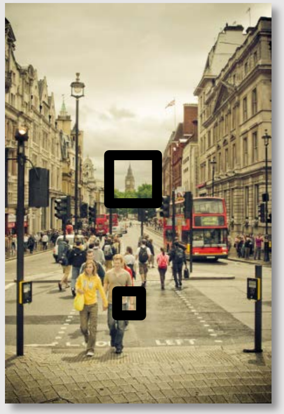
Image Interpolation

Today: back to images

- This photo is too small:



Might need this for forensics:

Zooming

- First consider a black and white image

  (one intensity value per pixel)

- We want to blow it up to poster size (say, zoom in by a factor of 16)

- First try: repeat each row 16 times, then repeat each column 16 times

Zooming: First attempt

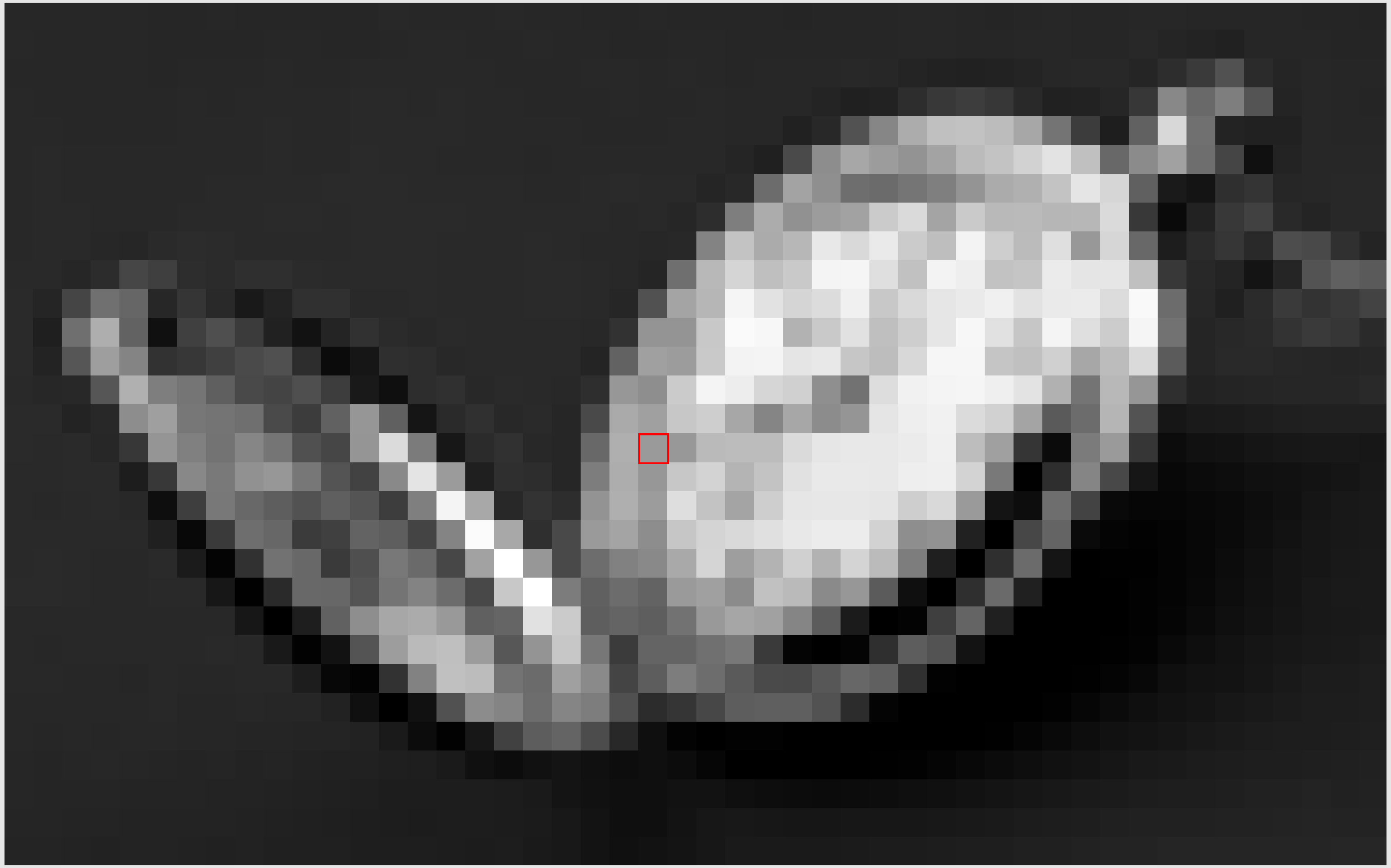


Image Interpolation

Interpolation — Process of using known data to estimate unknown values

e.g., zooming, shrinking, rotating, and geometric correction

Interpolation (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

Introduction

Image interpolation

An image f(x,y) tells us the intensity values at the integral lattice locations, i.e., when x and y are both integers

- Image interpolation refers to the “guess” of intensity values at missing locations, i.e., x and y can be arbitrary
- Note that it is just a guess (Note that all sensors have finite sampling distance)

Motivations

Why do we need image interpolation?

