Image Model

$f(x,y)$ can be characterized by two components:

1. The amount of source light incident on the scene being viewed
   Illumination component = $\boldsymbol{i(x,y)}$
2. Amount of light reflected by the objects in the scene
   Reflectance component = $\boldsymbol{r(x,y)}$

$$\boxed{\begin{aligned} &f(x,y) = i(x,y)r(x,y) \\ &0 \le i(x,y) < \infty \\ &0 \le r(x,y) \le 1 \end{aligned}}$$

Image Model

Reflectance is bounded by 0 (total absorption) and 1 (total reflectance)

The nature of $i(x,y)$ is determined by the light source and $r(x,y)$ is determined by the characteristics of the objects in a scene.

***i*(*x,y*)**:
Clear Day – 90,000 lm/m^2
Cloudy day – 10,000 lm/m^2
Full moon – 0.01 lm/m^2
Commercial office - 1000 lm/m^2

Typical values of ***r*(*x,y*)**
Black Velvet - 0.01
Stainless Steel - 0.65
Flat-White wall paint - 0.80
Silver plated metal - 0.90
Snow - 0.93

Gray Scale

Intensity of monochrome image f at coordinates (x,y) $\quad l = f(x_0, y_0)$

$$L_{\min} \leq l \leq L_{\max}$$

$$L_{\min} = i_{\min} r_{\min} \qquad\qquad L_{\max} = i_{\max} r_{\max}$$

The interval: $[L_{\min} \quad L_{\max}]$ is called the gray scale

Commonly, [0, L-1] interval is used.

0 > Black, & L-1 > White

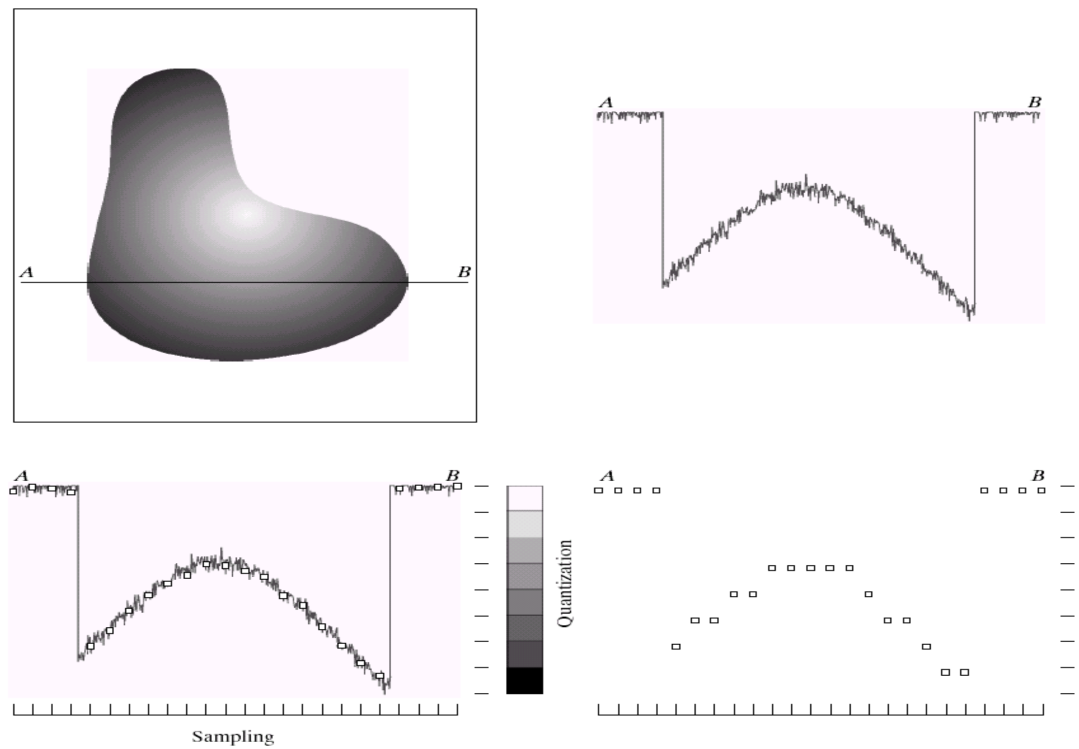
Image Sampling and Quantization

For computer processing, an image function needs to be digitized both spatially and in amplitude.

