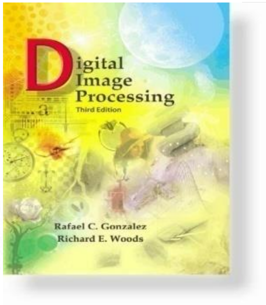


- Fundamentals of Spatial Filtering:

$$g(x,y)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)f(x+s,y+t)$$

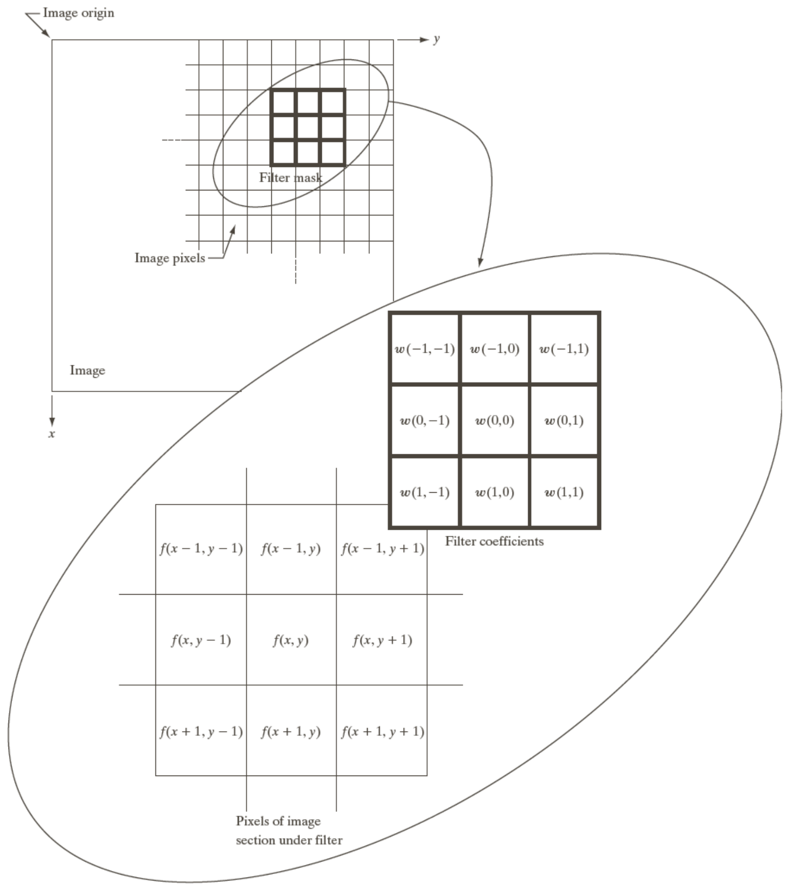
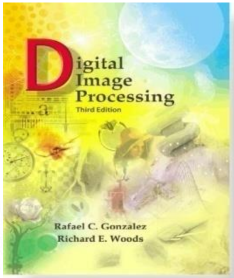


FIGURE 3.28 The mechanics of linear spatial filtering using a 3 × 3 filter mask. The form chosen to denote the coordinates of the filter mask coefficients simplifies writing expressions for linear filtering.



Intensity Transformations and Spatial Filtering

- Spatial Correlation (☆) and Convolution (★)

Correlation		Convolution	
(a)	Origin f: 0 0 0 1 0 0 0 0 w: 1 2 3 2 8	Origin f: 0 0 0 1 0 0 0 0 w rotated 180°: 8 2 3 2 1	(i)
(b)	0 0 0 1 0 0 0 0 1 2 3 2 8 Starting position alignment	0 0 0 1 0 0 0 0 8 2 3 2 1	(j)
(c)	Zero padding 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(k)
(d)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8 Position after one shift	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(l)
(e)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8 Position after four shifts	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(m)
(f)	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 2 3 2 8 Final position	0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 8 2 3 2 1	(n)
(g)	Full correlation result 0 0 0 8 2 3 2 1 0 0 0 0	Full convolution result 0 0 0 1 2 3 2 8 0 0 0 0	(o)
(h)	Cropped correlation result 0 8 2 3 2 1 0 0	Cropped convolution result 0 1 2 3 2 8 0 0	(p)

FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

Intensity Transformations and Spatial Filtering

- Spatial Correlation (☆) and Convolution (★)

Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

(a)

Padded f

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(b)

Initial position for w

1	2	3	0	0	0	0	0	0
4	5	6	0	0	0	0	0	0
7	8	9	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c)

Full correlation result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	9	8	7	0	0	0
0	0	0	6	5	4	0	0	0
0	0	0	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d)

Cropped correlation result

0	0	0	0	0
0	9	8	7	0
0	6	5	4	0
0	3	2	1	0
0	0	0	0	0

(e)

Rotated w

9	8	7	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(f)

Full convolution result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	0	0	0
0	0	0	4	5	6	0	0	0
0	0	0	7	8	9	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

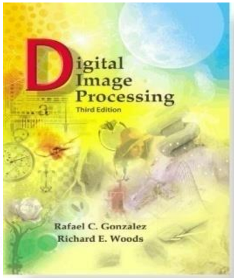
(g)

Cropped convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

(h)

FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.



