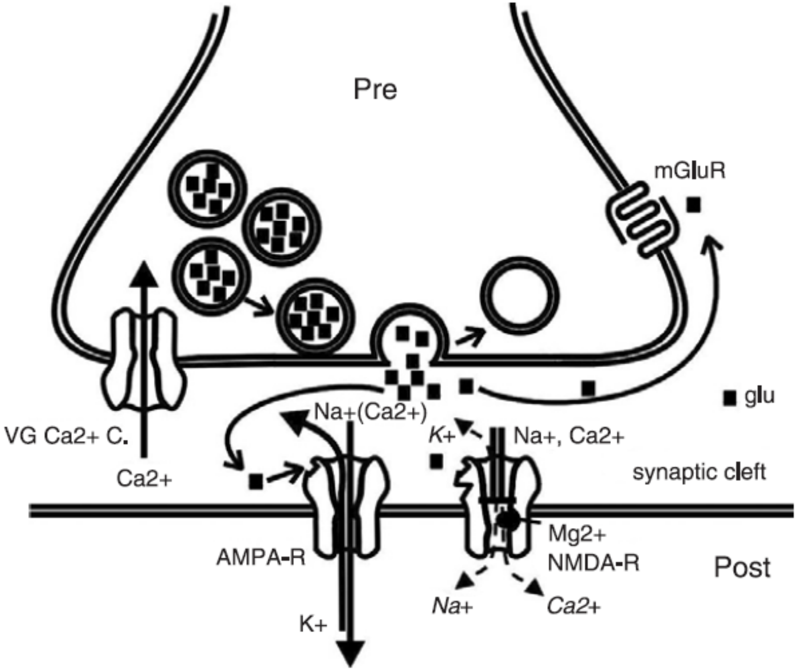
兴奋性神经递质：

- 乙酰胆碱 (ACh)
- 儿茶酚胺 (catecholamines)
- 谷氨酸 (glutamate)
- 组胺 (histamine)
- 5-羟色胺 (serotonin)
- 某些神经肽类 (some of neuropeptides)

抑制性神经递质：

- GABA
- 甘氨酸 (glycine)
- 某些神经肽类 (some of peptides)

Biochemical Classification of Neurotransmitters

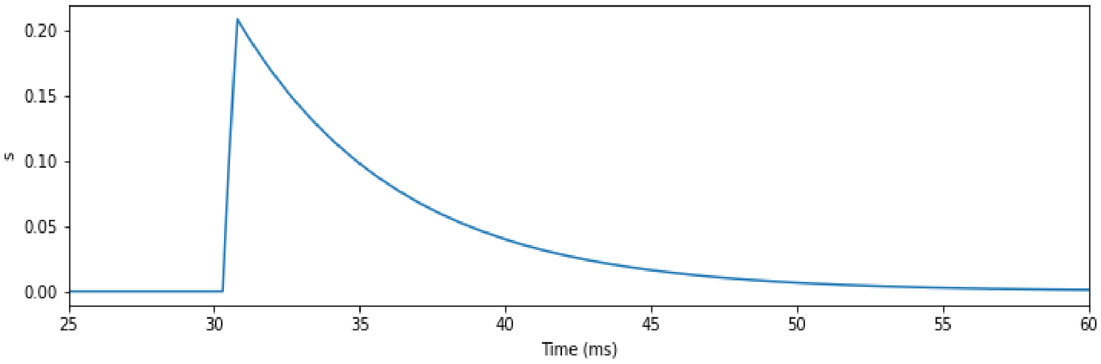


Overview of glutamergic synaptic transmission

Phenomenological Models

Phenomenological Models

The aim of a synapse model is to describe accurately the postsynaptic response generated by the arrival of an action potential at a presynaptic terminal.

