The caret Package

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The caret package (short for classification and regression training) contains functions to streamline the model training process for complex regression and classification problems. The package utilizes a number of R packages but tries not to load them all at package start-up¹. The package "suggests" field includes: ada, affy, caTools, class, e1071, earth (\geq 2.2-3), elasticnet, ellipse, fastICA, foba, foreach, gam, GAMens (\geq 1.1.1), gbm, glmnet, gpls, grid, hda, HDclassif, ipred, kernlab, klaR, lars, LogicForest, logicFS, LogicReg, MASS, mboost, mda, mgcv, mlbench, neuralnet, nnet, nodeHarvest, pamr, partDSA, party, penalized, pls, proxy, quantregForest, randomForest, rda, relaxo, rocc, rpart, rrcov, RWeka (\geq 0.4-1), sda, SDDA, sparseLDA (\geq 0.1-1), spls, stepPlr, superpc, vbmp. caret loads packages as needed and assumes that they are installed. Install caret using

```
install.packages("caret", dependencies = c("Depends", "Suggests"))
```

to ensure that all the needed packages are installed.

1 Model Training and Parameter Tuning

caret has several functions that attempt to streamline the model building and evaluation process.

The train function can be used to

- evaluate, using resampling, the effect of model tuning parameters on performance
- choose the "optimal" model across these parameters
- estimate model performance from a training set

More formally:

```
1 Define sets of model parameter values to evaluate
2 for each parameter set do
      for each resampling iteration do
3
         Hold-out specific samples
4
         Fit the model on the remainder
5
         Predict the hold-out samples
6
7
      end
      Calculate the average performance across hold-out predictions
8
10 Determine the optimal parameter set
11 Fit the final model to all the training data using the optimal parameter set
```

¹By adding formal package dependencies, the package startup time can be greatly decreased

First, a specific model must be chosen. Currently, 102 are available using train; see Table 1 for details.

In Table 1, there is a list of tuning parameters that can potentially be optimized. The first step in tuning the model (line 1 in Algorithm 1) is to choose a set of parameters to evaluate. For example, if fitting a Partial Least Squares (PLS) model, the number of PLS components to evaluate must be specified.

Once the model and tuning parameter values have been defined, the type of resampling should be also be specified. Currently, k-fold cross-validation, leave-one-out cross-validation and bootstrap resampling methods can be used by train. After resampling, the process produces a profile of performance measures is available to guide the user as to which tuning parameter values should be chosen. By default, the function automatically chooses the tuning parameters associated with the best value, although different algorithms can be used (see Section 2.5).

1.1 An Example

As an example, the multidrug resistance reversal (MDRR) agent data is used to determine a predictive model for the "ability of a compound to reverse a leukemia cell's resistance to adriamycin" (Svetnik et al, 2003). For each sample (i.e. compound), predictors are calculated that reflect characteristics of the molecular structure. These molecular descriptors are then used to predict assay results that reflect resistance.

The data are accessed using data(mdrr). This creates a data frame of predictors called mdrrDescr and a factor vector with the observed class called mdrrClass.

To start, we will:

- use unsupervised filters to remove predictors with unattractive characteristics (e.g. spare distributions or high inter-predictor correlations)
- split the entire data set into a training and test set
- center and scale the training and test set using the predictor means and standard deviations from the training set

See the package vignette "caret Manual – Data and Functions" for more details about these operations.

> print(ncol(mdrrDescr))

[1] 342

```
> nzv <- nearZeroVar(mdrrDescr)</pre>
> filteredDescr <- mdrrDescr[, -nzv]</pre>
> print(ncol(filteredDescr))
[1] 297
> descrCor <- cor(filteredDescr)</pre>
> highlyCorDescr <- findCorrelation(descrCor, cutoff = .75)</pre>
> filteredDescr <- filteredDescr[,-highlyCorDescr]
> print(ncol(filteredDescr))
[1] 50
> set.seed(1)
> inTrain <- sample(seq(along = mdrrClass), length(mdrrClass)/2)</pre>
> trainDescr <- filteredDescr[inTrain,]</pre>
> testDescr <- filteredDescr[-inTrain,]</pre>
> trainMDRR <- mdrrClass[inTrain]</pre>
> testMDRR <- mdrrClass[-inTrain]</pre>
> print(length(trainMDRR))
[1] 264
> print(length(testMDRR))
[1] 264
> preProcValues <- preProcess(trainDescr)
> trainDescr <- predict(preProcValues, trainDescr)</pre>
> testDescr <- predict(preProcValues, testDescr)</pre>
```

1.2 Basic Parameter Tuning

To estimate model performance across the tuning parameters "leave group out cross–validation" (LGOCV) can be used. This technique is repeated splitting of the data into training and test sets (without replacement). If the resampling method is not specified, simple bootstrapping is used. To train a support vector machine classification model (radial basis function kernel) on these multidrug resistance reversal agent data, we can first setup a control object² that specifies the type of resampling used, the number of data splits (30), the proportion of data in the sub–training sets (75%) and whether the iterations should be printed as they occur. In this case, we need to specify the proportion of samples used in each resampled training set. We also set the seed.

²This is optional; to use the default specifications, the control object does not need to be specified

More information about trainControl is given in Section 2.2.

The first two arguments to train are the predictor and outcome data objects, respectively. The third argument, method, specifies the type of model (see Table 1). For this model, the tuning parameters are the cost value (the C argument in kernlab's ksvm function) and the radius of the RBF (the sigma argument to the kernel function). The tuneLength argument sets the size of the grid used to search the tuning parameter space and trControl is the control parameter for the train function.

```
> svmFit <- train(trainDescr, trainMDRR,
                  method = "svmRadial",
                  tuneLength = 4,
                  trControl = fitControl)
> svmFit
Call:
train.default(x = trainDescr, y = trainMDRR, method = "svmRadial",
    trControl = fitControl, tuneLength = 4)
264 samples
50 predictors
summary of leave group out cross-validation (30 reps) sample sizes:
    198, 198, 198, 198, 198, 198, ...
LGOCV resampled training results across tuning parameters:
  C
       sigma
               Accuracy
                         Kappa
                                 Accuracy SD
                                              Kappa SD
                                                         Selected
  0.1 0.0222 0.581
                         0.0522
                                 0.0212
                                              0.0533
       0.0222 0.837
                                              0.0846
                         0.665
                                 0.0406
  10
       0.0222 0.807
                         0.609
                                 0.0562
                                              0.114
  100 0.0222 0.807
                         0.609
                                 0.0533
                                              0.109
```

Accuracy was used to select the optimal model using the largest value.

The final values used in the model were C = 1 and sigma = 0.0222.

There are two tuning parameters for this model: sigma is a parameter for the kernel function that can be used to expand/contract the distance function and C is the cost parameter that can be used as a regularization term that controls the complexity of the model. For this model, the function sigest in the kernlab package is used to provide a good estimate of the sigma parameter, so that only the cost parameter is tuned. This tuning scheme is the default, but can be modified (details are below).