- We want BIG images
  - When we see a video clip on a PC, we like to see it in the full screen mode
- We want GOOD images
  - If some block of an image gets damaged during the transmission, we want to repair it
- We want COOL images
  - Manipulate images digitally can render fancy artistic effects as we often see in movies

Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Basic Relationships Between Pixels

Neighbors of a pixel p at coordinates (x,y)

- **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
  (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

- **4 diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
  (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

- **8 neighbors of p**, denoted $\mathbf{N_8(p)}$
  $N_8(p) = N_4(p) \cup N_D(p)$

Basic Relationships Between Pixels

Adjacency

Let V be the set of intensity values

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

Adjacency

Let V be the set of intensity values

- **m-adjacency**: Two pixels p and q with values from V are m-adjacent if

(i) q is in the set $N_4(p)$, or

(ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Basic Relationships Between Pixels

Path

- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

  $$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

  Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

- Here *n* is the *length* of the path.

- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.

- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

V = {1, 2}

0	1	1		0	1	1		0	1	1
0	2	0		0	2	0		0	2	0
0	0	1		0	0	1		0	0	1

Examples: Adjacency and Path

V = {1, 2}

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

8-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

m-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0_{1,1} & 1_{1,2} & 1_{1,3} \\ 0_{2,1} & 2_{2,2} & 0_{2,3} \\ 0_{3,1} & 0_{3,2} & 1_{3,3} \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent

m-adjacent

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

Connected in S

Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.

- If S has only one connected component, then S is called ***Connected Set*.**

- We call R a **region** of the image if R is a connected set

- Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not to be adjacent are said to be ***disjoint.***

Basic Relationships Between Pixels

Boundary (or border)

- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

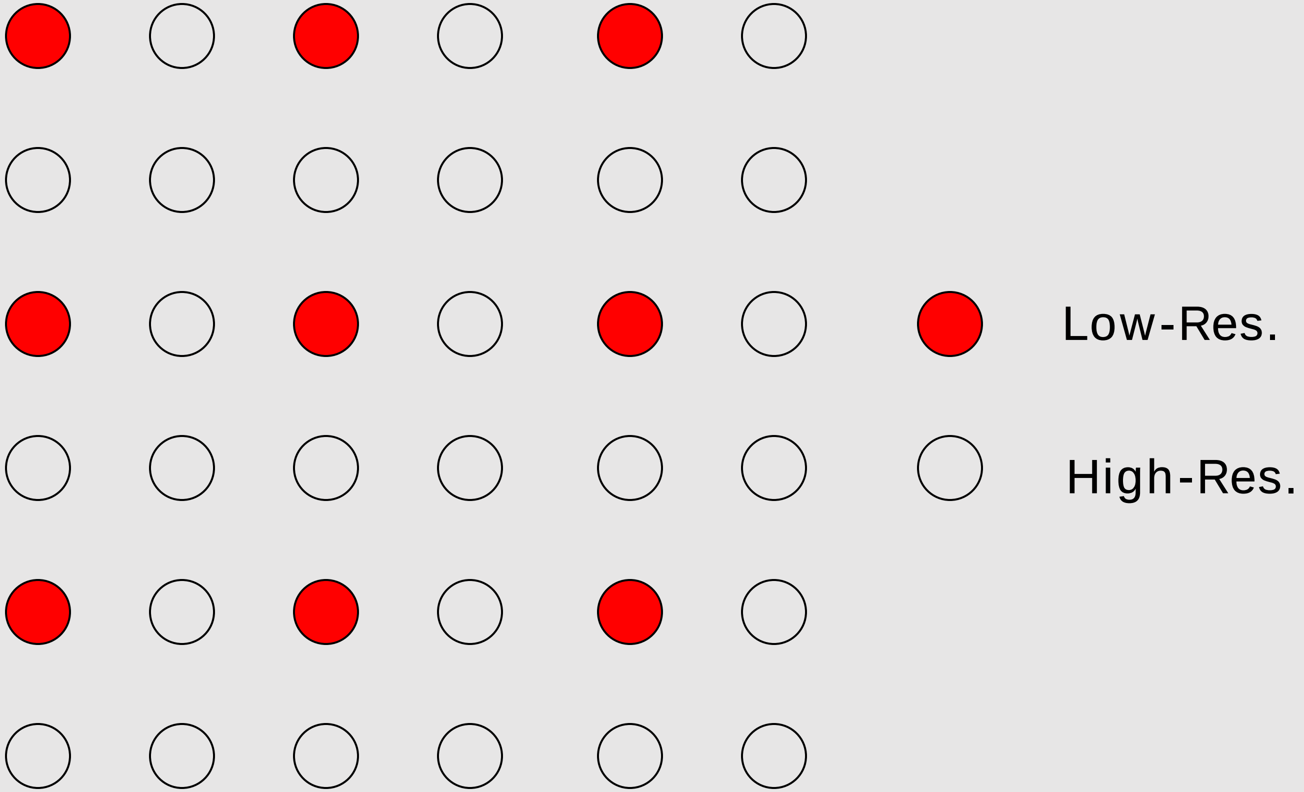
Foreground and background

- An image contains K disjoint regions, R_k, $k = 1, 2, \ldots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

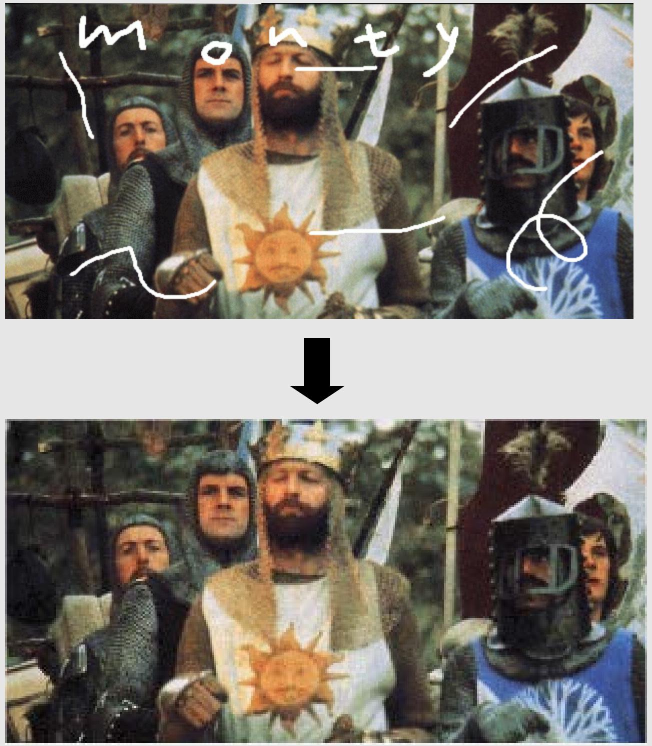
  All the points in R_u is called **foreground**;

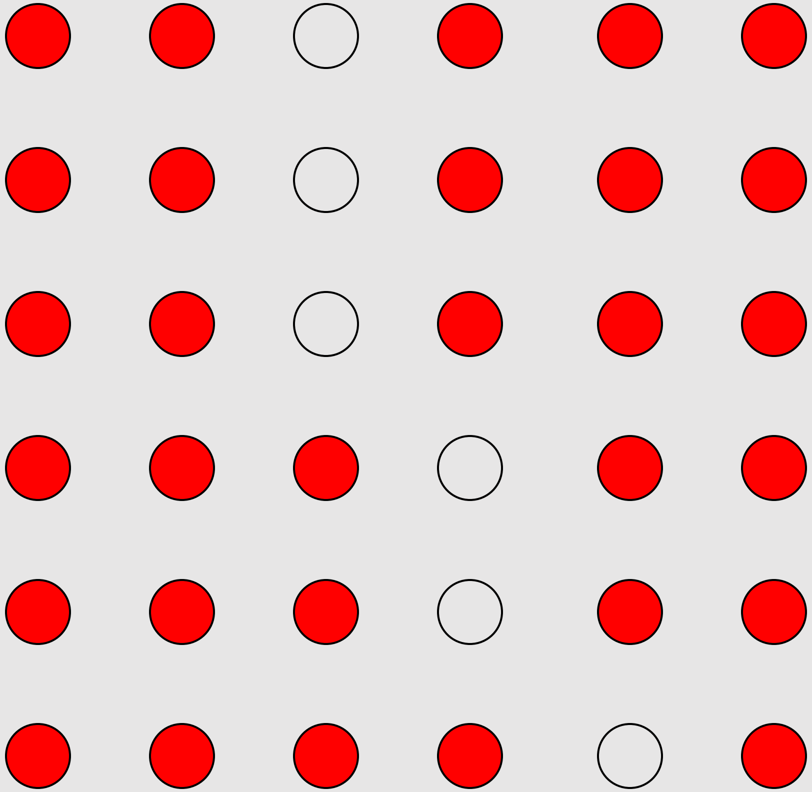
  All the points in $(R_u)^c$ is called **background**.

