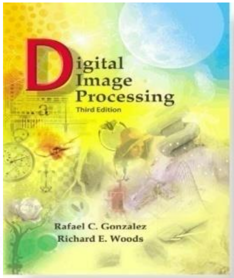


- Histogram Processing:
  - Enhancement based on statistical Properties:
    - Local
    - Global
  - Histogram Definition:

$$h(r_k) = n_k, \quad r_k \in [0, L-1], \quad n_k \in [0, M \times N]$$

$$p(r_k) = \frac{n_k}{n} = \frac{1}{M \times N} n_k$$

Intensity Transformations and Spatial Filtering

- Histogram Visual Meaning:
  - Dark
  - Light
  - Low Contrast
  - High Contrast

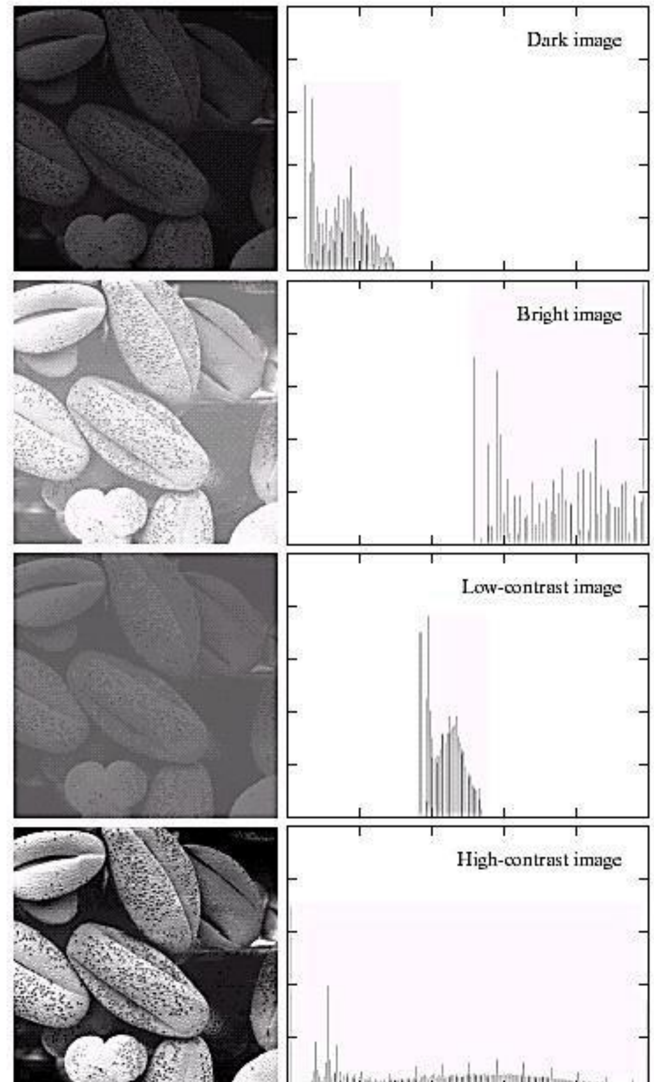
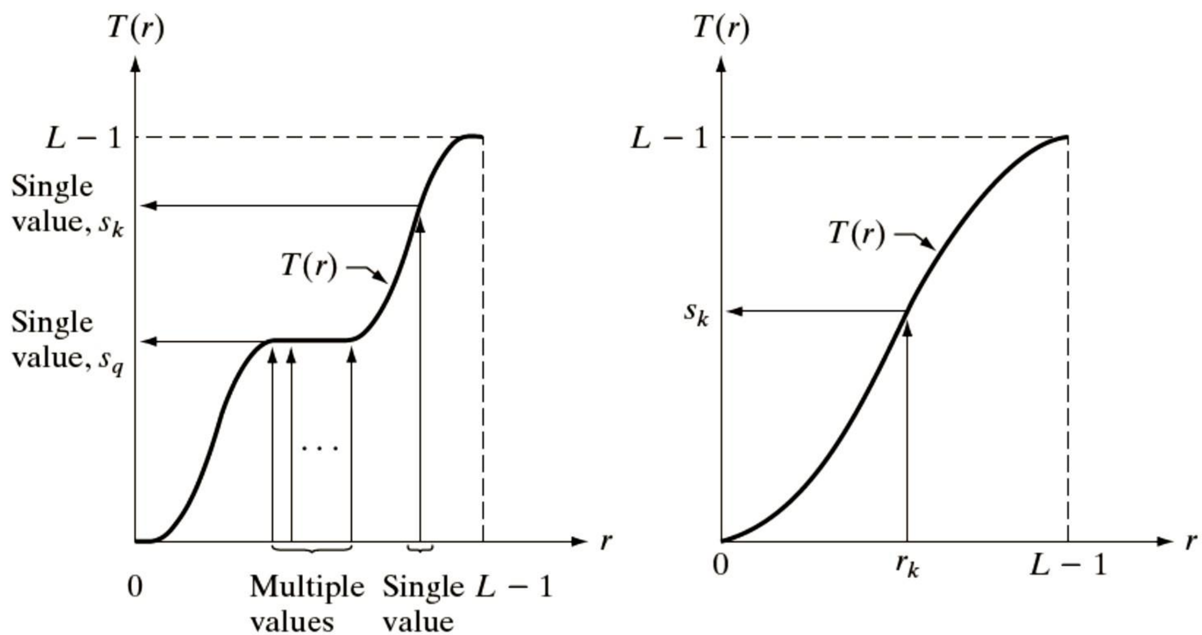


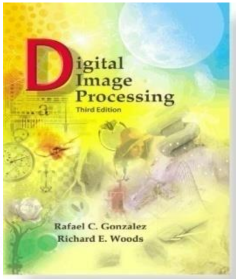
FIGURE 3.16 Four basic image types: dark, light, low contrast, high contrast, and their corresponding histograms.

