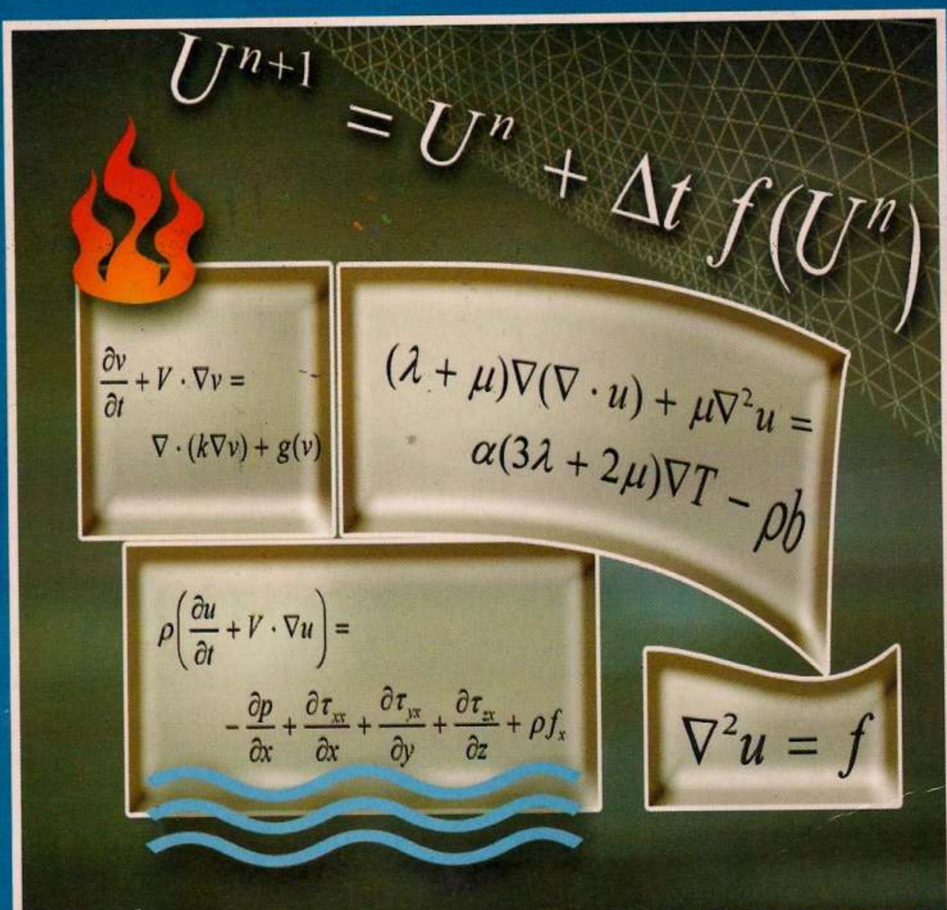
Numerical Methods by Vedamurthy pdf

Numerical Analysis (University of Engineering and Technology Lahore)

NUMERICAL METHODS



EXERCISE 1.1

1. The law of machine is $P = aW + b$, where P is the effort and W, the load in lb. Sketch a graph showing the relation between P and W, given

P	60	75	100	125	145
W	225	300	430	560	600

Find P when $W = 500$.

2. R is the resistance to motion of a train at speed V. Find a law of the type $R = aV^2 + b$ connecting R and V using the following data

R kg/ton	8	10	15	21	30
V (km/hr)	10	20	30	40	50

3. The resistance R of a carbon filament lamp was measured at various values of voltage V and the following observations were made.

V	62	70	78	84	92
R	73	70.7	69.2	67.8	66.3

(*Ranchi B.Tech 1986*)

Assuming a law of the form $R = a/V + b$, find by graphical method the best values of a and b.

4. μ, the co-efficient of friction between a belt and pulley and v, the velocity of the belt in ft/min, are connected as shown in the following table:

v	500	1000	2000	4000	6000
μ	0.29	0.33	0.38	0.45	0.51

The probable law is $\mu = a + b\sqrt{v}$. Test graphically the accuracy of this law and if it is true, find the values of a and b.

5. Fit a curve of the form $y = ae^{bx}$ to the following data:

x	1	2	3	4	5	6
y	14	27	40	55	68	300

6. The following observations are corresponding to pressure and specific volume of dry saturated steam. Fit a curve of the form $PV^n = k$ by graphical method.

V	38.4	20	8.51	4.44	3.03	2.31
P	10	20	50	100	150	200

ANSWERS

1. $P = 0.21W + 12,\ P = 117$
2. a = 0.0085, b = 7.35
3. a = 1120, b = 55.1
4. a = 0.2 b = 0.0044
5. $y = 7.943\, e^{0.5419x}$
6. PV [illegible]

EXERCISE 1.2

1. The weights of a calf taken at weekly intervals are given below. Fit a straight line using the method of group averages.

Age in weeks	1	2	3	4	5	6	7	8	9	10
weight	52.5	58.7	65	70.2	75.4	81.1	87.2	95.5	102.2	108.4

2. Fit a curve of the form $y = ax^n$ to the following set of observations by the method of group averages:

x	10	20	30	40	50	60	70	80
y	1.06	1.33	1.52	1.68	1.81	1.91	2,01	2.11

3. Fit a curve of the form $y = ab^x$ using the method of group averages for the following data:

x	2	4	6	8	10	12
y	7.32	8.24	9.20	10.19	11.01	12.05

4. Convert the equation $y = b/x(x - a)$ to a linear form and hence, determine a and b which will fit the following data using the method of group averages:

x	8	10	15	20	30	40
y	13	14	15.4	16.3	17.2	17.8

5. The following data represent test values obtained while testing a centrifugal pump. Assuming the relation to be $H = a + bQ + cQ^2$, where Q is the discharge in liter per second and H, head in meter of water, find the relation by the method of group averages.

Q	2	2.5	3	3.5	4	4.5	5	5.5	6
H	18	17.8	17.5	17	15.8	14.8	13.3	11.7	9

(M.U. B.E., 1971)

6. The temperature θ of a vessel of cooling water and the time t in minutes since the beginning of observation are connected by the law of the form $\theta = ae^{bt} + c$. The corresponding values of t and θ are given by:

t	0	1	2	3	5	7	10	15	20
θ	52.2	48.8	46.0	43.5	39.7	36.5	33.0	28.7	26.0

Find the best values of a, b and c using the method of group averages.

7. Fit a curve of the form $y = a + bx^c$ to the following data using the method of group averages.

x	1	2	4	6	10	16
y	15	45	165	364	1004	2564

8. Fit a curve of the form $y = a + bc^x$ to the following data using the method of group averages.

x	0	1	2	3	4	5	6	7	8
y	2.4	3.2	3.7	5.1	7.8	13.2	23.6	44.8	87

ANSWERS

1. $y = 46.048 + 6.104\,x$
2. $y = 0.4851\,x^{0.3354}$
3. $y = (6.7468)(1.0505)^x$
4. $a = 0.2039,\ b = 0.051$
5. $H = 1.58 + 2.1\,Q - 0.5\,Q^2$
6. $a = 29.53$[illegible], $b = -0.09968$, $c = 21.98$
7. $y = 5 + 10x^2$
8. $y = 2.26 +$ [illegible]$3)(2.07)^x$

EXERCISE 1.3

1. A simply supported beam carries a concentrated load P (lb) at its midpoint. Corresponding to various values of P, the maximum deflection Y (in) is measured. The data are given below. Find a law of the type $Y = a + bP$ by the method of least squares.

P	100	125	140	160	180	200
Y	0.45	0.55	0.60	0.70	0.80	0.85

(*Shivaji B.E., 1984*)

2. In the following table, y is the weight of potassium bromide which will dissolve in 100 gm of water at temperature x°C. Find a linear law between x and y using least square method.

x(°C)	0	10	20	30	40	50	60	70
y(gm)	53.5	59.5	65.2	70.6	75.5	80.2	85.5	90

3. By the method of least squares, find the curve $y = ax + bx^2$ that best fits the following data :

x	1	2	3	4	5
y	1.8	5.1	8.9	14.1	19.8

4. Find the parabola of the form $y = a + bx + cx^2$ which fits most closely with the following observations by the method of least squares.

x	-3	-2	-1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

(*Kerala B.E., 1985*)

5. By the method of least squares, fit a second degree curve $y = a+bx+cx^2$ to the following data :

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

6. By the method of least squares, fit a parabola $y = a + bx + cx^2$ to the following data.

x	2	4	6	8	10
y	3.07	12.85	31.47	57.38	91.29

(*Mangalore B.E.. 1985*)

7. Fit an equation of the form $y = ae^{bx}$ to the following data by the method of least squares.

x	1	2	3	4
y	1.65	2.7	4.5	7.35

8. The voltage v across a capacitor at time t seconds is given by the following table. Use the principle of least squares to fit a curve of the form $v = a\, e^{bt}$ to the data:

t	0	2	4	6	8
v	150	63	28	12	5.6

9. Fit a curve of the form $y = ae^{bx}$ to the following data in least square sense:

x	0	2	4
y	5.012	10	31.62

10. Fit a curve of the form $y = ax^b$ to the data given below in square sense:

x	1	2	3	4	5
y	7.1	27.8	62.1	10	[illegible]

11. Fit a curve of the form $y = ax^b$ in least square sense to the following observations:

x	1	2	3	4	5
y	0.5	2	4.5	8	12.5

(*Calicut B.E., 1988*)

12. Fit a curve of the form $y = ab^x$ in least square sense to the data given below:

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

(*Karnataka B.E., 1993*)

13. Fit a curve of the form $y = ab^x$ in least square sense to the data given below:

x	1	2	3	4
y	4	11	35	100

14. Fit a straight line $y = ax + b$ and also a parabola $y = ax^2 + bx + c$ to the following set of observations:

x	0	1	2	3	4
y	1	5	10	22	38

Calculate the sum of squares of the residuals in each case and test which curve is more suitable to the data.

ANSWERS

1. $Y = 0.004P + 0.048$
2. $y = 54.35 + 0.5184x$
3. $y = 1.37x + 0.53x^2$
4. $y = 1.243 - 0.004x + 0.22x^2$
5. $y = -1 + 3.55x - 0.27x^2$
6. $y = 0.34 - 0.78x + 0.99x^2$
7. $y = e^{0.5x}$
8. $v = 146.3\, e^{-0.4118t}$
9. $y = 4.642\, e^{0.46x}$
10. $y = 7.173\, x^{1.952}$
11. $y = 0.5012\, x^{1.9977}$
12. $y = 99.86\,(1.2)^x$
13. $y = 1.33\,(2.95)^x$
14. $y = 9.1\,x - 3$; $y = 2.2x^2 + 0.3x + 1.4$

    $E_1 = 70.7$, $E_2 = 2.5$, $E_2 < E_1$, parabola is the best curve of fit.

$$27.14 = 10a + 30.3333b + 102.5c$$
$$101.14 = 30.3333a + 102.5b + 369.05c$$

Solving, we get

$$a = 1.399,\ b = -1.7856 \text{ and } c = 0.6567$$

$\therefore$ From (i) the required parabola is

$$y = 1.399 - 1.7856x - 0.6567x^2$$

EXERECISE 1.4

1. Use the method of moments to fit a straight line to the data given below :

x	1	3	5	7	9
y	1.5	2.8	4.0	4.7	6.0

(*M.K.U., 1976*)

2. Fit a parabola of the form $y = ax^2 + bx + c$ to the data

x	1	2	3	4
y	1.7	1.8	2.3	3.2

by the method of moments. (*Coimbatore, B.E., 1988*)

ANSWERS

1 $y = 1.1845 + 0.5231x$

2. $y = 0.74x^2 + 0.063x + 1.53$

EXERCISE 2.1

1. Solve $x^3 + 6x + 20 = 0$, one root being -2.
2. Solve $x^3 - 12x^2 + 39x - 28 = 0$, whose roots are in arithmetic progression. *(M.U., B.E. 1995)*
3. Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetic progression.
4. Solve $27x^3 + 42x^2 - 28x - 8 = 0$, the roots of which are in geometric progression. *(M.U., B.E. 1994)*
5. Solve $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$, whose roots are in geometric progression.
6. Solve the equation $6x^3 - 11x^2 - 3x + 2 = 0$ whose roots are in harmonic progression.

7. Solve $15x^4 - 8x^3 - 14x^2 + 8x - 1 = 0$, whose roots are in harmonic progression.
8. Solve $x^3 - 8x^2 + 9x + 18 = 0$ given that two of its roots are in the ratio 1 : 2.
9. The equation $x^4 - 4x^3 + px^2 + 4x + q = 0$ has two pairs of equal roots. Find the values of p and q.
10. Solve the equation $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$, given that the sum of two of the roots is equal to the sum of the other two.
11. Solve $x^4 - 8x^3 + 23x^2 - 28x + 12 = 0$, given that the difference of two roots is equal to the difference of the other two.
12. Solve the equation $x^4 - 8x^3 + 7x^2 + 36x - 36 = 0$, given that product of two roots is negative of the product of the remaining two.
13. Solve $x^3 - 4x^2 - 20x + 48 = 0$, given that the relationship between two roots, α and β, is $\alpha + 2\beta = 0$.
14. Find the conditions in which the cubic $x^3 + px^2 + qx + r = 0$ should have its roots in

    (i) arithmetical progression (*M.U B.E., 1993, 1994*) (ii) geometrical progression and (iii) harmonic progression.
15. Solve the equation $3x^3 - 4x^2 + x + 88 = 0$, given that $2 - i\sqrt{7}$ is a root.
16. Solve $3x^5 - 4x^4 - 42x^3 + 56x^2 + 27x - 36 = 0$, given that $\sqrt{2} + \sqrt{5}$, $-\sqrt{2} - \sqrt{5}$ are two roots.

