

Exercise 2.4

Q#01

(Row-echelon form)

$$A = \begin{bmatrix} -1 & 2 & -5 \\ 2 & -1 & 6 \\ 2 & -2 & 7 \end{bmatrix}$$

$$-(R_1) \rightarrow \begin{bmatrix} 1 & -2 & 5 \\ 2 & -1 & 6 \\ 2 & -2 & 7 \end{bmatrix}$$

$R_2 - R_3$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 2 & -2 & 7 \end{bmatrix}$$

~~$R_3 - R_1$~~ $R_3 - 2R_1$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

$R_3/3$

~~$R_3 - R_2$~~

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix}$$

R3/-3

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 5 & 6 & -3 \\ -2 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 5 & 6 & -3 \\ -2 & -2 & 2 \end{bmatrix}$$

$R_2 - 3R_1, R_3 - 5R_1, R_4 + 2R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$3 - 3(1)$

$4 - 3(1)$

$-1 - 3(-1)$

$-2 + 3$

$5 - 5(1)$

$6 - 5(1)$

$-3 - 5(-1)$

QUESTION NO: 02

$$A = \begin{bmatrix} -1 & 1 & -1 & 0 & 3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{bmatrix}$$

$$-R_1 \rightarrow \begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{bmatrix}$$

$$R_2 + 3R_1, R_3 - 4R_1$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & -2 & -8 & -2 & -14 \end{bmatrix}$$

$$\begin{array}{r} 14 \\ 12 \\ \hline 26 \end{array}$$

$$R_3 + 2R_2$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -4 \\ -2 & -1 & 10 \\ 4 & 3 & -12 \end{bmatrix}$$

$$\rightarrow R_2 + 2R_1, R_3 - 4R_1$$

$$\begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix}$$

$$R_3 + R_2$$

$$\begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

$$R_3/6$$

$$\begin{bmatrix} 1 & 1 & -4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Q#02 row echelon form

$$A = \begin{bmatrix} -1 & 1 & -1 & 0 & 3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{bmatrix}$$

$-R_1$

$$\begin{bmatrix} 1 & -1 & +1 & 0 & -3 \\ -3 & 4 & 1 & 1 & 10 \\ 4 & -6 & -4 & -2 & -14 \end{bmatrix}$$

$R_2 + 3R_1, R_3 - 4R_1$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & -2 & -8 & -2 & -2 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

-3+
4+
1+
1+
10+
4.
-6
-4-
-2.
-10
-10

Q #03:

RREF

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$1 - 2(0) =$$

$$2 - 2(1) =$$

$$4 - 2(-2) =$$

$$R_1 - 8R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 + 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & 5 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - 3R_2$$

$$\begin{bmatrix} 1 & 4 & 0 & 17 \\ 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} 1 &= 3(0) \\ 4 &= 3(-4) \\ 3 &= 3(1) \\ 5 &= 3(-4) \end{aligned}$$

$$R_1 - 17R_3, R_2 + 4R_3$$

$$\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q#04.

$$A = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 + 3R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} 1 &+ 3(0) \\ 0 &+ 3(0) \\ -3 &+ 3(1) \\ 2 &+ 3(2) \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \leftarrow 8R_3 \rightarrow R_2 - R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$1 \neq 8(0)$$

$$0 \neq 8(1)$$

$$0 = 8(0)$$

$$8 = 8(1)$$

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 & 4 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

$$R_1 - 3R_2$$

$$\begin{bmatrix} 1 & 0 & 0 & -4 & 4 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

Q#05

RRREF

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 9 \\ 3 & 2 & 4 \end{bmatrix}$$

$R_2 + 2R_1$, $R_3 - 3R_1$

$$\begin{aligned} -2 + 2(1) &= 0 \\ 1 + 2(0) &= 1 \\ 9 + 2(-2) &= 5 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 2 & 10 \end{bmatrix}$$

$R_3 - 2R_1$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & -2 \\ 0 & 1 & 0 \\ -2 & 7 & 5 \end{bmatrix} \begin{matrix} \\ = R_2 + R_1 \\ R_4 + 2R_1 \end{matrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 7 & 7 \end{bmatrix}$$

$$R_4/7 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = R_2 - R_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_3 - R_4$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_1 + R_3, R_2 + R_3, R_4 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 2 \end{bmatrix}$$

$-R_3$ and R_4

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$R_4 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & -5 \\ 2 & -1 & 6 \\ 2 & -2 & 7 \end{bmatrix}$$

$$-R_1$$

$$\begin{bmatrix} 1 & -2 & 5 \\ 2 & -1 & 6 \\ 2 & -2 & 7 \end{bmatrix}$$

$$R_2 - 2R_1, R_3 - 2R_1$$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 3 & -4 \\ 0 & 2 & -3 \end{bmatrix}$$

$$2 - 2(1) = 0$$

$$-2 - 2(-2) = 2$$

$$4 - 2(5) = -6$$

$$R_2 - R_3$$

$$\begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix}$$

$$R_1 + R_3$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 2 & -3 \end{bmatrix},$$

$$R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 5 & 6 & -3 \\ -2 & -2 & 2 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 2 & 2 & -2 \\ -2 & -2 & 2 \end{bmatrix}$$

$R_4 + R_3$

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & -1 \\ 2 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 - 3R_1, R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$