2012 Contest Problems

MCM PROBLEMS

PROBLEM A: The Leaves of a Tree

"How much do the leaves on a tree weigh?" How might one estimate the actual weight of the leaves (or for that matter any other parts of the tree)? How might one classify leaves? Build a mathematical model to describe and classify leaves. Consider and answer the following:

• Why do leaves have the various shapes that they have?

• Do the shapes “minimize” overlapping individual shadows that are cast, so as to maximize exposure? Does the distribution of leaves within the “volume” of the tree and its branches effect the shape?

• Speaking of profiles, is leaf shape (general characteristics) related to tree profile/branching structure?

• How would you estimate the leaf mass of a tree? Is there a correlation between the leaf mass and the size characteristics of the tree (height, mass, volume defined by the profile)?

In addition to your one page summary sheet prepare a one page letter to an editor of a scientific journal outlining your key findings.

2012 美赛 A 题：一棵树的叶子

（数学中国翻译）

“一棵树的叶子有多重？”怎么能估计树的叶子（或者树的任何其它部分）的实际重量？怎样对叶子进行分类？建立一个数学模型来对叶子进行描述和分类。模型要考虑和回答下面的问题：

• 为什么叶子具有各种形状？

• 叶子之间是要将相互重叠的部分最小化，以便可以最大限度的接触到阳光吗？树叶的分布以及树干和枝杈的体积影响叶子的形状吗？

• 就轮廓来讲，叶形（一般特征）是和树的轮廓以及分枝结构有关吗？

• 你将如何估计一棵树的叶子质量？叶子的质量和树的尺寸特征（包括和外形轮廓有关的高度、质量、体积）有联系吗？

除了你的一页摘要以外，给科学杂志的编辑写一封信，阐述你的主要发现。

PROBLEM B: Camping along the Big Long River

Visitors to the Big Long River (225 miles) can enjoy scenic views and exciting white water rapids. The river is inaccessible to hikers, so the only way to enjoy it is to take a river trip that requires several days of camping. River trips all start at First Launch and exit the river at Final Exit, 225 miles downstream. Passengers take either oar- powered rubber rafts, which travel on average 4 mph or motorized boats, which travel on average 8 mph. The trips range from 6 to 18 nights of camping on the river, start to finish.. The government agency responsible for managing this river wants every trip to enjoy a wilderness experience, with minimal contact with other groups of boats on the river. Currently, *X* trips travel down the Big Long River each year during a six month period (the rest of the year it is too cold for river trips). There are *Y* camp sites on the Big Long River, distributed fairly uniformly throughout the river corridor. Given the rise in popularity of river rafting, the park managers have been asked to allow more trips to travel down the river. They want to determine how they might schedule an optimal mix of trips, of varying duration (measured in nights on the river) and propulsion (motor or oar) that will utilize the campsites in the best way possible. In other words, how many more boat trips could be added to the Big Long River's rafting season? The river managers have hired you to advise them on ways in which to develop the best schedule and on ways in which to determine the carrying capacity of the river, remembering that no two sets of campers can occupy the same site at the same time. In addition to your one page summary sheet, prepare a one page memo to the managers of the river describing your key findings.

2012 美赛 B 题：沿着“大长河”露营

【数学中国翻译】

游客在“大长河”(225 英里)可以享受到秀丽的风光和令人兴奋的白色湍流。这条河对于背包客来说是进不去的， 因此畅游这条长河的唯一办法就是在这条河上露营上几天。这次旅行从开始的下水点到最终结束点，共 225 英里，且是顺流而下的。乘客可以选择平均 4 英里/小时的以浆作为动力的橡胶筏或者平均 8 英里/小时的机动帆船旅行。整个旅行从开始到结束会经历 6 至 18 个夜晚。负责管理这条河的政府机构希望到这里的每一次旅行都能够享受到野外经历， 以最少的接触到在河上其它的船只。目前， 每年在六个月期间(一年的其余部分的天气对于河流旅行来说太冷)，共有 X 次旅行，有 Y 处露营地，露营地均匀的分布整个河道。由于漂流的受欢迎程度的上升，公园管理者已经被要求允许更多的旅行次数。所以他们想确定怎样可能安排一个最优的混合的旅行方案，不同的时间(单位为夜)和推动方式(马达或浆)，最大限度的利用露营地。换句话说，在长河的漂流季，将会有多少更多的乘船旅行可以加进来?河流的管理者现在雇佣你，为他们提出最佳排程方式和河流承载能力的建议，记住两个露营者不能在同一时

间内占据同一个露营地。除了你的一页摘要，准备一页备忘录，对河流的管理者描述你的主要发现。

ICM PROBLEM

PROBLEM C: Modeling for Crime Busting

Click the title below to download a ZIP file containing the 2012 ICM Problem.

Your ICM submission should consist of a 1 page Summary Sheet and your solution cannot exceed 20 pages for a maximum of 21 pages.

Modeling for Crime Busting