CSC101
Introduction to ICT

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Lecture - 7

Computer Arithmetic

Binary vs Decimal

- Information is handled in a computer by electronic components
- Electronic components operate in binary mode (can have only two states 'on' or 'off')
- The binary number system has only two digits (0 and 1) and is best suitable for expressing these two possible states
  - Simple circuit design
  - Less expensive
  - More reliable circuits

- Binary Arithmetic
  - Binary arithmetic is simple to learn as binary number system has only two digits
  - There are rules defined for the four basic arithmetic operations (+, –, x and ÷) using the binary numbers

Binary Addition

0	+	0	=	0	
0	+	1	=	1	
1	+	0	=	1	
1	+	1	=	0	Plus a carry of 1 to the next higher column

Examples:

		1	1	
1	0	0	1	1
+	1	0	0	1
1	1	1	0	0

	1	1	1	1	1	
	1	0	0	1	1	1
	+	1	1	0	1	1
1	0	0	0	0	1	0

* The addition of three or more 1s can be broken up into further steps. First, we add first two digits. The third digit is added to the result of first addition and so on.

Binary Subtraction

0	-	0	=	0				
0	-	1	=	1	With a borrow from the next column			
1	-	0	=	1				
1	-	1	=	0				

Examples:

			0	$^{1}0$		
	1	0	$\cancel{1}$	$\cancel{1}$	$^{1}0$	1
-	1	0	0	1	1	1
	0	0	0	1	1	0

	0	1	1				
	$\cancel{1}$	$^{1}\cancel{0}$	$^{1}\cancel{0}$	$^{1}0$	1	0	1
-		1	0	1	1	0	0
	0	0	1	1	0	0	1

Binary Multiplication

0	x	0	=	0
0	x	1	=	0
1	x	0	=	0
1	x	1	=	1

Example:

			1	0	1	0
		x	1	0	0	1
			1	0	1	0
		0	0	0	0	-
	0	0	0	0	-	-
1	0	1	0	-	-	-
1	0	1	1	0	1	0

Binary Division

0	÷	0	=	Divide by zero error
0	÷	1	=	0
1	÷	0	=	Divide by zero error
1	÷	1	=	1

Examples:

```
          0 1 1
       ________
 1 1  | 1 0 1 1
          1 1
        -----
          1 0 1
            1 1
          -----
            1 0
```

```
              1 0 0 1 1 0
            _________________
  1 1 0    | 1 1 1 0 0 1 1 0
             1 1 0
             -----
                 1 0 0 1
                   1 1 0
                 -------
                     1 1 1
                     1 1 0
                     -----
                         1 0
                     -----
```

Negative Numbers in Binary

- We have seen positive numbers representation till now, how would negative numbers be represented in the binary number system?

- To represent negative number, in the binary number system, three methods;
  - Signed Magnitude
  - One's Complement
  - Two's Complement

Negative Numbers in Binary

Signed Magnitude

- Use the leftmost digit as a sign indication, and treat the remaining bits as if they represented an unsigned number
- The convention is that, if the leftmost digit (most significant bit) is 0 the number is positive, if it's 1 the number is negative

Example:

$(00001101)_2 = (13)_{10}$

$(10001101)_2 = (-13)_{10}$

Negative Numbers in Binary

Signed Magnitude

- A major drawback --- it doesn't support binary arithmetic

Example:

- Let's add $(10)_{10}$ and $(-10)_{10}$ represented by 'Signed Magnitude' in the binary number system

$$
\begin{array}{r}
00001010 \\
+\ 10001010 \\
\hline
10010100
\end{array}
= (-20)_{10} \ \text{✗}
$$

Negative Numbers in Binary

One's Complement

- To obtain one's complement, simply flip all the bits (digits) of a binary number
- Suppose we have a binary number, 00001010
- It's one complement would be, 11110101
- Notice that the complement is $(245)_{10}$, which is $(255)_{10} - (10)_{10}$.