- Histogram Equalization:
  - Continuous Case.
  - Seek for a suitable transform (Except for negative):



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.



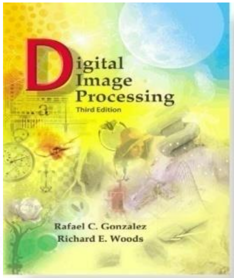
Intensity Transformations and Spatial Filtering

- Discrete Case

$$p_r(r_k) = \frac{n_k}{MN} = \frac{n_k}{n}, \quad k = 0, 1, 2, \cdots, L-1$$

$$S_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{L-1}{MN}\sum_{J=0}^{k} n_k, \quad k = 0, 1, \cdots, L-1$$

- Perfect equalization is NOT possible



- Numerical Example:

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$S_k = 7\sum_{j=0}^{k} p_r\left(r_j\right)$$

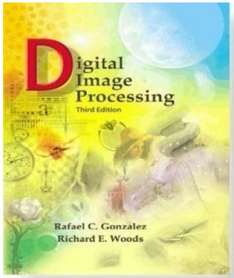
S_k	S_k
1.33	1
3.08	3
4.55	5
5.67	6
6.23	6
6.65	7
6.86	7
7.00	7

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7p_r(r_0) = 1.33$$

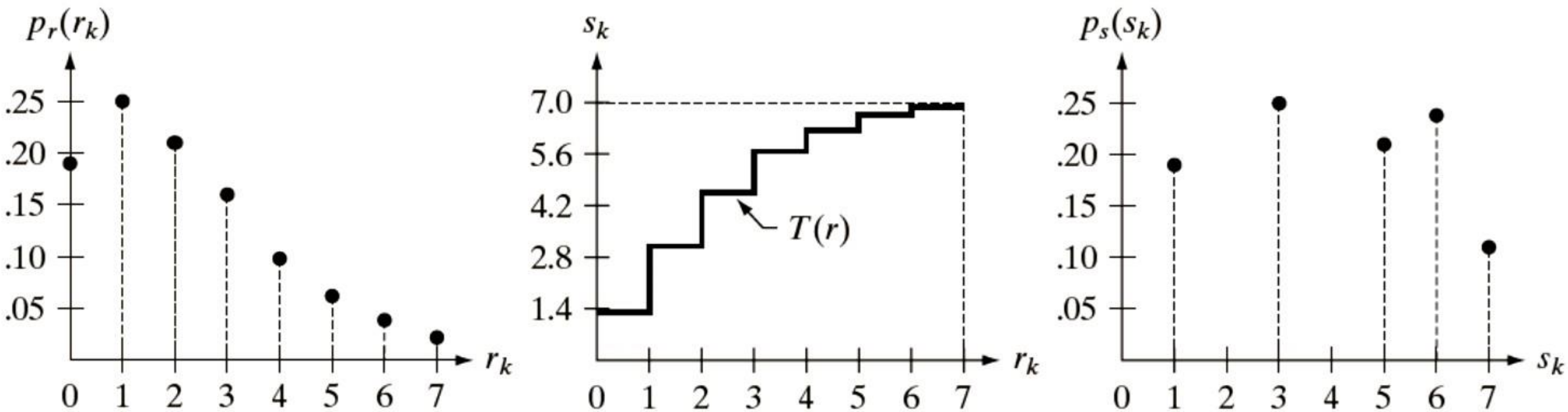
Similarly,

$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7p_r(r_0) + 7p_r(r_1) = 3.08$$

and $s_2 = 4.55$, $s_3 = 5.67$, $s_4 = 6.23$, $s_5 = 6.65$, $s_6 = 6.86$, $s_7 = 7.00$.



