Negative Binomial Distribution

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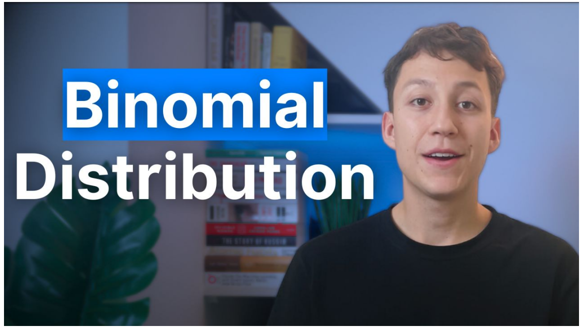
Introduction

Perhaps you have heard of the binomial distribution, but have you heard of its cousin the negative binomial distribution?

This discrete probability distribution is applied in numerous industries such as insurance and manufacturing (mainly count-based data), hence is a useful concept for Data Scientists to understand.



Intuition

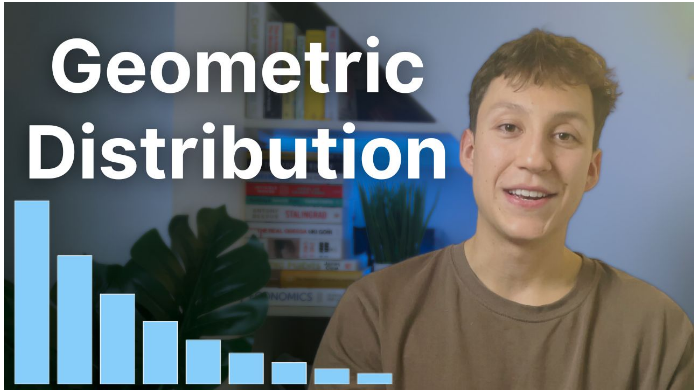
The binomial distribution measures the probability of measuring a certain number of successes, ***x***, in a given number of trials, ***n***. The trials in this case are [Bernoulli]() trials, where every outcome is binary (success or failure)

The negative binomial distribution flips this and models the number of trials, ***x***, needed to reach a certain number of successes, ***r***. This is why it is known as ‘negative’ because it is inadvertently modeling the number of failures before the certain number of successes.

Probability of the “r” success happening on the “x”trial

Intuition

A special case of the negative binomial distribution is the geometric distribution. This models the number of trials needed before we get our first success.



Formula & Derivation

Let’s say we have:

- ***p***: *probability of success*
- ***1-p***: *the probability of failure*
- ***x***: *number trials for* ***r*** *success*
- ***r***: *number of successes for* ***x*** *trials*

Consequently, we must have ***r-1*** successes in ***x-1*** trials and the probability of this is simply the Binomial distribution **probability mass function**:

$$P(X = x-1) = \binom{x-1}{r-1} p^{r-1}(1-p)^{(x-1)-(r-1)}$$

Equation generated by author in LaTeX.

Formula & Derivation

The next bit of information we have is that the ***r*** success must occur on ***x*** trial, and it will have a probability of ***p***. Therefore, we simply multiply the above formula by ***p***:

$$P(X = x) = \binom{x-1}{r-1} p^{r-1} (1-p)^{(x-1)-(r-1)} \times p$$
$$= \binom{x-1}{r-1} p^{r} (1-p)^{x-r}$$

Equation generated by author in LaTeX.

That's the negative binomial distribution PMF!

Example Problem

What would be the probability of rolling a second 4 on the 6th roll?

- *p = ⅙*
- *r = 2*
- *x = 6*

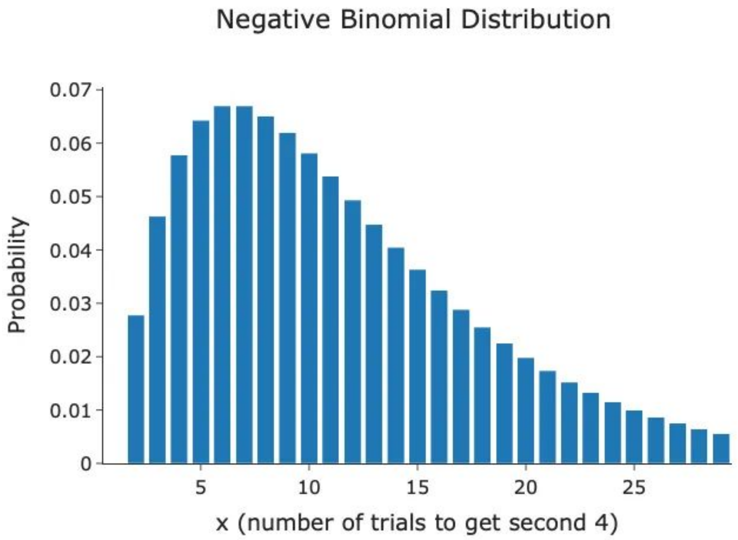
$$P(X=6) = \binom{6-1}{2-1}\frac{1}{6}^{2}(1-\frac{1}{6})^{6-2} \approx 0.067$$

Equation generated by author in LaTeX.

So, it is quite unlikely that we will get our second 4 on the 6th roll

Plot

What about if we want to know the probability of rolling our second 4 on other rolls? Well, to do this we need to plot the second roll as a function of the number of rolls, ***x***:




```
import plotly.graph_objects as go
from math import comb

# Parameters
r = 2
p = 1 / 6


# PMF
def neg_binomial_pmf(x, r, p):
    if x < r:
        return 0
    q = 1 - p
    return comb(x - 1, r - 1) * (p ** r) * (q ** (x - r))


# Values
x = list(range(1, 30))
probs = [neg_binomial_pmf(k, r, p) for k in x]

# Plot
fig = go.Figure(data=[go.Bar(x=x, y=probs, marker_color='rgba(176, 224, 230)')])
fig.update_layout(title="Negative Binomial Distribution",
                  xaxis_title="x (number of trials to get second 4)",
                  yaxis_title="Probability",
                  template="simple_white",
                  font=dict(size=16),
                  title_x=0.5,
                  width=700,
                  height=500)
fig.show()
```

Applications

- ***Time until an event***: *This is useful for churn models, where we want to predict when a customer may cancel their subscription.*
- ***Defect prediction***: *Predicting the number of defects in a manufactured product before it becomes fully functional.*
- ***Sports Analytics***: *There are several examples such as predicting after how many missed shots will a footballer score a goal.*
- ***Marketing***: *Determine how many advertisements to show a customer before they convert onto a subscription or click on the website.*
- ***Epidemiology***: *Estimating the volume of endangered species and how the environment is affecting their numbers.*

Thanks

Failures, Trials, and Successes: The Negative Binomial Distribution Explained

A dive into one of the lesser known probability distributions

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