ANSWERS

1. $1 \pm 3i, 2$
2. $1, 4, 7$
3. $-4, -1, 2, 5$
4. $-2/9, -2/3, -2$
5. $-2, -4, -1, -8$
6. $-1/2, 2, 1/3$
7. $-1, 1, 1/3, 1/5$
8. $3, 6, -1$
9. $p = 2, q = 1$
10. $-1, 1, 3, 5$
11. $1, 2, 2, 3$
12. $3, -2, 1, 6$
13. $-7, 2, 6$
14. (i) $2p^3 - 9pq + 27r = 0$ (ii) $p^3 r = q^3$ (iii) $2q^3 - 9pqr + 27r^2 = 0$
15. $2 \pm i\sqrt{7}, -8/3$
16. $\sqrt{2} \pm \sqrt{5}, -\sqrt{2} \pm \sqrt{5}, 4/3$

EXERCISE 2.2

1. If α, β, and γ are the roots of the $x^3 + px + q = 0$, then find

   (i) $\sum \alpha^3$ (*M.U., B.E., 1994*) (ii) $\sum \alpha^2\beta$ and
   (iii) $\sum \alpha^4$

2. If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, find the values of

   (i) $\sum 1/\alpha$ (ii) $\sum \alpha^3$
   (iii) $\sum \alpha^3\beta$ (iv) $\sum (\beta^2 + \beta\gamma + \gamma^2)$
   (v) $\sum (\beta + \gamma - \alpha)^3$, and (vi) $\sum (\alpha^2 + \beta\gamma)/(\beta + \gamma)$

3. If α, β and γ are the roots of $x^3 + px^2 + qx + r = 0$, form the equation whose roots are

   (i) $\alpha^2, \beta^2, \gamma^2$ (ii) $\alpha\beta, \beta\gamma, \alpha\gamma$
   (iii) $\alpha(\beta + \gamma), \beta(\gamma + \alpha), \gamma(\alpha + \beta)$ and
   (iv) $\alpha + 1/\beta\gamma, \beta + 1/\alpha\gamma, \gamma + 1/\alpha\beta$.

4. If α, β and γ are the roots of the equation $x^2 + px + q = 0$, obtain the equation whose roots are

   (i) $\alpha + \beta - \gamma, \beta + \gamma - \alpha, \gamma + \alpha - \beta$ (*M.U., B.E., 1993*)
   (ii) $(\alpha + \beta)(\gamma + \alpha), (\beta + \gamma)(\alpha + \beta), (\gamma + \alpha)(\beta + \gamma)$

5. If α, β, and γ are the roots of $x^3 - 7x + 6 = 0$, form an equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$. (*Raipur B.E., 1987*)

6. If α, β, and γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$, obtain an equation whose roots are $(1 - \alpha)^{-1}, (1 - \beta)^{-1}, (1 - \gamma)^{-1}$. (*Kerala B. Tech, 1988*)

7. If α, β, and γ are the roots of $x^3 - 3x + 1 = 0$, form the equation whose roots are $\dfrac{(\alpha - 2)}{(\alpha + 2)}, \dfrac{(\beta - 2)}{(\beta + 2)}, \dfrac{(\gamma - 2)}{(\gamma + 2)}$.

8. If θ is a root of $x^3 + x^2 - 2x - 1 = 0$, then prove that $\theta^2 - 2$ is also a root. (*M.U, B.E., 1993*)

9. If α, β, and γ are the roots of $x^3 + 2x^2 + 3x + 3 = 0$, prove that

$$\frac{\alpha^2}{(\alpha + 1)^2} + \frac{\beta^2}{(\beta + 1)^2} + \frac{\gamma^2}{(\gamma + 1)^2} = 13.$$

10. Find the equation whose roots are –3 times those of $x^4 - 3x^3 + x^2 - 6x + 4 = 0$.

11. Find the equation whose roots are with opposite signs to those of $x^5 - 4x^4 + 3x^3 - 5x^2 + x - 11 = 0$.

12. Find the equation whose roots are reciprocal of the roots of $x^5 - 11x^4 + 7x^3 - 8x^2 + 6x - 13 = 0$.

13. Diminish by 3 the roots of $x^4 + 3x^3 - 2x^2 - 4x - 3 = 0$.

14. Diminish the equation $x^4 - 8x^3 + 19x^2 - 12x + 2 = 0$ by 2 and hence solve it. *(M.U., B.E., 1996)*

15. Increase the roots of $3x^4 + 2x^3 - 10x^2 + 15x - 9 = 0$ by 4.

16. Find the equation whose roots are the roots of the eqution $x^3 - 4x^2 - 3x - 2 = 0$ increased by 2. *(M.U., B.E., 1995)*

17. Remove the second term in $x^4 - 8x^3 - x^2 + 68x + 60 = 0$ and solve it.

18. Diminish the roots of the equation $x^4 - 4x^3 - 7x^2 + 22x + 24 = 0$ by 1 and solve it.

19. Increase the roots of the equation $x^4 - 2x^3 - 10x^2 + 6x + 21 = 0$ by 2 and solve it.

20. Solve $x^3 - 4x^2 + 5x - 2 = 0$, given that it has a double root.

ANSWERS

1. (i) $-3q$ (ii) $3q$
   (iii) $2p^2$

2. (i) $-(q/r)$ (ii) $pq - 3r - p^3$
   (iii) $p^2q - 2q^2 - pr$ (iv) $2p^2 - 3q$
   (v) $24r - p^3$

3. (i) $y^3 + (2q - p^2)y^2 + (q^2 - 2pr)y - r^2 = 0$
   (ii) $y^3 - qy^2 + pry - r^2 = 0$
   (iii) $y^3 - 2qy^2 + (pr + q^2)y + (r^2 - prq) = 0$
   (iv) $y^3 + 2py^2 + (p^2 + q)y + pq - r = 0$

4. (i) $y^2 - 2py + 4q = 0$ (ii) $y^3 - py^2 - q^2 = 0$

5. $y^3 - 42y^2 + 441y - 400 = 0$ 6. $3y^3 - 11y^2 + 9y - 2 = 0$

7. $y^3 + 33y^2 + 27y + 3 = 0$

10. $y^4 + 9y^3 + 9y^2 + 162y + 324 = 0$

11. $y^5 + 4y^4 + 3y^3 + 5y^2 + y + 11 = 0$

12. $13y^5 - 6y^4 + 8y^3 - 7y^2 + 11y - 1 = 0$

13. $y^4 + 15y^3 + 79y^2 + 173y + 129 = 0$

14. $y^4 + 5y^2 + 6 = 0, 2 \pm \sqrt{3}, 2 \pm \sqrt{2}$

15. $3y^4 - 46y^3 + 254y^2 - 577y + 411 = 0$

16. $y^3 - 10y^2 + 31y - 32 = 0$

17. $-1, -2, 5, 6$ 18. [illegible]

19. $1 \pm 2\sqrt{2}, \pm\sqrt{3}$ 20. $1, 1, 2$

EXERCISE 2.3

1. Solve $x^5 + 4x^4 + x^3 + x^2 + 4x + 1 = 0$
2. Solve $x^5 + x^4 + x^3 + x^2 + x + 1 = 0$
3. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$ *(M.U, B.E., 1987, 1990, 1996)*
4. Solve $2x^6 - 9x^5 + 10x^4 - 3x^3 + 10x^2 - 9x + 2 = 0$
5. Solve $x^4 - 2x^3 + 3x^2 - 2x + 1 = 0$ *(M.U, B.E., 1986)*
6. Solve $2x^4 + x^3 - 6x^2 + x + 2 = 0$ *(M.U, B.E., 1986)*
7. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$
8. Solve $x^4 + 6x^3 - 5x^2 + 6x + 1 = 0$ *(M.U, B.E., 1988)*
9. Solve $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$ *(M.U, B.E., 1991)*
10. Solve $x^5 - x^4 + x^3 - x^2 + x - 1 = 0$
11. Solve $x^6 + 2x^5 + 2x^4 - 2x^2 - 2x - 1 = 0$ *(M.U, B.E., 1994)*
12. Solve $6x^6 - 35x^5 + 56x^4 - 56x^2 + 35x - 6 = 0$ *(M.U, B.E., 1986)*
13. Solve $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$
14. Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ transforms into a reciprocal equation by diminishing the root by 1. Hence solve it. *(M.U, B.E., 1990)*
15. Show that $x^4 - 10x^3 + 23x^2 - 6x - 15 = 0$ can be transformed into a reciprocal equation by diminishing the roots by 2. Hence solve it. *(M.U, B.E., 1993, Coimbatore B.E., 1988)*

ANSWERS

1. $-1, \dfrac{1\pm\sqrt{3}i}{2}, -2\pm\sqrt{3}i$

2. $-1, \dfrac{-1\pm\sqrt{3}i}{2}, \dfrac{-1\pm\sqrt{3}i}{2}$

3. $-1, 2, 1/2, -3, -1/3$

4. $2, 1/2, \dfrac{3\pm\sqrt{5}}{2}, \dfrac{-1\pm\sqrt{3}i}{2}$

5. $\dfrac{1\pm\sqrt{3}i}{2}, \dfrac{1\pm\sqrt{3}i}{2}$

6. $1, 1, -2, -1/2$

7. $2\pm\sqrt{3}, 3\pm 3\sqrt{2}$

8. $\dfrac{-7\pm 3\sqrt{5}}{2}, \dfrac{1\pm\sqrt{3}i}{2}$

9. $1, 2, 1/2, -3, -1/3$

10. $1, \dfrac{1\pm\sqrt{3}i}{2}, \dfrac{-1\pm\sqrt{3}i}{2}$

11. $\pm 1, \dfrac{-1\pm\sqrt{3}i}{2}, \dfrac{-1\pm\sqrt{3}i}{2}$

12. $\pm 1, 2, 1/2, 3, 1/3$

13. $\pm 1, -3, -1/3, \dfrac{3\pm\sqrt{5}}{2}$

14. $\dfrac{\sqrt{5}+3\pm\sqrt{-10+2\sqrt{5}}}{4}, \dfrac{-\sqrt{5}+3\pm\sqrt{-10+2\sqrt{5}}}{4}$

15. $\dfrac{9\pm\sqrt{21}}{2}, \dfrac{1\pm\sqrt{5}}{2}$

EXERCISE 3.1

1. Find a root of the following equations correct to three decimal places, using the Bisection method.

   (i) $x^3 - x^2 + x - 7 = 0$

   (ii) $x^3 - 2x - 5 = 0$

   (iii) $x^3 - 3x - 5 = 0$ (*Bangalore, B.E., 1989*)

   (iv) $x^3 - 4x - 9 = 0$ (*Mysore, B.E., 1987*)

   (v) $x^4 - x - 10 = 0$ (*S. Gujarat B.E., 1990*)

   (vi) $x - \cos x = 0$ (*B.U, B.E., 1995*)

   (vii) $3x - e^x = 0$

   (viii) $3x = \sqrt{(1 + \sin x)}$

   (ix) $x \log_{10} x - 1.2 = 0$

2. Using Bisection method find the negative root of $x^3 - 4x + 9 = 0$, correct to three decimal places.

3. Find a root of the following equations correct to three decimal places, using Iteration method.

(i) $x^3 + x^2 - 100 = 0$ (ii) $x = \frac{1}{2} + \sin x$

(iii) $3x - 6 = \log_{10} x$ (iv) $xe^x - \cos x = 0$

(v) $\sin x = e^x - 3x$ (vi) $2x - 7 - \log_{10} x = 0$

(vii) $1 - x + \dfrac{x^2}{(2!)^2} - \dfrac{x^3}{(3!)^2} + \dfrac{x^4}{(4!)^2} - \dfrac{x^5}{(5!)^2} + \ldots = 0$

4. Find a negative root of $x^3 - 2x + 5 = 0$, correct to three decimal places, using Successive Approximation method.

5. Find a root of the following equations correct to four decimal places using the method of False Position (*regula false* method).

(i) $x^3 - 4x - 9 = 0$ (ii) $x^3 + 2x^2 + 10x - 20 = 0$

(iii) $x^3 - 4x - 1 = 0$ (iv) $x^6 - x^4 - x^3 - 1 = 0$

(v) $xe^x = 2$ (vi) $e^x \sin x = 1$

(vii) $x = \cos x$ (viii) $x \tan x = -1$ in (2.5, 3)

(ix) $x \log_{10} x = 1.2$

ANSWERS

1. (i) 2.105 (ii) 2.095 (iii) 2.280 (iv) 2.706 (v) 1.813
(vi) 0.739 (vii) 0.619 (viii) 0.392 (ix) 2.740

2. – 2.706

3. (i) 4.331 (ii) 1.497 (iii) 2.108 (iv) 0.518 (v) 0.360
(vi) 3.789 (vii) 1.445

4. – 2.095

5. (i) 2.7065 (ii) 1.3688 (iii) 0.2541 (iv) [illegible] (v) [illegible]8526
(vi) 0.5885 (vii) 0.7391 (viii) 2.7981 (ix) 2.7406

EXERCISE 3.2

Using Newton–Raphson method, find a root correct to three decimal places of the following :

1. $x^3 - 3x^2 + 7x - 8 = 0$ (*M.U, B.E., 1992*)
2. $x^3 - 3x - 5 = 0$ (*Kerala B.Tech 1989*)
3. $x^3 - 5x + 3 = 0$ (*Gulbarga B.E, 1993*)
4. $x^4 - x - 10 = 0$
5. $x^4 - x - 13 = 0$
6. $e^x = 1 + 2x$
7. $xe^x - \cos x = 0$ (*Gujarat B.E., 1990; B.U, B.E., 1995*)
8. $e^x \sin x = 1$
9. $x^x = 1000$
10. $3x - 1 = \cos x$ (*B.R, B.E., 1993*)
11. $\sin x = 1 - x$
12. $x^2 + 4 \sin x = 0$
13. $2x \tan x = 1$
14. $x (1 - \log_e x) = 0.5$ (*M.U, B.E., 1987*)
15. $3x - e^x + \sin x = 0$
16. $x \sin x + \cos x = 0$ near $x = \pi$ (*Karnataka B.E., 1993*)

17. Find the interative formulae for finding $1/\sqrt{N}$, $\sqrt[3]{N}$, $\sqrt[4]{N}$ where N is a positive real number, using Newton's method. Hence evaluate $1/\sqrt{17}$, $\sqrt[3]{10}$, $\sqrt[4]{25}$.