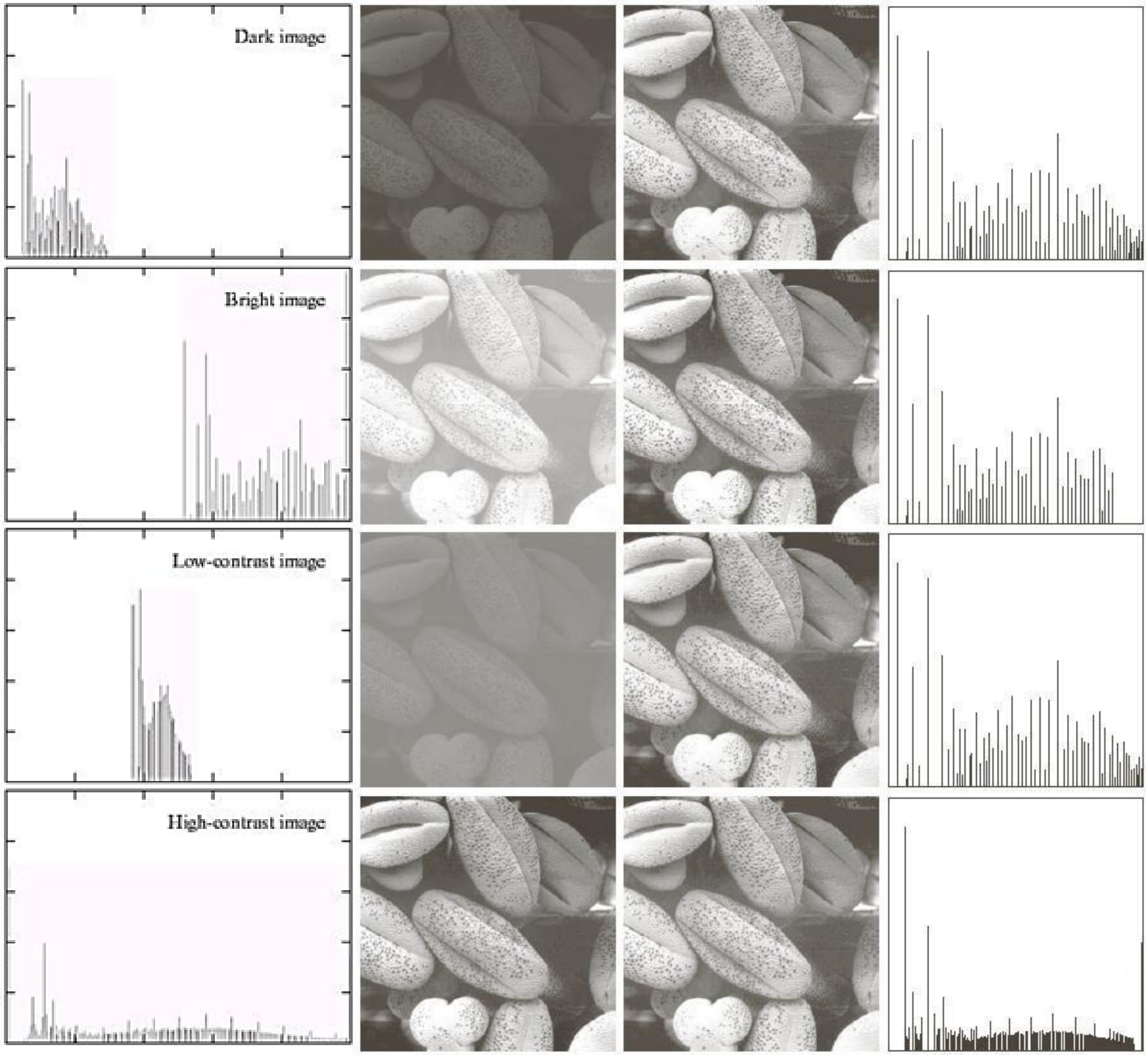
- Numerical Examples (Cont.)

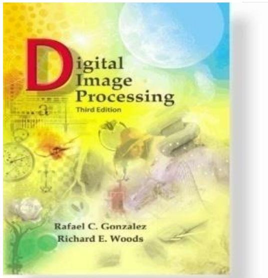


a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

- Real Experiment:





- Gray-Level Transfer Function

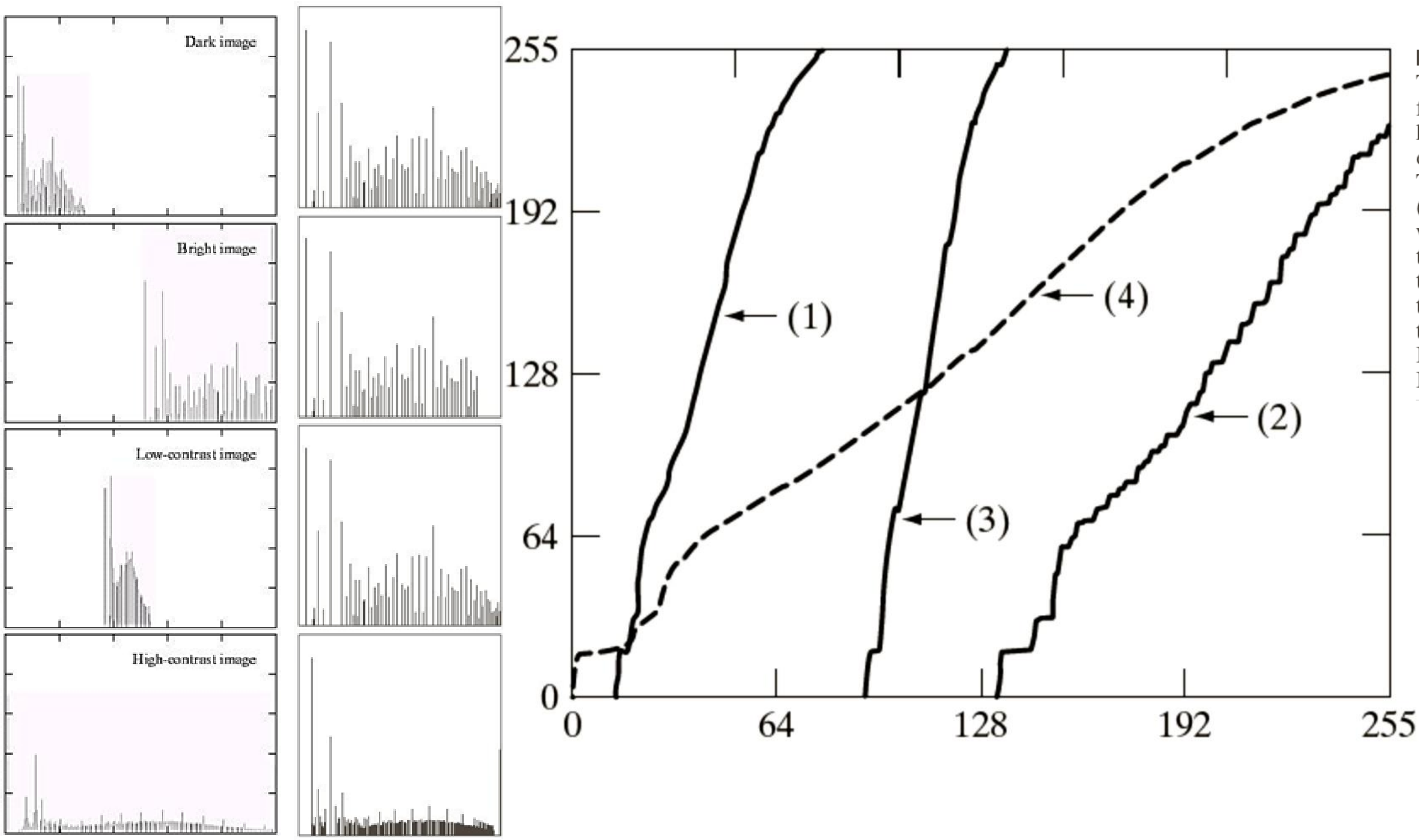
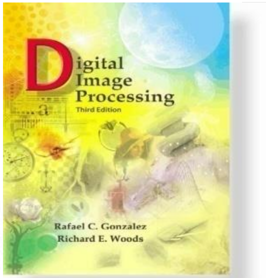


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).



- Histogram Matching and Modification:
  - Goal: Specify the shape of the histogram:

$$p_r(r) \xrightarrow{?} p_z(z)$$

$$\left.\begin{array}{l} s = T(r) = (L-1)\displaystyle\int_0^r p_r(w)\,dw \\ \\ G(z) = (L-1)\displaystyle\int_0^z p_z(t)\,dt \end{array}\right\} \Rightarrow z = G^{-1}\left[T(r)\right] = G^{-1}[s]$$

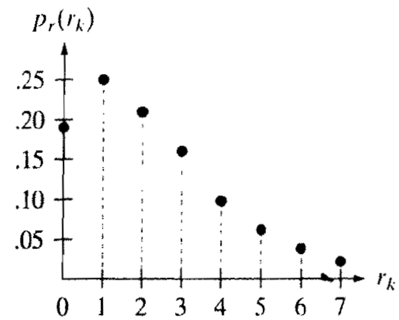
  - Example: Pages: 133-136

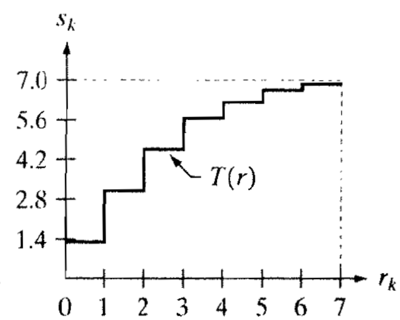
- Histogram Matching and Modification:
  - Procedure:

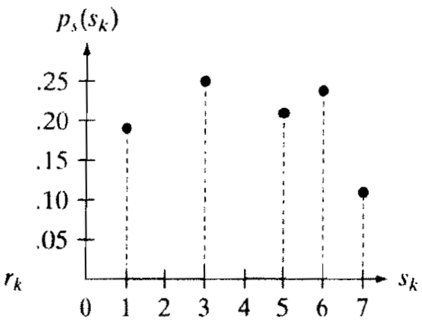
1. Compute the histogram $p_r(r)$ of the given image, and use it to find the histogram equalization transformation in Eq. (3.3-13). Round the resulting values, s_k, to the integer range $[0, L-1]$.
2. Compute all values of the transformation function G using the Eq. (3.3-14) for $q = 0, 1, 2, \ldots, L-1$, where $p_z(z_i)$ are the values of the specified histogram. Round the values of G to integers in the range $[0, L-1]$. Store the values of G in a table.
3. For every value of s_k, $k = 0, 1, 2, \ldots, L-1$, use the stored values of G from step 2 to find the corresponding value of z_q so that $G(z_q)$ is closest to s_k and store these mappings from s to z. When more than one value of z_q satisfies the given s_k (i.e., the mapping is not unique), choose the smallest value by convention.
4. Form the histogram-specified image by first histogram-equalizing the input image and then mapping every equalized pixel value, s_k, of this image to the corresponding value z_q in the histogram-specified image using the mappings found in step 3. As in the continuous case, the intermediate step of equalizing the input image is conceptual. It can be skipped by combining the two transformation functions, T and G^{-1}, as Example 3.8 shows.