The column labeled "Accuracy" is the overall agreement rate averaged over cross-validation iterations. The agreement standard deviation is also calculated from the cross-validation results. The column "Kappa" is Cohen's (unweighted) Kappa statistic averaged across the resampling results.

For regression models (i.e. a numeric outcome), a similar table would be produced showing the average root mean squared error and average R^2 value statistic across tuning parameters, otherwise known as Q^2 (see the note below related to this calculation).

caret works with specific models (see Table 1). For these models, train can automatically create a grid of tuning parameters. By default, if p is the number of tuning parameters, the grid size is 3^p . For example, regularized discriminant analysis (RDA) models have two parameters (gamma and lambda), both of which lie on [0,1]. The default training grid would produce nine combinations in this two-dimensional space.

1.3 Notes

- There is a formula interface (e.g. train(y ., data = someData) that can be used. One of the issues with a large number of predictors is that the objects related to the formula which are saved can get very large. In these cases, it is best to stick with the non-formula interface described above.
- The function determines the type of problem (classification or regression) from the type of the response given in the y argument.
- The ... option can be used to pass parameters to the fitting function. For example, in random forest models, you can specify the number of trees to be used in the call to train. In Section 2.1, there is an the example using gradient boosting machines (GBM) where the default trace for a gbm model was turned off using the verbose argument to gbm.
- For regression models, the classical R^2 statistic cannot be compared between models that contain an intercept and models that do not. Also, some models do not have an intercept only null model.

To approximate this statistic across different types of models, the square of the correlation between the observed and predicted outcomes is used. This means that the R^2 values produced by train will not match the results of lm and other functions.

Also, the correlation estimate does not take into account the degrees of freedom in a model and thus does not penalize models with more parameters. For some models (e.g random forests or on–linear support vector machines) there is no clear sense of the degrees of freedom, so this information cannot be used in \mathbb{R}^2 if we would like to compare different models.

- The nearest shrunken centroid model of Tibshirani et al (2003) is specified using method = "pam". For this model, there must be at least two samples in each class. train will ignore classes where there are less than two samples per class from every model fit during bootstrapping or cross—validation (this model only).
- For recursive partitioning models, an initial model is fit to all of the training data to obtain the possible values of the maximum depth of any node (maxdepth). The tuning grid is created based on these values. If tuneLength is larger than the number of possible maxdepth values determined by the initial model, the grid will be truncated to the maxdepth list.

The same is also true for nearest shrunken centroid models, where an initial model is fit to find the range of possible threshold values, and MARS models (see the details below).

• For multivariate adaptive regression splines (MARS), the earth package is used with a model type of mars or earth is requested. The tuning parameters used by train are degree and nprune. The parameter nk is not automatically specified and, if not specified, the default in the earth function is used.

For example, suppose a training set with 40 predictors is used with degree = 1 and nprune = 20. An initial model with nk = 41 is fit and is pruned down to 20 terms. This number includes the intercept and may include "singleton" terms instead of pairs.

Alternate model training schemes can be used by passing nk and/or pmethod to the earth function.

Also, there may be cases where the message such as "specified 'nprune' 29 is greater than the number of available model terms 24, forcing 'nprune' to 24" show up after the model fit. This can occur since the earth function may not actually use the number of terms in the initial model as specified by nk. This may be because the earth function removes terms with linear dependencies and the forward pass counts as if terms were added in pairs (although singleton terms may be used). By default, the train function fits and initial MARS model is used to determine the number of possible terms in the training set to create the tuning grid. Resampled data sets may produce slightly different models that do not have as many possible values of nprune.

• For the glmboost and gamboost functions from the mboost package, an additional tuning parameter, prune, is used by train. If prune = "yes", the number of trees is reduced based on the AIC statistic. If "no", the number of trees is kept at the value specified by the mstop parameter. See the mboost package vignette for more details about AIC pruning.

- The partitioning model of Molinaro et al. (2010) has a tuning parameter that is the number of partitions in the model. The R function partDSA has the argument cut.off.growth which is described as "the maximum number of terminal partitions to be considered when building the model." Since this is the maximum, the user might ask for a model with X partitions but the model can only predict Y < X. In these cases, the model predictions will be based on the largest model available (Y).
- For some models (blackboost, earth, enet, foba, gamboost, gbm, glmboost, glmnet, lars, lars2, lasso, logitBoost, pam, partDSA, pcr, pls, relaxo, rpart, scrda, superpc), the train function will fit a model that can be used to derive predictions for some sub-models. For example, for MARS (via the earth function), for a fixed degree, a model with a maximum number of terms will be fit and the predictions of all of the requested models with the same degree and smaller number of terms will be computed using update.earth instead of fitting a new model. When the verboseIter option is used, a line is printed for the "top-level" model (instead of each model in the tuning grid).
- There are print and plot methods. The plot method visualizes the profile of average resampled performance values across the different tuning parameters using scatter plots or level plots. See Figures 1 and 2 for examples. Functions that visualize the individual resampling results for train objects are discussed in Section 5.1.
- Using the first set of tuning parameters that are optimal (in the sense of accuracy or mean squared error), train automatically fits a model with these parameters to the entire training data set. That model object is accessible in the finalModel object within train. For example, svmFit\$finalModel is the same object that would have been produced using a direct call to the ksvm function with the final tuning parameters.

There is additional functionality in train that is described in the next section.

Table 1: Models used in train

Model	method Value	Package	Tuning Parameters
	$"Dual-Use\ Models"$	odels"	
Generalized linear model	glm glmStepAIC	stats	None None
Recursive Partitioning	rpart ctree	rpart party	maxdepth mincriterion maxdepth
Boosted Trees	gbm blackboost ada	gbm mboost ada	n.trees, shrinkage interaction.depth maxdepth, mstop maxdepth.iter.nu
Other Boosted Models	glmboost gamboost	mboost	mstop mstop
Random Forests	rf parRF cforest	<pre>randomForest randomForest, foreach party</pre>	mtry mtry mtry
Bagging	treebag bag logicBag	ipred caret logicFS	None vars ntrees, nleaves
Other Trees	nodeHarvest partDSA	nodeHarvest partDSA	<pre>maxinter, mode cut.off.growth, MPD</pre>
Multivariate Adaptive Regression Splines Bagged MARS	earth, mars bagEarth	earth caret, earth	degree, nprune degree, nprune
Logic Regression Elastic Net (glm)	logreg glmnet	LogicReg glmnet	ntrees, treesize alpha, lambda
Neural Networks	nnet pcaNNet	<pre>nnet caret, nnet</pre>	decay, size decay, size
Partial Least Squares Sparse Partial Least Squares Support Vector Machines	pls spls svmLinear	pls, caret spls, caret kernlab	ncomp K, eta, kappa none

(continued on next page)