Scenario I: Resolution Enhancement



Scenario II: Image Inpainting





Non-damaged Damaged

Scenario III: Image Warping

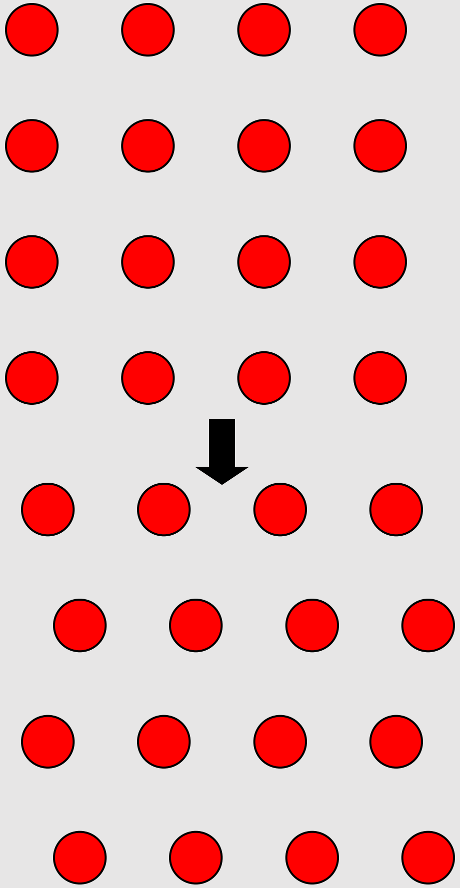


Image Interpolation – Method 1

Nearest Neighbors Interpolation

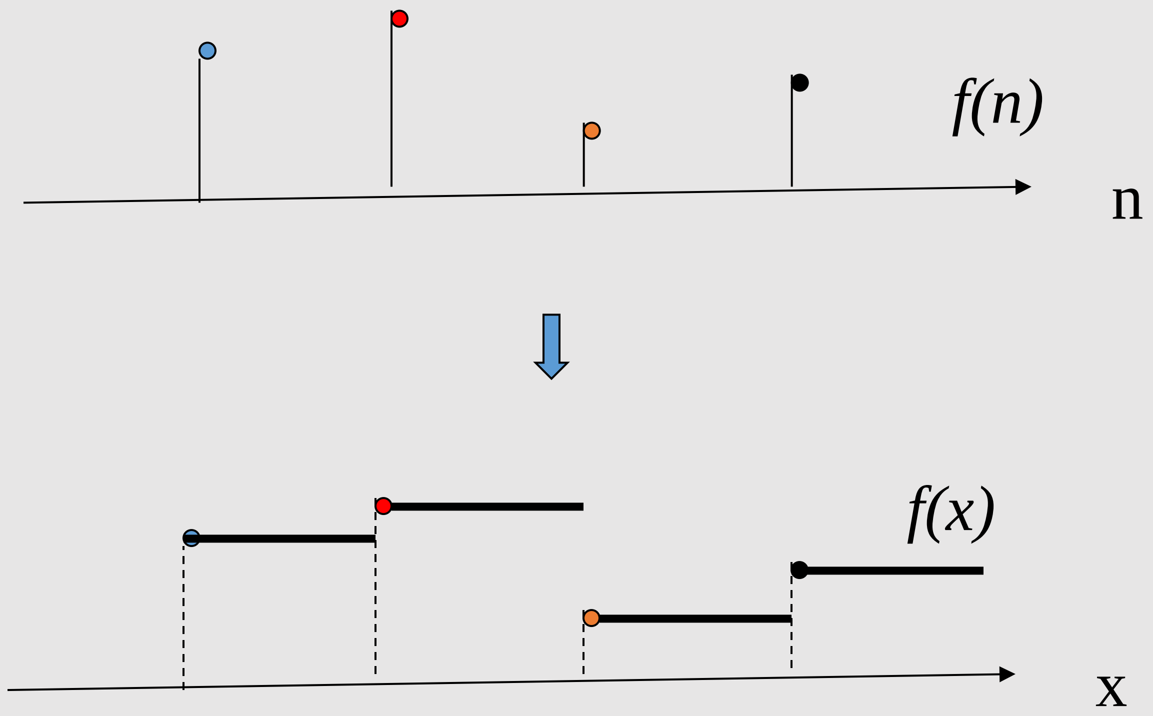


Image Interpolation – Method 1

Nearest Neighbors Interpolation

- Also Known as “**Pixel Replication”** and **“1D Zero-order”**

(Original image rows * zooming factor, Original Image cols * zooming factor)