- Spatial Filters

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

FIGURE 3.31 Another representation of a general 3×3 filter mask.

Linear Filters (averaging, lowpass)

$\frac{1}{9} \times$

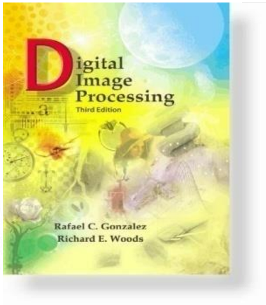
1	1	1
1	1	1
1	1	1

$\frac{1}{16} \times$

1	2	1
2	4	2
1	2	1

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.

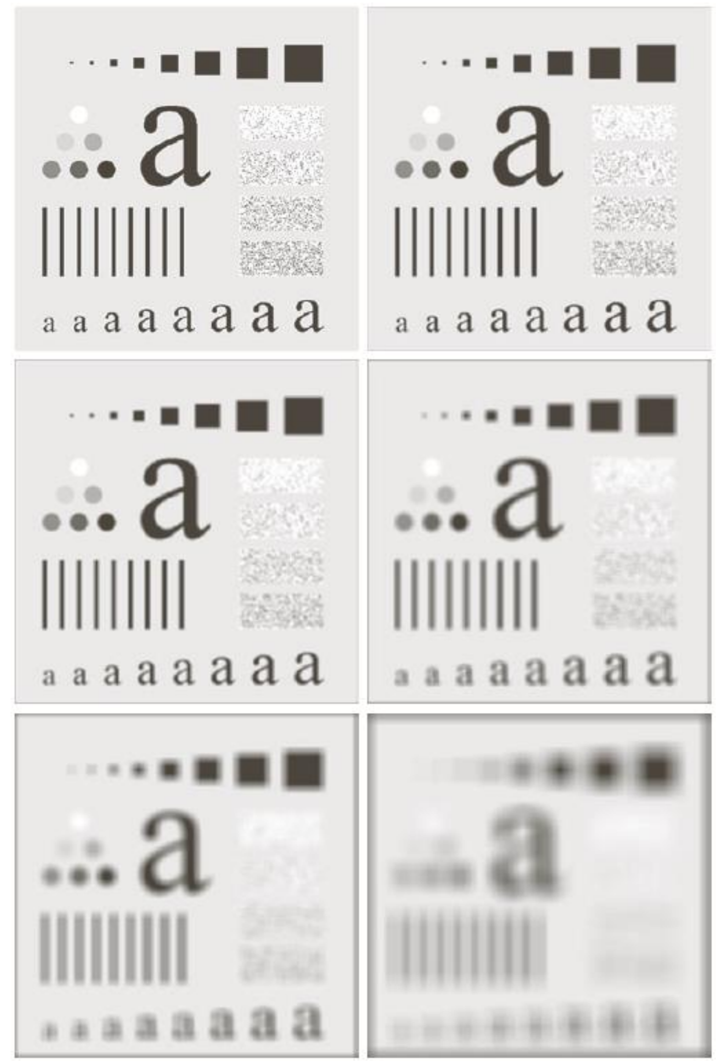


Intensity Transformations and Spatial Filtering

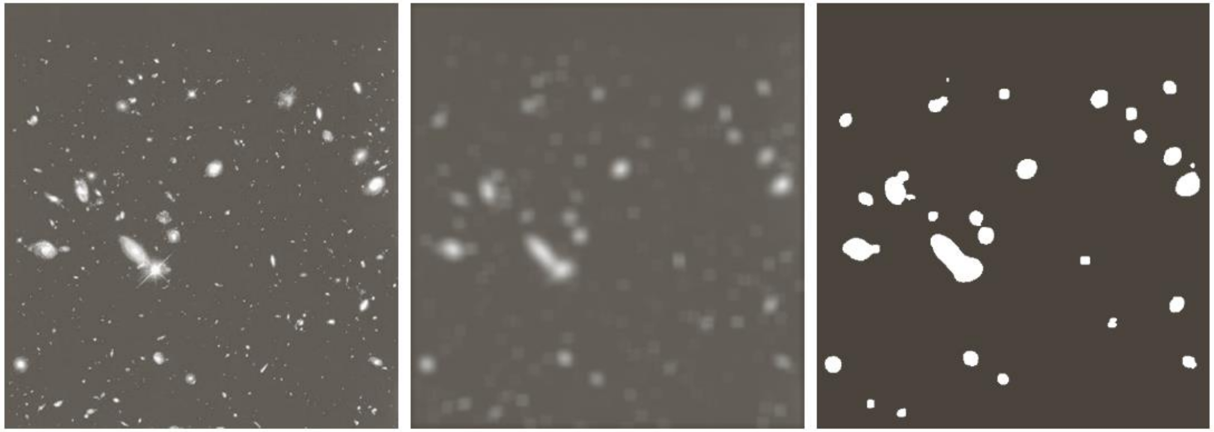
Square averaging Filter:

