Department Of Computer Science, CUI Lahore Campus

CSC102 - Discrete Structures

By

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Course Outline

- Sets
  - Set Operations
  - Inclusion-Exclusion Principle of Sets
  - Set Identities
  - Membership Tables

Set Operations

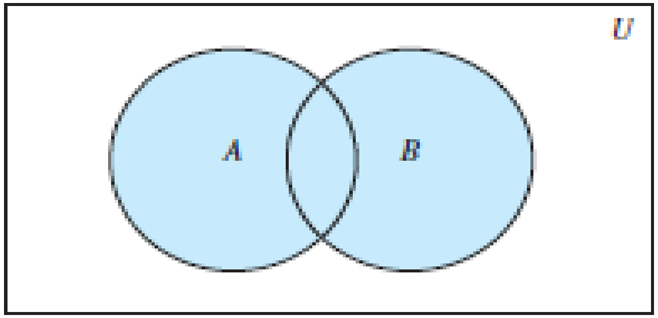
- Two sets can be combined in many different ways.
- Set operations can be used to combine sets.

Union

- Let A and B be sets.

- The **union** of A and B, denoted by **A∪B**, is the set containing those elements that are either in A or in B, or in both.

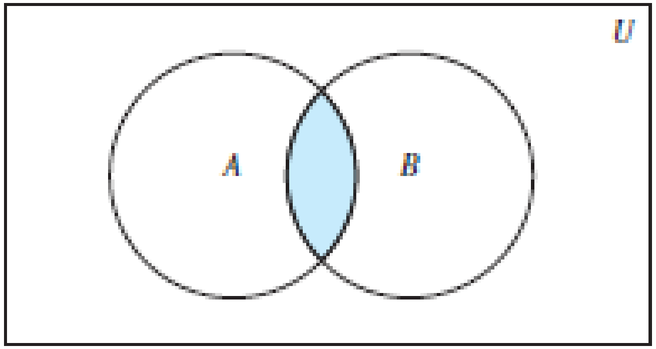
- $A \cup B = \{x \mid x \in A \vee x \in B\}$



$A \cup B$ is shaded.

Intersection

- Let A and B be sets.
- The **intersection** of A and B, denoted by **A ∩ B**, is the set containing those elements in both A and B.
- $A \cap B = \{x \mid x \in A \wedge x \in B\}$



A ∩ B is shaded.

Union (example)

- Let $A = \{1,2,3\}$
  $B = \{2,4,6,8\}$
  $A \cup B = \{1,2,3,4,6,8\}$

- Let $A = \{x \mid x \in \mathbf{Z} \wedge x \text{ is even}\}$
  $B = \{x \mid x \in \mathbf{Z} \wedge x \text{ is odd}\}$
  $A \cup B = \mathbf{Z}$

Intersection (example)

- Let $A = \{1,2,3\}$
  $B = \{2,4,6,8\}$
  $A \cap B = \{ 2 \}$

- Let $A = \mathbf{Z}$
  $B = \{x \mid x \in \mathbf{Z} \wedge x \text{ is odd}\}$
  $A \cap B = \{x \mid x \in \mathbf{Z} \wedge x \text{ is odd}\}$

Disjoint Sets

- Two sets are called **disjoint** if their intersection is empty.

- Let $A = \{x \mid x \in \mathbf{Z} \wedge x \text{ is even}\}$

  $B = \{x \mid x \in \mathbf{Z} \wedge x \text{ is odd}\}$

  $A \cap B = \varnothing$

The Cardinality of the Union of Sets

- **Principle of inclusion-exclusion**
- $|\boldsymbol{A} \cup \boldsymbol{B}| = |\boldsymbol{A}| + |\boldsymbol{B}| - |\boldsymbol{A} \cap \boldsymbol{B}|$

The Cardinality of the Union of Sets

- Find $|A \cup B| =?$

Solution:

- Let $A = \{1,2,3\}$

  $B = \{2,3,4\}$

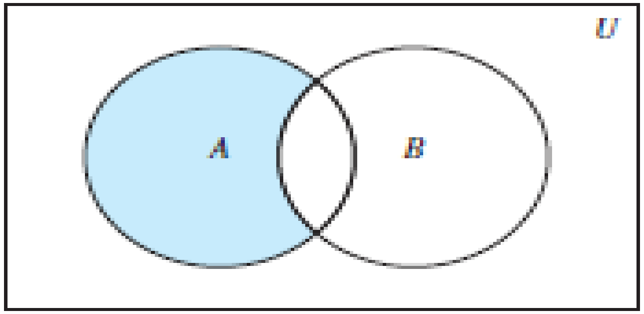
  $A \cap B = \{2,3\}$

  $A \cup B = \{1,2,3,4\}$

- $|\boldsymbol{A}| = \mathbf{3}$ $|\boldsymbol{B}| = \mathbf{3}$ $|\boldsymbol{A} \cap \boldsymbol{B}| = \mathbf{2}$
- $|\boldsymbol{A} \cup \boldsymbol{B}| = \mathbf{4}$

Difference

- Let A and B be sets.
- The **difference** of A and B, denoted by **A - B**, is the set containing those elements that are in A but not in B. (also called **complement of B with respect to A**).
- $A - B = \{x \mid x \in A \land x \notin B\}$



$A - B$ is shaded.

Difference (example)

- Let $A = \{1,2,3\}$

  $B = \{2,4\}$

  $A - B = \{1,3\}$

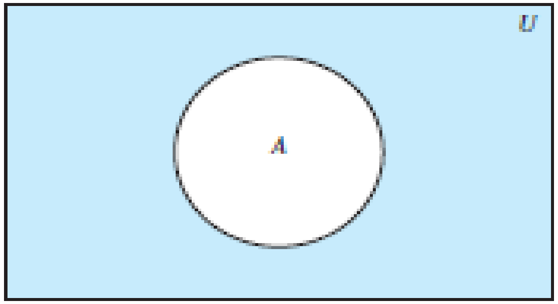
- Let $A = \mathbf{Z}$

  $B = \{ x \mid x \in \mathbf{Z} \wedge x \text{ is odd} \}$

  $A - B = \{ x \mid x \in \mathbf{Z} \wedge x \text{ is even} \}$

Complement

- Let U be the universal set and A be a set.
- The **complement** of A, denoted by $\bar{\mathbf{A}}$, is the complement of A with respect to U (which is U - A).
- $\bar{\boldsymbol{A}} = \{\, \mathbf{x} \mid \boldsymbol{x} \notin \mathbf{A} \,\}$



$\bar{A}$ is shaded.

Complement (example)

- Let A = { a, b, c, d } and

  U is the set of English alphabet

  $\bar{A}$ = { e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z }

- Let A = { x | x ∈ **Z** ∧ x is odd } and

  U is **Z**

  $\bar{A}$ = { x | x ∈ **Z** ∧ x is even }

Summary Set Operations

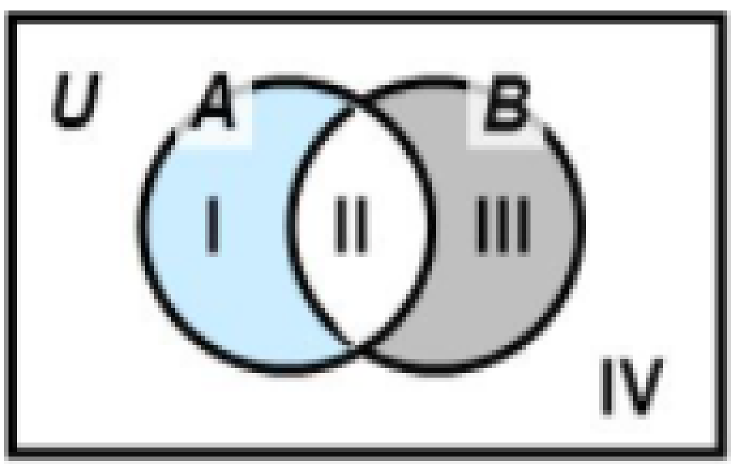
Operation	Notation
Union	$A \cup B = \{x \mid x \in A \vee x \in B\}$
Intersection	$A \cap B = \{x \mid x \in A \wedge x \in B\}$
Difference	$A - B = \{x \mid x \in A \wedge x \notin B\}$
Complement (U - A)	$\overline{A} = \{ x \mid x \notin A \}$