Image Interpolation – Method 1

Nearest Neighbors Interpolation

Original Image

1	2
3	4

Row wise replication

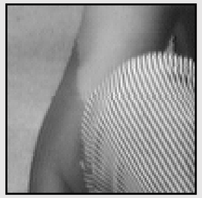
1	1	2	2
3	3	4	4

Column wise replication

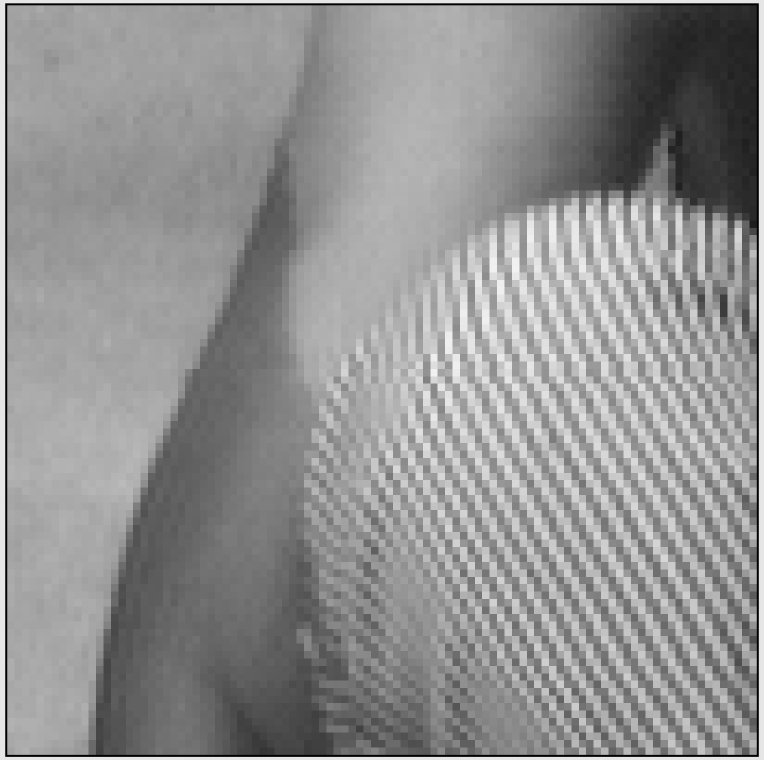
1	1	2	2
1	1	2	2
3	3	4	4
3	3	4	4

Image Interpolation – Method 1

Nearest Neighbors Interpolation



Original Image



Nearest Neighbors Interpolation

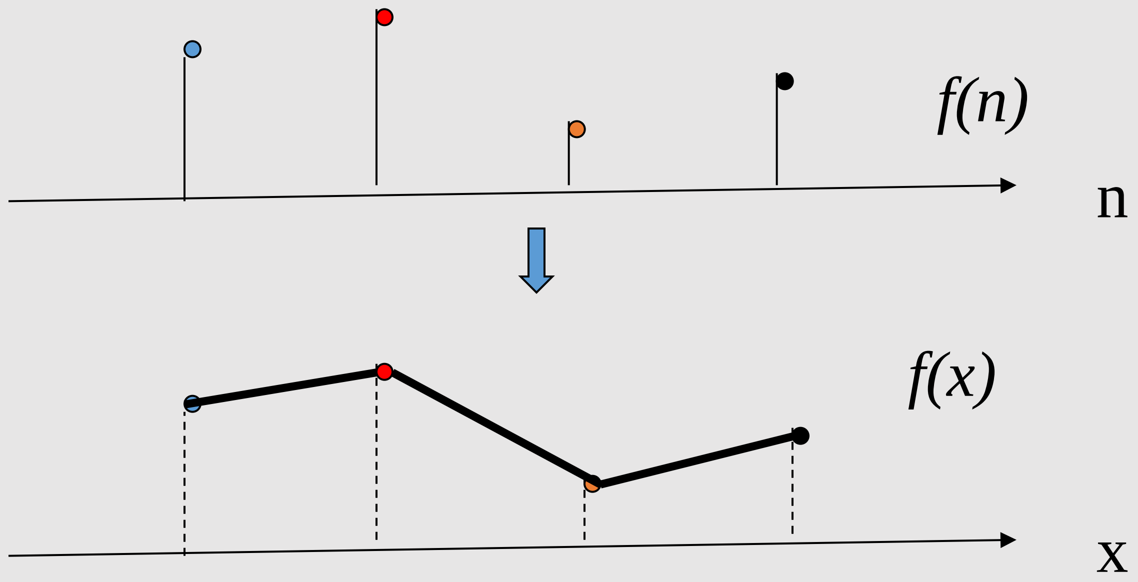
Image Interpolation – Method 1

Nearest Neighbors Interpolation

- **Advantage:**
  - Fast
  - Simple
- **Disadvantage:**
  - Jagged results
  - Result in fully blurred image

Image Interpolation – Method 2

1D First-order Interpolation (Linear)

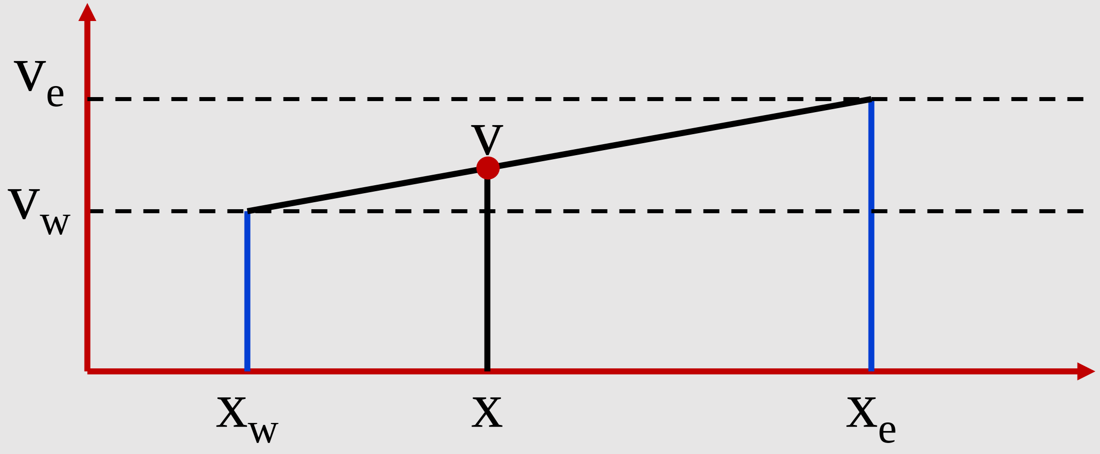


Heuristic: the closer to a pixel, the higher weight is assigned
Principle: line fitting to polynomial fitting (analytical formula)

Image Interpolation – Method 2

1D First-order Interpolation (Linear)

$$\frac{x - x_w}{x_e - x_w} = \frac{v - v_w}{v_e - v_w}$$



- Isolating v in the above equation:

$$\boxed{v = \alpha v_e + (1 - \alpha) v_w}$$

$$where \quad \alpha = \frac{x - x_w}{x_e - x_w}$$

Note: when $\alpha = 0.5$, we simply have the average of two

Image Interpolation – Method 2

1D First-order Interpolation (Linear)

f(n)=[0,120,180,120,0]

↓ Interpolate at 1/2-pixel

f(x)=[0,60,120,150,180,150,120,60,0], x=n/2

↓ Interpolate at 1/3-pixel

f(x)=[0,20,40,60,80,100,120,130,140,150,160,170,180,…], x=n/6

Image Interpolation – Method 2

Bilinear interpolation

- What about in 2D?
  - Interpolate in x, then in y

- Example
  - We know the red values
  - Linear interpolation in x between red values gives us the blue values
  - Linear interpolation in y between the blue values gives us the green value