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $m = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a b
c d
e f



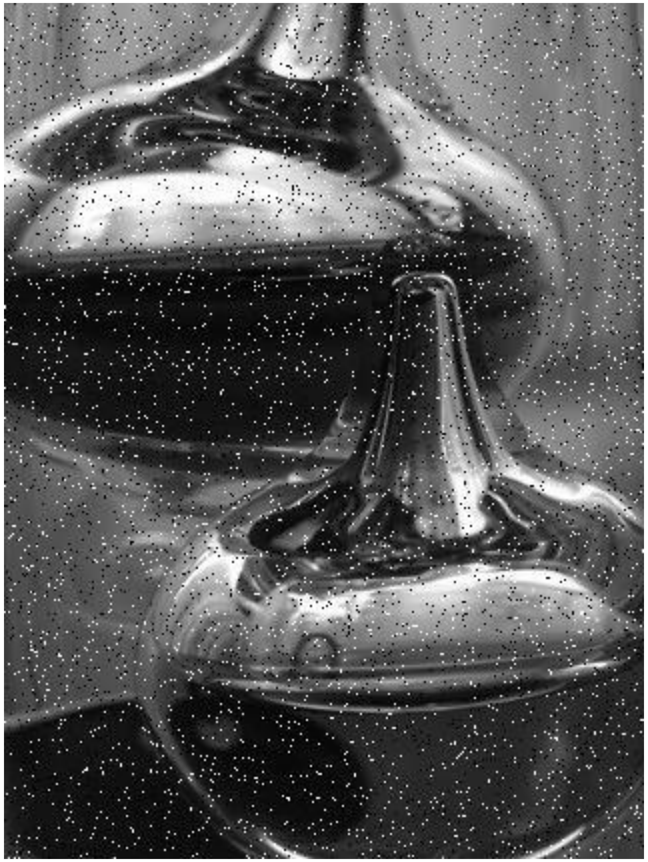
- Blurring Usage:
  - Delete unwanted (small) subjects.



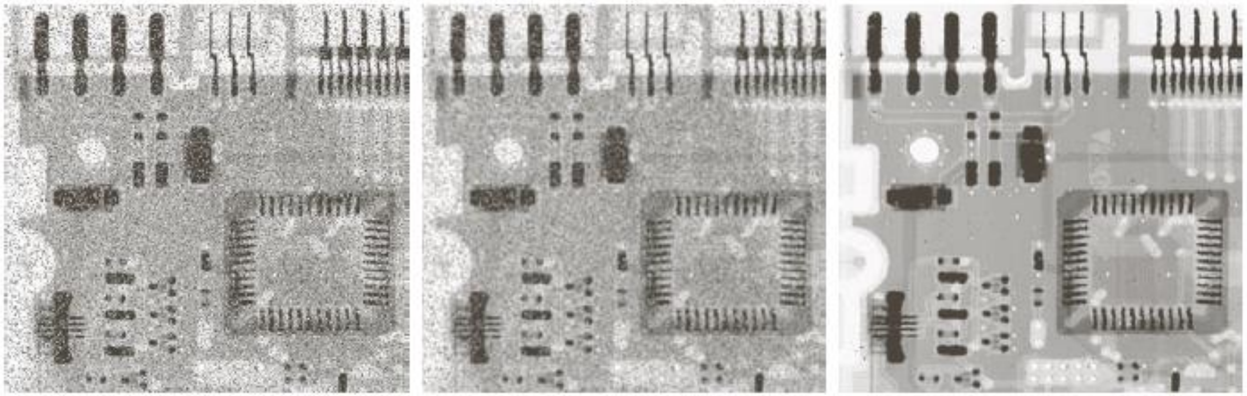
a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

- Order Statistics Filters:
  - Impulsive noise:
    - Mono Level : Salt, Pepper noises
    - Bi Level: Salt-Pepper noises
  - Filter
    - Median
    - Max
    - Min

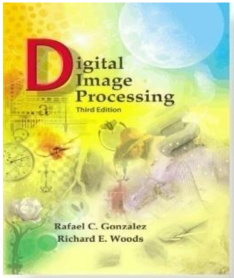


- Example:



a b c

FIGURE 3.35 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

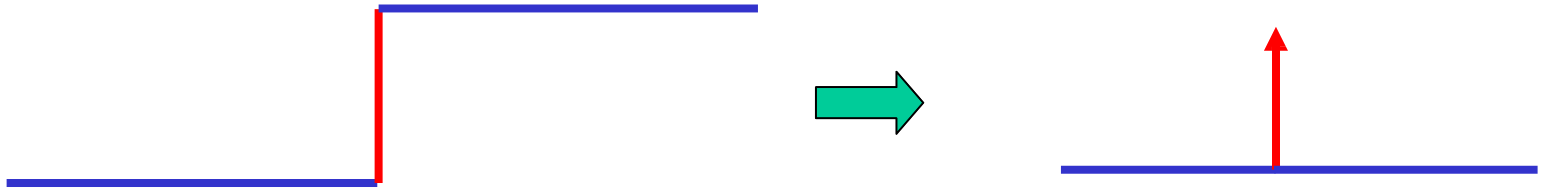


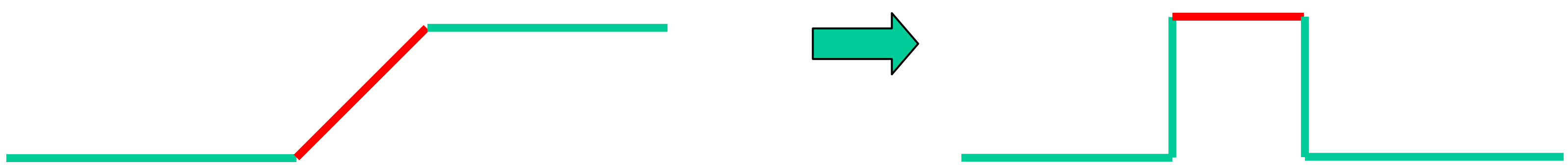
- Sharpening Spatial Filter
  - Highlights Intensity Transitions
  - First and Second order Derivatives

$$\frac{\partial f}{\partial x} \approx \begin{cases} f(x+1,y)-f(x,y) \\ f(x,y)-f(x-1,y) \\ 0.5\big(f(x+1,y)-f(x-1,y)\big) \end{cases}$$

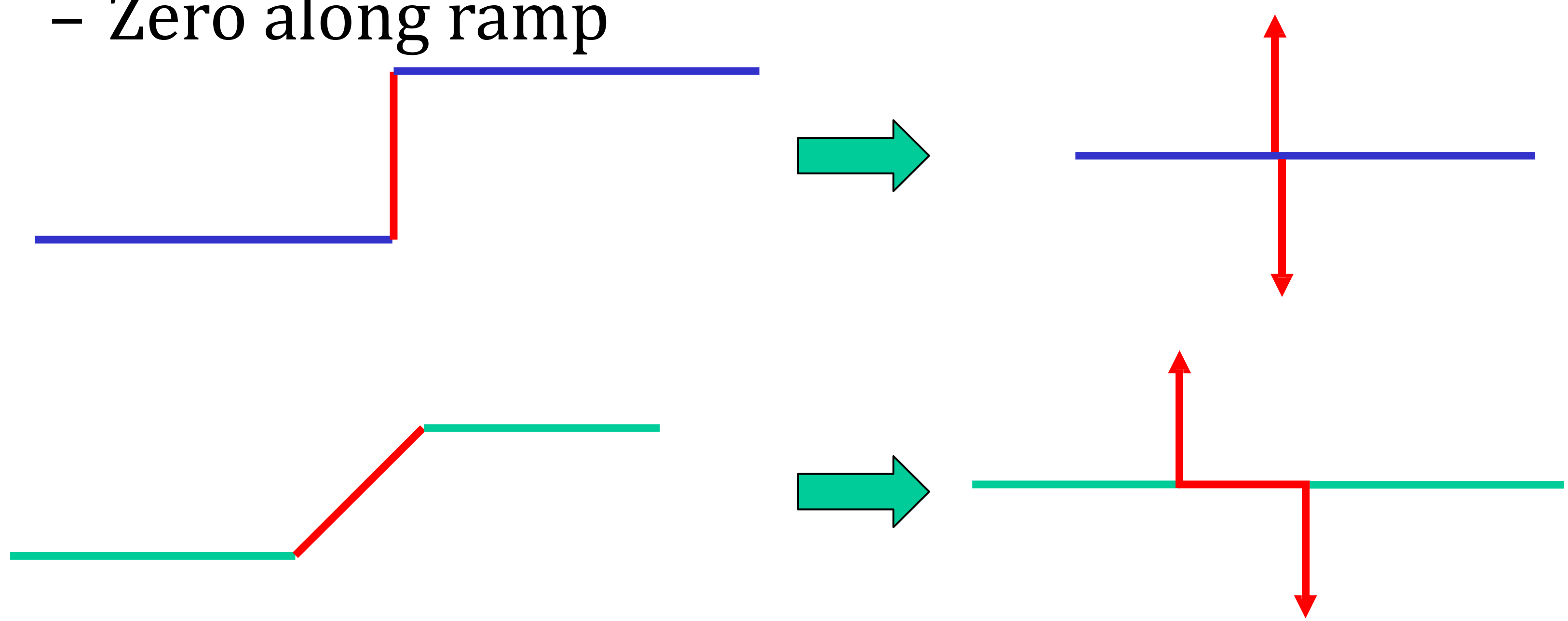
$$\frac{\partial^2 f}{\partial x^2} \approx f(x+1,y)-2f(x,y)+f(x-1,y)$$

