Discretization for 1D Hydrodynamic Solver with Deforming Mesh

Discretization of fluid equations:

$$\frac{v_i^{n+1/2} - v_i^{n-1/2}}{\Delta t^n} = -\frac{p_{i+1/2}^n - p_{i-1/2}^n + q_{i+1/2}^n - q_{i-1/2}^n}{\Delta m_i} \quad (1)$$

$$\frac{x_i^{n+1} - x_i^n}{\Delta t^{n+1/2}} = v_i^{n+1/2} \quad (2)$$

$$1/\rho_{i+1/2}^{n+1} = \frac{x_{i+1}^{n+1} - x_i^{n+1}}{\Delta m_{i+1/2}} \quad (3)$$

$$e_{i+1/2}^{n+1} - e_{i+1/2}^n = -(p_{i+1/2}^n + q_{i+1/2}^n)(1/\rho_{i+1/2}^{n+1} - 1/\rho_{i+1/2}^n) \quad (4)$$

These are explicit, so the "new" time can be solved for on the LHS. We're going to set up the grid so Δm is constant in each cell (notice it is not "evolved" above). Δt is the timestep:

$$v_i^{n+1/2} = v_i^{n-1/2} - \Delta t^n \, \frac{p_{i+1/2}^n - p_{i-1/2}^n + q_{i+1/2}^n - q_{i-1/2}^n}{\Delta m_i} \quad (5)$$

$$x_i^{n+1} = x_i^n + \Delta t^{n+1/2} \, v_i^{n+1/2} \quad (6)$$

$$\rho_{i+1/2}^{n+1} = \frac{\Delta m_{i+1/2}}{x_{i+1}^{n+1} - x_i^{n+1}} \quad (7)$$

$$e_{i+1/2}^{n+1} = e_{i+1/2}^n - (p_{i+1/2}^n + q_{i+1/2}^n)(1/\rho_{i+1/2}^{n+1} - 1/\rho_{i+1/2}^n) \quad (8)$$

Assuming an adiabatic equation of state, the new pressure can be obtained from the energy:

$$p_{i+1/2}^{n+1} = e_{i+1/2}^{n+1} \rho_{i+1/2}^{n+1} (\gamma - 1) \quad (9)$$

An alternative discretization of energy gives:

$$e_{i+1/2}^{n+1} = [e_{i+1/2}^n - (p_{i+1/2}^n/2 + q_i^n)(1/\rho_{i+1/2}^{n+1} - 1/\rho_{i+1/2}^n)]/(1 + \rho_{i+1/2}^{n+1}(\gamma - 1)\frac{1}{2}(1/\rho_{i+1/2}^{n+1} - 1/\rho_{i+1/2}^n)] \quad (10)$$

For the artificial viscosity use the form:

$$q_{i+1/2}^n = \left[q_0 (u_{i+1}^n - u_i^n)^2 - q_1 (u_{i+1}^n - u_i^n)\right] \frac{c_{s,j+1/2}}{\bar{\rho}} \quad \left(\frac{u_{i+1}^n - u_i^n}{x_{i+1}^n - x_i^n}\right) < 0 \quad (11)$$

$$q_{i+1/2}^n = 0 \quad \left(\frac{u_{i+1}^n - u_i^n}{x_{i+1}^n - x_i^n}\right) > 0 \quad (12)$$

where $\bar{\rho} = \frac{1}{2}\left(\frac{1}{\rho_{i+1/2}^{n+1}} + \frac{1}{\rho_{i+1/2}^n}\right)$, $q_0 = 4$ and $q_1 = 0.5$. Here we're going to lag the viscosity so calculate it at the beginning of the step (before eq 1) using the old values.

Code flow: Your code flow should go something like this:
1. Define variable arrays (e.g. array of N=200 points). Defining "New" (n+1) and "Old" (n) arrays for each helps keep things orderly.
2. Set the initial values for all variables using the initial conditions. Set the cell mass Δm_i, $\Delta m_{i+1/2}$, where $\Delta m_{i+1/2} = \rho_i (x_{max} - x_{min})/N$. All values at time $n+1$ are 0.

3. Calculate the timestep, Δt, using the CFL condition.
4. Apply boundary conditions in the ghost cells (e.g., $u[0] = 0$). These cells are used to advance the neighboring cells, but never appear on the LHS of the differenced equations. (Note there should be no activity at or near the boundary for our Sod problem so these quantities should remain at their initial values.)
5. Update each variable at the new time following the order above:
   Solve for the new velocity
   Compute the new grid positions.
   Compute the new density and sound speed.
   Compute the new artificial viscosity.
   Compute the new energy.
   Compute the new pressure.
6. Loop repeats starting at step 3. The time is now $t = t + \Delta t$.