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Digital Logic Design

bse 133

Assignment # 02

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(2.2)

Minimize literals:

a) $xy + xy'$

$= x(y + y')$

$= x(1)$

$\Rightarrow x$

b) $(x+y)(x+y')$

$= x + xy' + xy + yy'$

$= x + xy' + xy + 0$

$= x + x(y+y')$

$= x + x(1)$

$= x$

c) $xyz + x'y + xyz'$

$= xy(z+z') + x'y$

$= xy + x'y$

$= y(x+x')$

$= y$

d) $(x+y)'(x'+y')'$

Apply De-morgan's law

$= (x'y')(xy)$

$= 0$

② BSE-13

e) $(a+b+c')(a'b'+c)$

$= aa'b' + ac + a'bb' + cc'$

$= (0)b' + ac + a'(0) + 0$

$\Rightarrow ac$

f) $a'bc + abc' + abc + a'bc'$

$= a'bc + ab(c'+c) + a'bc'$

$= ab + a'bc + a'bc'$

$\cancel{ab + bc + a}$

$= ab + a'b(c+c')$

$= ab + a'b$

$= (a+a')b$

$\Rightarrow b$

2.4

Reduce to ~~f~~ minimum number of literals

a) $x'y'z' + y + xy'z'$

$= (x'+x)\,y'z' + y$

$\Rightarrow y'z + y$

$\cancel{x'yz + xz}$

b) $x'y(x'+z') + x'y + xyz$

$= xy + x'yz' + x'y + xyz$ BSE-133

$= (x+x')y + x'yz' + xyz$

$= y + y(x'z' + xz)$

$\Rightarrow y$

Verification by K-maps

$x \backslash yz$	00	01	11	10
0	0 (0)	0 (1)	1 (3)	1 (2)
1	0 (4)	0 (5)	1 (7)	1 (6)

$= y$

x	y	z	F	
0	0	0	0	
0	0	1	0	
0	1	0	1	2
0	1	1	1	3
1	0	0	0	
1	0	1	0	
1	1	0	1	6
1	1	1	1	7

c)

$(x+yz)' + (x+y'z')'$

$= (x')(y'+z') + (x')(y+z)$

$= x'y' + x'z' + x'y + x'z$

$= x'(y'+y) + x'z' + x'z$

$= x' + x'z' + x'z$

$= x'(1+z'+z)$

$= x'$

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d) $(w'+x)(w+y)(x'+y)(w+xyz)$

$= (w'\cancel{w} + w'y + xw + xy)(x'+y)(w+xyz)$

$= (x'w'y + w'y + x\cancel{x'}w + xwy + x\cancel{x'}y + xy)(w+xyz)$

$= (x'w'y + w'y + xwy + xy)(w+xyz)$

$= x'w'\cancel{w}y + xx'\cancel{w'}yz + w\cancel{w'}y + w'yz + xwy$
$+ xwyz + xwy + xyz$

$= w'yz + xwy + xwyz + xyz$

e) $wxy'z' + wy' + wx'y'z'$

$= wy'z'(x+x') + wy'$

$= wy'z' + wy'$

(2.6)

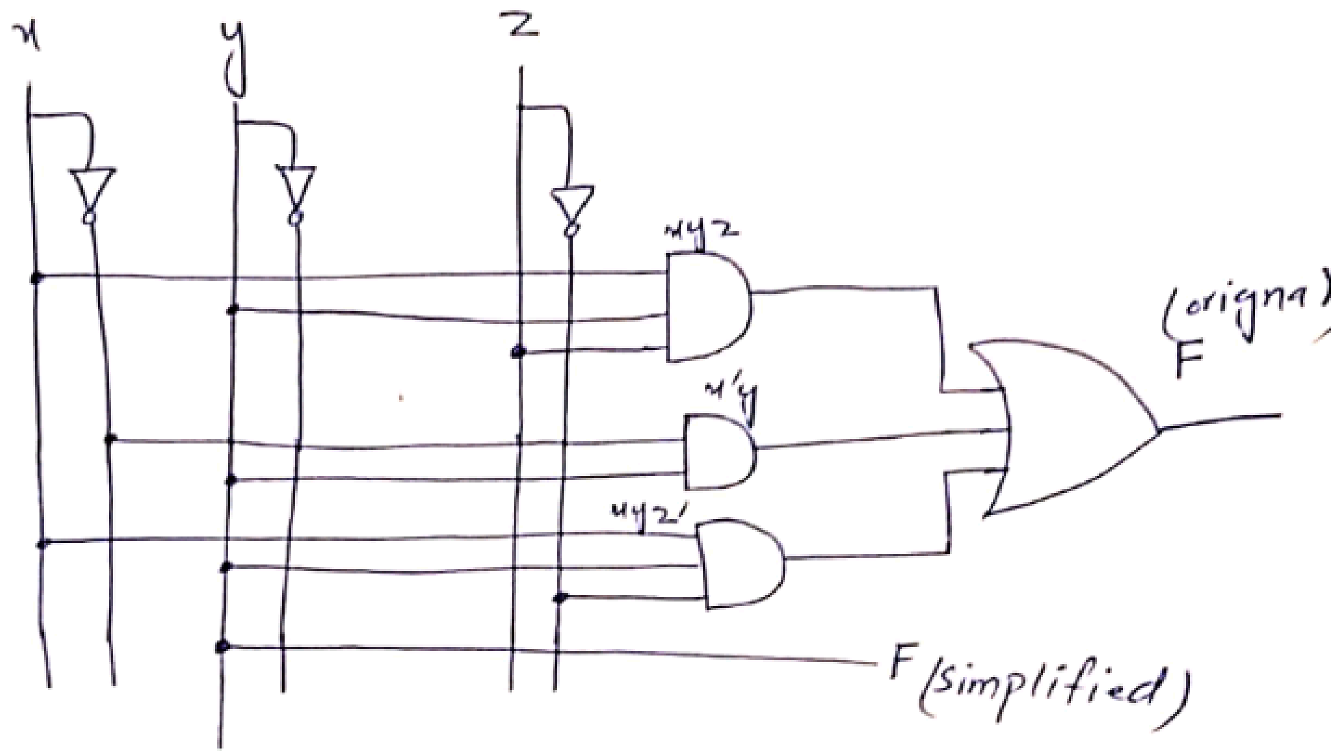
Draw circuit of simplified equations in 2.3.