Table 1: Models used in train

Model	method Value	Package	Tuning Parameters
v.	svmRadial	kernlab	sigma, C
v.	svmPoly	kernlab	scale, degree, C
Gaussian Processes	gaussprLinear	kernlab	none
عادات	gaussprRadial	kernlab	sigma
3)	gaussprPoly	kernlab	scale, degree
k Nearest Neighbors	knn	caret	ম
	Regression Only Models	Models	
Linear Least Squares	lm	stats	None
1	${\tt lmStepAIC}$	MASS	None
	pcr	pls	ncomp
Independent Component Regression	icr	caret	n.comp
egression	rlm	MASS	None
Neural Networks n	neuralnet	neuralnet	layer1, layer2, layer3
Forests	qrf	quantregForest	mtry
Rule-Based Models	M5Rules	RWeka	pruned
Projection Pursuit Regression	ppr	stats	nterms
Penalized Linear Models F	penalized	penalized	lambda1, lambda2
1	lars	lars	fraction
	lars2	lars	step
9	enet	elasticnet	lambda, fraction
1	lasso	elasticnet	fraction
Ŧ	foba	foba	lambda, k
Relevance Vector Machines	rvmLinear	kernlab	none
I	rvmRadial	kernlab	sigma
	rvmPoly	kernlab	scale, degree
Supervised Principal Components	superpc	superpc	n.components, threshold

Classification Only Models

(continued on next page)

Table 1: Models used in train

Model	method Value	Package	Tuning Parameters
Linear Discriminant Analysis	lda	MASS	None
٠	Linda	rrcov	None
Quadratic Discriminant Analysis	qda	MASS	None
	QdaCov	rrcov	None
Stabilized Linear Discriminant Analysis	slda	ipred	diagonal
Heteroscedastic Discriminant Analysis	hda	hda	newdim, lambda, gamma
Shrinkage Linear Discriminant Analysis	sda	sda	diagonal
Sparse Linear Discriminant Analysis	sparseLDA	sparseLDA	NumVars, lambda
Stepwise Discriminant	stepLDA,	klaR	None
Stepwise Diagonal Discriminant Analysis	sddaLDA, sddaQDA	SDDA	None
Regularized Discriminant Analysis	rda	klaR	lambda, gamma
Mixture Discriminant Analysis	mda	mda	subclasses
Sparse Mixture Discriminant Analysis	smda	sparseLDA	NumVars, R, lambda
Penalized Discriminant Analysis	pda	mda	lambda
	pda2	mda	df
High Dimensional Discriminant Analysis	hdda	HDclassif	model, threshold
Flexible Discriminant Analysis (MARS basis)	fda	mda, earth	degree, nprune
Bagged FDA	bagFDA	caret, earth	degree, nprune
Logistic/Multinomial Regression	multinom	nnet	decay
	plr	stepPlr	lambda, cp
LogitBoost	logitBoost	caTools	nIter
Logistic Model Trees	LMT	RWeka	iter
Rule–Based Models	J48	RWeka	Ö
	OneR	RWeka	None
	PART	RWeka	threshold, pruned
	JRip	RWeka	NumOpt
Logic Forests	logforest	LogForest	None
Bayesian Multinomial Probit Model	vbmpRadial	dmdv	estimateTheta
Least Squares Support Vector Machines	lssvmRadial	kernlab	sigma

(continued on next page)

Table 1: Models used in train

Model	method Value	Package	Tuning Parameters
Nearest Shrunken Centroids	pam scrda	pamr rda	threshold alpha, delta
Naive Bayes	qu	klaR	usekernel
Generalized Partial Least Squares	gpls	gpls	K.prov
Learned Vector Quantization	lvq	class	አ
ROC Curves	rocc	rocc	xgenes

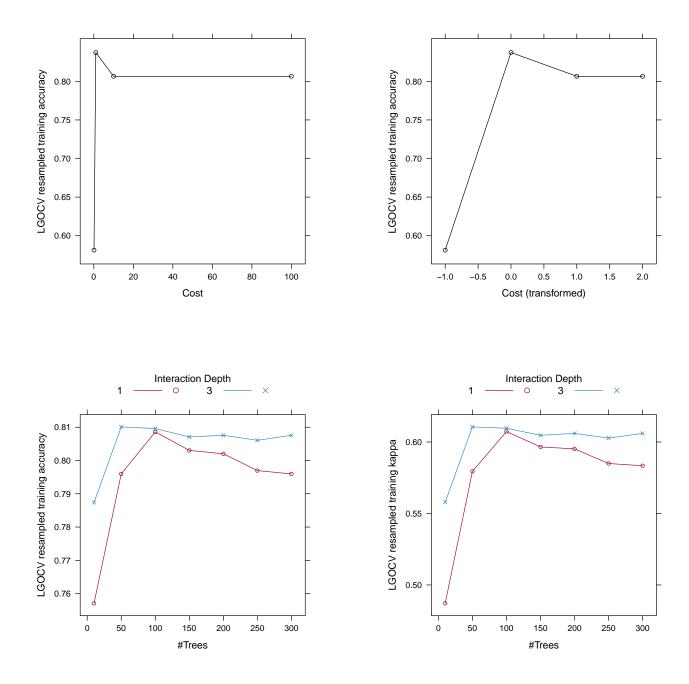


Figure 1: Examples of output from plot.tain. top left a plot produced using plot(svmFit) showing the relationship between SVM cost parameter and the resampled classification accuracy. Although this model has two tuning parameters, a constant value for the parameter sigma was used. top right the same plot but the xTrans argument was used to log-transform the cost parameter. bottom left a plot produced using plot(gbmFit) showing the relationship between the number of boosting iterations, the interaction depth and the resampled classification accuracy bottom right the same plot, but the Kappa statistic is plotted using plot(gbmFit metric = "Kappa")

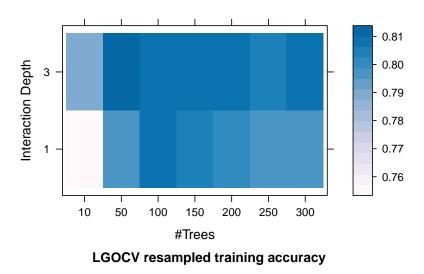


Figure 2: For the boosted tree example in Section 2.1, using plot(gbmFit metric = "Kappa", plotType = "level") shows the relationship (using a levelplot) between the number of boosting iterations, the interaction depth and the resampled estimate of the Kappa statistic.

2 Customizing the Tuning Process

There are a few ways to customize the process of selecting tuning/complexity parameters and building the final model.

2.1 Alternate Tuning Grids

The tuning parameter grid can be specified by the user. The argument tuneGrid can take a data frame with columns for each tuning parameter (see Table 1 for specific details). The column names should be the same as the fitting function's arguments with a period preceding the name. For the previously mentioned RDA example, the names would be .gamma and .lambda. train will tune the model over each combination of values in the rows.