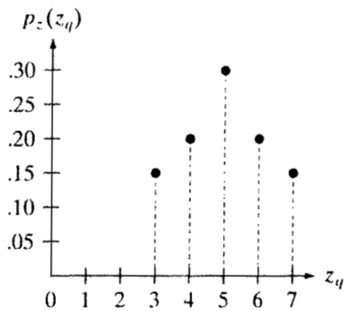
- Histogram Matching and Modification

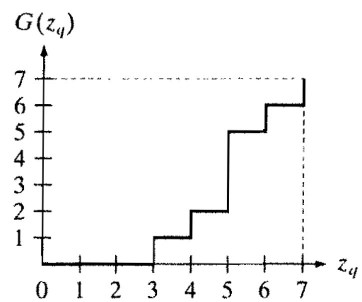
Step 1











Step 2

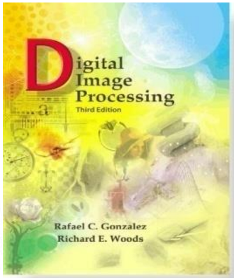
z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7

s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

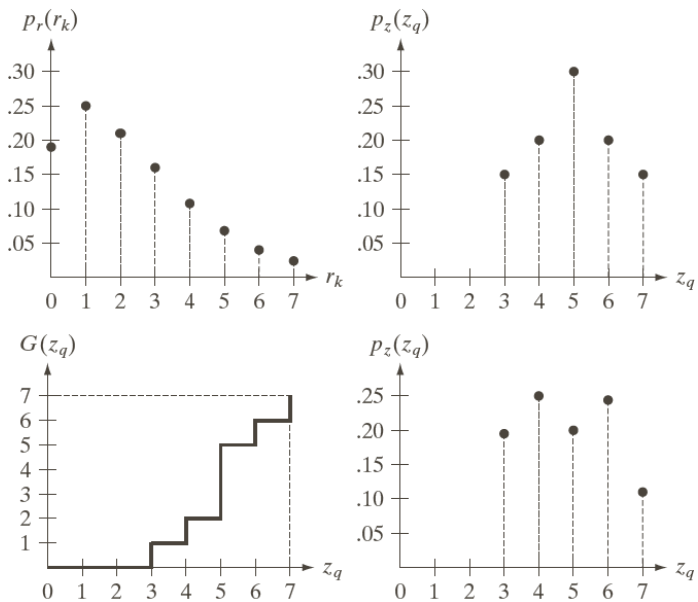
TABLE 3.4 Mappings of all the values of s_k into corresponding values of z_q.

Step 3

Step 4 = $G^{-1}(z_q)$



- Histogram Matching and Modification:

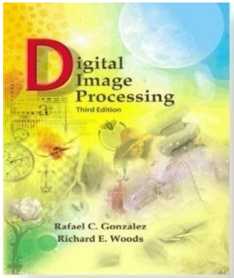


a b
c d

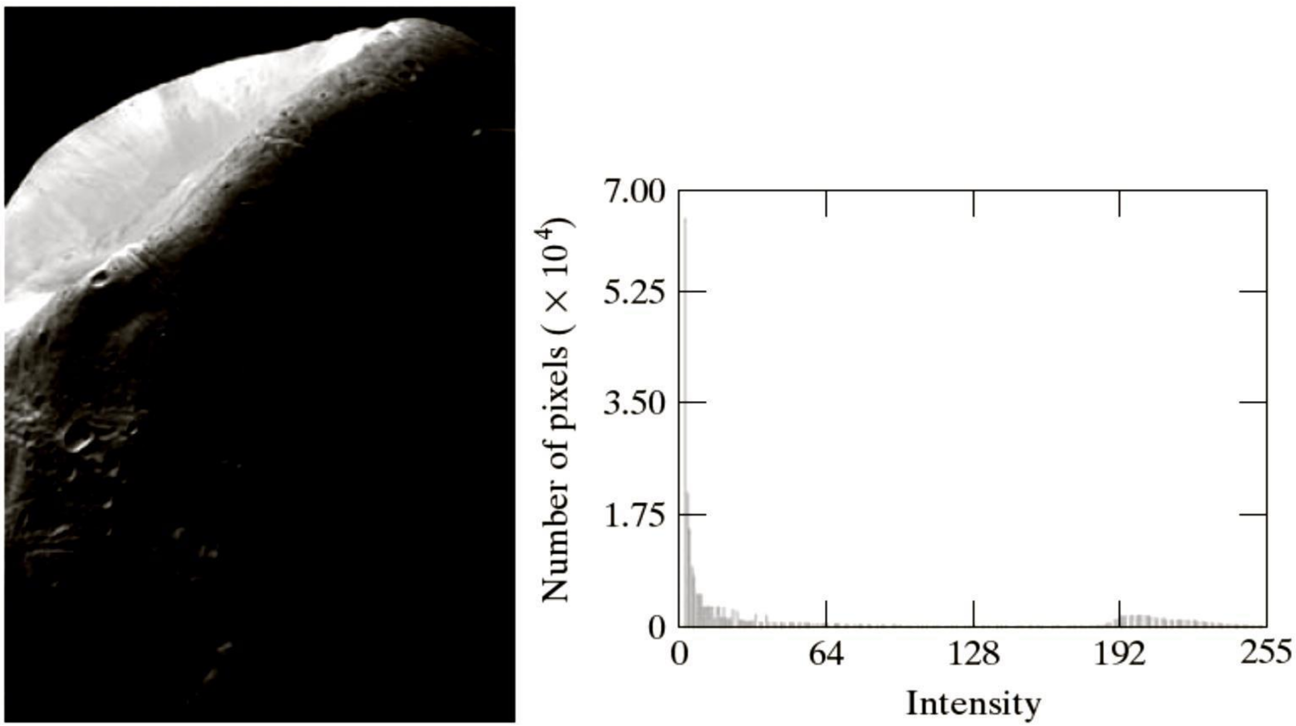
FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