Image Sampling – Digitization of the spatial coordinates (*x*,*y*)

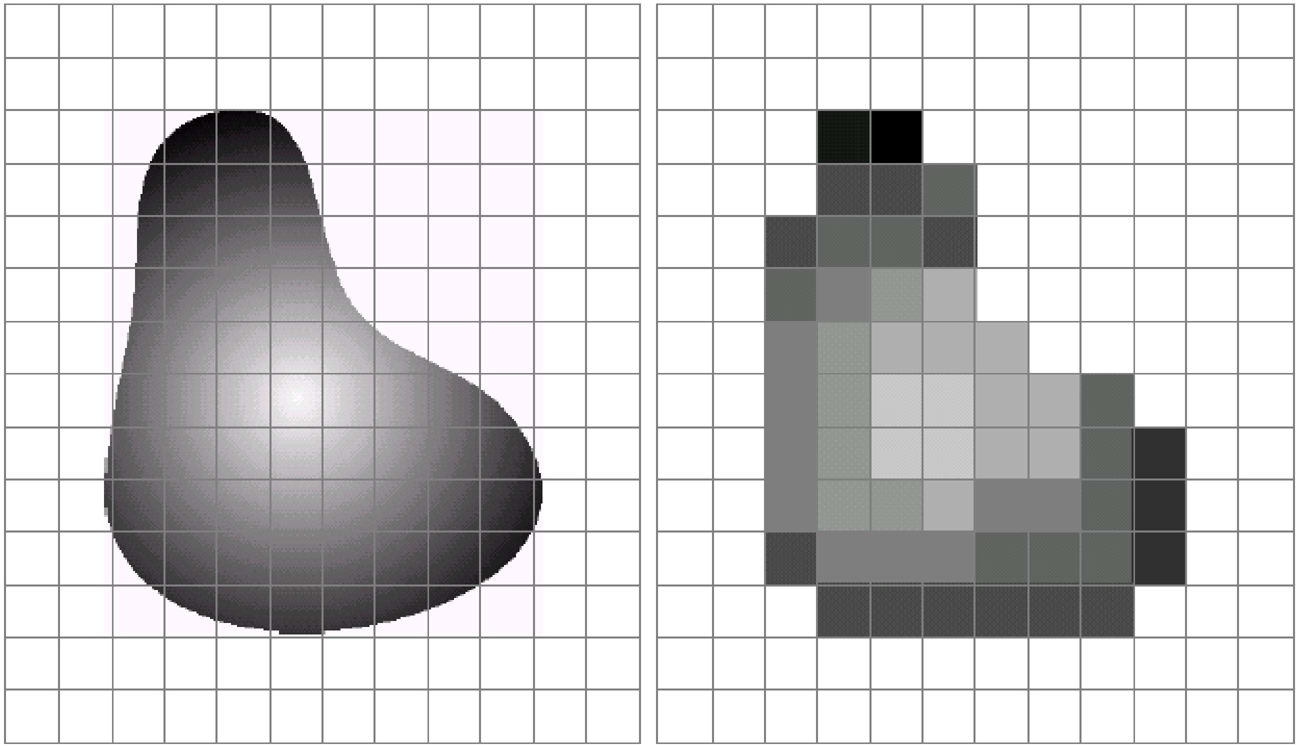
Gray-level quantization – Digitization of Amplitude

Scan Line



a b
c d

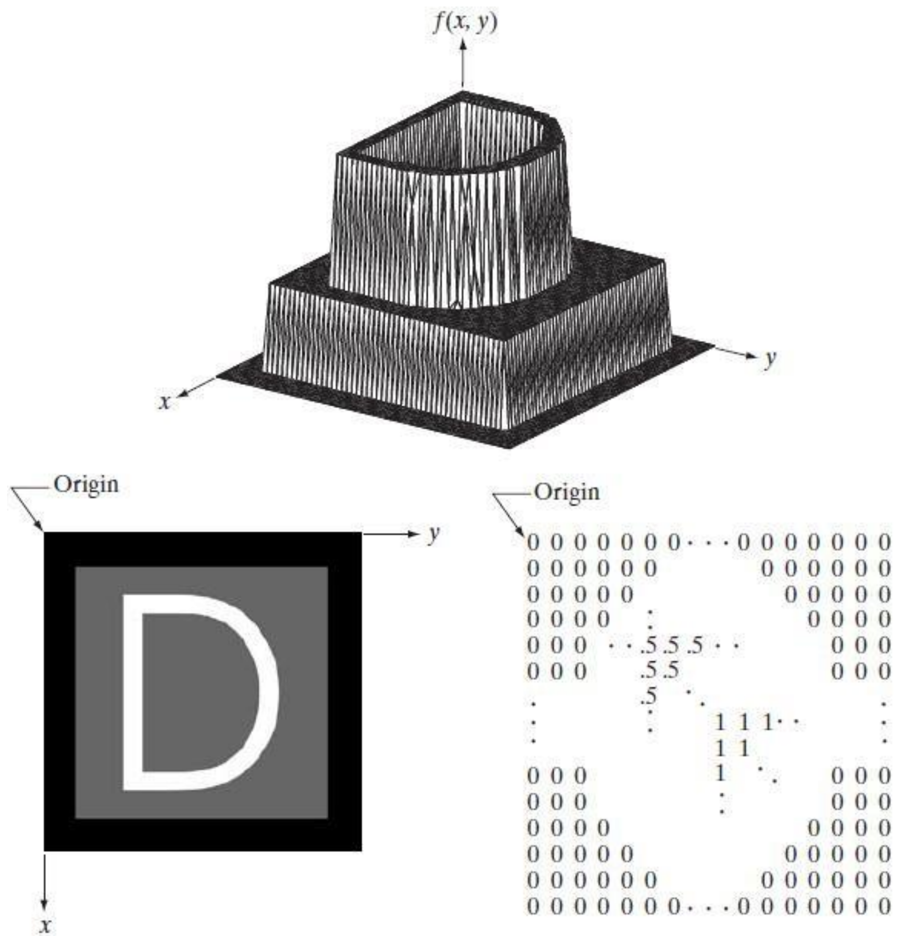
FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Digital Image Representation