- That's no co-incidence
- In general, complement of a number is the largest number represented with the number of bits available minus the number itself
- Since we are using 8 bits here, the maximum number represented is 255 (232 – 1), so the complement of $(10)_{10}$ will be $(245)_{10}$

Negative Numbers in Binary

One's Complement

- One's complement can be used to represent negative numbers in binary number system
- The one's complement form of a negative binary number is the complement of its positive counterpart, which can be obtained by applying the NOT to the positive counterpart

- One's complement has two representations of 0
    - 00000000 (+0) and 11111111 (–0)

Negative Numbers in Binary

One's Complement

Example:

- Let's add $(10)_{10}$ and $(-10)_{10}$ in binary number system using one's complement

$(00001010)_2 = (10)_{10}$

one's complement of $(00001010)_2 = (11110101)_2 = (-10)_{10}$

$$
\begin{array}{r}
0\,0\,0\,0\,1\,0\,1\,0 \\
+\,1\,1\,1\,1\,0\,1\,0\,1 \\
\hline
1\,1\,1\,1\,1\,1\,1\,1
\end{array}
= (0)_{10} \checkmark
$$

Negative Numbers in Binary

One's Complement

Example:

- Let's add $(69)_{10}$ and $(-38)_{10}$ in binary number system using one's complement

$(01000101)_2 = (69)_{10}$

$(00100110)_2 = (38)_{10}$

one's complement of $(00100110)_2 = (11011001)_2 = (-38)_{10}$

$$
\begin{array}{r}
01000101 \\
+\,11011001 \\
\hline
100011110
\end{array}
= (286)_{10} \;\; ✗
$$

Negative Numbers in Binary

One's Complement

Example:

- Let's add $(3)_{10}$ and $(-2)_{10}$ in binary number system using one's complement

$(00000011)_2 = (3)_{10}$

$(00000010)_2 = (2)_{10}$

one's complement of $(00000010)_2 = (11111101)_2 = (-2)_{10}$

$$
\begin{array}{r}
0\,0\,0\,0\,0\,0\,1\,1 \\
+\,1\,1\,1\,1\,1\,1\,0\,1 \\
\hline
1\,0\,0\,0\,0\,0\,0\,0\,0
\end{array}
= (256)_{10} \quad \times
$$

Negative Numbers in Binary

One's Complement

- So, it partially solves the binary arithmetic problem but there are some special cases left

Two's Complement

- Two's complement representation allows the use of binary arithmetic operations on signed integers, yielding the correct results
- Two's complement of a binary number can be obtained by adding 1 to the one's complement of that number

$(12)_{10} = (00001100)_2$

one's complement of $(00001100)_2$ = $(11110011)_2$

$$
\begin{array}{r}
1\,1\,1\,1\,0\,0\,1\,1 \\
+\,0\,0\,0\,0\,0\,0\,0\,1 \\
\hline
1\,1\,1\,1\,0\,1\,0\,0
\end{array}
$$

(two's complement of $(00001100)_2$)

Negative Numbers in Binary

Two's Complement

Example:

- Let's add $(12)_{10}$ with $(-5)_{10}$ in binary number system using two's complement

$(12)_{10} = (00001100)_2$

$(5)_{10} = (00000101)_2$

two's complement of $(00000101)_2 = (11111011)_2$

$$
\begin{array}{r}
00001100 \\
+\,11111011 \\
\hline
00000111
\end{array}
= (7)_{10}
$$

* How to add two negative numbers? e.g., $(-5)_{10} + (-2)_{10}$

Negative Numbers in Binary

Subtraction using one's complement

- Find the one's complement of the number you are subtracting
- Add this to the number from which you are taking away

- Two scenarios;
    1. If there is a carry of 1, add it to obtain the result
    2. if there is no carry, re-complement the sum and attach a negative sign

Negative Numbers in Binary

Scenario 1: If there is a carry

Example:

Subtract $(0111000)_2$ from $(1011100)_2$

One's complement of $(0111000)_2$ is $(1000111)_2$

	1	0	1	1	1	0	0
+	1	0	0	0	1	1	1
1	0	1	0	0	0	1	1
+							1
	0	1	0	0	1	0	0

Negative Numbers in Binary

Scenario 2: If there is no carry

Example:

Subtract $(100011)_2$ from $(010010)_2$

One's complement of $(100011)_2$ is $(011100)_2$

	0	1	0	0	1	0
+	0	1	1	1	0	0
	1	0	1	1	1	0

- Re-complement the answer

  one's complement of $(101110)_2$ is $(010001)_2$

- Append a negative sign to the final answer

  result = – 010001

THANK YOU