For a gradient boosting machine (GBM) model, there are three main tuning parameters:

- number of iterations, *i.e. trees*, (called n.trees in the gbm function)
- complexity of the tree, called interaction.depth
- learning rate: how quickly the algorithm adapts, called shrinkage

We can fix the learning rate and evaluate more than three values of n.trees:

```
> gbmGrid <- expand.grid(.interaction.depth = c(1, 3),
+ .n.trees = c(10, 50, 100, 150, 200, 250, 300),
+ .shrinkage = 0.1)</pre>
```

LGOCV resampled training results across tuning parameters:

Selected								*					
Kappa SD	0.0746	0.0857	0.0936	0.0883	0.0823	0.0857	0.0796	0.0759	0.0798	0.0815	0.0866	0.0739	0.0914
Accuracy SD	0.0368	0.0426	0.0464	0.0439	0.0403	0.0422	0.0374	0.0377	0.0391	0.0398	0.0421	0.0361	0.0449
Kappa 0 487	0.579	0.607	0.597	0.595	0.585	0.583	0.558	0.611	0.61	0.605	909.0	0.603	909.0
Accuracy	0.796	0.809	0.803	0.802	767.0	962.0	0.787	0.81	0.81	0.807	0.808	908.0	0.808
inkage	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
n.trees	50	100	150	200	250	300	10	20	100	150	200	250	300
interaction.depth	. ←1	1	₩	₽	₩	₩	က	က	က	က	က	က	က

Accuracy was used to select the optimal model using the largest value.

The final values used in the model were interaction.depth = 3, n.trees = 50 and shrinkage = 0.1.

2.2 The trainControl Function

The function trainControl, generates parameters that further control how models are resampled with possible values:

- method: The resampling method: boot, cv, LOOCV, LGOCV and oob. The last value, out-of-bag estimates, can only be used by random forest, bagged trees, bagged earth, bagged flexible discriminant analysis, or conditional tree forest models. GBM models are not included (the gbm package maintainer has indicated that it would not be a good idea to choose tuning parameter values based on the model OOB error estimates with boosted trees). Also, for leave-one-out cross-validation, no uncertainty estimates are given for the resampled performance measures.
- number: Either the number of folds or number of resampling iterations
- verboseIter: A logical for printing a training log.
- returnData: A logical for saving the data into a slot called trainingData.
- p: For leave-group out cross-validation: the training percentage
- classProbs: a logical value determining whether class probabilities should be computed for held—out samples during resample. Examples of using this argument are given in Section 2.4.
- index: a list with elements for each resampling iteration. Each list element is the sample rows used for training at that iteration. When these values are not specified, caret will generate them.
- summaryFunction: a function to compute alternate performance summaries. See Section 2.3 for more details.
- selectionFunction: a function to choose the optimal tuning parameters. See Section 2.5 for more details and examples.
- returnResamp: a character string containing one of the following values: "all", "final" or "none". This specifies how much of the resampled performance measures to save.
- workers, computeFunction, and computeArgs: these are options for parallel processing and are discussed in Section 2.6

2.3 Alternate Performance Metrics

The user can change the metric used to determine the best settings. By default, RMSE and R^2 are computed for regression while accuracy and Kappa are computed for classification. Also by default, the parameter values are chosen using RMSE and accuracy, respectively for regression and

classification. The metric argument of the train function allows the user to control which the optimality criterion is used. For example, in problems where there are a low percentage of samples in one class, using metric = "Kappa" can improve quality of the final model.

If none of these parameters are satisfactory, the user can also compute custom performance metrics. The trainControl function has a argument called summaryFunction that specifies a function for computing performance. The function should have these arguments:

- data is a reference for a data frame or matrix with columns called obs and pred for the observed and predicted outcome values (either numeric data for regression or character values for classification). Currently, class probabilities are not passed to the function. The values in data are the held—out predictions (and their associated reference values) for a single combination of tuning parameters. If the classProbs argument of the trainControl object is set to TRUE, additional columns in data will be present that contains the class probabilities. The names of these columns are the same as the class levels.
- lev is a character string that has the outcome factor levels taken from the training data. For regression, a value of NULL is passed into the function.
- model is a character string for the model being used (i.e. the value passed to the method value of train).

The output to the function should be a vector of numeric summary metrics with non–null names.

As an example, classification accuracy in two–class problems can be decomposed into sensitivity and specificity. We can use these values to tune the parameters using the following function:

To rebuild the support vector machine model using this criterion, we can see the relationship between the tuning parameters and sensitivity/specificity via the following code:

```
> fitControl$summaryFunction <- newSummary
> set.seed(2)
> svmNew <- train(trainDescr, trainMDRR,
+ method = "svmRadial",
+ metric = "Spec",</pre>
```

```
tuneLength = 4,
trControl = fitControl)
> svmNew

Call:
train.default(x = trainDescr, y = trainMDRR, method = "svmRadial",
    metric = "Spec", trControl = fitControl, tuneLength = 4)

264 samples
50 predictors

summary of leave group out cross-validation (30 reps) sample sizes:
    198, 198, 198, 198, 198, ...
```

LGOCV resampled training results across tuning parameters:

```
С
     sigma
              Sens
                     Spec
                              Sens SD
                                        Spec SD
                                                  Selected
0.1
     0.0222
                     0.0471
                                        0.0483
              1
1
     0.0222
             0.904
                     0.753
                              0.053
                                        0.0825
10
     0.0222
             0.812
                     0.8
                              0.0686
                                        0.0884
     0.0222
             0.812
                                        0.0837
100
                     0.8
                              0.0613
```

Spec was used to select the optimal model using the largest value.

The final values used in the model were C = 100 and sigma = 0.0222.

2.4 Performance Class Probabilities: ROC Curves

By default, train evaluate classification models in terms of the predicted classes. Optionally, class probabilities can also be used to measure performance. To obtain predicted class probabilities within the resampling process, the argument classProbs in trainControl must be set to TRUE. This merges columns of probabilities into the predictions generated from each resample (there is a column per class and the column names are the class names).

As shown in the last section, custom functions can be used to calculate performance scores that are averaged over the resamples. Another built—in function, twoClassSummary, will compute the sensitivity, specificity and area under the ROC curve (see Section 4.2 for details).

For example:

```
> fitControl <- trainControl(method = "LGOCV",
+ p = .75,</pre>
```

```
number = 30,
                              classProbs = TRUE,
                              summaryFunction = twoClassSummary,
                              returnResamp = "all",
                              verboseIter = FALSE)
> set.seed(2)
 svmROC <- train(trainDescr, trainMDRR,</pre>
                  method = "svmRadial",
                   tuneLength = 4,
                  metric = "ROC",
                   trControl = fitControl)
> svmROC
Call:
train.default(x = trainDescr, y = trainMDRR, method = "svmRadial",
    metric = "ROC", trControl = fitControl, tuneLength = 4)
264 samples
50 predictors
summary of leave group out cross-validation (30 reps) sample sizes:
    198, 198, 198, 198, 198, 198, ...
```

LGOCV resampled training results across tuning parameters:

C	sigma	Sens	Spec	ROC	Sens SD	Spec SD	ROC SD	Selected
0.1	0.0222	1	0.0471	0.864	0	0.0483	0.04	
1	0.0222	0.904	0.753	0.895	0.053	0.0825	0.037	*
10	0.0222	0.812	0.8	0.87	0.0686	0.0884	0.0457	
100	0.0222	0.812	0.8	0.868	0.0613	0.0837	0.0452	

ROC was used to select the optimal model using the largest value.