- First Order Derivative:
  - Zero in flat region
  - Non-zero at start of step/ramp region
  - Non-zero along ramp

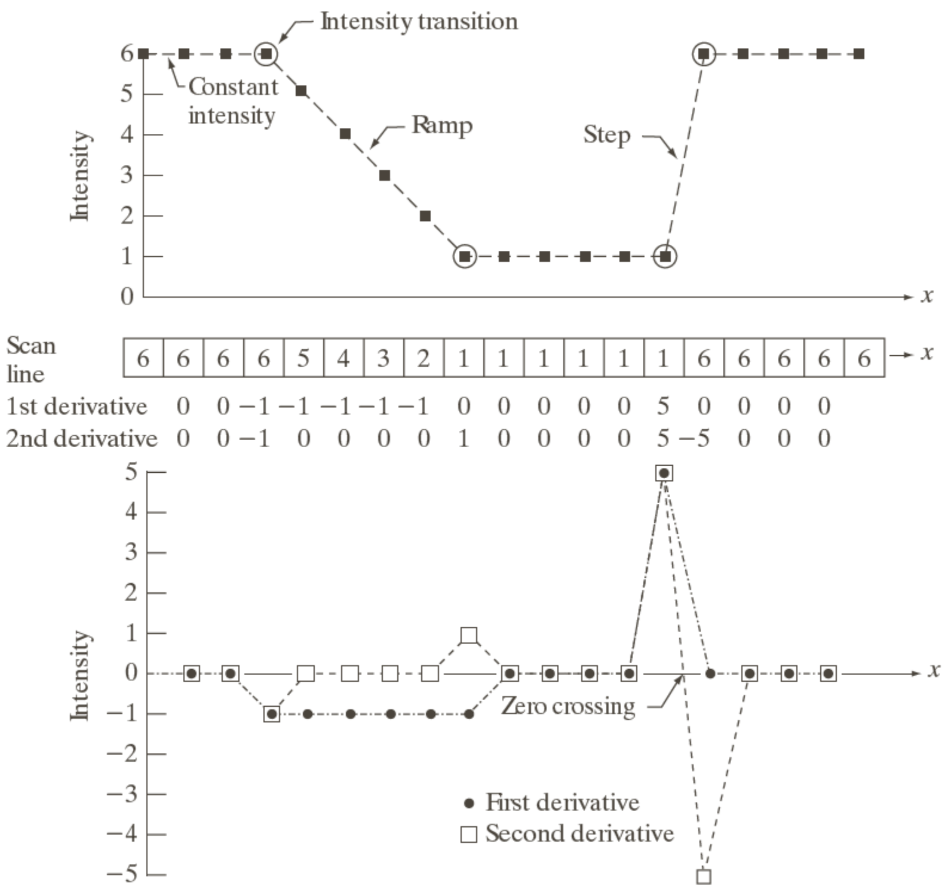




- Second Order Derivative:
  - Zero in flat region
  - Non-zero at start/end of step/ramp region
  - Zero along ramp

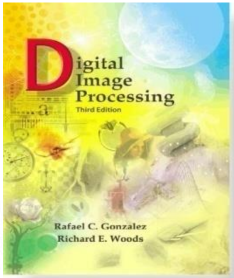


- Comparison:



a
b
c

FIGURE 3.36 Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.



- 1^{st} and 2^{nd} Order Derivative Comparison:
  - First Derivative:
    - Thicker Edge;
    - Strong Response for step changes;
  - Second Derivative:
    - Strong response for fine details and isolated points;
    - Double response at step changes.

- Laplacian as an isotropic Enhancer:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Discrete Implementation:

$$\nabla^2 f = \left[f(x+1, y) + f(x, y+1) + f(x-1, y) + f(x, y-1) - 4f(x, y) \right]$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \; 90^\circ \; isotropic \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} \; 45^\circ \; isotropic$$

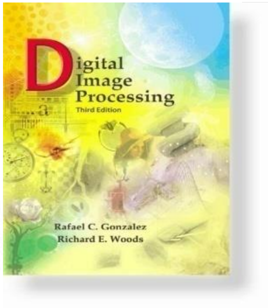
- Laplacian Masks

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1
0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

Practically use:

a b
c d

FIGURE 3.37 (a) Filter mask used to implement Eq. (3.6-6). (b) Mask used to implement an extension of this equation that includes the diagonal terms. (c) and (d) Two other implementations of the Laplacian found frequently in practice.