a) $xyz + x'y + xyz' \rightarrow$ simplified : y

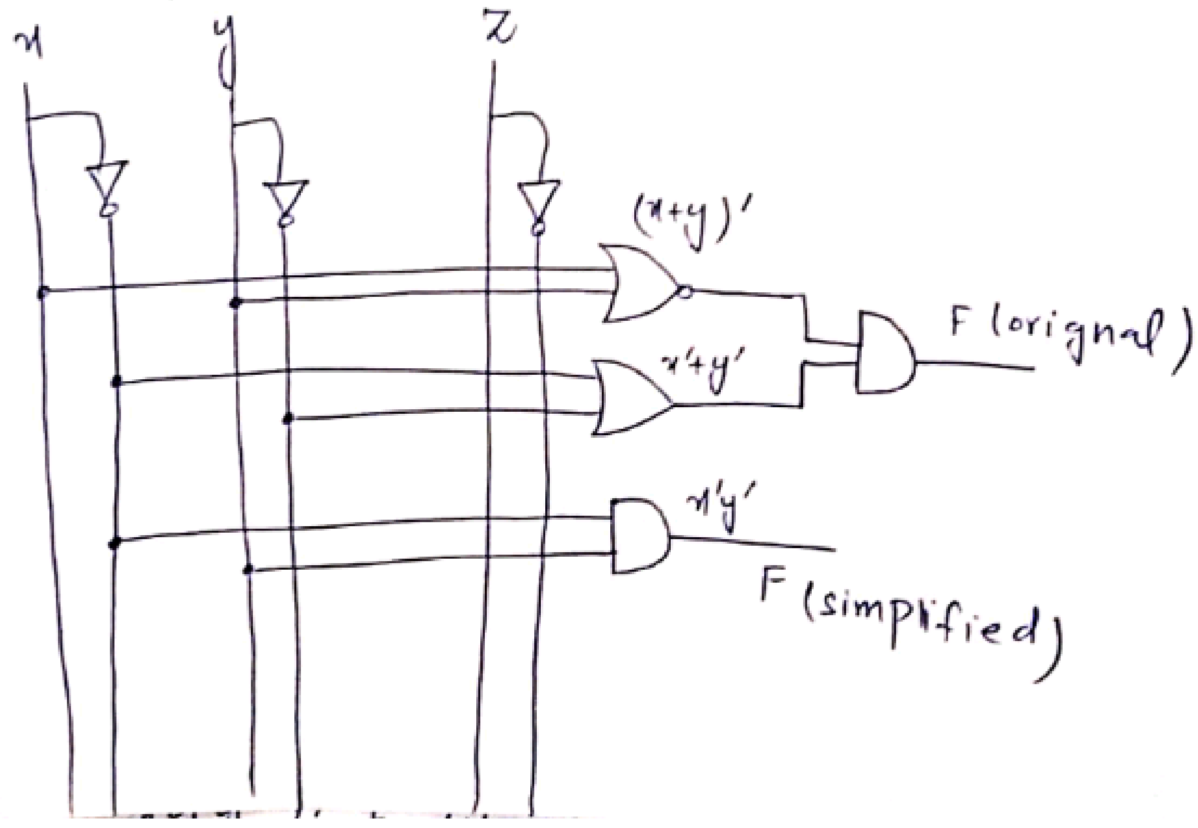
c) $x'y' \leftarrow (x+y)'(x'+y')$

f) $(x'+z')(x+y'+z')$

a)

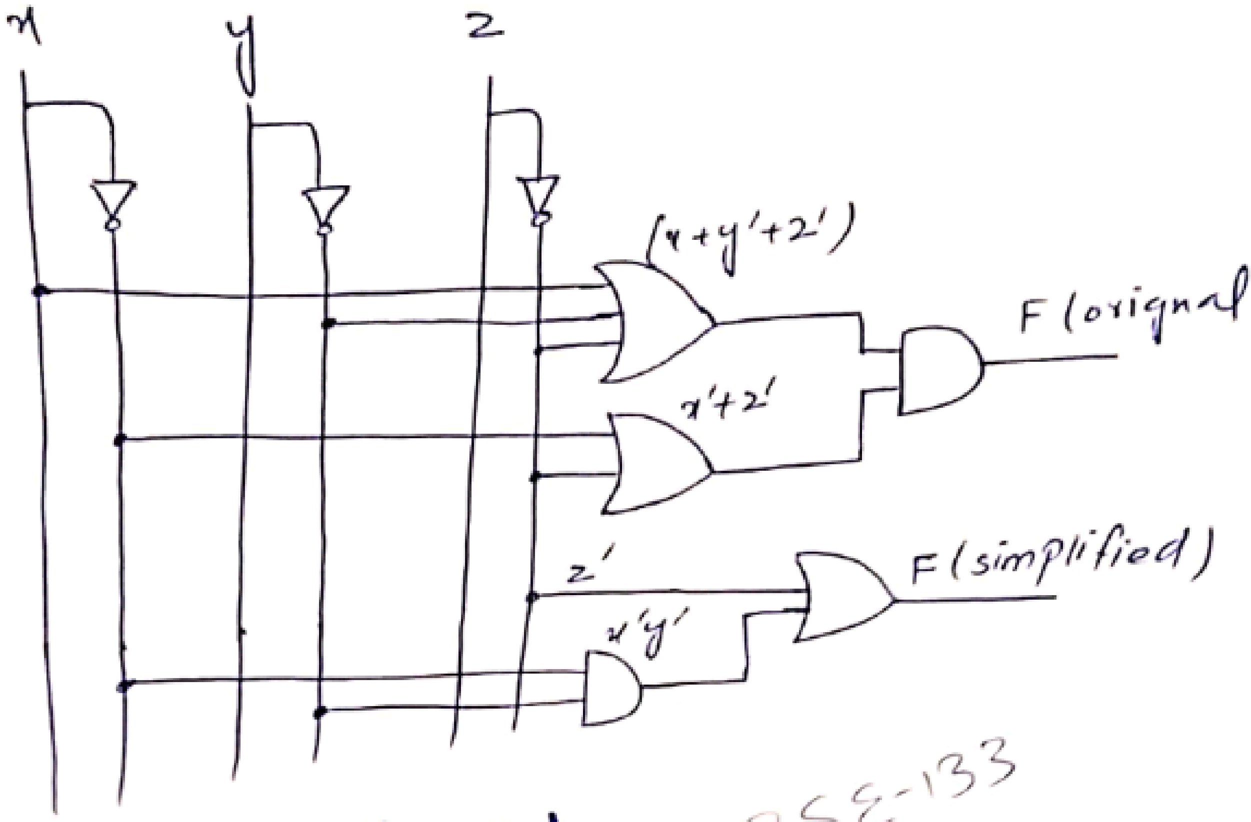


b)



f) $(x+y'+z')(x'+z') = $ orignal

$z'+x'y' = $ simplified



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(2·8)