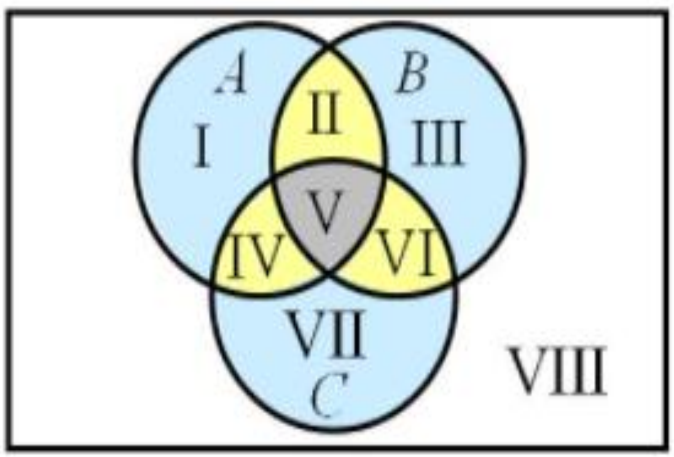
Inclusion-Exclusion Principle of Sets

2 and 3 - Set Venn Diagram

|A ∪ B| = |A| + |B| - |A ∩ B|

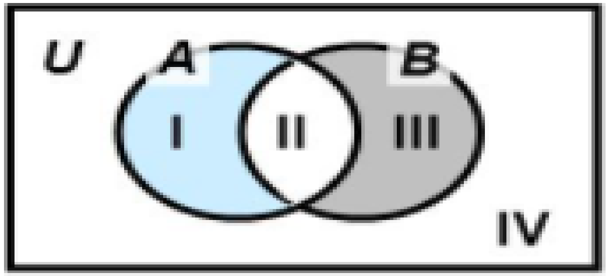
|A ∪ B ∪ C | = |A| + |B| +|C| - |A ∩ B| - |A ∩ C| - |B ∩ C| + |A ∩ B ∩ C |





2 - Set Venn Diagram

- **Region I** represents the elements in set A that are not in set B.
- **Region II** represents the elements in both sets A and B. **Region III** represents the elements in set B that are not in set A.
- **Region IV** represents the elements in the universal set that are in neither set A nor set B.



Example

Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists.

Find the number of students:

- only on list A
- only on list B
- on list A or B (or both)
- on exactly one list.

Example

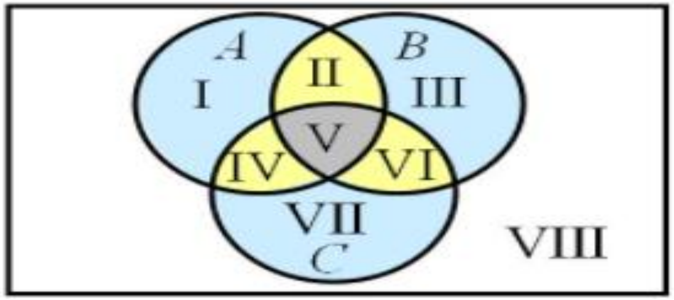
Solution:

- $30 - 20 = 10$ names are only on list A.
- $35 - 20 = 15$ are only on list B.
- $|A \cup B| = |A| + |B| - |A \cap B| = 30 + 35 - 20 = 45.$
- $10 + 15 = 25$ names are only on one list; that is,

  $|A \oplus B| = 25.$

3 - Set Venn Diagram

- **Region I** represents the elements in set A but not in set B or set C.
- **Region II** represents the elements in set A and set B but not in set C.
- **Region III** represents the elements in set B but not in set A or set C.
- **Region IV** represents the elements in sets A and C but not in set B.
- **Region V** represents the elements in sets A, B, and C.
- **Region VI** represents the elements in sets B and C but not in set A.
- **Region VII** represents the elements in set C but not in set A or set B.
- **Region VIII** represents the elements in the universal set U, but not in set A, B, or C.