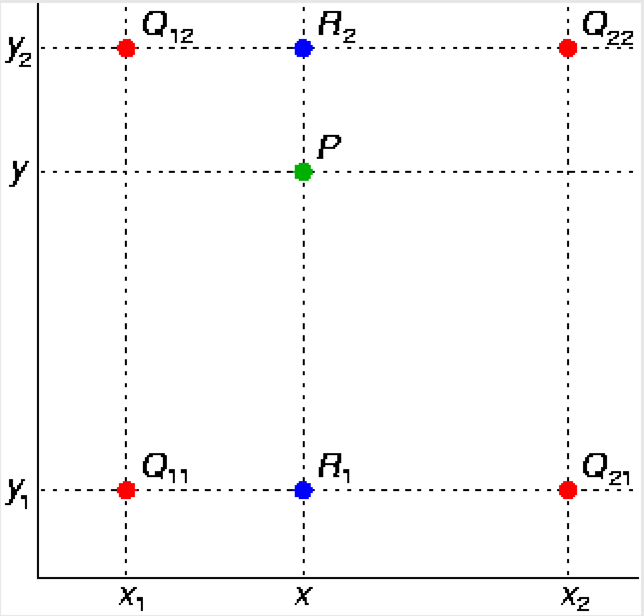
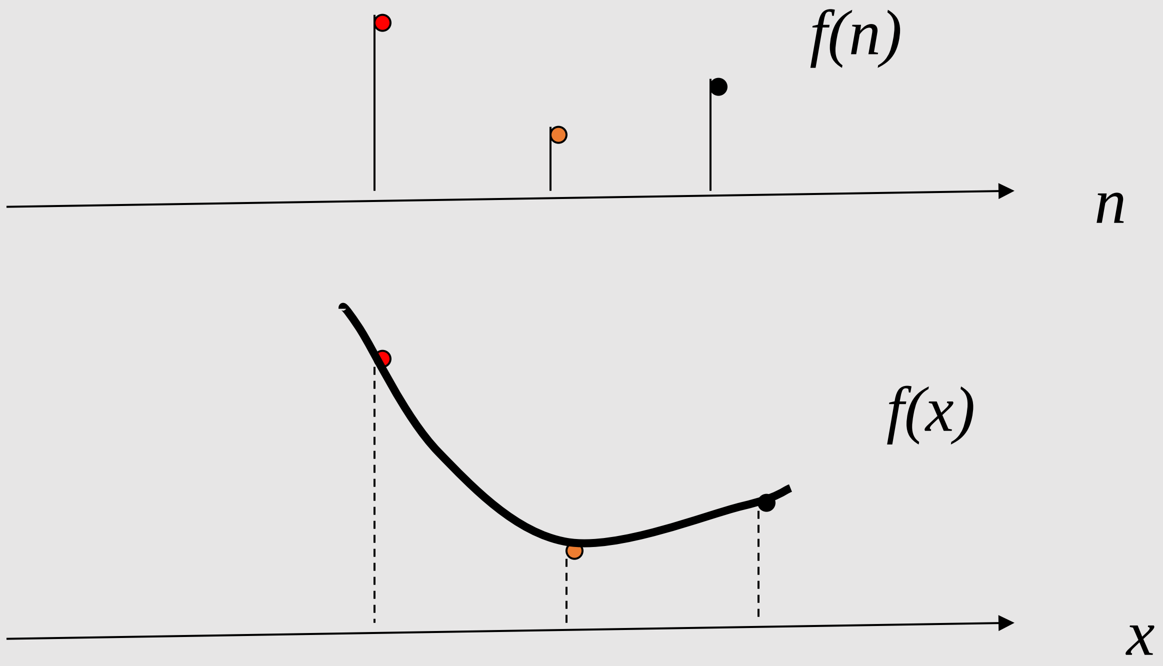