18. Find a negative root of the following equations using Newton's method.

(i) $x^3 - x^2 + x + 100 = 0$ (ii) $x^3 - 21x + 3500 = 0$

19. Find by Horner's method the root of the following equations correct to three decimal places.

(i) $x^3 + 3x^2 - 12x - 11 = 0$ (ii) $x^3 + x^2 + x - 100 = 0$

(iii) $x^3 - 6x - 13 = 0$ (iv) $x^3 - 3x + 1 = 0$

(v) $x^3 - 30 = 0$ (vi) $x^4 + x^3 - 4x^2 - 16 = 0$

20. A sphere of pine wood, 2 metres in diameter, floating in water sinks to the depth of h metre, given by the equation $h^3 - 3h^2 + 2.5 = 0$. Find h correct to two decimal places using Horner's method.

21. Find a negative root of $x^3 - 2x + 5 = 0$ correct to two decimal places using Horner's method.

22. Find all the roots of the following equations by Graeffe's method squaring thrice.

(i) $x^3 - 4x^2 + 5x - 2 = 0$ (ii) $x^3 - 2x^2 - 5x + 6 = 0$

(iii) $x^3 - 5x^2 - 17x + 20 = 0$ *(M.U, B.E., 1991)*

(iv) $x^3 - 9x^2 + 18x - 6 = 0$

(v) $x^3 - x - 1 = 0$

ANSWERS

1. 1.674
2. 2.279
3. 1.834
4. 1.856
5. 1.961
6. 1.256
7. 0.518
8. 0.589
9. 3.592
10. 0.607
11. 0.511
12. –1.934
13. 0.653
14. 0.187
15. 0.360
16. 2.798
17. 0.24246, 2.15466, 2.236
18. (i) –4.264 (ii) –16.56
19. (i) 2.769 (ii) 4.264 (iii) 3.177 (iv) 1.532 (v) 3.107 (vi) 2.231
20. 1.17
21. – 2.094
22. (i) 2,1,1 (ii) 3, –2, 1 (iii) 7.018, – 2.974, 0.958 (iv) 6.3, 2.3, 0.4 (v) 1.3247, – 0.6624, ± 0.5622i

EXERCISE 4.1

Solve the following equations by Gauss elimination method

1. $3x + 4y - z = 8,\ -2x + y + z = 3,\ x + 2y - z = 2$
2. $x - y + z = 1,\ -3x + 2y - 3z = -6,\ 2x - 5y + 4z = 5$
3. $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$

   (M.U, B.E., 1991)
4. $2x - y + 2z = 2,\ x + 10y - 3z = 5,\ x - y - z = 3$

   (Ranchi, B. Tech, 1987)
5. $10x_1 + x_2 + x_3 = 18.141, x_1 + x_2 + 10x_3 = 38.139, x_1 + 10x_2 + x_3 = 28.140$

   (M.U, B.E., 1991)
6. $x + y + z = 6.6,\ x - y + z = 2.2,\ x + 2y + 3z = 15.2$

   (North Bengal B Tech, 1987)
7. $2x + 4y + 2z = 15,\ 2x + y + 2z = -5,\ 4x + y - 2z = 0$

   (M.U, B.E., 1989)

8. $2x + y + z = 10,\ 3x + 2y + 3z = 18,\ x + 4y + 9z = 16$

(*Bangalore, B.E., 1990*)

9. $2x_1 + 2x_2 + x_3 = 12,\ 3x_1 + 2x_2 + 2x_3 = 8,\ 5x_1 + 10x_2 - 8x_3 = 10$

(*S.Gujarat, B.E., 1990*)

10. $x + 2y - 12z + 8w = 27,\ 5x + 4y + 7z - 2w = 4,$
$-3x + 7y + 9z + 5w = 11,\ 6x - 12y - 8z + 3w = 49$

(*M.U, B.E., 1987*)

Solve the following equations by Gauss – Jordan method

11. $2x - 3y + z = -1,\ x + 4y + 5z = 25,\ 3x - 4y + z = 2$

(*M.U, B.E., 1993*)

12. $2x + y + z = 12,\ 3x + 2y + 3z = 24,\ x + 4y + 9z = 34$

13. $10x + y + z = 12,\ x + 10y + z = 12,\ x + y + 10z = 12$

(*Bhopal B.E., 1991*)

14. $x + 2y + z = 8,\ 2x + 3y + 4z = 20,\ 4x + 3y + 2z = 16$

(*Punjab, B.E., 1987*)

15. $4x - y - z = -7,\ x - 5y + z = -10,\ x + 2y + 6z = 9$

16. $2x + 2y - z + t = 4,\ 4x + 3y - z + 2t = 6,$
$8x + 5y - 3z + 4t = 12,\ 3x + 3y - 2z + 2t = 6$

17. $5x_1 + x_2 + x_3 + x_4 = 4\ ;\ x_1 + 7x_2 + x_3 + 4x_4 = 2$
$x_1 + x_2 + 6x_3 + x_4 = -5\ ;\ x_1 + x_2 + x_3 + x_4 = -6$

Find the inverse of the following matrices using Gauss elimination method.

18. $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ 19. $\begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ 20. $\begin{bmatrix} 3 & -1 & 10 & 2 \\ 5 & 1 & 20 & 3 \\ 9 & 7 & 39 & 4 \\ 1 & -2 & 2 & 1 \end{bmatrix}$

ANSWERS

1.	1, 2, 3	2.	– 2, 3, 6
3.	1, 1, 1	4.	2, 0, – 1
5	1.234, 2.348, 3.455	6.	[illegible], 2.2, 3.2
7.	– 3.0556, 6.6667, – 2.778	8.	7, [illegible] 9, 5

EXERCISE 4.2

Solve the following equations by Factorisation (or Triangularisation) method.

1. $3x + y + 2z = 16;\ 2x - 6y + 8z = 24;\ 5x + 4y - 3z = 2$
2. $3x + 2y + 7z = 32;\ 2x + 3y + z = 40;\ 3x + 4y - z = 56$
3. $10x + y + z = 12;\ 2x + 10y + z = 13;\ x + y + 5z = 7$

   (M.U, B.E., 1991)

4. $28x + 4y - z = 32;\ x + 3y + 10z = 24;\ 2x + 17y + 4z = 35$
5. $2x - y + z = 0.3; -4x + 3y - 2z = -1.4;\ 3x - 8y + 3z = 0.1$
6. $10x + 7y + 8z + 7w = 32;\ 7x + 5y + 6z + 5w = 23,$
   $8x + 6y + 10z + 9w = 33\ ;\ 7x + 5y + 9z + 10w = 31$

Solve the following equations by Crout's method

7. $x + 3y + 8z = 4, x + 4y + 3z = -2;\ x + 3y + 4z = 1$ *(M.U, B.E., 1991)*
8. $2x - 6y + 8z = 24, 5x + 4y - 3z = 2;\ 3x + y + 2z = 16$*(M.U, B.E., 1993)*
9. $10x + y + 2z = 13; 3x + 10y + z = 14, 2x + 3y + 10z = 15$

   (Madurai, B.E., 1987)

10. $9x - 2y + z = 50,\ x + 5y - 3z = 18,\ -2x + 2y + 7z = 19$
11. $10x + y + z = 12,\ 2x + 10y + z = 13,\ 2x + 2y + 10z = 14$

   (Coimbatore, B.Tech., 1988)

12. $3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x - 2y + z = 4$

13. $10x_1 + 9x_2 + 6x_3 + x_4 = 26,\ 11x_1 + 6x_2 - x_3 + 2x_4 = 18$
$x_1 - 7x_2 + 3x_3 + 6x_4 = 3,\ 7x_1 + x_2 + x_3 + x_4 = 10$

14. $5x + y + z + w = 4,\ x + 7y + z + 4w = 12,$
$x + y + 6z + w = -5,\ x + y + z + 4w = -6$

Find the inverse of the following matrices using Crout's method.

(15) $\begin{bmatrix} -2 & 4 & 8 \\ -4 & 18 & -16 \\ -6 & 2 & -20 \end{bmatrix}$

(16) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

(17) $\begin{bmatrix} 13 & 14 & 6 & 4 \\ 8 & -1 & 13 & 9 \\ 6 & 7 & 3 & 2 \\ 9 & 5 & 16 & 11 \end{bmatrix}$

ANSWERS

(1) 1, 3, 5

(2) 7, 9, – 1

(3) 1, 1, 1

(4) 0.996, 1.5070, 1.8485

(5) 1.6, – 0.8, – 3.71

(6) 1, 1, 1, 1

(7) $\dfrac{19}{4}, -\dfrac{9}{4}, \dfrac{3}{4}$

(8) 1, 3, 5

(9) 1, 1, 1

(10) 6.13, 4.31, 3.23

(11) 1, 1, 1

(12) 1, 2, – 1

(13) 1, 1, 1, 1

(14) 1, 2, – 1, – 2

(15) $\dfrac{1}{190}\begin{bmatrix} -41 & 12 & -26 \\ 2 & 11 & -8 \\ 12.5 & -2.5 & -2.5 \end{bmatrix}$

(16) $\dfrac{1}{12}\begin{bmatrix} -5 & 3 & 4 \\ -7 & 3 & -8 \\ 1 & -3 & 4 \end{bmatrix}$

(17) $\begin{bmatrix} 1 & 0 & -2 & 0 \\ -5 & 1 & 11 & -1 \\ 287 & -67 & -630 & 65 \\ -416 & 97 & 913 & -94 \end{bmatrix}$

EXERCISE 4.3

Solve the following system of linear equations by (i) Gauss and (ii) Gauss Seidel iteration method.

1. $2x + y + z = 4, x + 2y + z = 4; x + y + 2z = 4$
2. $8x + y + z = 8; 2x + 4y + z = 4; x + 3y + 5z = 5$
3. $5x + 2y + z = 12, x + 4y + 2z = 15, x + 2y + 5z = 20$

4. $9x + 2y + 4z = 20, x + 10y + 4z = 6, 2x - 4y + 10z = -15$
5. $54x + y + z = 110, 2x + 15y + 6z = 72, -x + 6y + 27z = 85$ *(M.U, B.E., 1993)*
6. $28x - 4y - z = 32, x + 3y + 10z = 24, 2x + 17y + 4z = 35$
7. $5x - y + z = 10, 2x + 4y = 12, x + y + 5z = -1$ *(Bangalore, B.E., 1990)*
8. $10x_1 - 5x_2 - 2x_3 = 3, 4x_1 - 10x_2 + 3x_3 = -3, x_1 + 6x_2 + 10x_3 = -3$
9. $10x + 2y + z = 9, 2x + 20y - 2z = -44, -2x + 3y + 10z = 22$
10. $10x_1 + 7x_2 + 8x_3 + 7x_4 = 32, 7x_1 + 5x_2 + 6x_3 + 5x_4 = 23;$
    $8x_1 + 6x_2 + 10x_3 + 9x_4 = 33, 7x_1 + 5x_2 + 9x_3 + 10x_4 = 31$

Solve by relaxation method the following equations:

11. $9x + 2y + z = 50, x + 5y - 3z = 18, -2x + 2y + 7z = 19$
12. $3x + 9y - 2z = 11, 4x + 2y + 13z = 24, 4x - 4y + 3z = -8$ *(M.U, B.E., 1993)*
13. $4.215x - 1.212y + 1.105z = 3.216$
    $-2.120x + 3.505y - 1.632z = 1.247$
    $1.122x - 1.313y + 3.986z = 2.112$
14. $10x - 2y - 3z = 305, -2x + 10y - 2z = 154, -2x - y + 10z = 120$
15. $8x_1 + x_2 + x_3 + x_4 = 14; 2x_1 + 10x_2 + 3x_3 + x_4 = -8$
    $x_1 - 2x_2 - 20x_3 + 3x_4 = 111, \; 3x_1 + 2x_2 + 2x_3 + 19x_4 = 53$

ANSWERS

1. $x = 1, y = 1, z = 1$
2. $x = 0.876, y = 0.919, z = 0.574$
3. $x = 0.996, y = 1.95, z = 3.16$
4. $x = 2.733, y = 0.986, z = -1.652$
5. $x = 1.926, y = 3.573, z = 2.425$
6. $x = 0.994, y = 1.507, z = 1.849$
7. $x = 2.556, y = 1.722, z = -1.055$
8. $x_1 = 0.342, x_2 = 0.285, x_3 = -0.505$
9. $x = 1.013, y = -1.996, z = 3.001$
10. $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1$
11. $x = 6.13, y = 4.31, z = 3.23$
12. $x = 1.35, y = 2.103, z = 2.845$
13. $x = 0.943, y = 1.239, z = 0.673$
14. $x = 32, y = 26, z = 21$
15. $x_1 = 2, x_2 = 0, x_3 = -5, x_4 = 3$

EXERCISE 5.1

1. Tabulate the forward differences for the given data:

x	1	2	3	4	5	6	7	8	9
y	1	8	27	64	125	216	343	512	729

2. Form a table of backward differences of the function

$$f(x) = x^3 - 3x^2 - 5x - 7 \text{ for } x = -1, 0, 1, 2, 3, 4, 5.$$

3. Form the difference table of $f_x = x^4 - 5x^3 + 6x^2 + x - 2$ for the values of $x = -3, -2, -1, 0, 1, 2, 3$. Extend the table in both directions to give f_{-4}, f_{-5}, f_4, f_5.

4. Show that

(i) $y_3 = y_2 + \Delta y_1 + \Delta^2 y_0 + \Delta^3 y_0$

(ii) $\nabla^2 y_8 = y_8 - 2y_7 + y_6$ (iii) $\delta^2 y_5 = y_6 - 2y_5 + y_4$

5. If $y_0 = 3$, $y_1 = 12$, $y_2 = 81$, $y_3 = 2000$, $y_4 = 100$ show that $\Delta^4 y_0 = -7459$.

6. If the interval of differencing is unity, prove that

(i) $\Delta \sin x = 2\sin\frac{1}{2} \cos(x + \frac{1}{2})$

(ii) $\Delta f(x) = \dfrac{-\Delta f(x)}{f(x) f(x+1)}$

(iii) $\Delta \tan^{-1}\left(\dfrac{n-1}{n}\right) = \tan^{-1} \dfrac{1}{2n^2}$

(iv) $\Delta \dfrac{2^x}{x!} = \dfrac{2^x(1-x)}{(x+1)!}$

(v) $\Delta[x(x+1)(x+2)(x+3)] = 4(x+1)(x+2)(x+3)$

(vi) $\Delta^2\left[\dfrac{5x+12}{x^2+5x+6}\right] = \dfrac{10x+32}{(x+2)(x+3)(x+4)(x+5)}$

(vii) $\Delta^n e^x = (e-1)^n e^x$

(viii) $\Delta^n(1/x) = \dfrac{(-1)^n n!}{x(x+1)x+2)\ldots(x+n)}$

7. If h is the interval of differencing, prove that

(i) $\Delta^2\cos 2x = -4\sin^2 h\cos 2(x+h)$ (*Kerala B.E., 1989, M.U, B.E., 1996*)

(ii) $\Delta^3 a^{cx+d} = (a^{ch}-1)^3 a^{cx+d}$

(iii) $\Delta^n\sin(ax+b) = 2\sin(ah/2)^n \sin\left(ax+b+\dfrac{nah+n\pi}{2}\right)$

8. Show that

(i) $\Delta^3[(1-x)(1-2x)(1-3x)] = -36$ if $h = 1$.