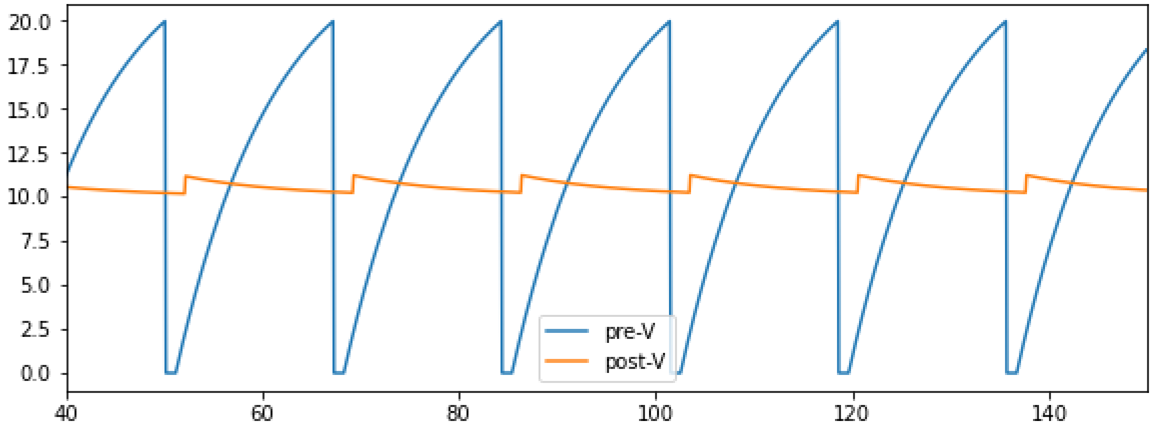


- Voltage jump model
- Exponential model
- Alpha function model
- Dual exponential model

Voltage Jump Model

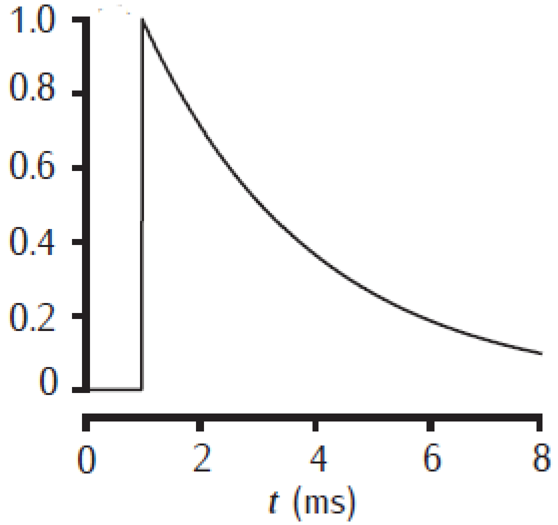
$$I = \sum_{j \in C} g\delta(t - t_j - D)$$

- g denotes the chemical synaptic strength
- t_j the spiking moment of the presynaptic neuron j
- C the set of neurons in the encoding layer
- D the transmission delay of chemical synapses



- Omit the rise and decay phases of post-synaptic currents!

Exponential Model



$$g_{\text{syn}}(t) = \bar{g}_{\text{syn}} e^{-(t-t_0)/\tau}$$

- τ is the time constant
- t_0 is the time of the pre-synaptic spike
- $\bar{g}_{syn}$ is the maximal conductance

↓ corresponding differential equation

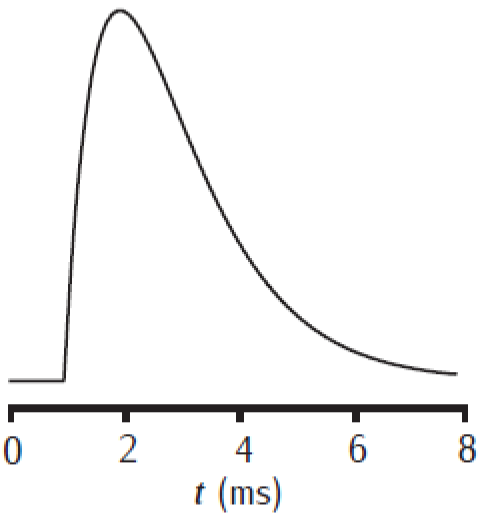
$$\tau \frac{dg_{\text{syn}}(t)}{dt} = -g_{\text{syn}}(t) + \bar{g}_{\text{syn}} \delta(t_0 - t)$$

Assumption:

- The release of neurotransmitter, its diffusion across the cleft, the receptor binding, and channel opening all happen very quickly, so that the channels instantaneously jump from the closed to the open state.

- Can fit with experimental data.
- A good approximation for $GABA_A$ and AMPA, because the rising phase is much shorter than their decay phase.