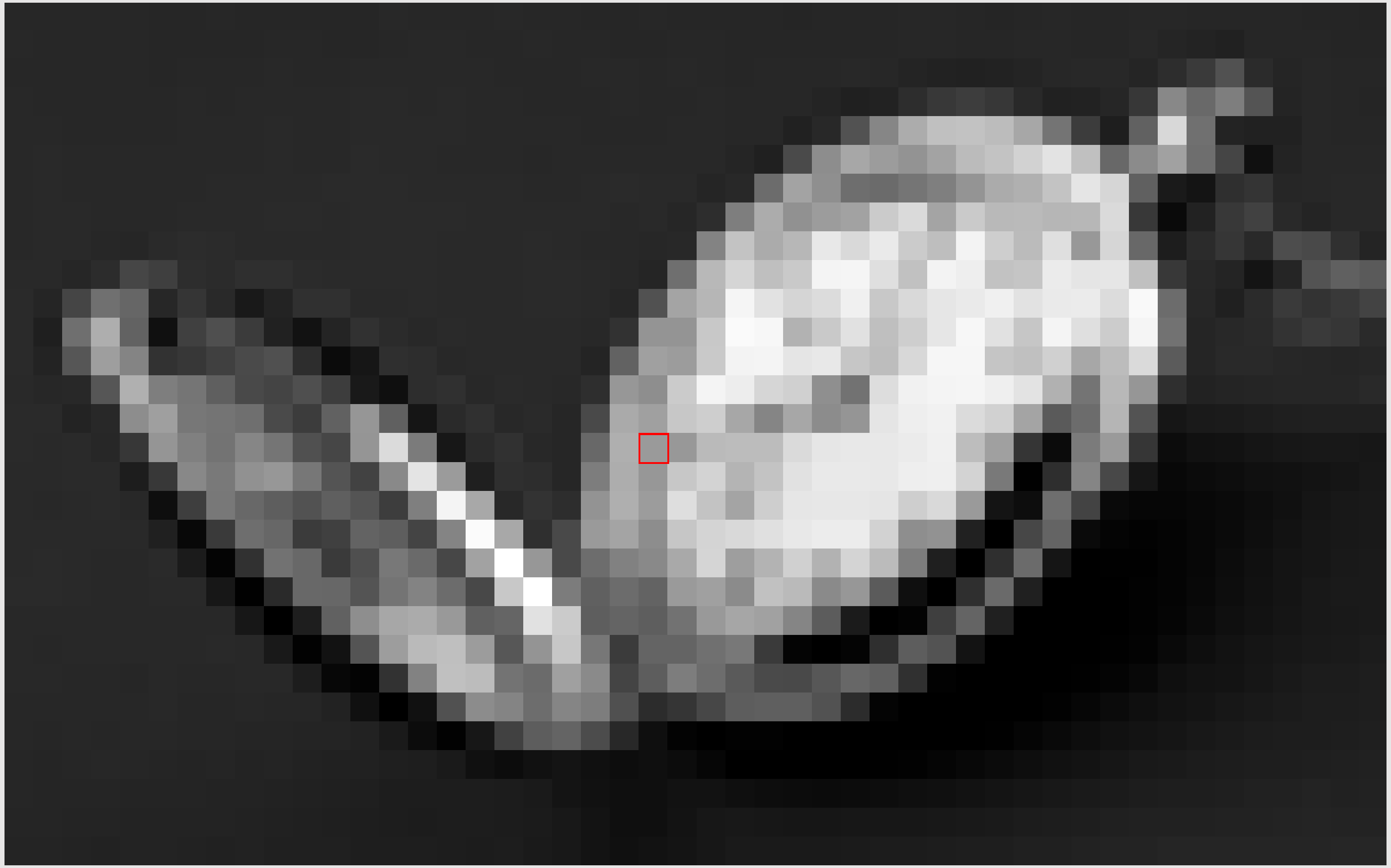
Image Interpolation – Method 3

1D Third order Interpolation (Cubic)



Cubic spline fitting

Nearest Neighbor Interpolation



Bilinear Interpolation

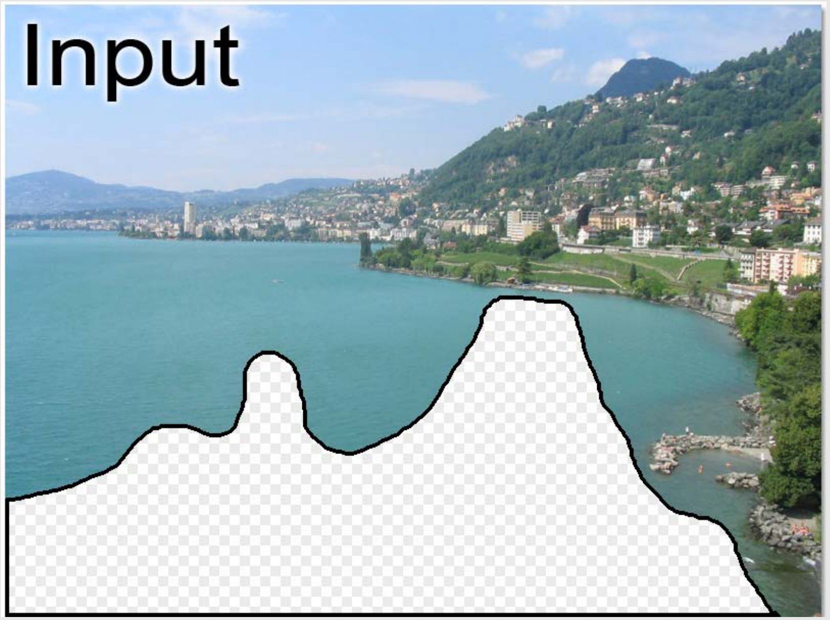
Bicubic Interpolation

Extrapolation

- Suppose you only know the values f(1), f(2), f(3), f(4) of a function
  - What is f(5)?

- This problem is called extrapolation
  - Much harder than interpolation: what is outside the image?

Image Extrapolation



Computed using a database of millions of photos