The final values used in the model were C = 1 and sigma = 0.0222.

In this case, the average area under the ROC curve was 0.895 across the 30 resamples.

2.5 Choosing the Final Model

Another method for customizing the tuning process is to modify the algorithm that is used to select the "best" parameter values, given the performance numbers. By default, the train function chooses

the model with the largest performance value (or smallest, for mean squared error in regression models). Other schemes for selecting model can be used. Breiman et al (1984) suggested the "one standard error rule" for simple tree—based models. In this case, the model with the best performance value is identified and, using resampling, we can estimate the standard error of performance. The final model used was the simplest model within one standard error of the (empirically) best model. With simple trees this makes sense, since these models will start to over-fit as they become more and more specific to the training data.

train allows the user to specify alternate rules for selecting the final model. The argument selectionFunction can be used to supply a function to algorithmically determine the final model. There are three existing functions in the package: best is chooses the largest/smallest value, oneSE attempts to capture the spirit of Breiman et al (1984) and tolerance selects the least complex model within some percent tolerance of the best value. See ?best for more details.

User-defined functions can be used, as long as they have the following arguments:

- x is a data frame containing the tune parameters and their associated performance metrics. Each row corresponds to a different tuning parameter combination
- metric a character string indicating which performance metric should be optimized (this is passed in directly from the metric argument of train.
- maximize is a single logical value indicating whether larger values of the performance metric are better (this is also directly passed from the call to train).

The function should output a single integer indicating which row in x is chosen.

As an example, if we chose the previous SVM model on the basis of specificity, we would choose a cost value of 100, the most complex model. Lower cost values would produce approximately the same performance with less complex models (with the exception of cost = 0.1). The tolerance function could be used to find a less complex model based on $(x - x_{best})/x_{best} \times 100$, which is the percent difference. For example, to select cost values based on 2% and 6% losses of performance:

SensSD

SpecSD

```
> whichTwoPct <- tolerance(svmNew$results, "Spec", 2, TRUE)
> cat("best model within 2 pct of best:\n")
best model within 2 pct of best:
> svmNew$results[whichTwoPct,]
```

Sens Spec

3 10 0.02222875 0.8117117 0.8 0.06862484 0.08844885

C

sigma

```
> whichSixPct <- tolerance(svmNew$results, "Spec", 6, TRUE)
> cat("\n\nbest model within 6 pct of best:\n")
best model within 6 pct of best:
> svmNew$results[whichSixPct,]

C sigma Sens Spec SensSD SpecSD
2 1 0.02222875 0.9036036 0.7528736 0.05296353 0.082542
```

The main issue with these functions is related to ordering the models from simplest to complex. In some cases, this is easy (e.g. simple trees, partial least squares), but in most cases, the ordering of models is subjective. For example, is a boosted tree model using 100 iterations and a tree depth of 2 more complex than one with 50 iterations and a depth of 8? The package makes some choices regarding the orderings. In the case of boosted trees, the package assumes that increasing the number of iterations adds complexity at a faster rate than increasing the tree depth, so models are ordered on the number of iterations then ordered with depth. See ?best for more examples for specific models.

2.6 Parallel Processing

If a model is tuned using resampling, the number of model fits can become large as the number of tuning combinations increases (see the two loops in Algorithm 1). To reduce the training time, parallel processing can be used. For example, to train the support vector machine model in Section 1.2, each of the 4 candidate models was fit to 30 separate resamples. Since each resample is independent of the other, these 120 models could be computed in parallel.

R has several packages that facilitates parallel processing when multiple processors are available (see Schmidberger et al., 2009). train can be used to build multiple models simultaneously. When a candidate model is resampled during parameter tuning, the resampled datasets are sent in roughly equal sized batches to different "workers," which could be processors within a single machine or across computers. Once their models are built, the results are returned to the original R session. Examples of R packages that can be used for parallel processing with train are nws and Rmpi, among others.

To run in parallel, any function that emulates lapply can be used. For example, the **snow** package has a parallel lapply function that uses MPI. We can write a wrapper for this function:

```
> mpiCalcs <- function(X, FUN, ...)
+ {
+ theDots <- list(...)
+ parLapply(theDots$cl, X, FUN)
+ }</pre>
```

To use this function, a cluster must be started using MPI:

```
> cl <- makeCluster(5, "MPI")
5 slaves are spawned successfully. 0 failed.</pre>
```

and to use this cluster, the trainControl object must be modified:

The last three options specify the number of workers, the function that emulates lapply and the extra arguments to this function (such as the cluster object). The documentation page for train has another example using the nws package.

This control object is passed into train with no other modifications. The command stopCluster(cl) will shut down the workers.

One common metric used to assess the efficacy of parallelization is $speedup = T_{seq}/T_{par}$, where T_{seq} and T_{par} denote the execution times to train the model serially and in parallel, respectively. Excluding systems with sophisticated shared memory capabilities, the maximum possible speedup attained by parallelization with P processors is equal to P. Factors affecting the speedup include the overhead of starting the parallel workers, data transfer, the percentage of the algorithm's computations that can be done in parallel, etc.

Figure 3 shows the results of a benchmarking study run on a 32 core machine. In the left panel, the actual training time for each of the models is shown. Irrespective of the number of processors used, the PLS model is much more efficient than the other models. This is most likely due to PLS solving straight-forward, well optimized linear equations. Unfortunately, partial least squares produces linear boundaries which may not be flexible enough for some problems. For the support vector machine and boosted tree models, the rate of decrease in training time appears to slow after 15 processors.

On the right-hand panel, the speedup is plotted. For each model, there is a decrease in the training time as more nodes are added, although there was little benefit of adding more than 15 workers. The support vector machine comes the closest to the theoretical speedup boundary when five or less workers are used. Between 5 and 15 workers, there is an additional speedup, but at a loss of efficiency. After 15 workers, there is a negligible speedup effect. For boosted trees, the efficiency



Figure 3: Training time profiles using parallel processing via train for a benchmarking data set run on a 32 core machine. The left panel shows the elapsed time to train various types of models using single or multiple processors. The panel on the right shows the "speedup," defined to be the time for serial execution divided by the parallel execution time. The reference line shows the maximum theoretical speedup.

of adding parallel workers was low, but there was more than a four-fold speedup obtained by using more than 10 workers. Although PLS benefits from parallel processing, it does not show significant gains in training time and efficiency.

One downside to parallel processing in this manner is that the dataset is held in memory for every node used to train the model. For example, if parallelism is used to compute the results from 50 bootstrap samples using P processors, P data sets are held in memory. For large datasets, this can become a problem if the additional processors are on the same machines where they are competing for the same physical memory. In the future, this might be resolved using specialized software that exploits systems with a shared memory architecture.