Example

Consider the following data for 120 mathematics students at a college concerning the languages French, German, and Russian:

65 study French, 45 study German,

42 study Russian , 20 study French and German,

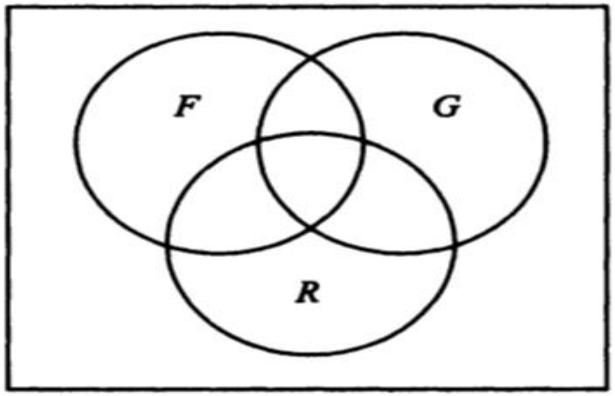
25 study French and Russian,

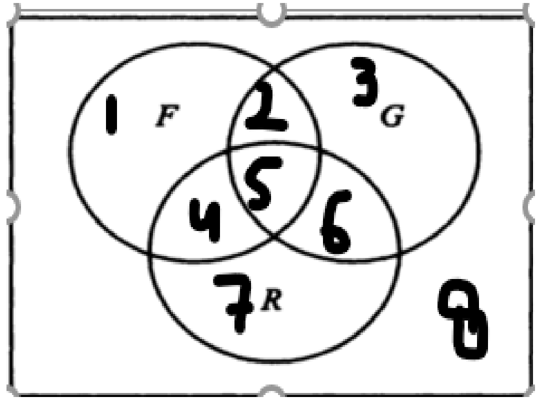
15 study German and Russian.

8 study all three languages.

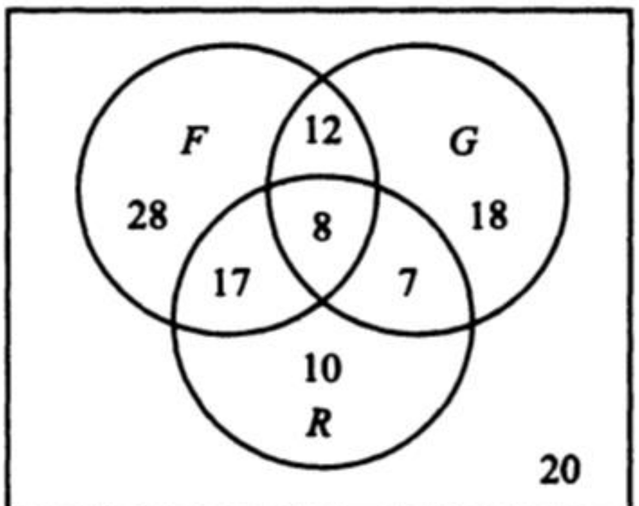
Determine how many students study exactly 1 language course and fill the correct numbers of students in each eight region of Venn diagram shown in figure.

Example





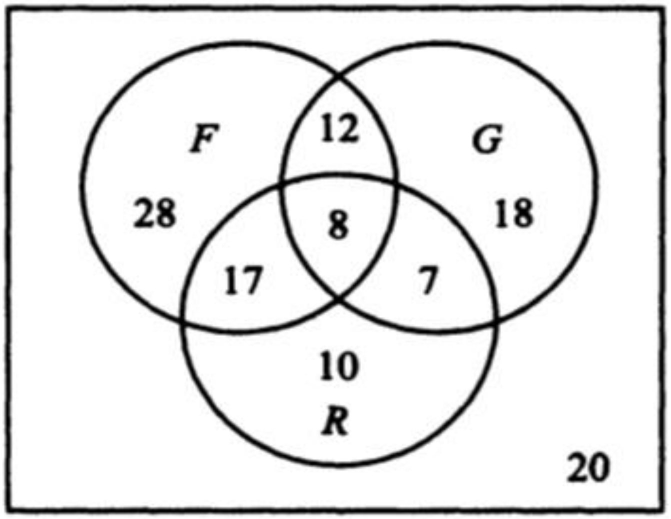
$$U = 120$$
$$F = 65$$
$$G = 45$$
$$R = 42$$
$$F \cap G = 20$$
$$F \cap R = 25$$
$$G \cap R = 15$$
$$F \cap G \cap R = 8$$