- Background Recovering:

$$g(x,y)=\begin{cases} f(x,y)-c[\nabla^2 f(x,y)] & c = -1 \\ f(x,y)+c[\nabla^2 f(x,y)] & c = +1 \end{cases}$$

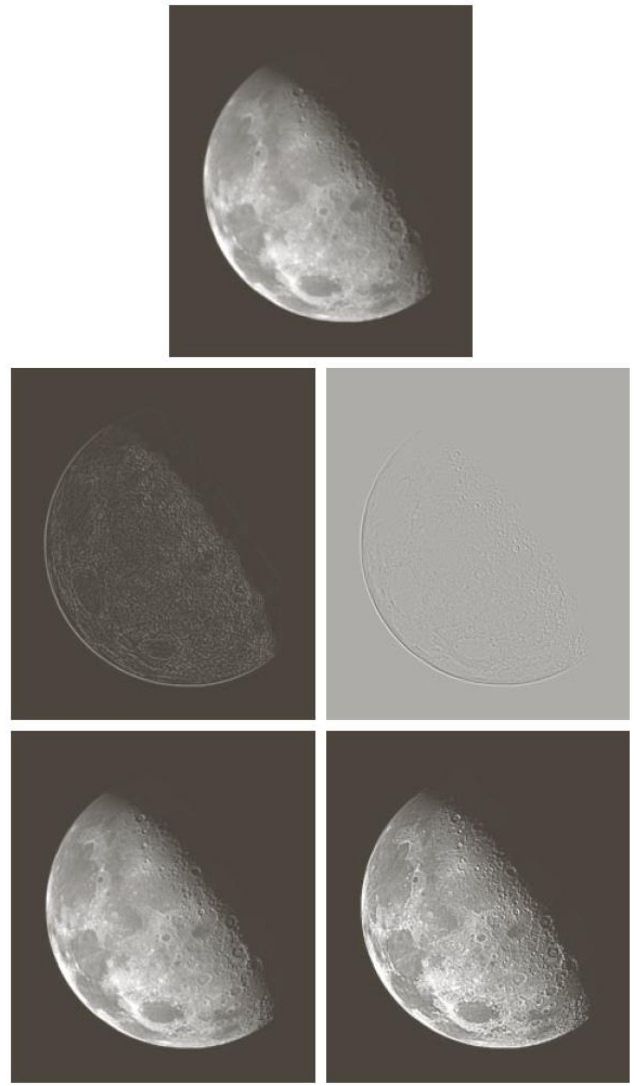
$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & +5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad 90^\circ \; isotropic \qquad \begin{bmatrix} -1 & -1 & -1 \\ -1 & +9 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad 45^\circ \; isotropic$$

Intensity Transformations and Spatial Filtering

- Example:

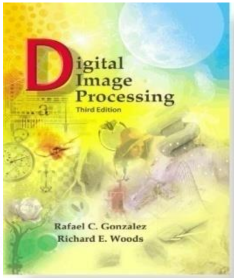
Non-Scaled and scaled Laplacian

Sharpened using 90º and 45º degree isotropic Laplacian



a
b c
d e

FIGURE 3.38 (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)



- Unsharp Masking and High-Boost Filtering:

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y), \quad \bar{f}(x,y): \text{Blurred image}$$

$$\mathrm{g}(x,y) = f(x,y) + k * g_{mask}(x,y)$$

- $k \geq 0$
  - $k=1$: Unsharp Masking
  - $k>1$: High Boost
- Another mask:
  - Laplacian and any highpass filter

- One Dimensional Illustration

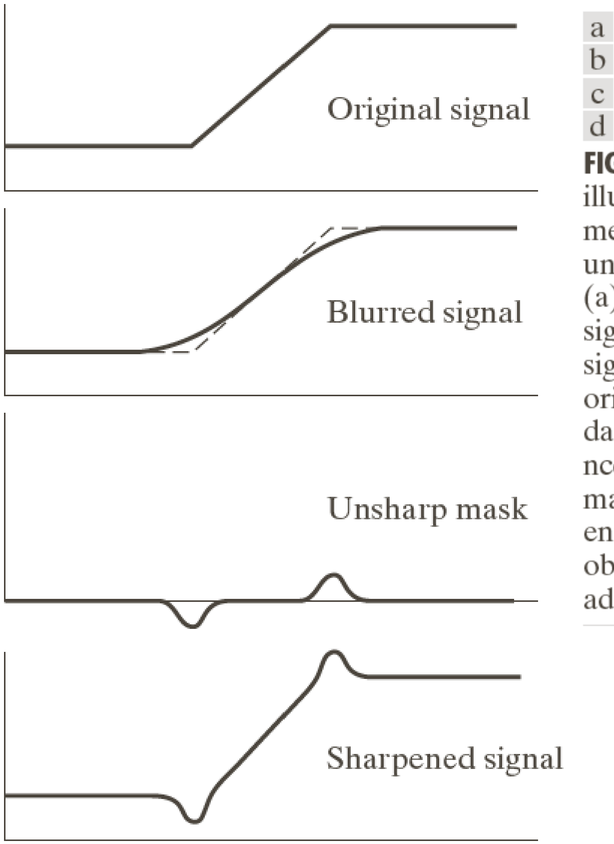


FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

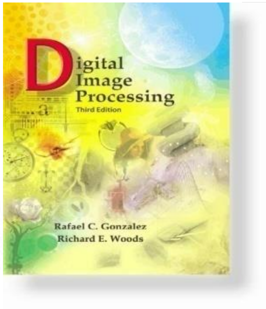
- Two Dimensional Example

Blurred, 5×5, σ=3

Unsharp mask

Unsharp masking , k=1

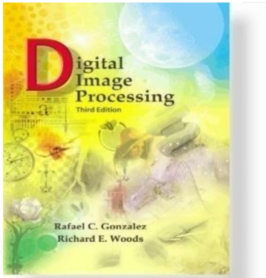
Highboost, k=4.5



- First Derivative - Gradient:

$$\nabla f = \begin{bmatrix} G_x & G_y \end{bmatrix}^T = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}^T$$

$$|\nabla f| = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2} \approx \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right|$$