Find complement

$F = x'y + yz'$

show that $FF' = 0$ & $F+F' = 1$

$F' = (x'y+yz')'$

$= (x'y)'(yz')' \Rightarrow (x+y')(y'+z)$

① $FF' = (x'y+yz')(x+y')(y'+z)$

$= x'xy + x'yy' + xyz' + yy'z'$

$= xyz'(y'+z)$

$= xyy'z' + xyzz' \Rightarrow 0+0 = 0$

② $F+F'=1$

$F+F' = x'y + yz' + (x+y')(y'+z)$

$= x'y + yz' + xy' + xz + y' + y'z$

$= y(1+z') + z(x+y') + x'y + xy'$

$= y + x'y + xy' + xz + xy'$

$= y + x'y + xy' + xz$

or

$= x'y + yz' + (x'y + yz')'$

$= 1 + 0$

or

$= 0 + 1$

$\boxed{F+F' = 1}$

(2.10)

Show that

$E = F_1 + F_2$

$G = F_1 F_2$

a) Let $F_1 = x'yz' + xyz + x'yz = \Sigma(2,3,7)$

$F_2 = xyz' + x'yz + x'y'z' = \Sigma(0,3,6)$

$E = F_1 + F_2 = x'yz' + xyz + x'yz + xyz' + x'yz + x'y'z'$

$= x'y'z + x'yz' + x'yz + xyz' + xyz$

$= \Sigma(0,2,3,6,7) \rightarrow$ contains min terms of F_1 & F_2

b) $G = F_1 F_2 = (x'yz' + xyz + x'yz)$
$(xyz' + x'yz + x'y'z')$

$= x'xyz' + x'yzz' + x'yy'z' + xyzz' + xx'yz$
$+ xx'yy'zz' + xx'yzz' + x'yz + x'yy'zz'$

$= x'yz$

$= \Sigma(3) \rightarrow$ the common term in both Min term's.

(2.12)

Apply logical operators:

A = 11001010 , B = 10010011

a) AND b) OR c) XOR d) NOT A

A	B	AND	OR	XOR	NOT A
1	1	1	1	0	0
1	0	0	0	1	0
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
0	0	0	0	0	1
1	1	1	1	0	0
0	1	0	1	1	1

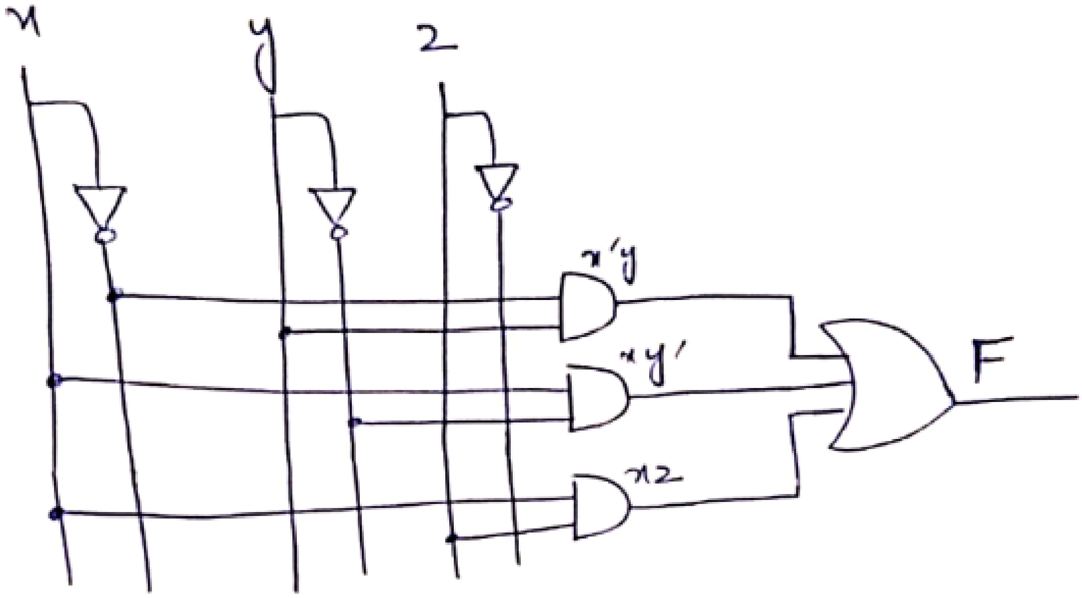
AND => 10000010
OR => 10011011
XOR => 01011001
NOT A => 00110101

(2.14)