TABLE 3.2 Specified and actual histograms (the values in the third column are from the computations performed in the body of Example 3.8).



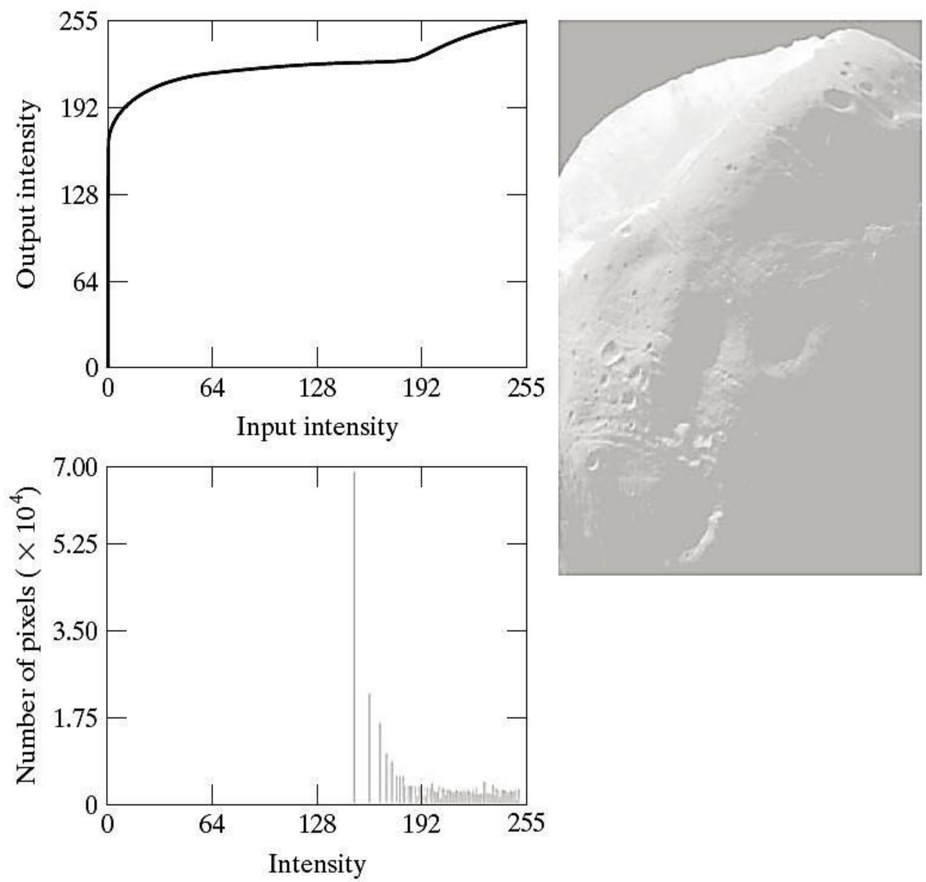
- Histogram Matching Example - Original Image



a b

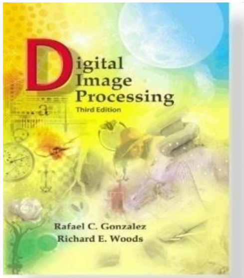
FIGURE 3.23
(a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

