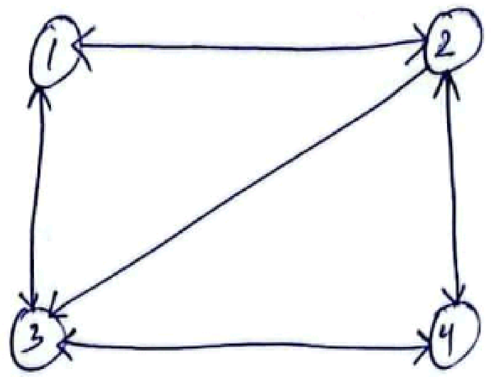
Travelling Salesman problem

Brute Force approach:

Dynamic programming
→ if subproblem exist multiple times



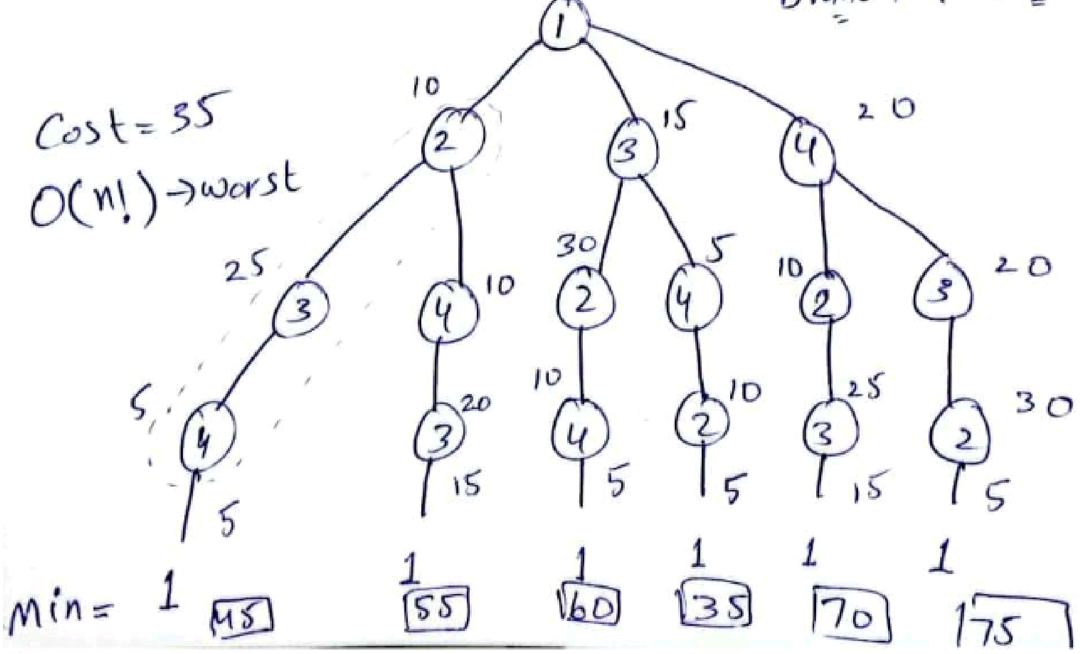
Cost-adjacency matrix:

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 5 & 0 & 25 & 10 \\ 3 & 15 & 30 & 0 & 5 \\ 4 & 5 & 10 & 20 & 0 \end{array}$$