Implement boolean function

$F = x'y + xy' + xz$

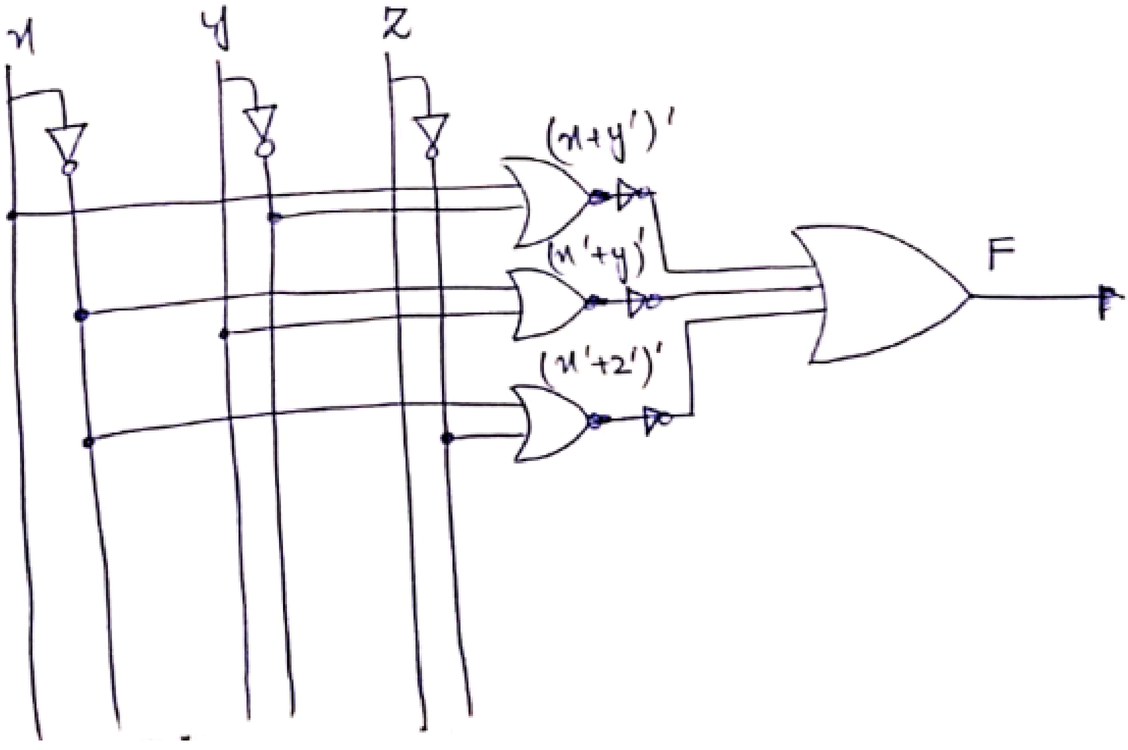
a) with AND, OR & inverter gate



b) With OR & inverter only

Apply De-morgan's law

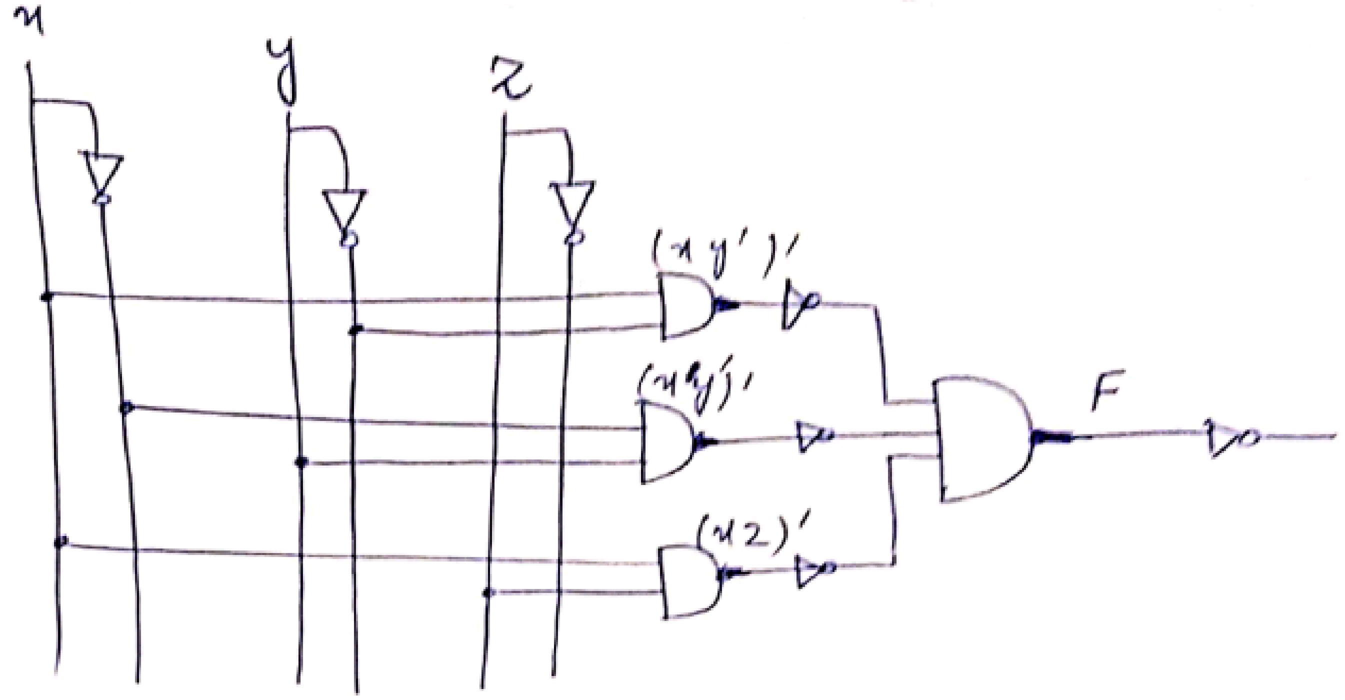
$= (x + y')' + (x' + y)' + (x' + z')'$



c) With AND and inverter

$$F = x'y + xy' + xz$$

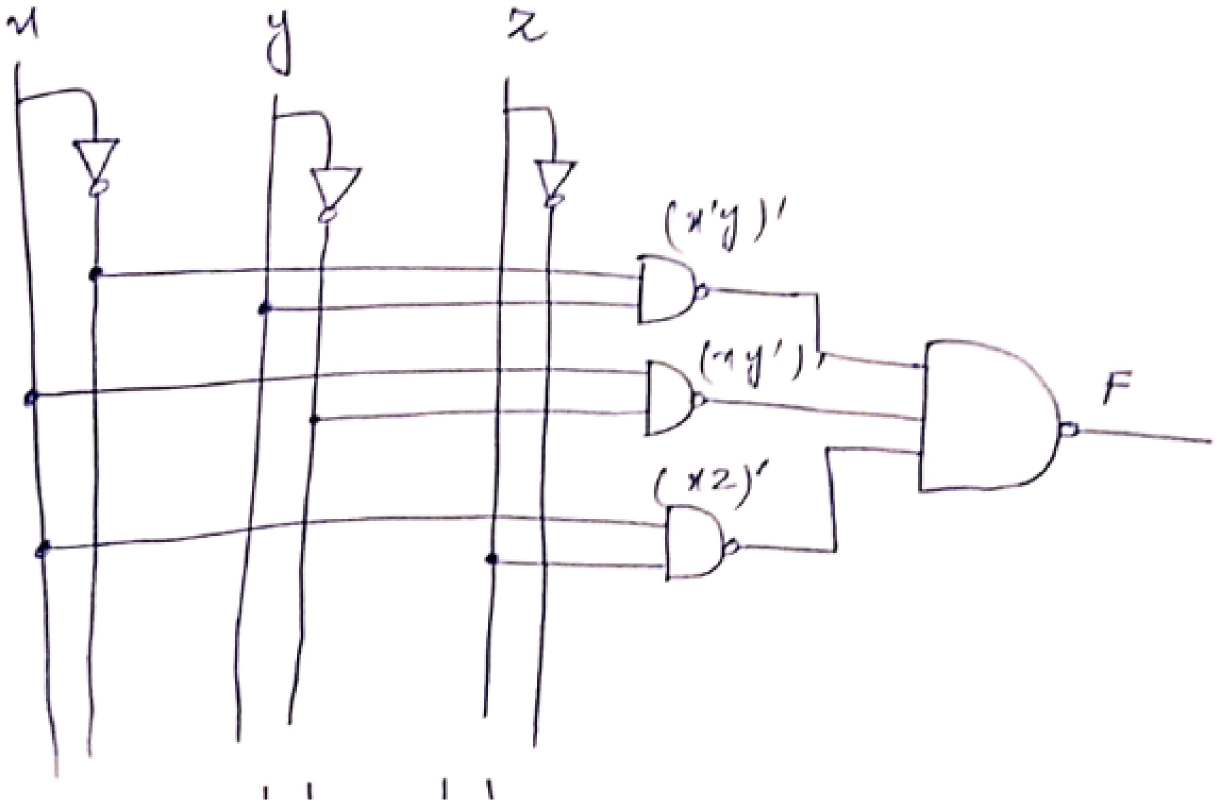
$$F = [(x'y)'(xy')'(xz)']'$$



d) with NAND and invertor

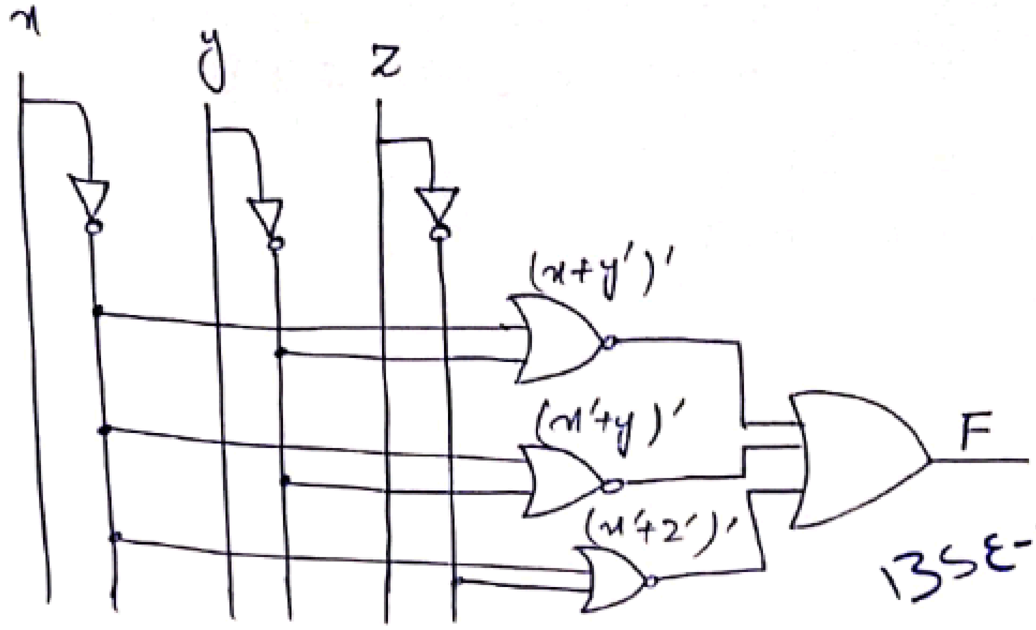
$$F = x'y + xy' + xz$$

$$F = [(x'y)'(xy')'(xz)']'$$



e) NOR & inverter

$F = x'y + xy' + xz = (x+y')' + (x'+y)' + (x'+z')'$



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(2.16)

Logical sum of all minterms = 1

a) prove when n = 3

$\Sigma m_i = 1$ where $i = 3$

Let input variables are x, y & z

$F(x,y,z) = x'y'z' + x'y'z + x'yz' + x'yz$
$+ xy'z' + xy'z + xyz' + xyz$
$= x'(y'z' + y'z + yz' + yz) + x'(y'z' + y'z + yz' + yz)$
$= (x'+x)(y'z' + y'z + yz' + yz)$
$= y'z' + y'z + yz' + yz$
$= y'(z'+z) + y(z'+z)$
$= (y'+y)(z'+z)$
$= 1$

c)

$$F(x_1, x_2, x_3, \ldots x_n) = \sum m_i$$

$\sum m_i$ has 2^n no. of terms

① $\sum m_i$ has $\left(\frac{2^n}{2}\right)$ minterms with x_1 & $\frac{2^n}{2}$ will have x_1' which can be factorize & remove.

② Remaining 2^{n-1} terms will have $\left(\frac{2^{n-1}}{2}\right)$ terms with x_2 & $\left(\frac{2^{n-1}}{2}\right)$ minterms will have x_2' which can also be factorize & remove. —
Continues this process until the last term is left and $x_n + x_n' = 1$

$$F = (x_n + x_n')G = 1$$

(2.18)