(ii) $\Delta^{10}[(1-x)(1-2x^2)(1-3x^3)(1-4x^4)] = 24 \times 2^{10} \times 10!$ if $h = 2$.

9. Find the seventh term of the sequence 2, 9, 28, 65, 126, . . . and also find the general term.

10. Evaluate $\Delta^2 f(x)$ if $f(x)$ is

(i) $\dfrac{1}{x(x+4)x+8)}$ (ii) $\dfrac{1}{(3x+1)(3x+4)(3x+7)}$

11. Find $\Delta^3 f(x)$ if $f(x)$ is $(3x+1)(3x+4)(3x+7)\ldots(3x+19)$

12. Express the following in factorial notation.

(i) $f(x) = 2x^3 - 3x^2 + 3x - 10$

(ii) $f(x) = x^3 - 2x^2 + x - 1$

(iii) $f(x) = 3x^4 - 4x^3 + 6x^2 + 2x + 1$

(iv) $f(x) = x^4 - 3x^3 - 5x^2 + 6x - 7$ and get their successive forward differences.

13. Obtain the function whose first difference is $x^3 + 3x^2 + 5x + 12$.

14. Express the following in factorial notation taking $h = 2$ and find their differences of second order.

(i) $f(x) = 7x^4 + 12x^3 - 6x^2 + 5x - 3$

(ii) $f(x) = x^3 - 3x^2 + 5x + 7$

15. Prepare a forward difference table for values (x_i, y_i), i = 1, 2, 3, . . 7. Indicate the propogation error ε introduced in the tabulated value of y_4.

16. The value of a polynomial of degree 5 are tabulated below. If $f(3)$ is known to be an error, find its correct value.

x	0	1	2	3	4	5	6
$f(x)$	1	2	33	254	1025	3126	7777

17. A polynomial function is given by the following table:

x	0	1	2	3	4	5	6
$f(x)$	0	3	14	39	84	155	258

Form a difference table and explain how the correctness of the arithmetic may be checked.

18. Find y_6 if $y_0 = 9, y_1 = 18, y_2 = 20, y_3 = 24$ and the third differences are constant.

19. Assuming that the following values of y_x belong to a polynomial of degre 4, compute the next three values.

x	0	1	2	3	4	5	6	7
$f(x)$	1	-1	1	-1	1	–	–	–

20. Find the missing term in the following table:

x	1	2	3	4	5	6	7
$f(x)$	2	4	8	–	32	64	128

21. Find and correct a single error in y in the following table:

x	0	1	2	3	4	5	6	7
$f(x)$	0	0	1	6	24	60	120	210

22. With the usual notations prove that

(i) $E\Delta = \Delta E$ (ii) $E\nabla = \nabla E = \Delta$

(iii) $E = (\Delta / \delta)^2$ (*M.U., B.E., 1996*) (iv) $\nabla = 1 - (1 + \nabla)^{-1}$

(v) $\Delta = \delta E^{1/2}$; $\nabla = \delta E^{-1/2}$ (vi) $(1 + \Delta)(1 - \nabla) = 1$

(vii) $\Delta^2 = (1+\Delta)\delta^2$ (viii) $\mu\,\delta = \frac{1}{2}\Delta E^{-1} + \frac{1}{2}\Delta$

(ix) $E^{1/2} = \mu + \frac{1}{2}\delta,\ E^{-1/2} = \mu - \frac{1}{2}\delta$ (x) $\delta = \Delta(1+\Delta)^{-1/2} = \nabla(1-\nabla)^{-1/2}$

(xi) $\mu\delta = \frac{1}{2}(\Delta + \nabla)$ (xii) $\mu = \dfrac{2+\Delta}{2\sqrt{1+\Delta}}$

(xiii) $\dfrac{\Delta^2}{E^2} = E^{-2} - 2E^{-1} + 1$ (xiv) $\mu^2 = 1 + \dfrac{1}{4}\delta^2$

(*Coimbatore B.E., 1985*)

(xv) $E = \sum_{i=0}^{\infty} \nabla_i$

(xvi) $\nabla^2 = h^2D^2 - h^3D^3 + \dfrac{7}{12}h^4D^4 + \cdots$ (*Madurai, B.E. 1989*)

(xvii) $\dfrac{\Delta}{\nabla} - \dfrac{\nabla}{\Delta} = E - E^{-1}$ (*M.U, B.E. 1997*)

(xviii) $(1+\Delta)(1-\nabla) = 1$ (*M.U. B.E. 1997*)

23) Show the following

(i) $\nabla^3 y_2 = \nabla^3 y_5$ (ii) $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

(iii) $(\Delta + \nabla)^2 (x^2 + x) = 8$ (iv) $(\Delta^2 E^{-1})\, x^3 = 6x$

(v) $\dfrac{\Delta^2}{E} \sin(x+h) + \dfrac{\Delta^2 \sin(x+h)}{E\sin(x+h)} = 2(\cosh - 1)\,[\sin(x+h) + 1]$

(vi) $\Delta f_k^2 = (f_k + f_{k+1})\,\Delta f_k$

(vii) $\dfrac{\Delta^2 x^2}{E(x + \log x)} = \dfrac{2}{x + 1 + \log(x+1)}$

24) Use the method of separation of symbols to prove that

$(u_1 - u_0) - x\,(u_2 - u_1) + x^2\,(u_3 - u_2) - \cdots$

$$= \frac{\Delta u_0}{1+x} - x\,\frac{\Delta^2 u_0}{(1+x)^2} + x^2\,\frac{\Delta^3 u_0}{(1+x)^3} - \cdots$$

25) $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^n u$

26) $y_x = y_n - {}^{n-x}C_1\,\Delta y_{n-1} + {}^{n-x}C_2\,\Delta^2 y_{n-2} - \cdots + (-1) \cdots \Delta \cdots y_x$

27) $u_0 + \dfrac{u_1 x}{1!} + \dfrac{u_2 x^2}{2!} + \cdots = e^x \left(u_0 + x\,\Delta u_0 + \dfrac{x^2}{2!}\,\Delta^2 u_0 + \cdots\right)$

28) $u_0 + {}^xC_1\,\Delta u_1 + {}^xC_2\,\Delta^2 u_2 + \cdots = u_x + {}^xC_1\,\Delta^2 u_{x-1} + {}^xC_2\,\Delta^4 u_{x-2} + \cdots$

29) Sum the series to n terms:

(i) $1.2.3 + 2.3.4 + 3.4.5 + \cdots$

(ii) $4.5.6. + 5.6.7 + 6.7.8 + \cdots$

(iii) $2.5 + 5.8 + 8.11 + \cdots$

30) Using the method of finite differences, find the sum to n terms of the series whose nth term is $n(n-1)(n-2)$.

31) Using the method of finite differences, find the sum of the first

(i) n squares and (ii) n cubes.

32) Sum the series using the identity of Example 5.15.

(i) $5 + \dfrac{4x}{1!} + \dfrac{5x^2}{2!} + \dfrac{14x^3}{3!} + \dfrac{37x^4}{4!} + \cdots$

(ii) $1 + \dfrac{4x}{1!} + \dfrac{10x^2}{2!} + \dfrac{20x^3}{3!} + \dfrac{35x^4}{4!} + \cdots$

33) Using Montmort's theorem, sum the series

$1.3 + 3.5x + 5.7x^2 + 7.9x^3 + \cdots$

ANSWERS

9) $344,\ (n+1)^3 + 1$

10) (i) $\Delta^2 f(x) = \dfrac{192}{x(x+4)(x+8)(x+12)(x+16)}$

(ii) $\Delta^2 f(x) = \dfrac{108}{(3x+1)(3x+4)(3x+7)(3x+10)(3x+13)}$

11) $\Delta^3 f(x) = 459270\,(3x+19)(3x+16)(3x+13)(3x+10)$

12) (i) $f(x) = 2x^{(3)} + 3x^{(2)} + 2x^{(1)} - 10$

(ii) $f(x) = x^{(3)} + x^{(2)} - 1;$

(iii) $f(x) = 3x^{(4)} + 14x^{(3)} + 15x^{(2)} + 7x^{(1)} + 1;$

(iv) $f(x) = x^{(4)} + 9x^{(3)} + 11x^{(2)} + 5x^{(1)} - 7$

13) $f(x) = \frac{1}{4}\ x^{(4)} + 2x^{(3)} + \frac{9}{2}x^{(2)} + 12x^{(1)}$ + constant

14) (i) $f(x) \quad = 7x^{(4)} + 96x^{(3)} + 262x^{(2)} + 97x^{(1)} - 3$

$\Delta f(x) = 56x^{(3)} + 576x^{(2)} + 1048x^{(1)} + 194$

$\Delta^2 f(x) = 336x^{(2)} + 2304x^{(1)} + 2096$

(ii) $f(x) \quad = x^{(3)} + 3x^{(2)} + 3x^{(1)} + 7$

$\Delta f(x) = 6x^{(2)} + 12x^{(1)} + 6$

$\Delta^2 f(x) = 24x^{(1)} + 24$

15) $f(x) = 244$, error $= -10$

18) $y_6 = 138$

19) 31, 129, 351

20) 16.1

21) Error is at $x = 2$; $y(2) = 0$

29) (i) $\frac{1}{4}n(n+1)\ (n+2)\ (n+4)$

(ii) $\frac{1}{4}[\ (n+6)\ (n+5)\ (n+4)\ (n+3) - 360]$

(iii) $n\ (3n^2 + 6n + 1)$

30) $\frac{1}{4}(n+1)\ (n)(n-1)\ (n-2)$

31) (i) $\frac{n(n+1)(2n+1)}{6}$ (ii) $\frac{n^2\ (n+1)^2}{4}$

32) (i) $e^x\ (x^3 + x^2 - x + 5)$ (ii) $e^x\left(1+3x+\frac{3x^2}{2}+\frac{x^3}{6}\right)$

33) $\frac{3+6x-x^2}{(1-x)^3}$

EXERCISE 6.1

1. From the following data find y at $x = 43$ using Newton's forward – interpolation formula.

x	40	50	60	70	80	90
y	184	204	226	250	176	304

2. The population of a town in decennial census was as given below. Estimate the population for the year 1895.

Years (x)	1891	1901	1911	1921	1931
Population (y) in thousands	46	66	81	93	101

3. Using Newton's forward interpolation formula find the value of $f(1.6)$ if

x	1	1.4	1.8	2.2
y	3.49	4.82	5.96	6.5

(*Bangalore, B.E., 1989*)

4. The following data gives the melting point of an alloy of lead and zinc, where t°C is the temperature and p is the percentage of lead in the alloy.

p	40	50	60	70	80	90
t	184	204	226	250	276	304

Using Newton's backward interpolation formula, find the melting point of the alloy containing 84% of lead.

5. The area A of a circle of diameter d is given for the following values

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

Calculate the area of a circle of diameter 105.

6. The following are the numbers of deaths in four successive ten-year age groups. Find the number of deaths at 45–50 and 50–55 age groups.

Age group	25–35	35–45	45–55	55–65
Death	13229	18139	24225	31496

7. From the following table, find y when $x = 1.85$ and $x = 2.4$ using Newton's interpolation formula.

x	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$y = e^x$	5.474	6.050	6.686	7.389	8.166	9.025	9.974

8. Estimate the values of $f(22)$ and $f(42)$ from the following data :

x	20	25	30	35	40	45
$f(x)$	354	332	291	260	231	204

(*Gulbarga B.E., 1993*)

9. Find a polynomial which takes the following values :

x	4	6	8	10
y	1	3	8	16

Hence calculate y at $x = 5$. (*M.U, B.E., 1989*)

10. Using Newton's backward interpolation formula, find the polynomial of degree four passing through (1, 1), (2, – 1), (3, 1), (4, – 1) and (5, 1). (*Karnataka, B.E., 1989*)

11. Obtain the estimate of the missing figure in the following table :

x	1	2	3	4	5
y	2	5	7	–	32

12. Interpolate the missing values in the following table of rice cultivation:

Year x	1911	1912	1913	1914	1915	1916	1917	1918	1919
Acres y (in millions)	76.6	78.2	–	77.7	78.7	–	80.6	77.6	78.6

13. Given $y_0 = 3, y_1 = 12, y_2 = 81, y_3 = 200, y_4 = 100$. Find $\Delta^4 y_0$ without forming the difference table. (*M.U, B.E., 1989*)

14. If $u_{-1} = 10$, $u_1 = 8$, $u_2 = 10$, $u_4 = 50$, find u_0 and u_3. (*M.U, B.E., 1993*)

15. If y_x is the value of y at x for which the [illegible] differences are constant and $y_1 + y_7 = -784$, $y_2 + y_6 = 686$, $y_3 + y_5$ [illegible] 1088, find y_4.

16. Determine the maximum step size that can be used in the tabulation of $f(x) = e^x$ in [0, 1], so that the error in linear interpolation be less than 5×10^{-4}.

17. Given $\sin 25° = 0.42262$, $\sin 26° = 0.43837$, $\sin 27° = 0.45399$, $\sin 28° = 0.46947$, $\sin 29° = 0.48481$ and $\sin 30° = 0.5$. Using Newton's interpolation formula find $\sin 28° 24'$. Estimate the error.