Alpha Function Model



$$g_{\mathrm{syn}}(t) = \bar{g}_{\mathrm{syn}} \frac{t - t_0}{\tau} e^{1-(t-t_0)/\tau}$$

- τ is the synaptic time constant
- t_0 is the time of the pre-synaptic spike
- $\bar{g}_{syn}$ is the maximal conductance

corresponding differential equation

$$g_{\mathrm{syn}}(t) = \bar{g}_{\mathrm{syn}} g$$

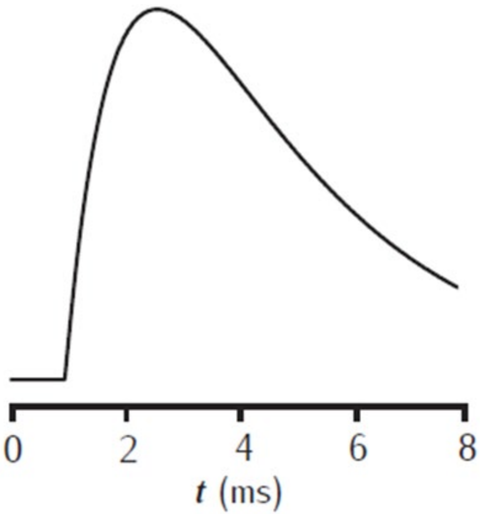
$$\frac{dg}{dt} = -\frac{g}{\tau} + h$$

$$\frac{dh}{dt} = -\frac{h}{\tau} + \delta(t_0 - t)$$

- For most synapses, the rising phase of synaptic conductance has a finite duration, which can have strong effects on network dynamics (van Vreeswijk et al., 1994).
- The alpha function can describe a conductance that has a rising phase that is not infinitely fast, but has a certain rise time.

- A good approximation for some synapses.

Dual Exponential Model



$$g_{\text{syn}}(t) = \bar{g}_{\text{syn}} \frac{\tau_1 \tau_2}{\tau_1 - \tau_2} \left(\exp\left(-\frac{t - t_0}{\tau_1} \right) - \exp\left(-\frac{t - t_0}{\tau_2} \right) \right)$$

- τ is the synaptic time constant
- t_0 is the time of the pre-synaptic spike
- $\bar{g}_{syn}$ is the maximal conductance

corresponding differential equation

$$g_{\text{syn}}(t) = \bar{g}_{\text{syn}} g$$

$$\frac{dg}{dt} = -\frac{g}{\tau_{\text{decay}}} + h$$

$$\frac{dh}{dt} = -\frac{h}{\tau_{\text{rise}}} + \delta(t_0 - t),$$

- In alpha synapse, the time courses of the rise and decay are correlated and cannot be set independently. In general this condition is not physiologically realistic.
- Dual exponential synapse provides a general way to describe the synaptic conductance with different rising and decay time constants.

- The time course of most synaptic conductance can be well described by this sum of two exponentials.

Current-based and Conductance-based Synapses

Conductance-base Synapse

Most synaptic ion channels, such as AMPA and GABA, display an approximately linear current-voltage relationship when they open.

$$I_{syn} = \boldsymbol{g_{syn}(t)}\,[\boldsymbol{V(t) - E_{syn}}]$$

Ohmic conductance   The driving force

- E_{syn} is the reversal potential
- $V(t)$ is the post-synaptic current

For example:

- The synapse is located on a thin dendrite, because the local membrane potential V changes considerably when the synapse is activated.

Conductance-base Synapse

In some case, we can also approximate the synapses as sources of current and not a conductance.

$$I_{syn} = \boldsymbol{g_{syn}(t)}\,[V_{rest} - E_{syn}]$$

$$I_{syn} = \boldsymbol{g_{syn}(t)}\,J$$

- J is a constant

For example:

- The excitatory synapse on a large compartment, because the depolarization of the membrane is small.