$$F = w'xy + w'yz + wy'z + w'y'z + xy \quad \text{SOP}$$

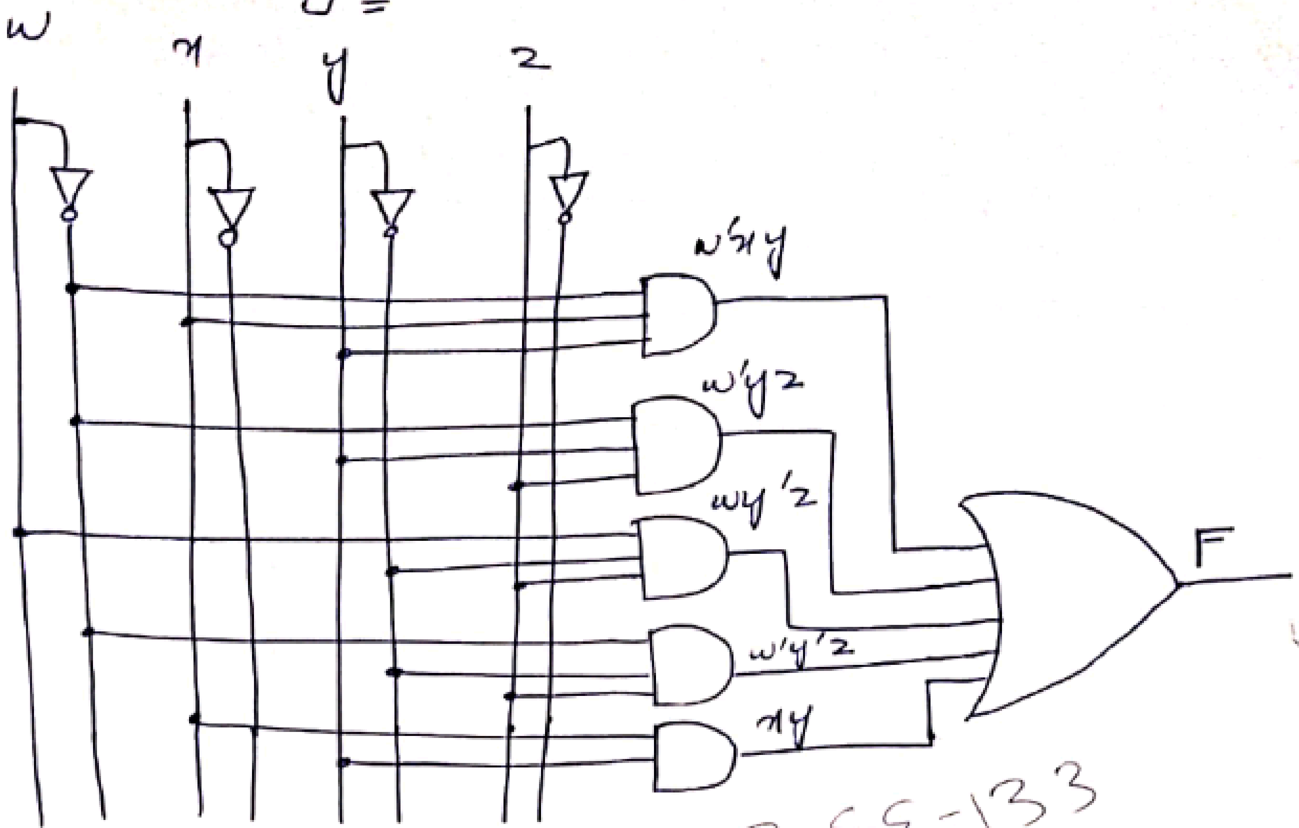
a) Truth table:

w	x	y	z	F	
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	0	2
0	0	1	1	1	3
0	1	0	0	0	4
0	1	0	1	1	5
0	1	1	0	1	6
0	1	1	1	1	7
1	0	0	0	0	8
1	0	0	1	1	9
1	0	1	0	0	10
1	0	1	1	0	11
1	1	0	0	0	12
1	1	0	1	1	13
1	1	1	0	1	14
1	1	1	1	1	15

$= \sum(1, 3, 5, 6, 7, 9, 13, 14, 15)$

$\Rightarrow m_1, m_3, m_5, m_6, m_7, m_9, m_{13}, m_{14}, m_{15}$

b) Logic diagram:



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c) Simplify:

$= w'xy + w'yz + wy'z + w'y'z + xy$

$= (1+w')xy + w'yz + wy'z + w'y'z$

$= xy + w'yz + wy'z + w'y'z$

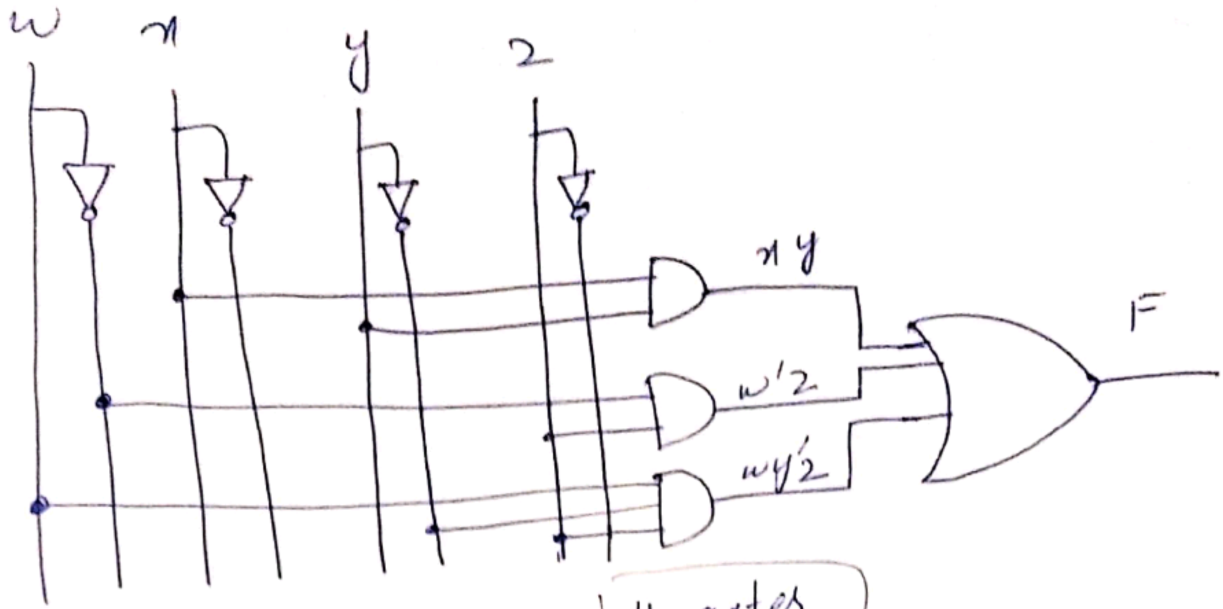
$= xy + z(w'y + wy' + w'y')$

$= xy + z(w'(y+y') + wy')$

$= xy + z(w' + wy')$

$\Rightarrow xy + w'z + wy'z$

e) $xy + w'z + wy'z$



$\boxed{4\text{-gates}}$

(2.20)

Express the complement in Sum of minterms (Σ).

a) $F(w,x,y,z) = \Sigma(1,5,7,11,12,14,15)$

$F'(w,x,y,z) = \Sigma(0,2,3,4,6,8,9,10,13)$

b) $F(x,y,z) = \Pi(2,4,5)$

$F'(x,y,z) = \Pi(0,1,3,6,7)$

$= \Sigma(2,4,5)$

(2.22)