ANSWERS

1. 189.70
2. 54.8528
3. 5.54
4. 287 (nearly)
5. 8666
6. 11278, 12947
7. 6.36, 11.02
8. 352, 219
9. $y = \frac{1}{8}(3x^2 - 22x + 48)$, 1.625
10. $y = \frac{1}{3}(2x^4 - 24x^3 + 100x^2 - 168x + 93)$
11. 14
12. $y_2 = 78.34$, $y_5 = 80.59$
13. -259
14. $u_0 = 10$, $u_3 = 22$
15. 571
16. $h = 0.3836$
17. 0.47562, -0.01714

EXERCISE 7.1

1. The values of annuities for certain ages are given for the following ages. Find the annuity at age 27½ using Gauss's forward interpolation formula.

Age	25	26	27	28	29
Annuity	16.195	15.919	15.630	15.326	15.006

2. Using Gauss's forward interpolation formula, find y at $x = 1.7489$ given that

x	1.72	1.73	1.74	1.75	1.76	1.77	1.78
y	0.1791	0.1773	0.1775	0.1738	0.1720	0.1703	0.1686

3. Find $\sqrt{12516}$ using Gauss's backward interpolation foumula given that $\sqrt{12500} = 111.8033$, $\sqrt{12510} = 111.8481$, $\sqrt{12520} = 111.8928$ and $\sqrt{12530} = 111.9374$.

4. Find $\sin 45°$ using Gauss's backward interpolation formula given that $\sin 20° = 0.342$, $\sin 30° = 0.502$, $\sin 40° = 0.642$, $\sin 50° = 0.766$, $\sin 60° = 0.866$, $\sin 70° = 0.939$, $\sin 80° = 0.984$.

5. Employ Bessel's formula to find the value of y at $x = 1.95$ given that

x	1.7	1.8	1.9	2.0	2.1	2.2	2.3
y	2.979	3.144	3.283	3.391	3.463	3.997	4.491

6. Use Bessel's formula to find the value of y when $x = 3.75$ given the table:

x	2.5	3.0	3.5	4.0	4.5	5.0
y	24.145	22.043	20.225	18.64[illegible]	[illegible]	[illegible]

7. Given cos (0.8050) = 0.6931, cos (0.8055) = 0.6928, cos (0.8060) = 0.6924, cos (0.8065) = 0.6920, cos (0.8070) = 0.6917, cos (0.8075) = 0.6913, and cos (0.8080) = 0.6909, find cos (0.806595) using Stirling's formula.

8. Apply Stirling's formula to find a polynomial of degree four which takes

x	1	2	3	4	5
y	1	–1	1	–1	1

9. Use Laplace–Everett's formula to find log 337.5 given that log 310 = 2.4913, log 320 = 2.5051, log 330 = 2.5185, log 340 = 2.5315, log 350 = 2.5441 and log 360 = 2.5563.

10. Find y at $x = 34$ using Laplace – Everett's formula given the table.

x	20	25	30	35	40
y	11.4699	12.7834	13.7648	14.4982	15.0463

11. The following table gives the values of the probability integral $f(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}\,dx$ for certain values of x. Find the value of their integral at $x = 0.5437$ using (i) Stirling's formula, (ii) Bessel's formula, and (iii) Everett's formula.

x	0.51	0.52	0.53	0.54	0.55	0.56	0.57
y	0.5292437	0.5378987	0.5464641	0.5549392	0.5633233	0.5716157	0.5798158

ANSWERS

1. 15.480
2. 0.1739
3. 111.8749
4. 0.707
5. 3.347
6. 19.4074
7. 0.6919
8. $\frac{1}{2}\{2(x-3)^4 - 8(x-3)^2 + 3\}$
9. 2.5283
10. 14.3684
11. 0.55805196, 0.55805196, 0.55805195.

EXERCISE 8.1

1. If $f(x) = x^{-2}$ show that $f(a, b, c, d) = -\dfrac{(abc + bcd + acd + abd)}{a^2b^2c^2d^2}$
2. If $f(x) = x^3$ show that $f(a^3, b^3, c^3) = a + b + c$.
3. If $f(x) = x^{-1}$ show that $f(x_0, x_1, \ldots, x_n) = \dfrac{(-1)^n}{x_0, x_1, \ldots, x_n}$.
4. If $f(x) = x^3 - 9x^2 + 17x + 6$, compute $f(-1, 1, 2, 3)$.
5. The following table gives some relation between steam pressure and temperature. Find the pressure at 372.1°

Temp.°C	361°	367°	378°	387°	399°
Pressure (kgf/cm²)	154.9	167.9	[illegible]	[illegible]	244.2

6. Using Newton's divided difference formula, evaluate $f(8)$ and $f(15)$ given that

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

7. The observed values of a function are 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the argument, respectively. What is the best estimate for the value of the function at position 6.

Fit a polynomial of third degree to the following data using Newton's divided difference method.

x	0	1	2	4	5	6
$f(x)$	1	14	15	5	6	9

9. If $f(0) = -18, f(1) = 0, f(3) = 0, f(5) = -248, f(6) = 0, f(9) = 13104$, find $f(x)$.

10. From the following table, obtain $f(x)$ as a polynomial in powers of $(x-5)$ using Newton's method.

x	0	2	3	4	5	6
$f(x)$	4	26	58	112	466	922

ANSWERS

4. 1 5. 177.84 6. 448, 3150 7. 147

8. $f(x) = x^3 - 9x^2 + 21x + 1$ 9. $f(x) = x^5 - 9x^4 + 18x^3 - x^2 + 9x - 18$

10. $f(x) = 194 + 98(x-5) + 17(x-5)^2 + (x-5)^3$

EXERCISE 8.2

1. Given that $\log_{10}300 = 2.4771$, $\log_{10}304 = 2.4829$, $\log_{10}305 = 2.4843$ and $\log_{10}307 = 2.4871$, find by using Lagrange's formula, the value of $\log_{10} 310$. (*Karnataka 1993*)

2. Given the values $f(14) = 68.7$, $f(17) = 64$, $f(31) = 44$ and $f(35) = 39.1$, find $f(27)$ using Lagrange's formula.

3. Given: $u_1 = 22$, $u_2 = 30$, $u_4 = 82$, $u_7 = 106$ and $u_8 = 206$. Find u_6 using Lagrange's interpolation formula.

4. The amount A of a substance remaining in a reacting system after an interval of time t in a certain chemical experiment is given by the following data:

t	2	5	8	14
A	94.8	87.9	81.3	68.7

Find the value of A at $t = 11$

5. The following table gives the viscosity of an oil as a function of temperature. Use Lagrange's formula to find the viscosity of oil at a temperature of 140°.

T°	110	130	160	190
Viscosity	10.8	8.1	5.5	4.8

6. The following are the measurements of t made on a curve recorded on an oscillograph representing a change in the conditions of electric current i.

t	1.2	2.0	2.5	3.0
i	1.36	0.58	0.34	0.20

Find the value of i at $t = 1.6$.

7. Using a polynomial of third degree, complete the record given below of the export of a certain commodity during five years.

Year	1917	1918	1919	1920	1921
Export (in tons)	443	384	–	397	467

8. The following data give the percentage of criminals for different age groups:

Age (less than x)	25	30	40	50
% of criminals	52	67.3	84.1	94.4

Using Lagrange's formula, find the percentage of criminals under the age of 35.

9. If $y_0, y_1, y_2, \ldots, y_6$ are the consecutive terms of a series, then using Lagrange's formula prove that $y_3 = 0.05\,(y_0 + y_6) - 0.3\,(y_1 + y_5) + 0.75\,(y_2 + y_4)$.

10. Given that $f(-1) = -2, f(0) = -1, f(2) = 1, f(3) = 4$, fit a polynomial of third degree.

11. Determine $f(x)$ as a polynomial in x for the following data:

x	–4	–1	0	2	5
$f(x)$	1245	33	5	9	1335

12. Find a polynomial of fifth degree from the following data:

x	0	1	3	5	6	9
$f(x)$	–18	0	0	–248	0	13104

(*M.U. 1991*)

13. Apply Lagrange's formula inversely to obtain the root of $f(x) = 0$, given that $f(30) = -30, f(34) = -13$ $f(38) = 3$ and $f(42) = 18$.

(*M.U. B.E, 1993*)

14. Given that $f(0) = 16.35, f(5) = 14.88, f(10) = 13.59$ and $f(15) =$ 12.46, find x when $f(x) = 14$.

15. Find x when $f(x) = 0.163$, given that

x	80	82	84	86	88
$f(x)$	0.134	0.154	0.176	0.200	0.221

16. Obtain the value of t when $A = 85$ in Problem 4.(*Madurai B.E, 1983*)

17. The following table gives the values of the probability integral

$P(x) = \dfrac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$. For what value of x, $P(x) = 0.5$?

x	0.45	0.46	0.47	0.48	0.49	0.50
$P(x)$	0.4755	0.4847	0.4937	0.5027	0.5117	0.5205

18. Given that $f(10) = 1754, f(15) = 2648, f(20) = 3564$, find the value of x when $f(x) = 3000$ by iterative method.

19. Given $\cosh x = 1.285$, find x by iterative method using the following data:

x	0.736	0.737	0.738	0.739	0.740	0.741
$\cosh x$	1.28330	1.28410	1.28490	1.28572	1.23652	1.28733

20. Solve the equation

(i) $x^3 - 6x - 11 = 0$ $(3<x<4)$ and

(ii) $x = \frac{1}{2} + \sin x$ by iterative method.

ANSWERS

1. 2.4786
2. 49.3
3. 83.515
4. 74.9
5. 7.03
6. 0.8908
7. 369
8. 77.4

10. $f(x) = \frac{1}{6}(x^3 - x^2 + 4x - 6)$
11. $f(x) = 3x^4 - 5x^3 + 6x^2 - 14x + 5$
12. $f(x) = x^5 - 5x^4 + 6x^3 - x^2 + 5x - 6$
13. 37.23
14. 8.34
15. 82.8
16. 6.5928
17. 0.477
18. 16.9
19. 0.73811
20. (i) 3.092 (ii) 1.4973

EXERCISE 9.1

1. Find the first and second derivatives of the function tabulated below at the point $x = 19$

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	0	0.128	0.544	1.296	2.432	4.00

(*Madras 1991*)

2. The following data gives corresponding values of pressure and specific volume of super-heated steam.

V	2	4	6	8	10
P	105	42.07	25.3	16.7	13

(i) Find the rate of change of pressure with respect to volume when $V = 2$.

(ii) Find the rate of change of volume with respect to pressure when $P = 105$.

3. Find $y'(0)$ and $y''(0)$ from the following table:

x	0	1	2	3	4	5
y	4	8	15	7	6	2

4. From the values in the table given below, find the value of sec 31° using numerical differentiation.

θ °	31	32	33	34
$\tan\theta$	0.6008	0.6249	0.6494	[illegible]

5. The table given below reveals velocity V of a body during time 't' specified. Find its acceleration at $t = 1.1$

t	1.0	1.1	1.2	1.3	1.4
v	43.1	47.7	52.1	56.4	60.8

6. A rod is rotating in a plane. The following table gives the angle θ (radians) through which the rod has turned for various values of time t in seconds.

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.122	0.493	0.123	2.022	3.200	4.61

Find the angular velocity and angular acceleration at $t = 0.6$.

7. From the following table of values of x and y, find y' (1.25) and y'' (1.25).

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	1.00000	1.02470	1.04881	1.07238	1.09544	1.11803	1.14017

8. Obtain the value of f' (0.04) using Bessel's formula given the table below:

x	0.01	0.02	0.03	0.04	0.05	0.06
$f(x)$	0.1023	0.1047	0.1071	0.1096	0.1122	0.1148

9. Use Stirling's formula to compute f' (0.5) from the following data:

x	0.35	0.40	0.45	0.50	0.55	0.60	0.65
$f(x)$	1.521	1.506	1.488	1.467	1.444	1.418	1.389

10. A slider in a machine moves along a fixed straight rod. Its distance x (cm) along the rod is given below for various values of time t seconds. Find the veocity of the slider and its acceleration when $t = 0.3$.

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

11. For the following pairs of values of x and y, find numerically the first derivative at $x = 4$.

x	1	2	4	8	10
y	0	1	5	21	27

12. Find the value of $f'(7.60)$ from the following table using Gauss's formula.

x	7.47	7.48	7.49	7.50	7.51	7.52	7.53
$f(x)$	0.193	0.195	0.198	0.201	0.203	0.206	0.208

13. Find the maximum and minimum values of the function from the following table:

x	0	1	2	3	4	5
$f(x)$	0	0.25	0	2.25	16.00	56.25

14. From the table below, for what value of x, y is maximum. Also find this value of y.

x	3	4	5	6	7	8
y	0.205	0.240	0.259	0.262	0.250	0.224

15. Given the following data, find the maximum value of y.

x	0	2	3	4	7	9
y	4	26	58	112	466	922

ANSWERS

1. 0.63, 6.6
2. –52.4, –0.01908
3. –27.9, 117.67
4. 1.17
5. 44.917
6. 3.82 rad/sec 6.75 rad/sec^2
7. 0.44733, 0.158332
8. 0.25625
9. –0.44
10. 5.33, –45.6
11. 2.8326
12. 0.223
13. Maximum value = 0.25 at $x = 1$; Minimum value 0 at $x = 0$ or 2
14. Minimum value = 0.2628 at $x = 5.6875$
15. No maximum or minimum value

EXERCISE 9.2

1. Evaluate $\int_0^2 y\,dx$ from the following table using Trapezoidal rule.

x	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y	1.21	1.37	1.46	1.59	1.67	2.31	2.91	3.83	4.01	4.79	5.31

2. Find an approximate value of $\log_e 5$ by calculating to four decimal places by Simpson's 1/3 rule the integral $\int_0^5 \frac{dx}{4x+5}$, dividing the range into 10 equal parts.

3. Apply Simpson's 3/8 rule to evaluate $\int_0^2 \frac{dx}{1+x^3}$ to two decimal places by dividing the range into nine ~~eight~~ equal parts.

4. Evaluate $\int_0^{10} e^x\,dx$ by Weddle's rule given that $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.60$, $e^5 = 148.41$, $e^6 = 403.43$, $e^7 = 1096.63$, $e^8 = 2980.96$, $e^9 = 8103.08$, $e^{10} = 22026.47$

5. Evaluate $\int_0^{\frac{\pi}{2}} \sin x\,dx$ by (i) Trapezoidal rule, (ii) Simpson's rule using 11 ordinates. Also estimate the errors by finding the value of the integral.