Mathematical Representation

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0, N-1) \\ f(1,0) & f(1,1) & \cdots & f(1, N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1, 0) & f(M-1, 1) & \cdots & f(M-1, N-1) \end{bmatrix}$$

Mathematical Representation

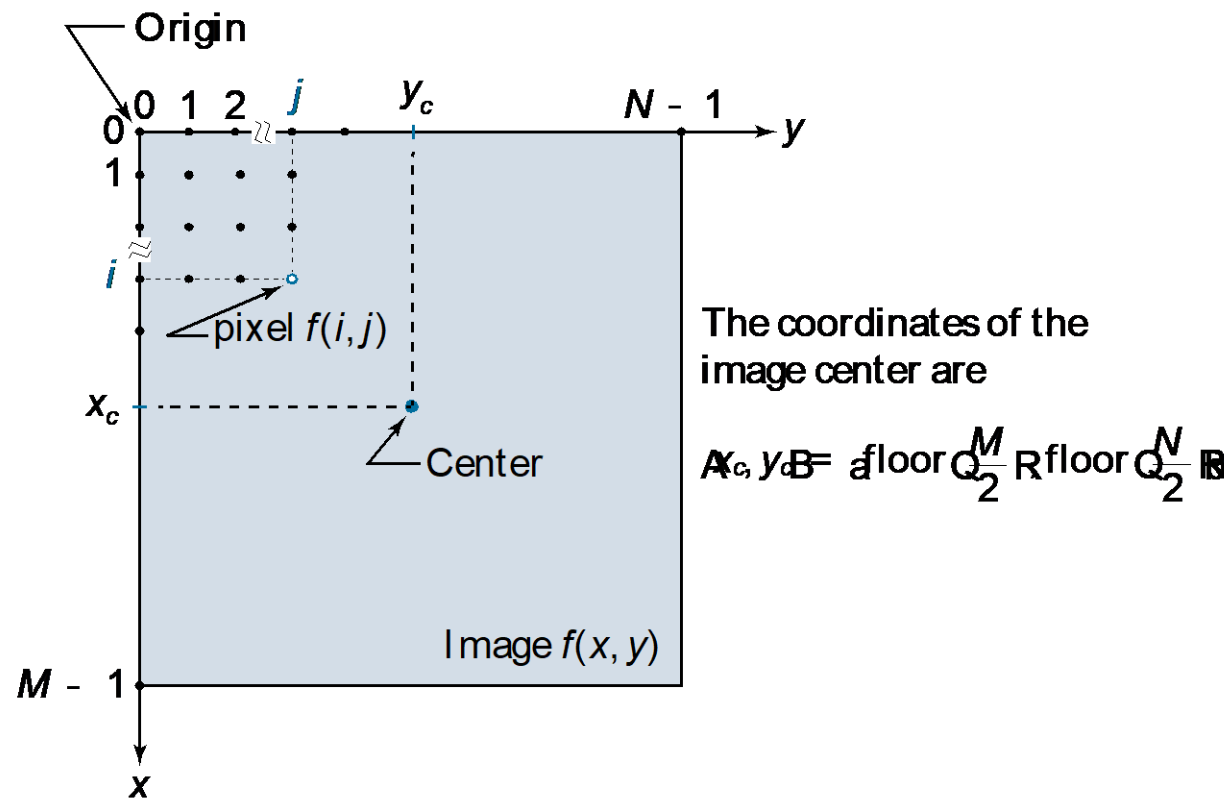


FIGURE 2.19 Coordinate convention used to represent digital images. Because coordinate values are integers, there is a one-to-one correspondence between *x* and *y* and the rows (*r*) and columns (*c*) of a matrix.