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Covert to Sop & POS:

a) $(w+xy')(x+y'z)$

$= wx + wy'z + xy' + xy'z$

$= wx + wy'z + x(y'+y'z)$

$= wx + wy'z + x(y'(1+z))$

$= wx + wy'z + xy'z$

$= wx + y'(wz + xz)$

Sop $= wx + wy'z + xy'z$

$= wx(y+y')(z+z') + wy'z(x+x') + xy'z(w+w')$

$= (wx+y)(wx+y')(wx+z)(wx+z') + (wy'z+x)$

$(wy'z+x')(xy'z+w)(wxy'z+w')$

$= (w+y)(x+y)(w+y')(x+y')(w+z)(x+z)(w+z')$

$(x+z')(w+x)(y'+x)(x+z)(x'+w)(x'+y')(x'+z)$

$(x+w)(y'+w)(w+z)(w'+x)(w'+y')(w'+z)$

POS $= (w+x)(w+y')(x+y')(x+z)$

b) $xy + (w'+y'z')(z'+x'y')$

$= xy + w'z' + w'x'y' + y'z' + x'y'z'$

$= xy + w'z' + x'(w'y' + y'z') + y'z'$

$= xy + w'z' + x'w'y' + y'z' + y'x'z'$

$= xy + w'z' + y'(w'x' + z' + x'z')$

$= xy + w'z' + y'(w'x' + z'(1 + x'))$

$= xy + w'z' + y'(w'x' + z'x')$

SoP $= xy + w'z' + w'x'y' + x'y'z'$

$= \cancel{xy(z+z')(w+w') + (w'+y')(w'+z')}(x'+z')$
$\cancel{(y'+z')}$

$= (\cancel{x+z) + (y+z')(w+w') + (w'+y')(w'+z'}$
$(\cancel{x'+z')(y'+z'})$

OR

$= (x + w' + y'z')(y + w' + y'z')(z' + x'y')$

$= (w'+x+y')(w'+x+z')(w'+y+y')(w'+y+z')$
$(x'+z')(y'+z')$

$= (w'+x+y')(w'+x+z')\,w'(w'+y+z')(x'+z')(y'+z')$

POS $\Rightarrow w'(w'+x+y')(w'+x+z')(w'+y+z')(x'+z')(y'+z')$

(2.24)

Find dual:

$F = x'y + (x+z)(x+y')$

Using De-morgan's rule

$= [x'y + (x+z)(x+y')]'$

$= (x'y)'[(x+z)(x+y')]'$

$= (x+y')(x+z)' + (x+y')'$

$= (x+y')(x'z' + x'y)$

Again Applying same rule

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Dual $\Rightarrow$ $[(x+y')(x'z' + x'y)]'$

$= (x+y')' + (x'z' + x'y)'$

$= x'y + ((x'z')'(x'y)')$

$= x'y + (x+z)(x+y')$

$F' =$ Dual of F

(2.26)

Show that +ive logic NAND gate is a -ive logic NOR gate.

Let two input A & B

(+ive NAND)

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0

(-ive NOR)

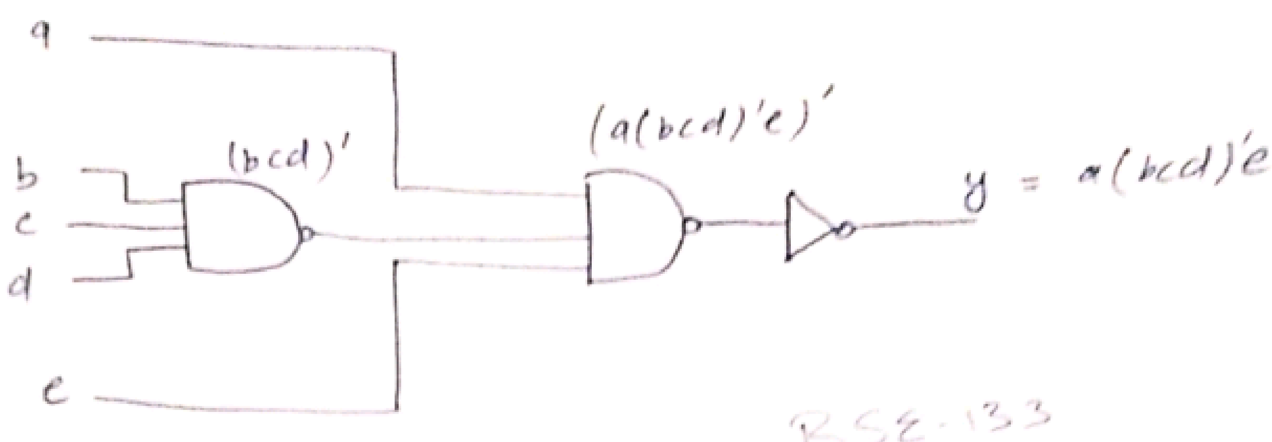
A	B	Output
1	1	0
1	0	0
0	1	0
0	0	1

Hence, proved by outputs.

(2·28)

write boolean expression & truth table.

(9)



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for $a=0$

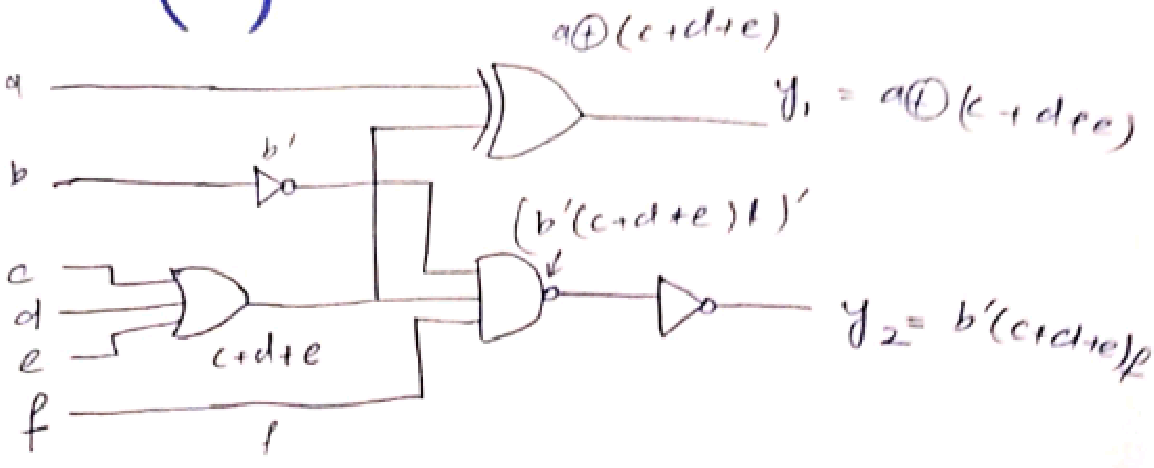
b	c	d	e	(bcd)'	y = a(bcd)'e
0	0	0	0	1	0
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	0	0
1	1	1	1	0	0