- Histogram Matching:
  - Equalized Image



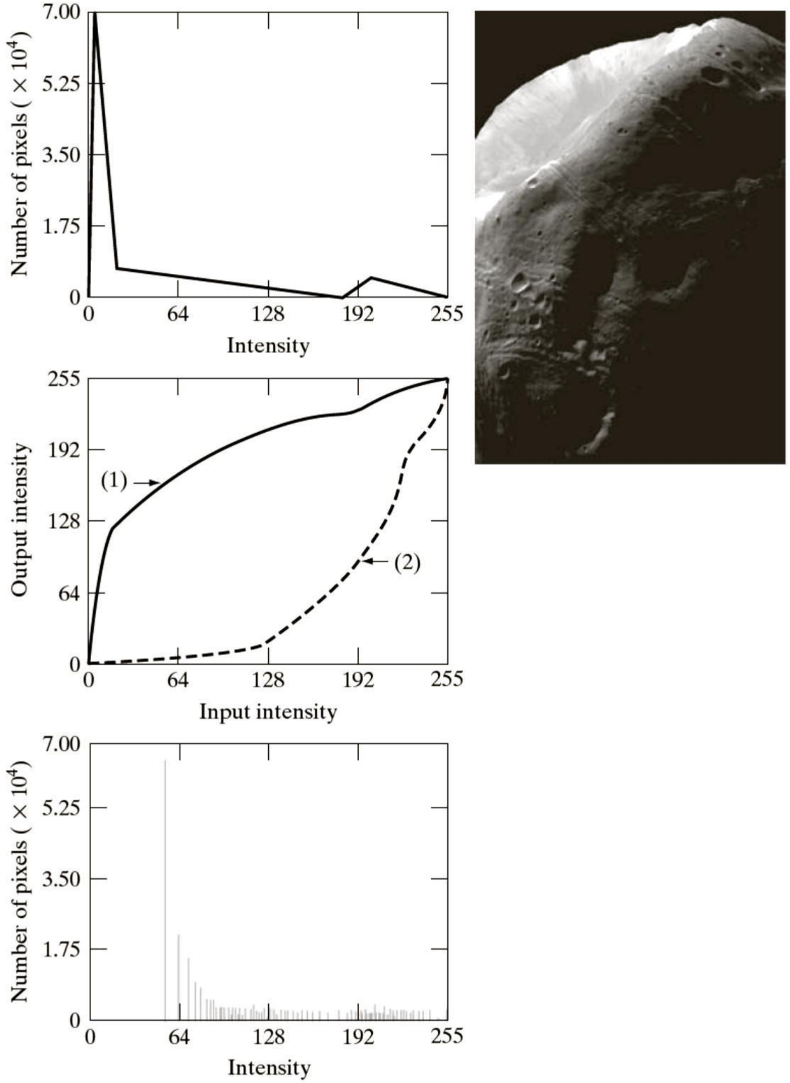
a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).



Intensity Transformations and Spatial Filtering

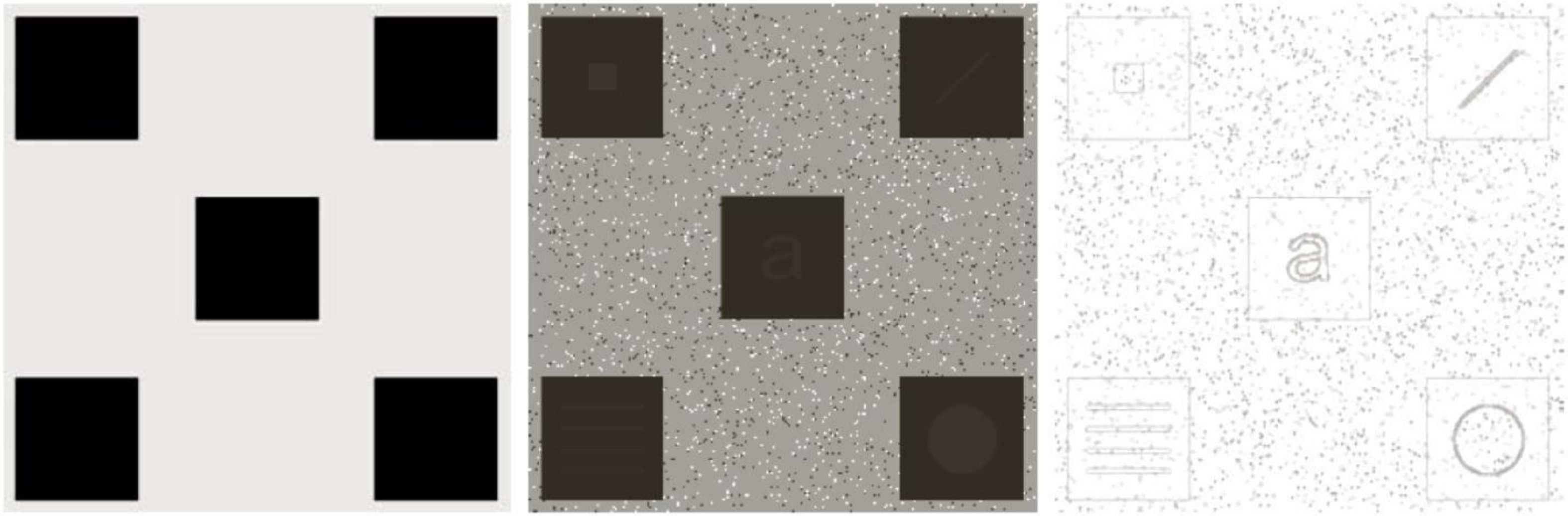
- Histogram Matching



a c
b
d

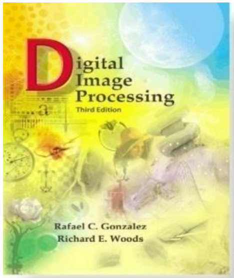
FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

- Local Histogram Enhancement



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.