Coding Example: Exponential Model

Pre-synaptic neuron

$$\frac{ds}{dt} = -\frac{s}{\tau_{decay}} + \boxed{\sum_k \delta(t - t_j^k)}$$

Synaptic Delay, like 2 ms

$$I_{syn} = \bar{g}_{syn} \boxed{s} (\boxed{V} - E_{syn})$$

Delayed state

Post-synaptic neuron

```python
[1]: class AMPA(bp.TwoEndConn):
         target_backend = ['numpy', 'numba']

         def __init__(self, pre, post, conn, delay=0, g_max=0.1, E=0, tau=2, **kwargs):
             # parameters
             self.g_max = g_max
             self.E = E
             self.tau = tau
             self.delay = delay

             # connections
             self.conn = conn(pre.size, post.size)
             self.pre_ids, self.post_ids = conn.requires('pre_ids', 'post_ids')
             self.size = len(self.pre_ids)

             # variables
             self.s = bp.backend.zeros(self.size)
             self.g = self.register_constant_delay('g', size=self.size, delay_time=delay)

             super(AMPA, self).__init__(pre=pre, post=post, **kwargs)
```

Initialize Parameters

pre ids	0	0	0	1	1	1	1	2	···
post ids	3	5	7	0	2	4	6	1	···
syn ids	0	1	2	3	4	5	6	7	···

Synaptic connectivity

Initialize Variables

Delay variable

Initialize Base Class

```python
    @staticmethod
    @bp.odeint(method='exponential_euler')
    def int_s(s, t, tau):
        return - s / tau

    def update(self, _t):

        for i in range(self.size):

            pre_i, post_i = self.pre_ids[i], self.post_ids[i]

            self.s[i] = self.int_s(self.s[i], _t, self.tau) + self.pre.spike[pre_i]

            self.g.push(i, self.g_max * self.s[i])

            self.post.input[post_i] -= self.g.pull(i) * (self.post.V[post_i] - self.E)
```

$$\frac{ds}{dt} = -\frac{s}{\tau_{decay}} + \sum_k \delta(t - t_j^k)$$

$$I_{syn} = \bar{g}_{syn} s(V - E_{syn})$$

In [12]:
```python
bp.backend.set('numpy')
```

In [13]:
```python
group = HH(10, monitors=['V', 'spike'])
syn = AMPA(pre=group, post=group, conn=bp.connect.All2All(), delay=1.5, monitors=['s'])
```

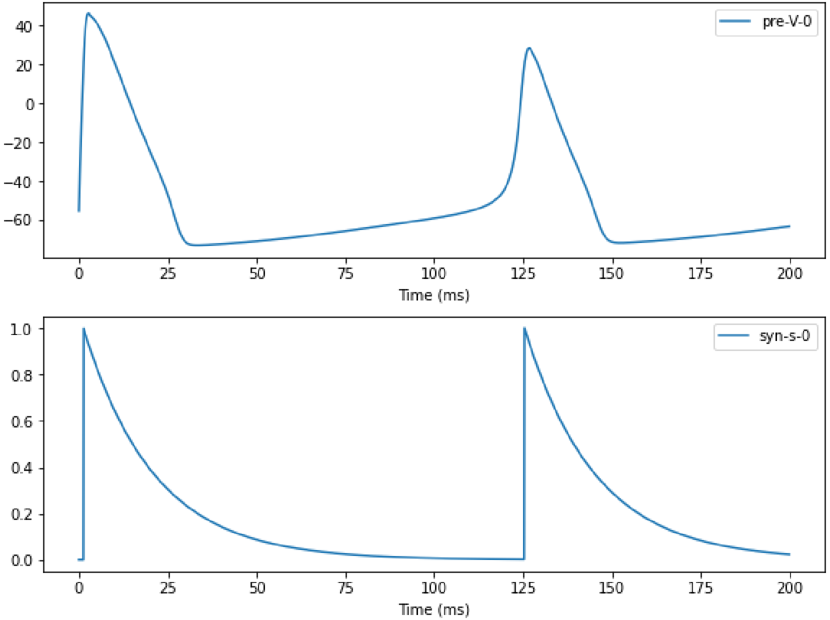
In [14]:
```python
net = bp.Network(group, syn)
net.run(duration=200., inputs=(group, "input", 20.), report=True)
```

```
Compilation used 0.3165 s.
Start running ...
Run 10.0% used 0.058 s.
Run 20.0% used 0.116 s.
Run 30.0% used 0.173 s.
Run 40.0% used 0.232 s.
Run 50.0% used 0.293 s.
Run 60.0% used 0.351 s.
Run 70.0% used 0.408 s.
Run 80.0% used 0.465 s.
Run 90.0% used 0.519 s.
Run 100.0% used 0.578 s.
Simulation is done in 0.578 s.
```