More research is needed to determine when it is advantageous to parallel process, given the type of model and the dimensions of the training set.

3 Extracting Predictions and Class Probabilities

As previously mentioned, objects produced by the train function contain the "optimized" model in the finalModel sub-object. Predictions can be made from these objects as usual. In some cases, such as pls or gbm objects, additional parameters from the optimized fit may need to be specified. In these cases, the train objects uses the results of the parameter optimization to predict new samples.

For example, we can load the Boston Housing data:

```
> library(mlbench)
> data(BostonHousing)
> # we could use the formula interface too
> bhDesignMatrix <- model.matrix(medv ~. - 1, BostonHousing)
split the data into random training/test groups:
> set.seed(4)
> inTrain <- createDataPartition(BostonHousing$medv, p = .8, list = FALSE, times = 1)
> trainBH <- bhDesignMatrix[inTrain,]</pre>
> testBH <- bhDesignMatrix[-inTrain,]</pre>
> preProc <- preProcess(trainBH)</pre>
> trainBH <- predict(preProc, trainBH)</pre>
> testBH <- predict(preProc, testBH)</pre>
> trainMedv <- BostonHousing$medv[inTrain]</pre>
> testMedv <- BostonHousing$medv[-inTrain]
fit partial least squares and multivariate adaptive regression spline models:
> set.seed(5)
> plsFit <- train(trainBH, trainMedv,
                   "pls",
                   tuneLength = 10,
                   trControl = trainControl(verboseIter = FALSE,
                                              returnResamp = "all"))
> set.seed(5)
> marsFit <- train(trainBH, trainMedv,
                    "earth",
                    tuneLength = 10,
                    trControl = trainControl(verboseIter = FALSE,
                                               returnResamp = "all"))
```

To obtain predictions for the PLS model, predict.mvr can be used. In this case, the number of components must be manually specified or all of the sub-models are predicted:

```
> plsPred1 <- predict(plsFit$finalModel, newdata = as.matrix(testBH))
> dim(plsPred1)

[1] 99 1 9
```

Alternatively, predict.train can be used to get a vector of predictions for the optimal model only:

```
> plsPred2 <- predict(plsFit, newdata = testBH)
> length(plsPred2)
```

[1] 99

For multiple models, the objects can be grouped using a list and predicted simultaneously:

In some cases, observed outcomes and their associated predictions may be needed for a set of models. In this case, extractPrediction can be used. This function takes a list of models and test and/or unknown samples as inputs and returns a data frame of predictions:

```
> allPred <- extractPrediction(bhModels,
                                testX = testBH,
                                testY = testMedv)
> testPred <- subset(allPred, dataType == "Test")
> head(testPred)
             pred model dataType object
     obs
408 34.7 30.15640
                             Test
                                     pls
                     pls
409 21.7 21.87263
                     pls
                             Test
                                     pls
410 20.2 16.06634
                    pls
                             Test
                                     pls
411 15.2 16.01122
                    pls
                             Test
                                     pls
412 15.6 15.80842
                             Test
                                     pls
                     pls
413 14.5 17.94325
                     pls
                             Test
                                     pls
```

The output of extractPrediction is a data frame with columns:

- obs, the observed data
- pred, the predicted values from each model
- model, a character string ("rpart", "pls" etc.)
- dataType, a character string for the type of data:
 - "Training" data are the predictions on the training data from the optimal model,
 - "Test" denote the predictions on the test set (if one is specified),
 - "Unknown" data are the predictions on the unknown samples (if specified). Only the predictions are produced for these data. Also, if the quick prediction of the unknowns is the primary goal, the argument unkOnly can be used to only process the unknowns.

Some classification models can produce probabilities for each class. The functions predict.train and predict.list can be used with the type = "probs" argument to produce data frames of class probabilities (with one column per class). Also, the function extractProbs can be used to get these probabilities from one or more models. The results are very similar to what is produced by extractPrediction but with columns for each class. The column pred is still the predicted class from the model.

4 Evaluating Test Sets

A function, postResample, can be used obtain the same performance measures as generated by train for regression or classification.

4.1 Confusion Matrices

caret also contains several functions that can be used to describe the performance of classification models. The functions sensitivity, specificity, posPredValue and negPredValue can be used to characterize performance where there are two classes. By default, the first level of the outcome factor is used to define the "positive" result (i.e. the event of interest), although this can be changed.

The function confusionMatrix can also be used to summarize the results of a classification model:

```
> mbrrPredictions <- extractPrediction(list(svmFit), testX = testDescr, testY = testMDRR)
> mbrrPredictions <- mbrrPredictions[mbrrPredictions$dataType == "Test",]</pre>
```

> sensitivity(mbrrPredictions\$pred, mbrrPredictions\$obs)

[1] 0.8066667

> confusionMatrix(mbrrPredictions\$pred, mbrrPredictions\$obs)

Confusion Matrix and Statistics

Reference

Prediction Active Inactive
Active 121 25
Inactive 29 89

Accuracy: 0.7955

95% CI: (0.7417, 0.8424)

No Information Rate : 0.5682 P-Value [Acc > NIR] : 6.255e-15

Kappa : 0.5849

Sensitivity: 0.8067 Specificity: 0.7807 Pos Pred Value: 0.8288 Neg Pred Value: 0.7542 Prevalence: 0.5682 Detection Rate: 0.4583

Detection Prevalence: 0.5530

'Positive' Class : Active

The "no-information rate" is the largest proportion of the observed classes (there were more actives than inactives in this test set). A hypothesis test is also computed to evaluate whether the overall

accuracy rate is greater than the rate of the largest class. Also, the prevalence of the "positive event" is computed from the data (unless passed in as an argument), the detection rate (the rate of true events also predicted to be events) and the detection prevalence (the prevalence of predicted events).

Suppose a 2×2 table with notation

	Reference					
Predicted	Event	No Event				
Event	A	В				
No Event	С	D				

The formulas used here are:

$$Sensitivity = \frac{A}{A+C}$$

$$Specificity = \frac{D}{B+D}$$

$$Prevalence = \frac{A+C}{A+B+C+D}$$

$$PPV = \frac{sensitivity \times prevalence}{((sensitivity \times prevalence) + ((1-specificity) \times (1-prevalence))}$$

$$NPV = \frac{specificity \times (1-prevalence)}{((1-sensitivity) \times prevalence) + ((specificity) \times (1-prevalence))}$$

$$Detection Rate = \frac{A}{A+B+C+D}$$

$$Detection Prevalence = \frac{A+B}{A+B+C+D}$$

When there are three or more classes, confusionMatrix will show the confusion matrix and a set of "one-versus-all" results. For example, in a three class problem, the sensitivity of the first class is calculated against all the samples in the second and third classes (and so on).

