CSC101
Introduction to ICT

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Lecture - 5

Number Systems

Number Systems

- Simply put, a number system is a way to represent numbers
- There are two types of number systems

<u>Positional number systems</u>

- Uses only a few symbols called digits
- These digits represents different values depending on the position they occupy in the number
- The value of each digit is determined by
  - the digit itself
  - the position of the digit in the number
  - the base of the number system
    - base is the total number of digits in the number system

<u>Non-positional number systems</u>

- Uses symbols such as I for 1, II for 2, III for 3 etc.
- Each symbol represents the same value regardless of its position in the number
- The symbols are simply added to find out the value of a particular number
- It is difficult to perform arithmetic with such a number system

Decimal Number System

- A positional number system which has 10 symbols or digits
- A total of 10 digits means base of decimal number system is 10

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- The maximum value of a single digit is 9 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (10)
- The most popular and used by us in our day-to-day life

Example

$(2586)_{10}$

$(163)_{10}$

$(981)_{10}$

Binary Number System

- A positional number system which has only two digits
- Base = 2, only two digits 0, 1
- The maximum value of a single digit is 1 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (2)
- Mostly is used in the field of computer science

Example

$(10101)_2$

$(101111010)_2$

$(1110101)_2$

Octal Number System

- A positional number system which has eight digits
- Base = 8, total digits 0, 1, 2, 3, 4, 5, 6, 7
- The maximum value of a single digit is 7 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (8)
- Not much used in the real-world mathematics

Example

$(2057)_8$

$(6605)_8$

$(321)_8$

Hexadecimal Number System

- A positional number system which has a total of 16 digits or symbols
- Base = 16, so total 16 symbols and digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
- The symbols A, B, C, D, E, F represents decimal values 10, 11, 12, 13, 14 and 15
- The maximum value of a single digit is 15 (one less than the value of the base)
- Each position of a digit represents a specific power of the base (16)
- Not much used in the real-world mathematics

Example

$(1AF)_{16}$

$(9EA)_{16}$

$(664BD)_{16}$

Conversion between Number Systems

- *Converting a number of another base to decimal number*

Example:

$$(4706)_8 = (4x8^3) + (7x8^2) + (0x8^1) + (6x8^0)$$
$$= (4x512) + (7x64) + (0x8) + (6x1)$$
$$= 2048 + 448 + 0 + 6$$
$$= (2502)_{10}$$

Conversion between Number Systems

- *Converting a decimal number to a number of another base*

Example:

$(952)_{10} = ?_8$

8	952	
	119	0
	14	7
	1	6
	0	1

Hence, $(952)_{10} = (1670)_8$

Conversion between Number Systems

- Converting a number of some base to a number of another base

- Convert the original number to a decimal base (10)
- Convert the decimal number obtained in the above step to the new base number

Examples:

$(545)_6 = ?_4$

$= (209)_{10} = (3101)_4$

$(545)_8 = ?_4$

$= (357)_{10} = (11211)_4$

Conversion between Number Systems

- Shortcut method for converting a binary number to an octal number

- Divide the binary digits into groups of three starting from the right
- Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion

Example: $(1101010)_2 = ?_8$

001 101 010

$(001)_2 = (0\times2^2) + (0\times2^1) + (1\times2^0) = (1)_{10}$

$(101)_2 = (1\times2^2) + (0\times2^1) + (1\times2^0) = (5)_{10}$

$(010)_2 = (0\times2^2) + (1\times2^1) + (0\times2^0) = (2)_{10}$

Hence, $(1101010)_2 = (152)_8$

Conversion between Number Systems

- Shortcut method for converting an octal number to a binary number

- Convert each octal digit (or its decimal equivalent) to a three digit binary number
- Combine all the individual groups of three binary digits into a single binary number

Example:

$(562)_8 = ?_2$

5 = 101 6 = 110 2 = 010

101 110 010

Hence, $(562)_8 = (101110010)_2$

Conversion between Number Systems

- *Shortcut method for converting a binary number to a hexadecimal number*

- *Divide the binary digits into groups of four starting from the right*
- *Convert each group of four binary digits to one hexadecimal digit using the method of binary to hexadecimal conversion*

Example: $(111101)_2 = ?_{16}$

0011 1101

$(0011)_2 = (0\times2^3) + (0\times2^2) + (1\times2^1) + (1\times2^0) = (3)_{10} = (3)_{16}$

$(1101)_2 = (1\times2^3) + (1\times2^2) + (0\times2^1) + (1\times2^0) = (13)_{10} = (D)_{16}$

Hence, $(111101)_2 = (3D)_{16}$

Conversion between Number Systems

- Shortcut method for converting a hexadecimal number to a binary number

- Convert the decimal equivalent of each hexadecimal digit to a four digit binary number
- Combine all the individual groups of four binary digits into a single binary number

Example:

$(2AB)_{16} = ?_2$

$(2)_{16} = (2)_{10} = 0010$ $(A)_{16} = (10)_{10} = 1010$ $(B)_{16} = (11)_{10} = 1011$

0010 1010 1011

Hence, $(2AB)_{16} = (001010101011)_2$

Fractional Numbers

- Fractional numbers are formed the same way
- In general, a number in a number system with base b would be written as;

  $a_n, a_{n-1}, \ldots\ldots\ldots\ a_0\ a_{-1}, a_{-2}\ \ldots\ldots\ldots\ a_{-m}$

- And would be interpreted as;

  $a_n \times b^n + a_{n-1} \times b^{n-1} + \ldots + a_0 \times b^0 + a_{-1} \times b^{-1} + a_{-2} \times b^{-2} + \ldots + a_{-m} \times b^{-m}$

- The symbols $a_n, a_{n-1}, \ldots, a_{-m}$ in the above representation should be one of the symbols allowed in the number system

Fractional Numbers

- Formatting of fractional numbers in binary number system

Position	4	3	2	1	0	.	-1	-2	-3	-4
Position Value	2^4	2^3	2^2	2^1	2^0		2^{-1}	2^{-2}	2^{-3}	2^{-4}
Quantity Represented	16	8	4	2	1		1/2	1/4	1/8	1/16

- Conversion of fractional numbers in binary number system

Example:

$(110.101)_2 = ?_{10}$

$(110.101)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 4 + 2 + 0 + 0.5 + 0 + 0.125$

$= (6.625)_{10}$

Fractional Numbers

- Formatting of fractional numbers in octal number system

Position	4	3	2	1	0	.	-1	-2	-3	-4
Position Value	8^4	8^3	8^2	8^1	8^0		8^{-1}	8^{-2}	8^{-3}	8^{-4}
Quantity Represented	4096	512	64	8	1		1/8	1/64	1/512	1/4096

- Conversion of fractional numbers in octal number system

Example:

$(127.54)_8 = ?_{10}$

$(127.54)_8 = 1 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 5 \times 8^{-1} + 4 \times 8^{-2}$

$= 64 + 16 + 7 + 5/8 + 4/64$

$= (87.6875)_{10}$

THANK YOU