6. Calculate the value of the following integrals by (i) Trapezoidal rule, (ii) Simpson's 1/3 rule, (iii) Simpson's 3/8 rule, and (iv) Weddle's rule. After finding the true value of the integral, compare the errors in the four cases

(i) $\int_4^{5.2} \log x\,dx$ (ii) $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\,dx$

7. A river is 80 feet wide. Depth d in feet at a distance of x feet from one bank is given by the following table

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of the cross-section.

8. Find the approximate distance travelled by a train between 11.50 a.m. and 12.30 p.m. from the following data using Simpson's 1/3 rule.

time	11.50 a.m.	12.00	12.10 p.m.	12.20 p.m.	12.30 p.m.
Speed m.p.h.	24.2	35.0	41.3	42.8	39.2

9. A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's 1/3 rule, find the velocity and height of the rocket at $t = 80$.

t (s)	0	10	20	30	40	50	60	70	80
a (m/s²)	30	31.63	33.64	35.47	37.75	40.33	43.25	46.69	50.67

10. When a train is moving at 30 miles an hour, steam is shut off and brakes are applied. The speed of the train in miles per hour after t seconds is given by

t	0	5	10	15	20	25	30	35	40
v	30	24	19.5	16	13.6	11.7	10.0	8.5	7.0

Determine how far the train has moved in the 40 seconds.

11. The speed of an electric train at various times after leaving one station until it stops at the next station are given in the following table:

Speed in m.p.h	0	13	33	39.5	40	40	36	15	0
Time in min	0	0.5	1	1.5	2	2.5	3	3.25	3.5

Find the distance between the two stations.

12. A solid of revolution is formed by rotating about the x-axis, the area between x-axis and lines $x = 0$ and $x = 1$, and a curve through the points with the following coordinates.

x	0	0.25	0.50	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Estimate the volume of the solid formed using Simpson's 1/3 rule.

13. A curve passes through the points (1, 0.2), (2, 0.7), (3, 1), (4, 1.3), (5, 1.5), (6, 1.7), (7, 1.9), (8, 2.1), (9, 2.3). Using Weddle's rule, estimate the volume generated by revolving the area between the curve x axis and the ordinates $x = 1$ and $x = 9$ about the x axis.

14. The table below gives the velocity v of a moving particle at time t. Find the distance covered by the particle in 12 s and also the acceleration at $t = 2$ s.

t	0	2	4	6	8	10	12
v	4	6	16	34	60	94	136

15. Estimate the length of the arc of the curve $3y = x^3$ from (0, 0) to (1, 3) using Simpson's 1/3 rule taking eight sub-intervals.

16. Apply Romberg's method to evaluate $\int_4^{5.2} \log x \; dx$ given that

$\log_e 4 = 1.3863$, $\log_e 4.2 = 1.4351$, $\log_e 4.4 = 1.4816$, $\log_e 4.6 = 1.526$, $\log_e 4.8 = 1.5686$, $\log_e 5 = 1.6094$, $\log_e 5.2 = 1.6486$.

17. Evaluate $\int_0^1 \frac{dx}{1+x}$ correct to three decimal places by Trapezoidal rule with $h = 0.5, 0.25,$ and 0.125. Use Romberg's method to get an accurate value for the definite integral. Hence find the value of $\log_e 2$.

18. A reservoir discharging water through sluices at a depth h feet below the water surface, has a surface area A for various values of h as given below.

h (ft)	10	11	12	13	14
A (in sq ft)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by

$dh/dt = \dfrac{-48\sqrt{h}}{A}$. Estimate the time taken for the water level to fall from 14 to 10 feet above the sluices.

ANSWERS

1. 5.44
2. 1.6101
3. 0.8687
4. 24256.53
5. 0.9981, 1.0006; 0.0019, –0.0006
6. (i) 1.8276551, 1.8278472, 1.827847, 1.8278475
   errors: 0.0001924, 0.0000003, 0.0000005, 0.0000001
   (ii) 4.05617, 4.05106, 4.05116, 4.05098
   errors: –0.00522, –0.00011, –0.00021, –0.00003
7. 710 sq. ft
8. 25.4 miles
9. 30.87 m/s, h = 112.75 km
10. 296.7 yards
11. 5/3 miles
12. 2.8192
13. 59.68 cu. units
14. 532 m, 3 m/s^2
15. 1.0893 units
16. 1.8278
17. 0.708, 0.697, 0.694, 0.6931
18. 29 min (approx.)

EXERCISE 10.1

Form the difference equations by eliminating arbitrary constants.

1. $y = C_1 3^x + C_2 8^x$
2. $y = C_1 x^2 + C_2 x + 9$
3. $y = (C_1 + C_2 n)(-2)^n$
4. $y = C_1 x^2 + C_2 x + C_3$

Solve the following difference equations.

5. $y_{n+3} - 2y_{n+2} - y_{n+1} + 2y_n = 0$
6. $y_{x+4} + y_{x+3} - 13y_{x+2} - y_{x+1} + 12y_x = 0$
7. $y_{n+2} + 2y_{n+1} + 4y_n = 0$
8. $\Delta^3 u_n - 5\Delta u_n + 4u_n = 0$
9. $u_{k+4} + 6u_{k+3} + 9u_{k+2} - 4u_{k+1} - 12u_k = 0$
10. $y_{x+4} - 9y_{x+3} + 30y_{x+2} - 44y_{x+1} + 24y_x = 0$
11. $y_{x+2} - y_{x+1} + y_x = 0$, given $y_0 = 1$ and $y_1 = \dfrac{1+\sqrt{3}}{2}$
12. $u_{x+4} - 5u_{x+2} + 8u_{x+1} - 4u_x = 0$, given $u_0 = 3$, $u_1 = 2$, and $u_4 = 22$
13. If y_k satisfies the difference equation $y_{k+1} - \alpha y_k + y_{k-1} = 0$, $k = 1, 2, 3$ and the end conditions $y_0 = y_4 = 0$, show that non-trivial solution exists when $\alpha = 0, \pm\sqrt{2}$.
14. If y_n satisfies $y_{n+1} - 2y_n \cos\alpha + y_{n-1} = 0$ for $n = 1,2,\ldots$ and if $y_0 = 0$, $y_1 = 0$, find y_2, y_3, y_4.

Solve the following difference equations.

15. $y_{x+2} - 6y_{x+1} + 8y_x = 4^x$
16. $y_{n+2} - 3y_{n+1} + 2y_n = 5^n + 2^n$
17. $u_{x+2} - 4u_{x+1} + 4u_x = 3.2^x + 5.4^x$
18. $u_{n+2} - 4u_{n+1} + 3u_n = 2^n + 3^n + 7$
19. $y_{x+2} - 5y_{x+1} + 6y_x = x^2 + x + 1$
20. $\Delta^2 u_x + 2\Delta u_x + u_x = 3x + 2$
21. $\Delta u_x + \Delta^2 u_x = \cos x$
22. $u(x+2) - 7u(x+1) + 12u(x) = \cos x$
23. $u_{n+2} - 7u_{n+1} - 8u_n = 2^n n^2$
24. $u_{x+2} - 2u_{x+1} + u_x = 2^x x^2$
25. $2u_{n+2} + 5u_{n+1} + 2u_n = 2^n + n^2$

ANSWERS

1. $y_{x+2} - 11y_{x+1} + 24y_x = 0$
2. $x(1+x)y_{x+2} - 2x(x+2)y_{x+1} + (x^2 + 3x + 2)y_x + 9 = 0$
3. $y_{n+2} + 4y_{n+1} + 4y_n = 0$

4. $y_{x+3} - 3y_{x+2} + 3y_{x+1} - y_x = 0$

5. $y_n = C_1 + C_2(-1)^n + C_3 2^n$

6. $y_x = C_1 + C_2(-1)^x + C_3 3^x + C_4(-4)^x$

7. $y_n = \left\{C_1 \cos\frac{2n\pi}{3} + C_2 \sin\frac{2n\pi}{3}\right\} 2^n$

8. $u_n = C_1 2^n + C_2\left[\frac{1+\sqrt{17}}{2}\right]^n + C_3\left[\frac{1-\sqrt{17}}{2}\right]^n$

9. $u_k = C_1 + C_2(-3)^k + (C_3 + C_4 k)(-2)^k$

10. $y_x = (C_1 + C_2 x + C_3 x^2)2^x + C_4(-3)^x$

11. $y_x = \cos\frac{n\pi}{3} + \sin\frac{n\pi}{3}$

12. $y_x = 6 + (-3 + n)2^n$

14. $y_2 = 2\cos\alpha$; $y_3 = 4\cos^2\alpha - 1$; $y_4 = 8\cos^3\alpha - 4\cos\alpha$

15. $y_x = C_1 2^x + C_2 4^x + \frac{x}{8} 4^x$

16. $y_n = C_1 + C_2 2^n + \frac{5^n}{12} - n\, 2^{n-1}$

17. $u_x = (C_1 + C_2 x)2^x + 3x(x-1)\, 2^{x-3} + 5 \cdot 4^{x-1}$

18. $u_n = C_1 + C_2 3^n - 2^n + \frac{n}{2} 3^{n-1} - \frac{7n}{2}$

19. $y_x = C_1 2^x + C_2 3^x + \frac{1}{4}(2x^2 + 8x + 15)$

20. $u_x = 3x - 4$

21. $u_x = C_1 + \frac{\cos(x-2) - \cos(x-1)}{2(1-\cos 1)}$

22. $u_x = C_1 4^x + C_2 3^x + \frac{\cos(x-2) - 7\cos(x-1) + 12\cos x}{24\cos 2 - 182\cos 1 + 194}$

23. $u_n = C_1 8^n + C_2(-1)^n - \frac{2^{n-1}}{9}\left(n^2 - \frac{2n}{3} + \frac{1}{3}\right)$

24. $u_x = C_1 + C_2 x + 2^x(x^2 - 8x + 20)$

25. $u_n = C_1(-2)^n + C_2\left(-\frac{1}{2}\right)^n + \frac{2n}{20} + \frac{1}{9}\Big(n$ [illegible] $\frac{5}{9}\Big)$

EXERCISE 11.1

1. Using first four terms of the Maclaurin's series find y at $x = 0.1(0.1)(0.6)$ given that $2y' = (1 + x)\, y^2$, $y\,(0) = 1$. Compare the values with the exact solution.
2. Find the first six terms of the power series solution of $y' = \sin x + y^2$ which passes through the point (0, 1).
3. Given $y' = 3x + \dfrac{y}{2}$ and $y(0) = 1$, find by Taylor's series $y(0.1)$ and $y(0.2)$.
4. Using Taylor's series method solve $y' = xy + y^2$, $y(0) = 1$ at $x = 0.1$, 0.2, 0.3.
5. Solve by Taylor's series method of third order, the problem $y' = (x^3 + xy^2)\, e^{-x}$, $y(0) = 1$ to find y for $x = 0.1$, 0.2, 0.3.

6. Employ Taylor's method to obtain the approximate value of y at $x = 0.2$ for $y' = 2y + 3e^x$, $y(0) = 0$. Compare the numerical solution obtained with exact solution.

7. Solve $y' = y^2 + x$, $y(0) = 1$ using Taylor's series method to compute $y(0.1)$ and $y(0.2)$.

8. Solve $\frac{dy}{dx} = z - x$, $\frac{dz}{dx} = y + x$ with $y(0) = 1$, $z(0) = 1$ to get $y(0.1)$ and $z(0.1)$, using Taylor's method.

9. Given $\frac{dx}{dt} - ty - 1 = 0$ and $\frac{dy}{dt} + tx = 0$, $t = 0$, $x = 0$, $y = 1$, evaluate $x(0.1)$, $y(0.1)$, $x(0.2)$ and $y(0.2)$.

10. Using Taylor's series method, obtain the values of y at $x = (0.1)(0.1)$ 0.3 to four significant figures if y satisfies the equation $\frac{d^2y}{dx^2} + xy = 0$ given that $\frac{dy}{dx} = \frac{1}{2}$ and $y = 1$ when $x = 0$.

11. Evaluate the integral of the following problem to four significant figures at $x = 1.1(0.1)\,1.3$ using Taylor's series expansion.

$$\frac{d^2y}{dx^2} + y^2\frac{dy}{dx} - x^3 = 0 ; \left.\frac{dy}{dx}\right]_{x=1} = 1 ; \; y(1) = 1$$

12. Using Picard's method find $y(0.2)$ given that $y' = x - y$; $y(0) = 1$.

13. Using Picard's method obtain a solution upto the fifth approximation to the equation $y' = y + x$, such that $y(0) = 1$. Check your answer by finding the exact particular solution. Also find $y(0.1)$ and $y(0.2)$.