[target, key, value, (ops)]，支持 +, -, *, /, = 赋值

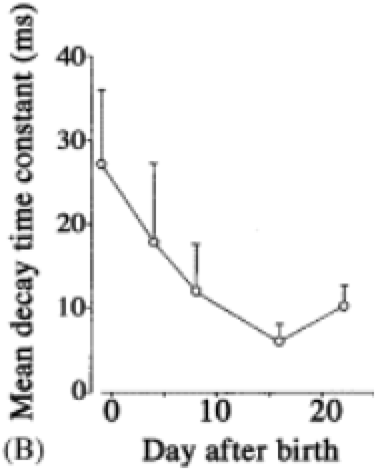
[15]:

```python
fig, gs = bp.visualize.get_figure(2, 1, 3, 8)

fig.add_subplot(gs[0, 0])
bp.visualize.line_plot(group.mon.ts, group.mon.V, legend='pre-V')

fig.add_subplot(gs[1, 0])
bp.visualize.line_plot(syn.mon.ts, syn.mon.s, legend='syn-s', show=True)
```



Time Constants

Synaptic Time Constants for AMPA, NMDA, and GABA$_A$



General property of the synaptic time constants:

- The time constants of synaptic conductance vary widely among synapse types.
- The synaptic kinetics tends to accelerate during development *(T. Takahashi, Neuroscience Research, 2005).*
- The synaptic kinetics becomes substantially faster with increasing temperature.

AMPA synapse:

- $t_{decay} = 0.18$ ms in the auditory system of the chick nucleus magnocellularis (Trussell, 1999).
- $t_{rise}= 0.25$ ms and τ_{decay}=0.77 ms in dentate gyrus basket cells (Geiger et al., 1997).
- $t_{rise}= 0.2$ ms and τ_{decay}=1.7 ms in in neocortical layer 5 pyramidal neurons (Hausser and Roth, 1997b).
- Reversal potential is nearly 0 mV.

NMDA synapse:

- The decay time constants (at near-physiological temperature): 19 ms in dentate gyrus basket cells (Geiger et al., 1997), 26 ms in neocortical layer 2/3 pyramidal neurons (Feldmeyer et al., 2002), 89 ms in CA1 pyramidal cells (Diamond, 2001).
- The rise time constants are about 2 ms (Feldmeyer et al., 2002).
- Reversal potential is nearly 0 mV.

GABA$_A$ synapse:

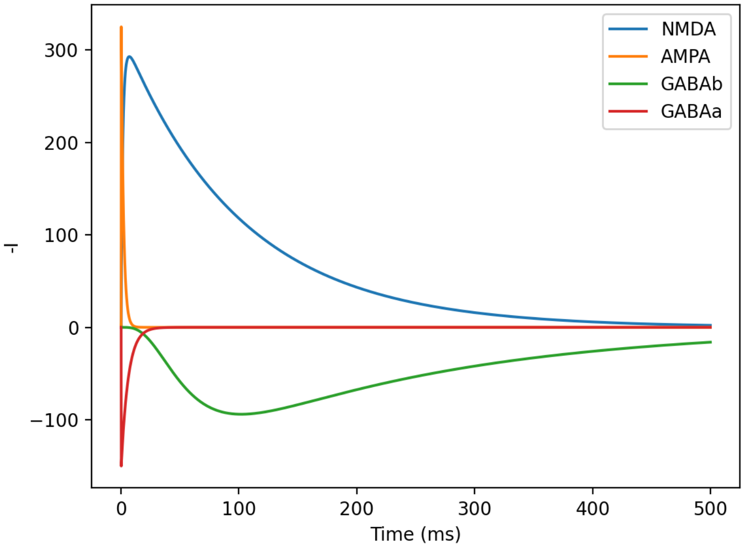
- GABAergic synapses from dentate gyrus basket cells onto other basket cells are faster: t_{rise} = 0.3 ms and t_{decay} = 2.5 ms (Bartos et al., 2001) than synapses from basket cells to granule cells: t_{rise} = 0.26 ms and t_{decay} = 6.5 ms (Kraushaar and Jonas, 2000).

- Reversal potential is nearly -80 mV.

GABA$_B$ synapse:

- Common models use models with a rise time of about 25-50ms, a fast decay time in the range of 100-300ms and a slow decay time of 500-1000ms.

- Reversal potential is nearly -90 mV.



Exercise

1. Implement exponential synapse model
2. Implement alpha synapse model
3. Implement dual Exponential synapse model