$$R1 - |F| = 65 - 12 - 8 - 17 = 28$$
$$R3 - |G| = 45 - 12 - 8 - 7 = 18$$
$$R7 - |R| = 42 - 17 - 8 - 7 = 10$$
$$R2 - |F \cap G| = 20 - 8 = 12$$
$$R4 - |F \cap R| = 25 - 8 = 17$$
$$R6 - |G \cap R| = 15 - 8 = 7$$
$$R5 - |F \cap G \cap R| = 8$$
$$|F \cup G \cup R| = 65+45+42-20-25-15+8=100$$
$$R8 - |U| - |F \cup G \cup R| = 120 - 100 = 20$$

Example

- Total number of students exactly registered in one course = 28+18+10=56



Set Identities

$A \cup \emptyset = A$ $A \cap U = A$	Identity Laws
$A \cup U = U$ $A \cap \emptyset = \emptyset$	Domination Laws
$A \cup A = A$ $A \cap A = A$	Idempotent Laws
$\overline{(\overline{A})} = A$	Complementation Law
$A \cup \overline{A} = U$ $A \cap \overline{A} = \emptyset$	Complement Laws

Set Identities

$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Laws

Set Identities

$\overline{A \cup B} = \overline{A} \cap \overline{B}$ De Morgan's Law

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A - B = A \cap \overline{B}$ Difference Law

How to Prove a Set Identity

- Four methods:
  - Use membership tables
  - Use the basic set identities
  - Prove each set is a subset of each other
  - Use set builder notation and logical equivalences

What is a membership table

- Membership tables show all the combinations of sets an element can belong to
  - 1 means the element belongs, 0 means it does not
- Consider the following membership table:

A	B	A ∪ B	A ∩ B	A - B
1	1	1	1	0
1	0	1	0	1
0	1	1	0	0
0	0	0	0	0

Membership Table

A	B	B-A	$\overline{A}$	$\overline{B}$	$\overline{A \cap B}$	$\overline{A} \cup \overline{B}$
1	1	0	0	0	0	0
1	0	0	0	1	1	1
0	1	1	1	0	1	1
0	0	0	1	1	1	1

Membership Table

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Distributive Law

A	**B**	**C**	**B ∩ C**	**A ∪ (B ∩ C)**	**A ∪ B**	**A ∪ C**	**(A ∪ B) ∩ (A ∪ C)**
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

Set Identities (example)

- Show $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$

Set Identities (example)

- Show $\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

Solution:

$\overline{A \cup (B \cap C)}$

$= \bar{A} \cap (\overline{B \cap C})$ (By DeMorgan′s Law)

$= \bar{A} \cap (\bar{B} \cup \bar{C})$ (By DeMorgan′s Law)

$= \bar{A} \cap (\bar{C} \cup \bar{B})$ (By CommutativeLaw)

$= (\bar{C} \cup \bar{B}) \cap \bar{A}$ (By Commutative Law)

Set Identities

$$A \cup (B - A) = A \cup B$$

$$L.H.S = A \cup (B - A)$$

$= A \cup (B \cap \overline{A})$ Difference Law

$= (A \cup B) \cap (A \cup \overline{A})$ Distribution Law

$= (A \cup B) \cap \bigcup$ Complement Law

$= A \cup B$ Identity Law

$= R.H.S$

Exercise Questions

Chapter # 2

Topic # 2.2

Question # 1, 2, 3,4,5,6,15,16,17,18, 19,20,21,22,23,24,25