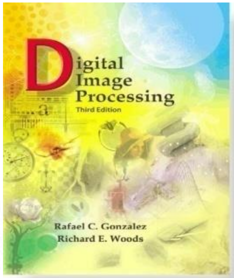


- Histogram Statistics For Image Enhancement:
  - Use of Global Statistical Measures

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n \, p(r_i) \approx \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} \left[f(x,y) - m \right]^n$$

$$m = \sum_{i=0}^{L-1} r_i \, p(r_i) \approx \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} f(x,y)$$

  - Gross adjustments in overall intensity (m) and contrast (μ_2)



- Histogram Statistics For Image Enhancement:
  - Local mean and local variance:

$$m_{S_{xy}}(x,y)=\sum_{i=0}^{L-1} r_i\, p_{S_{xy}}(r_i)\approx \frac{1}{|S_{xy}|}\sum_{(s,t)\in S_{xy}} f(s,t)$$

$$\sigma^2_{S_{xy}}(x,y)=\sum_{i=0}^{L-1}\left(r_i-m_{S_{xy}}(x,y)\right)^2 p_{S_{xy}}(r_i)\approx \frac{1}{|S_{xy}|}\sum_{(s,t)\in S_{xy}}\left[f(s,t)-m_{S_{xy}}(x,y)\right]^2$$

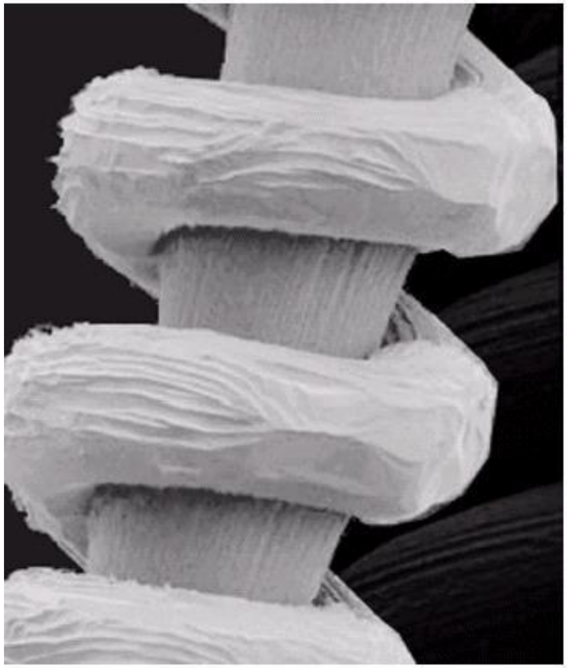
S_{xy} : Neighborhood centered on (x,y)

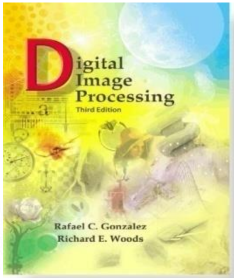
  - Local information intensity and contrast (edges)

- A simple enhancement algorithm for SEM image:

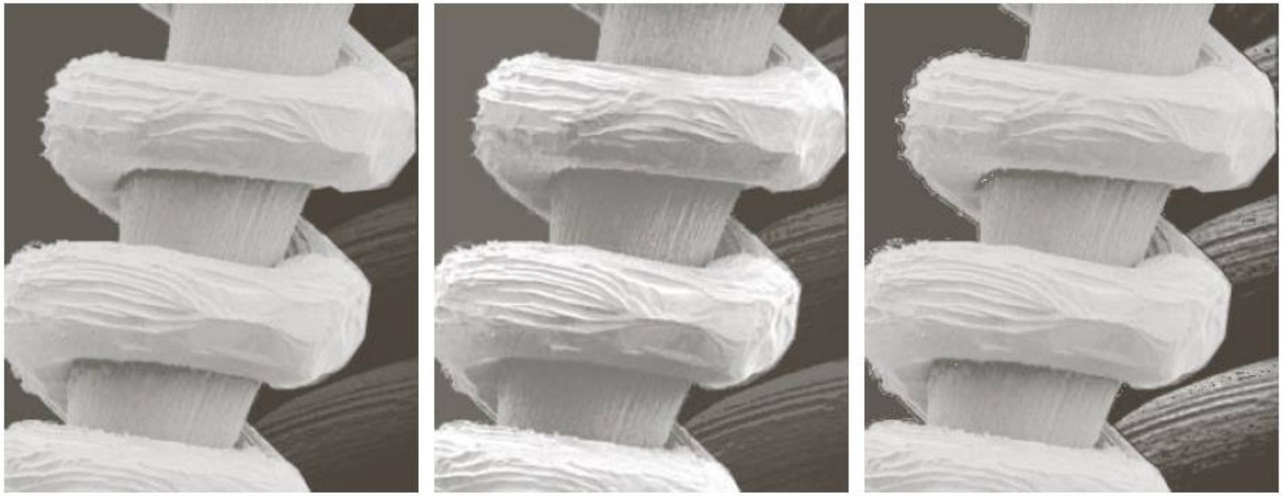
$$g(x,y)=\begin{cases} E.f(x,y) & m_S(x,y)\le k_0 m_G \text{ and } k_1\sigma_G \le \sigma_S(x,y)\le k_2\sigma_G \\ f(x,y) & O.W \end{cases}$$

$$E=4.0,\ k_0=0.4,\ k_1=0.02,\ k_2=0.4$$





- Graphical Illustration:



a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)