14. Using Picard's method find $y(0.2)$ and $y(0.4)$ given that $y' = 1 + y^2$ and $y(0) = 0$.

15. Use Picard's method to approximate the value of y when $x = 0.1$ given that $y(0) = 1$ and $y' = 3x + y^2$.

16. Using Picard's method find the approximate values of y and z corresponding to $x = 0.1$ given that $y(0) = 2$, $z(0) = 1$ and

$$\frac{dy}{dx} = x + z, \quad \frac{dz}{dx} = x - y^2.$$

17. Using Picards method obtain the second approximation to the solution to $y'' - x^3y' - x^3y = 0$ so that $y(0) = 1$, $y'(0) = \ldots$

ANSWERS

1.

x	0	0.1	0.2	0.3	0.4	0.5	0.6
Approx. value of y	1	1.055375	1.123	1.205125	1.304	1.421875	1.561
Exact value of y	1	1.055	1.124	1.209	1.316	1.455	1.64

2. $y = 1 + x + \frac{3x^2}{2} + \frac{4x^3}{3} + \frac{11x^4}{8} + \frac{23x^5}{15} + \cdots$
3. $y(0.1) = 1.0665$; $y(0.2) = 1.167196$
4. $y(0.1) = 1.1167$, $y(0.2) = 1.2767$, $y(0.3) = 1.5023$
5. $y(0.1) = 1.0047$, $y(0.2) = 1.01812$, $y(0.3) = 1.03995$
6. $y(0.2) = 0.811$, exact value of $y(0.2) = 0.8112$
7. $y(0.1) = 1.1164$, $y(0.2) = 1.2725$
8. $y(0.1) = 1.1003$, $z(0.1) = 1.1102$
9. $x(0.1) = 0.105$, $y(0.1) = 0.9997$
   $x(0.2) = 0.21998$, $y(0.2) = 0.9972$
10. $y(0.1) = 1.050$, $y(0.2) = 1.099$, $y(0.3) = 1.145$
11. $y(1.1) = 1.100$, $y(0.2) = 1.201$, $y(0.3) = 1.306$
12. $y(0.2) = 0.837$
13. $y(0.1) = 1.1103$; $y(0.2) = 1.2428$
14. $y(0.2) = 0.2027$, $y(0.4) = 0.4227$
15. $y(0.1) = 1.127$
16. $y(0.1) = 2.0845$; $z(0.1) = 0.5867$
17. $y_2 = 1 + \frac{1}{2}x + \frac{3}{40}x^5$

11.10 EULER'S METHOD

Consider the differential equation

$$\frac{dy}{dx} = f(x, y) \tag{11.32}$$

where $y(x_0) = y_0$

Suppose that we wish to find successively $y_1, y_2, \ldots, y_m$, where y_m is the value of y corresponding to $x = x_m$, where $x_m = x_0 + mh$, $m = 1, 2, \ldots$, h being small. Here, we use the property that in a small interval, a curve is nearly a straight line.

EXERCISE 11.2

1. Use Euler's method and Improved Euler's method to approximate y when $x = 0.1$, given that

$$\frac{dy}{dx} = \frac{y - x}{y + x}, \quad y(0) = 1 \text{ taking } h = 0.2.$$

2. Solve $y' = 3x^2 + y$ in $0 \le x \le 1$ by Euler's method taking $h = 0.1$ given that $y(0) = 4$.

3. Solve $y' = x + y$, $y(0)$ choosing the step length 0.2 for $y(1.2)$ by Euler's method.

4. Using Euler's method solve $y' = x + y$ in $0 \le x \le 1$ with $h = 0.1$, if $y(0) = 1$. Find the exact value of y at $x = 1$, using analytical method.

5. Using Euler's method find $y(0.6)$ of $y' = 1 - 2xy$, given that $y(0) = 0$ taking $h = 0.2$.

6. Solve $y' = -y$; $y(0) = 1$ by (i) Euler's method for $y(0.04)$ and (ii) Modified Euler's method for $y(0.6)$.

7. Solve $y' = x + y + xy$, $y(0) = 1$ for $y(0.1)$ taking $h = 0.025$, using Euler's method.

8. Given that $y' = \log(x + y)$ with $y(0) = 1$. Use (i) Improved Euler's method to find $y(0.2)$, $y(0.5)$; (ii) Modified Euler's method to find $y(0.2)$.

9. Use Euler's method and its Modified form to obtain $y(0.2)$, $y(0.4)$ and $y(0.6)$ correct to three decimal places given that $y' = y - x^2$, $y(0) = 1$.

10. Use Euler's modified method to get $y(0.25)$ given that $y' = 2xy$, $y(0) = 1$.

11. Using Improved Euler's method, solve
$y' = x + |\sqrt{y}|$, $y(0) = 1$ in the range $0 \le x \le 0.6$ taking $h = 0.2$.

12. Given that $y' = 2 + \sqrt{(xy)}$ and $y(1) = 1$. Find $y(2)$ in steps of 0.2 using Improved Euler's method.

13. Given $y^{(1)} = x^2 + y^2$, $y(0) = 1$, determine $y(0.1)$ and $y(0.2)$ by Modified Euler's method.

14. Solve $y^{(1)} = y + e^x$, $y(0) = 0$ for $y(0.2)$, $y(0.4)$ by Improved Euler's method.

15. Solve $y^{(1)} = y + x^2$, $y(0) = 1$ for $y(0.02)$, $y(0.04)$ and $y(0.06)$ using Euler's Modified method.

ANSWERS

1. 1.0928, 1.0932
2. 4.4, 4.843, 5.3393, 5.90023, 6.538253, 7.2670783, 8.1017861, 9.0589647, 9.1039647, 10.257361
3. 1.1831808
4. 1.1, 1.22, 1.362, 1.5282, 1.7210, 1.9431, 2.1974, 2.4871, 2.8158, 3.1873 ; exact solution = 3.4366
5. 0.4748
6. 0.9603 ; 0.551368
7. 1.1448
8. 1.0082, 1.0490 ; 1.0095
9. 1.2, 1.432, 1.686 ; 1.218, 1.467, 1.737
10. 1.0625
11. 1.2309, 1.5253, 1.8851
12. 5.051
13. 1.1105, 1.25026
14. 0.24214, 0.59116
15. 1.0202, 1.0408, 1.0619

11.13 RUNGE'S METHOD

Let the differential equation be

$$\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$$

EXERCISE 11.3

1. Solve $y' = x - y$ given that $y = 0.4$ at $x = 1$ for $y(1.6)$ using Runge's method.
2. Using Runge's method, find y at $x = 1.1$ given

$$\frac{dy}{dx} = 3x + y^2, \; y(1) = 1.2$$

3. Evaluate $y(0.8)$ using Runge's method given

$$y' = \sqrt{x + y} \; ; y = 0.41 \text{ at } x = 0.4$$

4. Using second order Runge–Kutta method, find y at $x = 0.1, 0.2$ and 0.3 given $2y' = (1 + x)y^2$; $y(0) = 1$.
5. Find $y(1.2)$ by Runge–Kutta method of fourth order given $y' = x^2 + y^2$; $y(1) = 1.5$. Take $h = 0.1$.
6. If $\dfrac{dy}{dx} = \dfrac{2xy + e^x}{x^2 + xe^x}$; $y(1) = 0$, solve for y at $x = 1.2, 1.4$ using Runge–Kutta method of fourth order.
7. Using Runge–Kutta method of fourth order, find y at $x = 1.1, 1.2$ given that

$$2y' = 2x^3 + y; \; y(1) = 2.$$

8. Find y at $x = 0.1, 0.2$ using fourth order Runge–Kutta algorithm given that

$y' - yx^2 = 0$; $y(0) = 1$.

9. Use Runge–Kutta method to evaluate y at $x = 0.2, 0.4, 0.6$ given that

$$\frac{dy}{dx} - xy = 1 \; ; \; y(0) = 2.$$

10. Using Runge–Kutta method of fourth order, find $y(0.1), y(0.2)$ given that

$$\frac{dy}{dx} - y = -x \; ; \; y(0) = 2.$$

11. Solve $10y' = x^2 + y^2$, $y(0) = 1$ to evaluate $y(0.2)$ and $y(0.4)$ by fourth order R–K algorithm.

12. Given $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{y^2 + x^2}$; $x_0 = 0$, $y_0 = 1$, $h = 0.2$, find y_1 and y_2 using Runge–Kutta method.

13. Solve $\dfrac{dy}{dx} = \dfrac{1}{x + y}$ for $x = 0.5$ to z using R–K method with $x_0 = 0$, $y_0 = 1$ (take $h = 0.5$).

14. Use Runge–Kutta method of order four to find y at $x = 0.1, 0.2$ given that

$x(dy + dx) = y(dx - dy)$; $y(0) = 1$.

15. Solve $y' = x + y$, $y(0) = 1$ to find y at $x = 0.1, 0.2, 0.3$ using R–K method.

16. Solve the following for $y(0.1), y(0.2)$ using Runge–Kutta algorithms of (i) second order, (ii) third order and (iii) fourth order.

(a) $\dfrac{dy}{dx} + y = 0$; $y(0) = 1$

(b) $\dfrac{dy}{dx} + 2y = x$; $y(0) = 1$

17. Use second order Runge–Kutta algorithm to solve $\dfrac{dy}{dx} + xz = 0$; $\dfrac{dy}{dx} - y^2 = 0$ at $x = 0.2, 0.4$ given that $y = 1$, $z = 1$ at $x = 0$.

18. Solve $\dfrac{dy}{dx} = 1 + xz$, $\dfrac{dz}{dx} = -xy$ for $x = 0.3, 0.6, 0.9$ given that $y = 0$, $z = 1$ at $x = 0$ by R–K method.

19. Solve $\dfrac{dy}{dx} = x + z,\ \dfrac{dz}{dx} = x - y^2$ for $y(0.1), z(0.1)$ given that $y(0) = 2$, $z(0) = 1$ by Runge – Kutta method.

20. Solve $y' = x + z,\ z' = x - y$ for $x = 0.1, 0.2$ given that $y = 0, z = 1$ at $x = 0$ by Runge–Kutta method.

21. Solve $\dfrac{d^2y}{dx^2} - x^2\dfrac{dy}{dx} - 2xy = 0$ given that $y(0) = 1,\ y'(0) = 0$ for $y(0.1)$ using Runge–Kutta method.

22. Use Runge–Kutta method to solve

$y'' - xy + 4y = 0$; $y(0) = 3$; $y'(0) = 0$ at $x = 0.1$.

23. Apply R–K algorithm to find y at $x = 0.1$ given $\dfrac{d^2y}{dx^2} = y^3$; $y(0) = 10$; $y'(0) = 5$.

y'

24. Solve $\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - y = 0$, $y(0) = 1, y'(0) = 0$ to find $y(0.2)$, $y'(0.2)$ using Runge–Kutta method.

25. Evaluate y(0.2) by R–K method given that $y'' - xy'^2 + y^2 = 0$; $y(0) = 1, y'(0) = 0$.

ANSWERS

1. 0.8176
2. 1.7278
3. 0.8481
4. 1.0552, 1.1230, 1.2073
5. 2.5505
6. 0.1402, 0.2705
7. 2.2213, 2.4914
8. 1.0053, 1.0227
9. 2.243, 2.589, 2.072
10. 2.20517, 2.42139
11. 1.0207, 1.038
12. $y_1 = y(0.2) = 1.19598$; $y_2 = y(0.4) = 1.3751$
13. 1.3571, 1.5837, 1.7555, 1.8956
14. 1.0911, 1.1678,
15. 1.1103, 1.2428, 1.3997
16. (a) 0.905, 0.81901 ; 0.91, 0.82337 ; 0.90484, 0.81873

    (b) 0.825, 0.6905 ; 0.8234, 0.6878 ; 0.82342, 0.6879
17. 0.978, 1.2 ; 0.9003 ; 1.382

EXERCISE 11.4

1. If $y' = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ $y(0.3) = 2.090$, find $y(0.4)$, $y(0.5)$ correct to three decimal places applying Milne's Predictor–Corrector method.

2. Solve $y' = x^2 - y$ given that $y(0) = 1$, $y(0.1) = 0.9052$, $y(0.2) = 0.8213$, for $y(0.5)$. Here, use Milne's method by computing $y(0.3) = 1$, using Taylor's method.

3. Tabulate the solution to $y' = x + y$ with the initial condition

   (i) $y(0) = 0$ for $0.4 < x 1 \le 1.0$, $h = 0.1$

   (ii) $y(0) = 1$ for $0.1 \le x 1 \le 0.3$, $h = 0.05$

   using Milne's predictor – Corrector method.

4. Using Taylor's series method, solve $y' = xy + y^2$, $y(0) = 1$ at $x = 0.1$, 0.2, 0.3. Continue the solution at $x = 0.4$ by Milne's method.

5. Solve $y' = 1 + xy^2$, for $y(0.4)$ by Milne's method given that $y(0) = 1$, $y(0.1) = 1.105$, $y(0.2) = 1.223$, $y(0.3) = 1.355$.

6. Use Milne's method to compute $y(0.3)$ from $y' = x^2 + y^2$, $y(0) = 1$. Find the initial values $y(-0.1)$, $y(0.1)$, $y(0.2)$ from the Taylor's series.

7. Solve $y' = x^2 + y^2 - 2$, using Milne's method for $x = 0.3$ given that $y = 1$ at $x = 0$. Compute $y(-0.1)$, $y(0.1)$ $y(0.2)$ using Runge–Kutta method of fourth order.

8. Given $y' + y = 1$, $y(0) = 0$, find $y(0.1)$ by using Euler's method, $y(0.2)$ by modified Euler's method, $y(0.3)$ by Improved Euler's method, and $y(0.4)$ by Milne's method.

9. Solve by Taylor's series of third order, the problem $y' = (x^3 + xy^2)e^{-x}$, $y(0) = 1$ to find y for $x = 0.1$, 0.2, 0.3. Continue the solution at $x = 0.4$ and $x = 0.5$ by Milne's method.

10. Using Adams–Bashforth predictor–corrector method, find $y(1.4)$ given that $x^2y' + xy = 1$; $y(1) = 1$, $y(1.1) = 0.996$, $y(1.2) = 0.986$, $y(1.3) = 0.972$.

11. Using Adams–Bashforth formulae, determine $y(0.4)$ given the equation $y' = 0.5\,xy$; $y(0) = 1$, $y(0.1) = 1.0025$, $y(0.2) = 1.0101$, $y(0.3) = 1.0228$.

12. Using Adams–Bashforth formulae, find $y(0.4)$, $y(0.5)$, if y satisfies $\frac{dy}{dx} = 3e^x + 2y$ with $y(0) = 0$. Compute y at $x = 0.1$, 0.2, 0.3 by means of Runge–Kutta method.

ANSWERS

1. 2.1621, 2.2546
2. 0.6435
3. (i) 0.0918, 0.1487, 0.2221, 0.3138, 0.4255, 0.5596, 0.7183

   (ii) 1, 1.0525, 1.1105, 1.2312, 1.2604, 1.3265
4. 1.8369
5. 1.45982
6. 1.4392
7. 0.61432
8. 0.3333
9. 1.0709, 1.1103
10. 0.94934
11. 1.0408
12. 2.2089, 3.20798

EXERCISE 12.1

1. Classify the following partial differential equations.

   (i) $u_{xx} - 2u_{xy} + u_{yy} + 3u_y - 4u_x = 3x - 2y$

   (ii) $(x + 1)\, u_{xx} - 2(x + 2)u_{xy} + (x + 3)u_{yy} = \cos(x - 2y)$

   (iii) $u_{xxx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = \sin(x + y)$

2. Solve $u_{xx} - u_{yy} = 0$ over the square mesh of side four units satisfying the following boundary conditions:

(i) $u(0, y) = 0$ for $0 \le y \le 4$ (ii) $u(4, y) = 12 + y$ for $0 \le y \le 4$

(iii) $u(x, 0) = 3x$ for $0 \le x \le 4$ (iv) $u(x, 4) = x^2$ for $0 \le x \le 4$

3. Solve for the following square mesh with boundary conditions as shown in Fig. 12.10. Iterate until the maximum difference between two successive values at any grid point is less than 0.005.