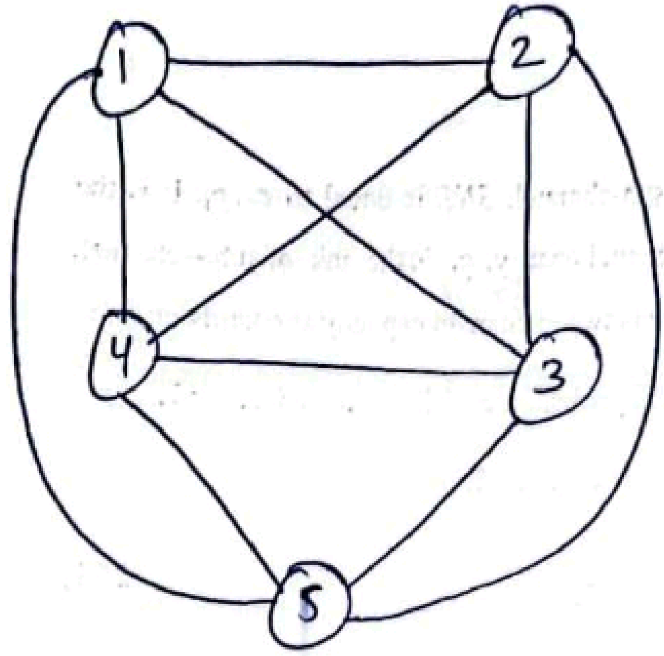
Branch & bound

Cost = 35

$O(n!)$ → worst



Dynamic programing approach



Cost Adjacency matrix

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \left[\begin{array}{ccccc} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 17 & 16 & \infty \end{array}\right] \end{array}$$

Step#01: Find minimum cost adjacency matrix for each node

Step#02: Draw State based tree & pick the node having minimum cost from its corresponding matrix

Step#03: Continue until reach leaf node

$$
\begin{array}{c|ccccc|c}
 & 1 & 2 & 3 & 4 & 5 & \text{Mininqum} \\
\hline
1 & \infty & 20 & 30 & 10 & 11 & 10 \\
2 & 15 & \infty & 16 & 4 & 2 & 2 \\
3 & 3 & 5 & \infty & 2 & 4 & 2 \\
4 & 19 & 6 & 18 & \infty & 3 & 3 \\
5 & 16 & 4 & 7 & 16 & \infty & 4
\end{array}
$$

1) Subtract each ~~minimum~~ cost from minimum of its row

$$
\begin{array}{c|ccccc|c}
 & & & & & & \\
\hline
 & \infty & 10 & 20 & 0 & 1 & 10 \\
 & 13 & \infty & 14 & 2 & 0 & 2 \\
 & 1 & 3 & \infty & 0 & 2 & 2 \\
 & 16 & 3 & 15 & \infty & 0 & 3 \\
 & 12 & 0 & 3 & 12 & \infty & 4 \\
\hline
\text{Minimum of each column} & 1 & 0 & 3 & 0 & 0 & 21
\end{array}
$$

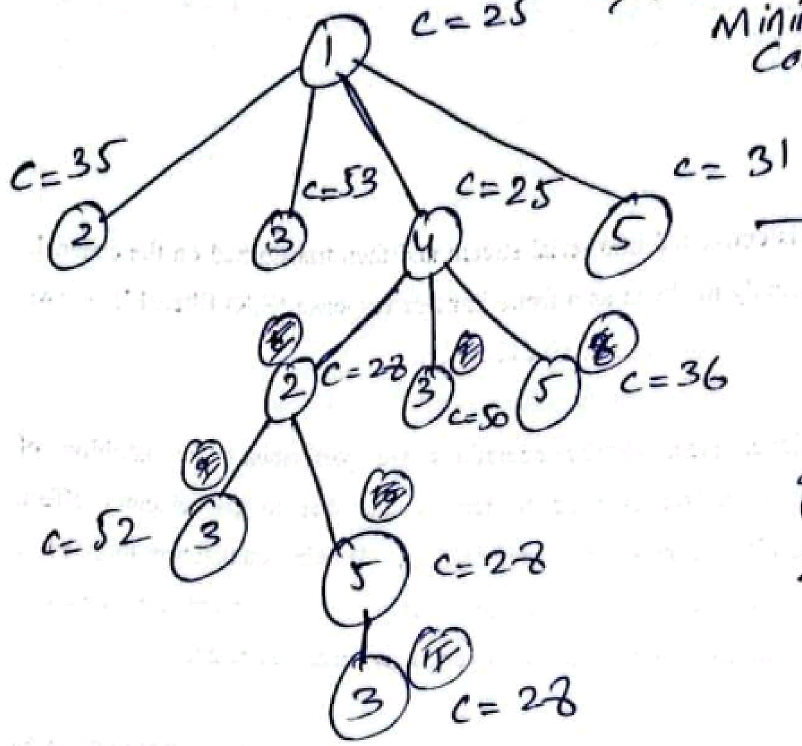
2) Subtract each column cost from its minimum