- Discrete Implementation

Roberts Cross Gradient

$$G_x = (z_9 - z_5) \qquad G_y = (z_8 - z_6)$$

Sobel Gradient

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

(a)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(b)

−1	0
0	1

(c)

0	−1
1	0

(d)

−1	−2	−1
0	0	0
1	2	1

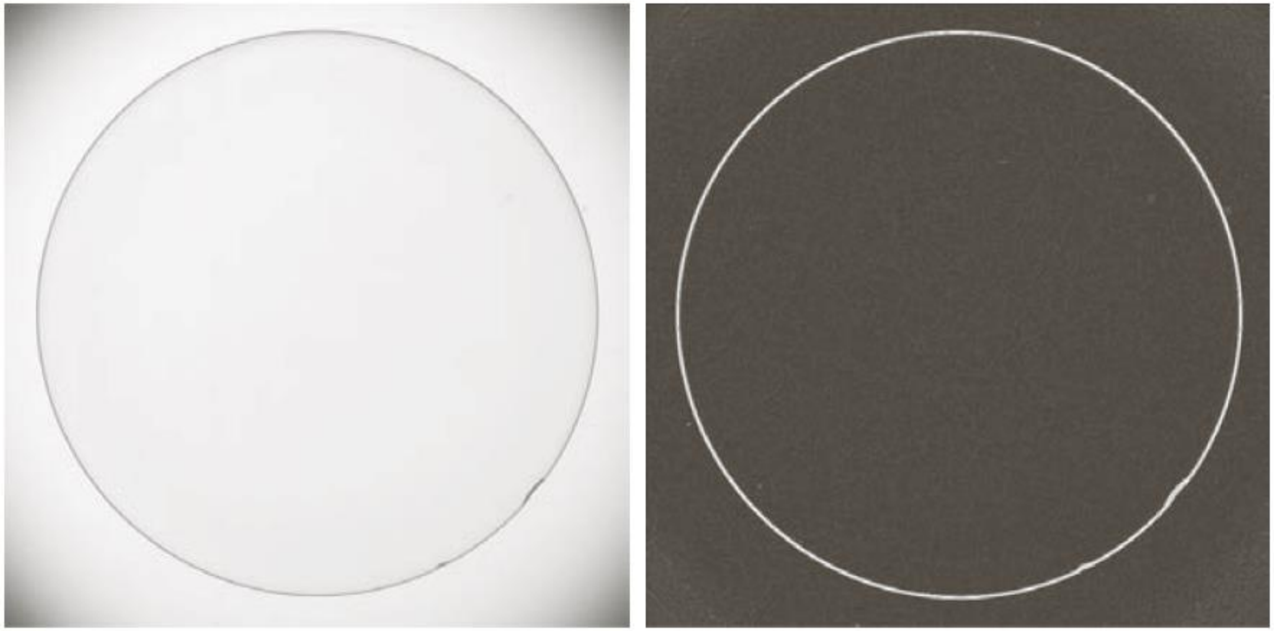
(e)

−1	0	1
−2	0	2
−1	0	1

a
b c
d e

FIGURE 3.41 A 3 × 3 region of an image (the zs are intensity values). (b)–(c) Roberts cross gradient operators. (d)–(e) Sobel operators. All the mask coefficients sum to zero, as expected of a derivative operator.

- Example (Sobel Mask):