4.2 ROC Curves

The function roc^3 can be used to calculate the sensitivity and specificity used in an ROC plot. For example, using the previous support vector machine fit to the MBRR data, the predicted class probabilities on the test set can used to create an ROC curve. The area under the ROC curve, via the trapezoidal rule, is calculated using the aucRoc function.

```
> mbrrProbs <- extractProb(list(svmFit), testX = testDescr, testY = testMDRR)
> mbrrProbs <- mbrrProbs[mbrrProbs$dataType == "Test",]</pre>
```

³I'm looking into using the ROCR package for ROC curves, so don't get too attached to these functions

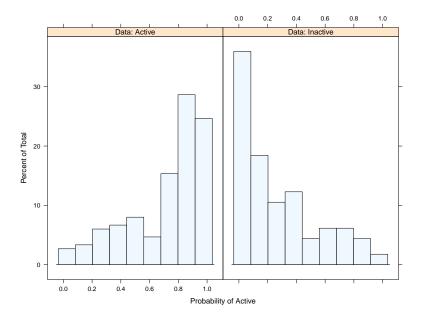


Figure 4: The predicted class probabilities from a support vector machine fit for the MBRR test set. This plot was created using plotClassProbs(mbrrProbs).

- > mbrrROC <- roc(mbrrProbs\$Active, mbrrProbs\$obs)
- > aucRoc(mbrrROC)

[1] 0.8724269

See Figure 5 for an example.

Plotting Predictions and Probabilities

Two functions, plot0bsVsPred and plotClassProbs, are interfaces to lattice to plot model results. For regression, plot0bsVsPred plots the observed versus predicted values by model type and data (e.g. test). See Figures 6 and 5 for examples. For classification data, plot0bsVsPred plots the accuracy rates for models/data in a dotplot.

To plot class probabilities, plotClassProbs will display the results by model, data and true class (for example, Figure 4).

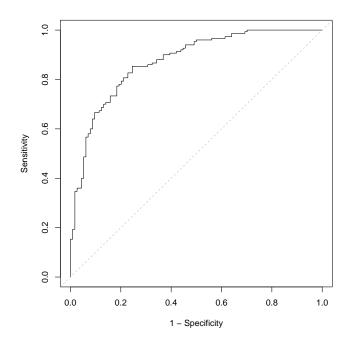


Figure 5: An ROC curve from the predicted class probabilities from a support vector machine fit for the MBRR test set.

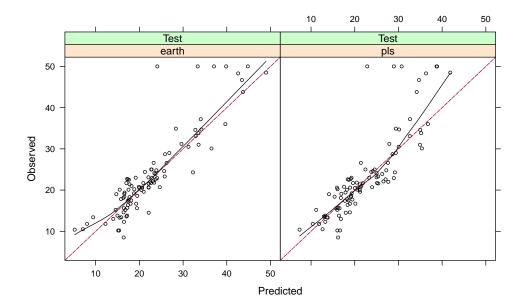


Figure 6: The results of using <code>plotObsVsPred</code> to show plots of the observed median home price against the predictions from two models. The plot shows the training and test sets in the same Lattice plot

5 Exploring and Comparing Resampling Distributions

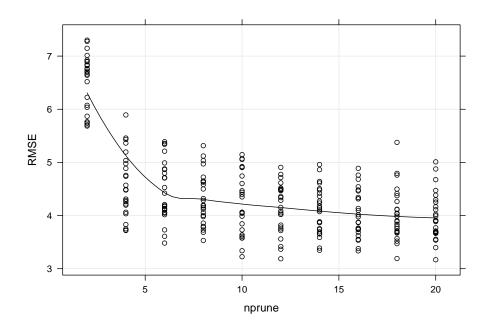
5.1 Within-Model

There are several Lattice functions than can be used to explore relationships between tuning parameters and the resampling results for a specific model:

- xyplot and stripplot can be used to plot resampling statistics against (numeric) tuning parameters.
- histogram and densityplot can also be used to look at distributions of the tuning parameters across tuning parameters.

For example, the following statements produces the images in Figure 7.

```
> xyplot(marsFit, type= c("g", "p", "smooth"), degree = 2) > densityplot(marsFit, as.table = TRUE, subset = nprune < 10)
```



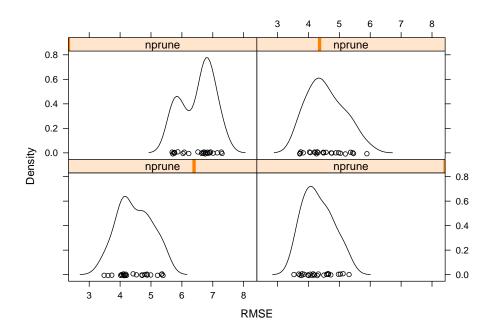


Figure 7: Scatter plots and density plots of the resampled RMSE by the number of retained terms for the MARS model fit to the Boston Housing data

5.2 Between-Models

caret also includes functions to characterize the differences between models (generated using train, sbf or rfe) via their resampling distributions. These functions are based on the work of Hothorn et al. (2005) and Eugster et al (2008).

Using the blood-brain barrier data (see ?BloodBrain), three regression models were created: an rpart tree, a conditional inference tree using ctree, M5 rules using M5Rules and a MARS model using earth. We ensure that the models use the same resampling data sets. In this case, 100 leave—group—out cross—validation was employed.

```
> data(BloodBrain)
> set.seed(1)
> tmp <- createDataPartition(logBBB, p = 0.8, times = 100)
 rpartFit <- train(bbbDescr, logBBB,</pre>
+
                     "rpart",
                     tuneLength = 16,
                     trControl = trainControl(method = "LGOCV", index = tmp))
Fitting: maxdepth=16
> ctreeFit <- train(bbbDescr, logBBB,
                    "ctree2",
                     tuneLength = 10,
                     trControl = trainControl(method = "LGOCV", index = tmp))
Fitting: maxdepth=1
Fitting: maxdepth=2
Fitting: maxdepth=3
Fitting: maxdepth=4
Fitting: maxdepth=5
Fitting: maxdepth=6
Fitting: maxdepth=7
Fitting: maxdepth=8
Fitting: maxdepth=9
Fitting: maxdepth=10
> earthFit <- train(bbbDescr, logBBB,
                     "earth",
+
                     tuneLength = 20,
                     trControl = trainControl(method = "LGOCV", index = tmp))
```

```
Fitting: degree=1, nprune=76
> m5Fit <- train(bbbDescr, logBBB,</pre>
+
                "M5Rules",
               trControl = trainControl(method = "LGOCV", index = tmp))
Fitting: pruned=Yes
Fitting: pruned=No
Given these models, can we make statistical statements about their performance differences? To do
this, we first collect the resampling results using resamples.
> resamps <- resamples(list(CART = rpartFit,
                          CondInfTree = ctreeFit,
+
                         MARS = earthFit,
                         M5 = m5Fit)
> resamps
Call:
resamples.default(x = list(CART = rpartFit, CondInfTree = ctreeFit,
                                                                 MARS = earthFit, M5
Models: CART, CondInfTree, MARS, M5
Number of resamples: 100
Performance metrics: RMSE, Rsquared
> summary(resamps)
Call:
summary.resamples(object = resamps)
Models: CART, CondInfTree, MARS, M5
Number of resamples: 100
RMSE
            Min. 1st Qu. Median
                                Mean 3rd Qu.
CART
          CondInfTree 0.4528 0.5934 0.6375 0.6427 0.6873 0.8685
MARS
          M5
```