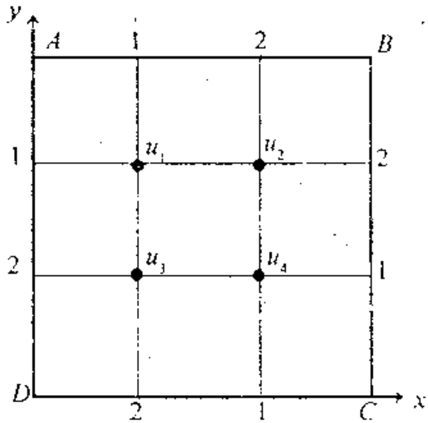


Fig. 12.10

4. Find the values of $u(x, y)$ satisfying the Laplace equation $\nabla^2 u = 0$, at the pivotal points of a square region, with boundary values as shown in (i) Fig.12.11 and (ii) Fig.12.12.

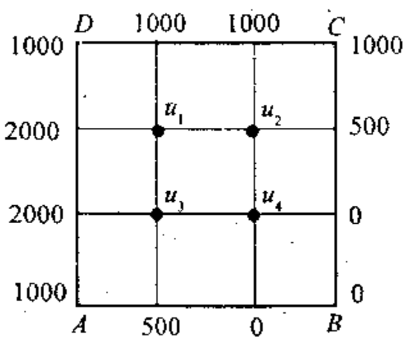


Fig. 12.11

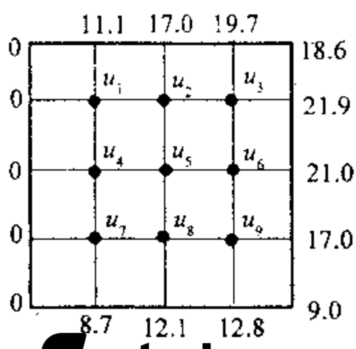


Fig. 12.12

5. Solve $u_{xx} + u_{yy} = 0$ for the following square meshes with boundary conditions as exhibited in Figures (i) 12.13 (ii) 12.14 (iii) 12.15 (iv) 12.16 and (v) 12.17.

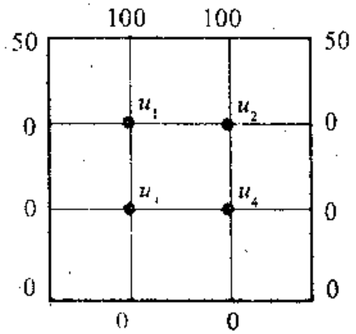


Fig. 12.13

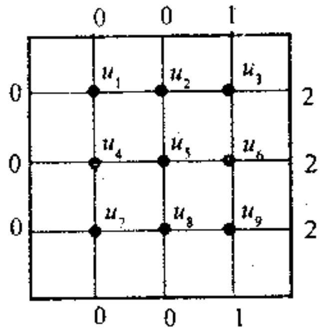


Fig. 12.14

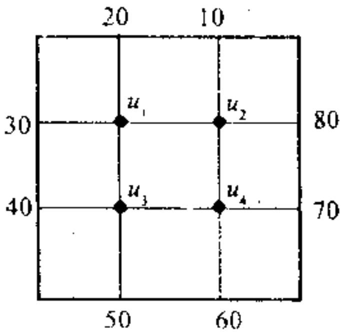


Fig. 12.15

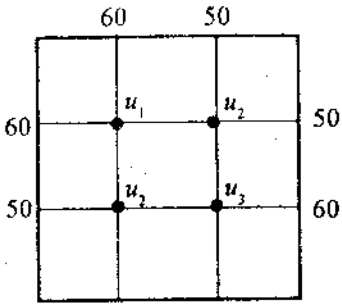


Fig. 12.16

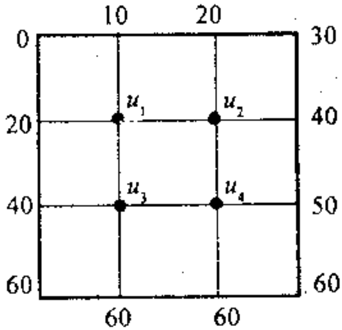


Fig. 12.17

6. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 8x^2y^2$ for square mesh given $u = 0$ on the four boundaries dividing the square into 16 sub-squares of length one unit.

ANSWERS

1. (i) Parabolic (ii) Hyperbolic

   (iii) Elliptic in the region outside the ellipse $\frac{x^2}{4} + \frac{y^2}{4} = 1$;

   Parabolic on the ellipse; hyperbolic inside the ellipse

2. 2.37, 5.59, 9.87, 2.88, 6.13, 9.88, 3.01, 6.16, 9.51

3. 1.333, 1.667, 1.667. 1.333

4. (i) 1208.3, 791.7, 1041.7, 458.4

   (ii) 7.9, 13.7, 17.9, 6.6, 11.9, 16.3, 6.6, 11.2, 14.3

5. (i) 37.5, 37.5, 12.5, 12.5

   (ii) 0.1875, 0.5000, 1.1875, 0.2500, 0.6250, 1.2500

   (iii) 34.986, 44.993, 44.993, 54.996

   (iv) 56.601, 52.051, 56.025

   (v) 26.65, 33.33, 43.32, 46.66

6. –3, –2, –3, –2, –2, –2, –3, –2, –3

EXERCISE 12.2

1. Given $u_t = 25u_{xx}$; $u(0, t) = 0 = u(10, t)$; $u(x, 0) = \frac{1}{25}x(10-x)$.
   Choosing $h = 1$ and k suitably, find u_{ij} for $0 \le i \le 9$, $1 \le j \le 4$.
2. Solve the equation $u_{xx} = u_t$ with the conditions $u(0, t) = 0$, $u(x, 0) = x(1-x)$; $u(1, t) = 0$. Assume that the region between $x = 0$ and $x = 1$ is divided into 10 equal parts of $h = 0.1$. Tabulate u for $t = k, 2k, 3k$, choosing an appropriate value of k.
3. Find the values of $u(x, t)$ satisfying the parabolic equation $u_t = 4u_{xx}$ and the boundary conditions $u(0, t) = 0 = u(8, t)$ and

   $u(x, 0) = 4x - \frac{1}{2}x^2$ at the points $x = i$; $i = 0, 1, 2, \ldots, 7$ and $t = \frac{1}{8}j$; $j = 0, 1, 2, \ldots, 5$.
4. Solve $u_t = 5u_{xx}$ with $u(0, t) = 0$; $u(5, t) = 60$ and

   $$u(x, 0) = 20x \text{ for } 0 < x \le 3$$
   $$= 60 \text{ for } 3 < x \le 5$$

   for five time steps having $h = 1$ by Schmidt method.
5. Compute u for one time step by Crank–Nicholson method if $ut = u_{xx}$; $0 < x < 5$, $t > 0$; $u(x, 0) = 20$; $u(0, t) = 0$ and $u(5, t) = 100$
6. Solve $u_t = u_{xx}$ subject to the conditions $u(x,0) = 0$; $u(0,t) = 0$ and $u(1,t)=1$. Compute u for $t = \frac{1}{8}$ in two steps, using Crank–Nicholson scheme.
7. Obtain the numerical solution to solve $u_t = u_{xx}$, $0 \le x \le 1$, $t \ge 0$, under the conditons that $u(0,t) = u(1,t) = 0$ and

$$u(x,0) = \begin{cases} 2x \text{ for } 0 \le x \le \frac{1}{2} \\ 2(1-x) \text{ for } \frac{1}{2} \le x \le \frac{1}{2} \end{cases}$$

ANSWERS

1.

$j \backslash i$	1	2	3	4	5	6	7	8	9
1	0.32	0.6	0.8	0.92	0.96	0.92	0.8	0.6	0.32
2	0.3	0.56	0.76	0.88	0.92	0.88	0.76	0.64	0.3
3	0.28	0.53	0.72	0.84	0.88	0.84	0.76	0.53	0.32
4	0.265	0.5	0.685	0.8	0.84	0.82	0.685	0.54	0.265

2.

j \ i	0	1	2	3	4	5	6	7	8	9	10
0	0	0.09	0.16	0.21	0.24	0.25	0.24	0.21	0.16	0.09	0
1	0	0.08	0.15	0.20	0.23	0.24	0.23	0.20	0.15	0.08	0
2	0	0.075	0.14	0.19	0.22	0.23	0.22	0.19	0.14	0.075	0
3	0	0.07	0.1325	0.18	0.21	0.22	0.21	0.18	0.1325	0.07	0

3.

j \ i	0	1	2	3	4	5	6	7	8
0	0	3.5	6	7.5	8	7.5	6	3.5	0
1	0	3	5.5	7	7.5	7	5.5	3	0
2	0	2.75	5	6.5	7	6.5	5	2.75	0
3	0	2.5	4.625	6	6.5	6	4.625	2.5	0
4	0	2.3125	4.25	5.5625	6	5.5625	4.25	2.3125	0
5	0	2.125	3.9375	5.125	5.5625	5.125	3.9375	2.125	0

4.

j \ i	0	1	2	3	4	5
0	0	20	40	60	60	60
0.1	0	20	40	50	60	60
0.2	0	20	35	50	55	60
0.3	0	17.5	35	45	55	60
0.4	0	17.5	31.25	45	52.5	60
0.5	0	15.625	31.25	41.875	52.5	60

5.

j \ i	0	1	2	3	4	5
0	0	20	20	20	20	100
1	0	9.80	20.19	30.72	59.92	100

6.

j \ i	0	0.25	0.5	0.75	1
0	0	0	0	0	0
$\frac{1}{16}$	0	0.00116	0.004464	0.01674	$\frac{1}{16}$
$\frac{1}{8}$	0	0.005899	0.019132	0.052771	$\frac{1}{8}$

7.

j \ i	0	0.2	0.4	0.6	0.8	1.0	0.8	0.6	0.4	0.2	0
0.1	0	0.1936	0.3689	0.5400	0.6461	0.6921	0.6461	0.5400	0.3689	0.1936	0
0.02	2	0.1989	0.3956	0.5834	0.7381	[illegible]	[illegible]	[illegible]	[illegible]	0.1989	0

EXERCISE 12.3

1. Evaluate the pivotal values for the following equation taking $h = 1$ and upto one half of the period of vibration.
$16u_{xx} = u_{tt}$, given that $u(0, t) = u(5, t) = 0$
$u(x, 0) = x^2(x - 5)$ and $u_t(x, 0) = 0$

2. Solve the hyperbolic partial differential equation (vibration of strings) for one half period of oscillation taking $h = 1$.
$u_{tt} = 25u_{xx}$, $u(0, t) = u(5, t) = 0$; $u_t(x, 0) = 0$

$$u(x, 0) = \begin{cases} 2x \text{ for } 0 \le x \le 2.5 \\ 10 - 2x \text{ for } 2.5 \le x \le 5 \end{cases}$$

3. Solve $u_{tt} = u_{xx}$ upto $t = 0.5$ with spacing of 0.1 given that
$u(0, t) = 0 = u(1, t)$; $u_t(x, 0) = 0$ and $u(x, 0) = 10 + x(1 - x)$

ANSWERS

1.

j \ i	0	1	2	3	4	5
0	0	4	12	18	16	0
1	0	4	12	18	16	0
2	0	8	10	10	2	0
3	0	6	6	−6	−6	0
4	0	−2	−10	−10	−8	0
5	0	−16	−18	−12	−4	0

2.

j \ i	0	1	2	3	4	5
0	0	2	4	4	2	0
0.2	0	2	4	4	2	0
0.4	0	2	2	2	2	0
0.6	0	0	0	0	0	0
0.8	0	−2	−2	−2	−2	0
1.0	0	−2	−4	−4	−2	0

3.

j \ i	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0	10.09	10.16	10.21	10.24	10.25	10.24	10.21	10.16	10.09	0
0.1	0	10.09	10.16	10.21	10.24	10.25	10.24	10.21	10.16	10.09	0
0.2	0	0.07	10.14	10.19	10.22	10.23	10.22	10.19	10.17	0.7	0
0.3	0	0.05	0.1	10.15	10.18	10.19	10.18	10.15	0.1	0.05	0
0.4	0	0.03	0.06	0.09	10.12	10.13	10.12	0.09	0.06	0.03	0
0.5	0	0.01	0.02	0.03	0.04	10.05	0.04	0.03	0.02	0.01	0

EXERCISE 12.4

1. Given that $u(x, y)$ satisfies the equation $\nabla^2 u = 0$ and the boundary conditions are $u(0, y) = 0$, $u(4, y) = 8 + 2y$, $u(x, 0) = \frac{1}{2} x^2$ and $u(x, 4) = x^2$, find the values of $u(i, j)$; $i = 1, 2, 3$; $j = 1, 2, 3$ by relaxation method.

2. Solve by relaxation method, the Laplace equation $\nabla^2 u = 0$ in the following square region starting with the values $u_1 = u_2 = u_3 = u_4 = 1$

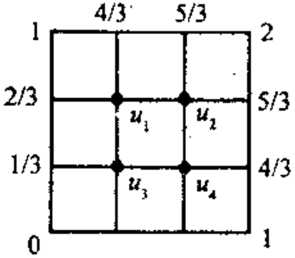


Fig 12.27

ANSWERS

1) $u_1 = 1.9$, $u_2 = 4.9$, $u_3 = 9.1$, $u_4 = 2.1$, $u_5 = 4.7$ $u_6 = 8.4$, $u_7 = 1.6$, $u_8 = 3.9$, $u_9 = 6.7$

2) $u_1 = 1$, $u_2 = 1.3$, $u_3 = 0.7$, $u_4 = 1$