$$
N_1 \quad
\begin{array}{ccccc|c}
1 & 2 & 3 & 4 & 5 & \\
\hline
\infty & 10 & 17 & 0 & 1 & 10 \\
12 & \infty & 11 & 2 & 0 & 2 \\
0 & 3 & \infty & 0 & 2 & 2 \\
15 & 3 & 12 & \infty & 0 & 3 \\
11 & 0 & 0 & 12 & \infty & 4 \\
\hline
1 & 0 & 3 & 0 & 0 & 21+4=25
\end{array}
$$

Node 1 minimum cost = 25

$\Rightarrow 1-4-2-5-3$

Minimum cost $=28$



$\rightarrow$ min $=25$

so go just after N4

I have named 2, 3, 5 as 6, 7, 8 for clarity

Now Find cost for each node n_2, n_3, n_4 & n_5

put infinity in parent node raw, For N_2, put 1st row & 2nd colum to ∞ also at $M(2,1)$

$$N_2 \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix} \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

$$\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \end{array}$$

cost of $N2 = 0$

$$\Rightarrow \underbrace{c(1,2)}_{} + \underbrace{r}_{\text{cost of parent node}} + \underbrace{\hat{r}}_{\text{reduced cost}}$$

$$\Rightarrow 10 + 25 + 0 \Rightarrow 35$$

Finding N3

$$\begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 15 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & 0 \end{bmatrix} \\ \text{Min col} & \begin{array}{ccccc} 11 & 0 & 0 & 0 & 0 \end{array} \end{array} \quad \begin{array}{c} \text{Min row} \\ \\ 0 \\ 0 \\ 0 \\ 0 \end{array}$$

Cost = 11

↓ Reduced matrix

$$N3: \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \\ 4 & 3 & \infty & \infty & 0 \\ 0 & 0 & \infty & 12 & 0 \end{bmatrix}$$

$= c(1,3) + r + \hat{r}$

$= 17 + 25 + 11$

$= 53$

Finding N4

$$N4 = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix}$$

$C = 25$

$$N_5 = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 10 & \infty & 9 & 0 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 12 & 0 & 9 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix}$$

$C = 31$

Take

$$N_6 = \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & \infty & 0 \\ 0 & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & 0 & \infty & \infty \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$

for ② of ④

$$\begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$\hat{r} = 0$

$$= c(2,4) + \underbrace{r}_{\text{cost of } N_4} + \underbrace{\hat{r}}_{\text{cost of } N_6}$$

$C = 3 + 25 + 0 = 28$

$$N_7 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & 1 & \infty & \infty & \infty & 0 \\ 3 & \infty & 1 & \infty & \infty & 0 \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & 0 & 0 & \infty & \infty & \infty \end{array}$$

$C = 50$

$$N_8 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & 1 & \infty & 0 & \infty & \infty \\ 3 & 0 & 3 & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & 0 & 0 & \infty & \infty \end{array}$$

$C = 36$

$$N_9 = \begin{array}{c|ccccc|c} & 1 & 2 & 3 & 4 & 5 & \\ \hline 1 & \infty & \infty & \infty & \infty & \infty & \\ 2 & \infty & \infty & \infty & \infty & \infty & \\ 3 & \infty & \infty & \infty & \infty & 0 & \\ 4 & \infty & \infty & \infty & \infty & \infty & 2 \\ 5 & 0 & \infty & \infty & \infty & \infty & 11 \end{array}$$

$\overline{\quad 13 \quad}$

$= c(2,3) + \underbrace{r}_{\text{cost of } N_6} + \underbrace{\hat{r}}_{\text{cost of } N_9}$

$= 11 + 28 + 13$

$= 52$

$$N_{10} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & \infty & \infty & \infty & \infty & \infty \\ 2 & \infty & \infty & \infty & \infty & \infty \\ 3 & 0 & \infty & \infty & \infty & \infty \\ 4 & \infty & \infty & \infty & \infty & \infty \\ 5 & \infty & \infty & 0 & \infty & \infty \end{array}$$

$C = 28$