Rsquared

```
Min. 1st Qu. Median
                                      Mean 3rd Qu.
                                                      Max.
CART
                     0.2749 0.3436 0.3455
            0.12070
                                            0.4049 0.5981
CondInfTree 0.07711
                     0.2852 0.3517 0.3449
                                            0.4099 0.6164
            0.18800
MARS
                     0.3381 0.4146 0.4141
                                            0.4939 0.6515
M5
            0.12260
                     0.3080 0.3910 0.3950
                                            0.4744 0.6286
```

There are several Lattice plot methods that can be used to visualize the resampling distributions: density plots, box—whisker plots, scatterplot matrices and scatterplots of summary statistics. In the latter case, the plot consists of the differences between two models on the y—axis and the average on the x—axis (See Figure 8). In Figure 9, density plots of the data are shown. In this figure, the R^2 distributions indicate that M5 rules and MARS appear to be similar to one another but different from the two tree—based models. However, this pattern is inconsistent with the root mean squared error distributions.

Since models are fit on the same versions of the training data, it makes sense to make inferences on the differences between models. In this way we reduce the within–resample correlation that may exist. We can compute the differences, then use a simple t–test to evaluate the null hypothesis that there is no difference between models.

```
> difValues <- diff(resamps)
> difValues

Call:
diff.resamples(x = resamps)

Models: CART, CondInfTree, MARS, M5
Metrics: RMSE, Rsquared
Number of differences: 6

> summary(difValues)
```

```
Call:
```

```
Upper diagonal: estimates of the difference
Lower diagonal: p-value for HO: difference = 0

RMSE

CART CondInfTree MARS M5

CART -0.009607 0.020238 0.002280
```

M5 0.8030070 0.1826212 0.0443769

summary.diff.resamples(object = difValues)

Rsquared

CART CondInfTree MARS M5

CART 0.0005807 -0.0685809 -0.0495256

CondInfTree 0.9558245 -0.0691617 -0.0501063

MARS 9.588e-09 1.090e-08 0.0190553

M5 0.0003594 0.0001707 0.1132682

Note that these results are consistent with the patterns shown in Figure 9; there are more differences in the R^2 distributions than in the error distributions.

Several Lattices methods also exist to plot the differences (density and box–whisker plots) or the inferential results (level and dot plots). Figures 10 and 11 show examples of level and dot plots.

6 Session Information

- R version 2.12.0 Under development (unstable) (2010-08-08 r52687), x86_64-apple-darwin9.8.0
- Locale: en_US/en_US/en_US/c/en_US/en_US

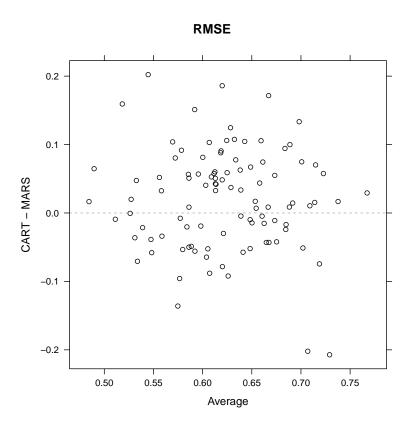


Figure 8: Examples of output from xyplot(resamps, models = c("CART", "MARS")). The averages and differences of the two models is shown for each resampling data set.

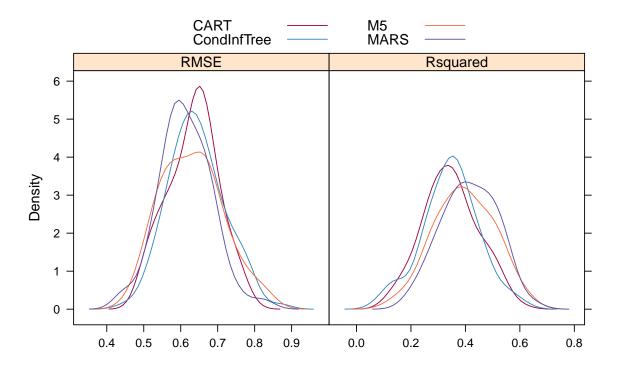


Figure 9: Examples of output from densityplot(resamps). Looking at \mathbb{R}^2 , M5 rules and MARS appear to be similar to one another but different from the two tree-based models. However, this pattern is inconsistent with the root mean squared error distributions.



Figure 10: Examples of output from levelplot(difValues, what = "differences"). The pairwise differences in RMSE are shown

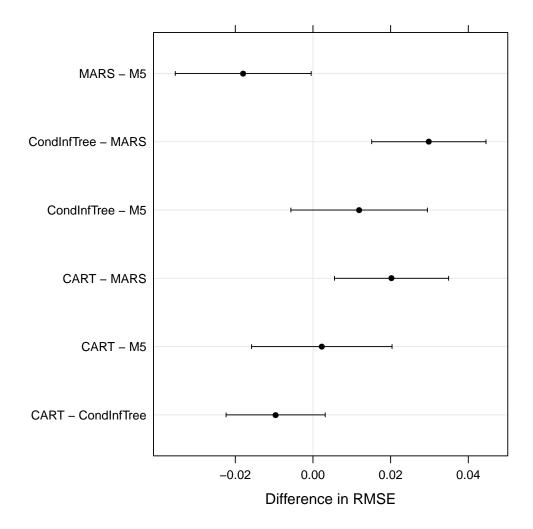


Figure 11: Examples of output from dotplot(difValues). The differences in RMSE and their associated confidence intervals are shown.

- Base packages: base, datasets, graphics, grDevices, grid, methods, splines, stats, stats4, utils
- Other packages: akima 0.5-4, caret 4.58, class 7.3-2, e1071 1.5-24, earth 2.4-0, ellipse 0.3-5, gam 1.03, gbm 1.6-3.1, Hmisc 3.8-2, ipred 0.8-8, kernlab 0.9-10, klaR 0.6-3, lattice 0.18-8, leaps 2.9, MASS 7.3-7, mlbench 2.0-0, modeltools 0.2-16, mvtnorm 0.9-92, nnet 7.3-1, pls 2.1-0, plyr 1.1, proxy 0.4-6, randomForest 4.5-36, reshape 0.8.3, rpart 3.1-46, survival 2.35-8
- Loaded via a namespace (and not attached): cluster 1.13.1, coin 1.0-12, colorspace 1.0-1, party 0.9-9997, rJava 0.8-4, RWeka 0.4-1, tools 2.12.0

7 References

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