or $a=1$

b	c	d	e	a	$(bcd)'$	$a(bcd)'e$
0	0	0	0	1	1	0
0	0	0	1	1	1	1
0	0	1	0	1	1	0
0	0	1	1	1	1	1
0	1	0	0	1	1	0
0	1	0	1	1	1	1
0	1	1	0	1	1	0
0	1	1	1	1	1	1
0	0	0	0	1	1	0
1	0	0	1	1	1	1
1	0	1	0	1	1	0
1	0	1	1	1	1	1
1	1	0	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	1	0	0
1	1	1	1	1	0	1

$1 \times 1 \times 0$

(b)



Expression: $y_1 = a \oplus (c+d+e)$

$y_2 = b'(c+d+e)f$

for $y_1 = a \oplus (c + de) \Rightarrow a \oplus (c+d+e)$

$2^4 = 8$ rows

a	c	d	e	or $c+d+e$	y_1 $a \oplus (c+d+e)$
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	1	1
0	0	1	1	1	1
0	1	0	0	1	1
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	1	0
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	1	0

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$y_2 = b'(c+d+e)f$

b	c	d	e	f	b'	$c+d+e$ ~~$b+c$~~	y_2 $b'(c+d+e)f$
0	0	0	0	0	1	0	0
0	0	0	0	1	1	0	0
0	0	0	1	0	1	1	0
0	0	0	1	1	1	1	1
0	0	1	0	0	1	1	0
0	0	1	0	1	1	1	1
0	0	1	1	0	1	1	0
0	0	1	1	1	1	1	1
0	1	0	0	0	1	1	0
0	1	0	0	1	1	1	1
0	1	0	1	0	1	1	0
0	1	0	1	1	1	1	1
0	1	1	0	0	1	1	0
0	1	1	0	1	1	1	1
0	1	1	1	0	1	1	0
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0
1	0	0	1	0	0	1	0
1	0	0	1	1	0	1	0
1	0	1	0	0	0	1	0
1	0	1	0	1	0	1	0
1	0	1	1	0	0	1	0
1	0	1	1	1	0	1	0
1	1	0	0	0	0	1	0
1	1	0	0	1	0	1	0
1	1	0	1	0	0	1	0
1	1	0	1	1	0	1	0
1	1	1	0	0	0	1	0
1	1	1	0	1	0	1	0
1	1	1	1	0	0	1	0
1	1	1	1	1	0	1	0

2.30

write in Sum of product form:

$$(w+x+y)(x'+y'+z')$$

$$= wx'+wy'+wz'+xx'+xy'+xz'+x'y+yy'+yz'$$

$$= wx'+wy'+wz'+xy'+xz'+x'y+yz'$$

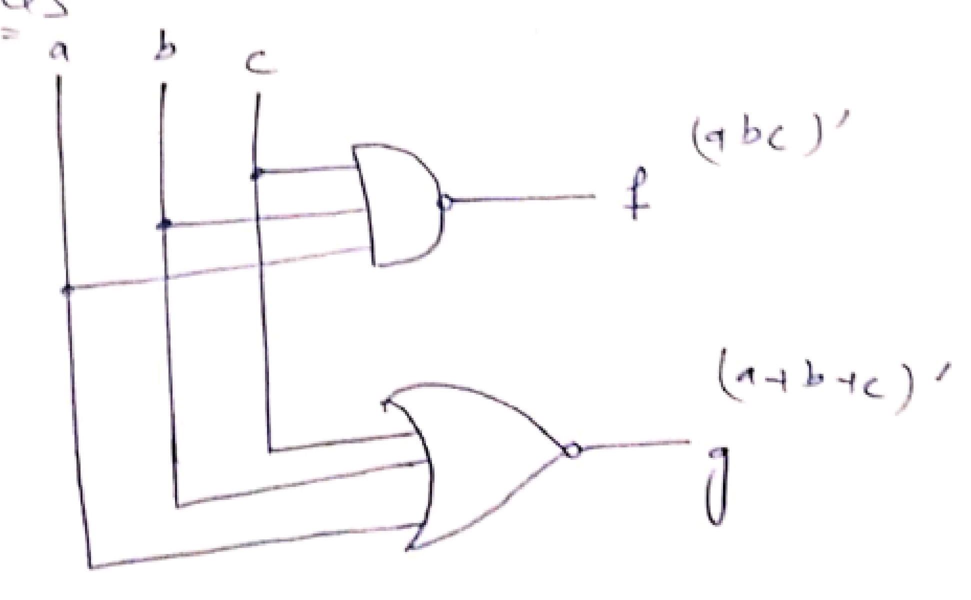
$$= wx'(y+y')(z+z')+wy'(x+x')(z+z')+ wz'(x+x')(y+y')+xy'(w+w')(z+z')+ xz'(w+w')(y+y')+x'y(w+w')(z+z') +yz'(x+x')(w+w')$$

$$= wx'y'z'+wx'yz+wxy'z+wx'y'z'+wxyz' + wx'y'z'+wxy'z+w'xyz'+wxyz'+w'xy'z' + wx'yz+w'x'yz'+wxyz'+wx'yz'$$

$$\Rightarrow wx'y'z'+wx'yz+wxy'z+wxyz'+w'xy'z' + wx'yz'$$

(2.32) BSSE-133

Show signals



Truth table:

$2^3 = 8$ row

a	b	c	abc	a+b+c	f $(abc)'$	g $(a+b+c)'$
0	0	0	0	0	1	1
0	0	1	0	1	1	0
0	1	0	0	1	1	0
0	1	1	0	1	1	0
1	0	0	0	1	1	0
1	0	1	0	1	1	0
1	1	0	0	1	1	0
1	1	1	1	1	0	0

The END

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