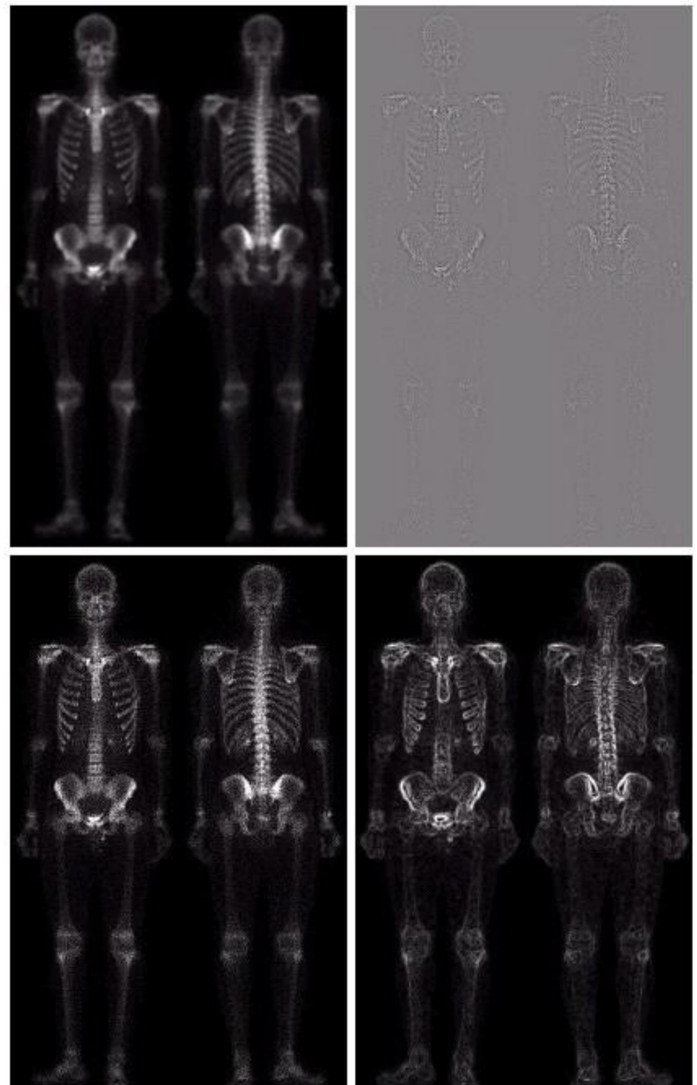
a b

FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)

- Combination:

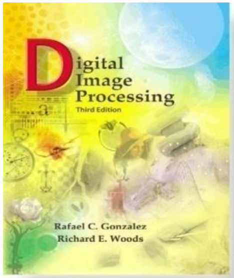
Bone Scan

Scaled Laplacian



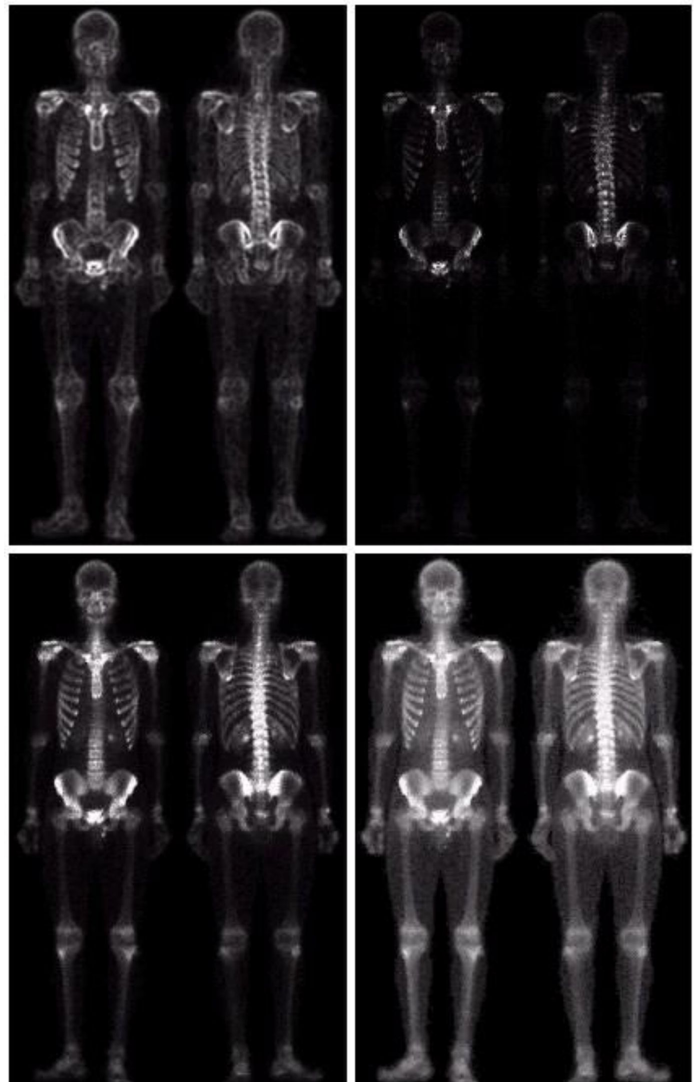
Original+Laplacian

Sobel of Original



Intensity Transformations and Spatial Filtering

- Combination:

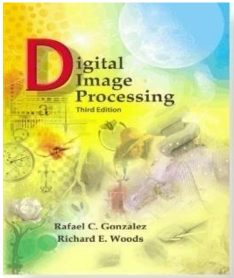


Smoothed Sobel

(Orig. + L.)*S.Sobel

Orig.+ (Orig.+L.)*S.Sobel

Apply Power-Law



Intensity Transformations and Spatial Filtering

Chapter # 3

- # Assignment statement:

  Apply the concepts of image enhancement in spatial domain to solve exercise problems related to spatial filtering for smoothing and sharpening and histogram processing.

- # End problems:

  All questions except: 2,3,8,9,11,14,17,22,27,30-34