Dynamic Range

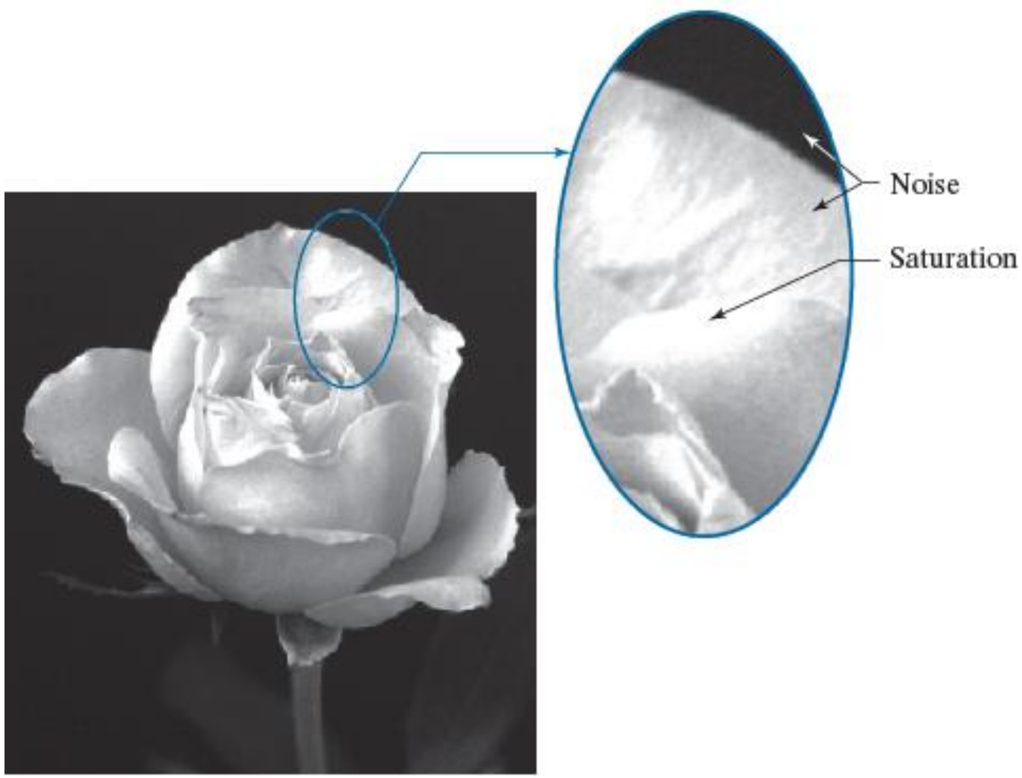
- $L = 2^k$ gray levels, gray scales [0,…,L-1]
- The dynamic range of an image
  - [min(image) max(image)]
  - Highest value beyond that all values are clipped (saturation)
  - texture region (noise).

If the dynamic range of an image spans a significant portion of the gray scale → high contrast

- Otherwise, low dynamic range results in a washed out gray look.

Saturation and Noise

FIGURE 2.20
An image exhibiting saturation and noise. Saturation is the highest value beyond which all intensity values are clipped (note how the entire saturated area has a high, constant intensity level). Visible noise in this case appears as a grainy texture pattern. The dark background is noisier, but the noise is difficult to see.



Representing Digital Images

Continuous image $f(x,y)$ – Equally spaced samples arranged as (N×M) array (matrix)

Each element in the array is a discrete quantity.

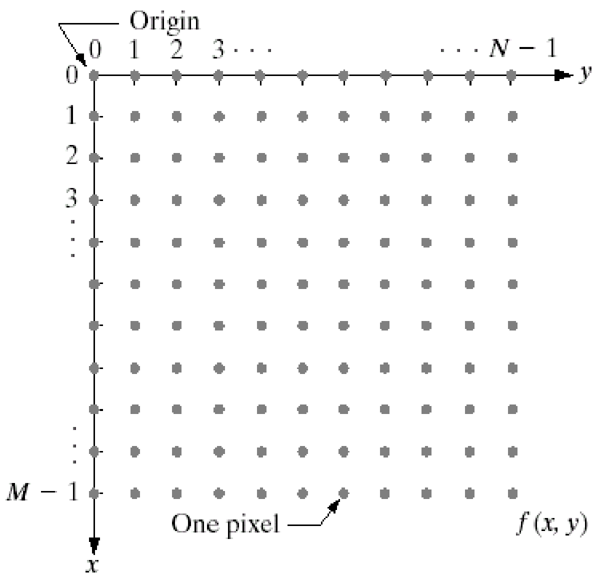


FIGURE 2.18 Coordinate convention used in this book to represent digital images.

Number of Storage bits for different choice of N and k (N=M)

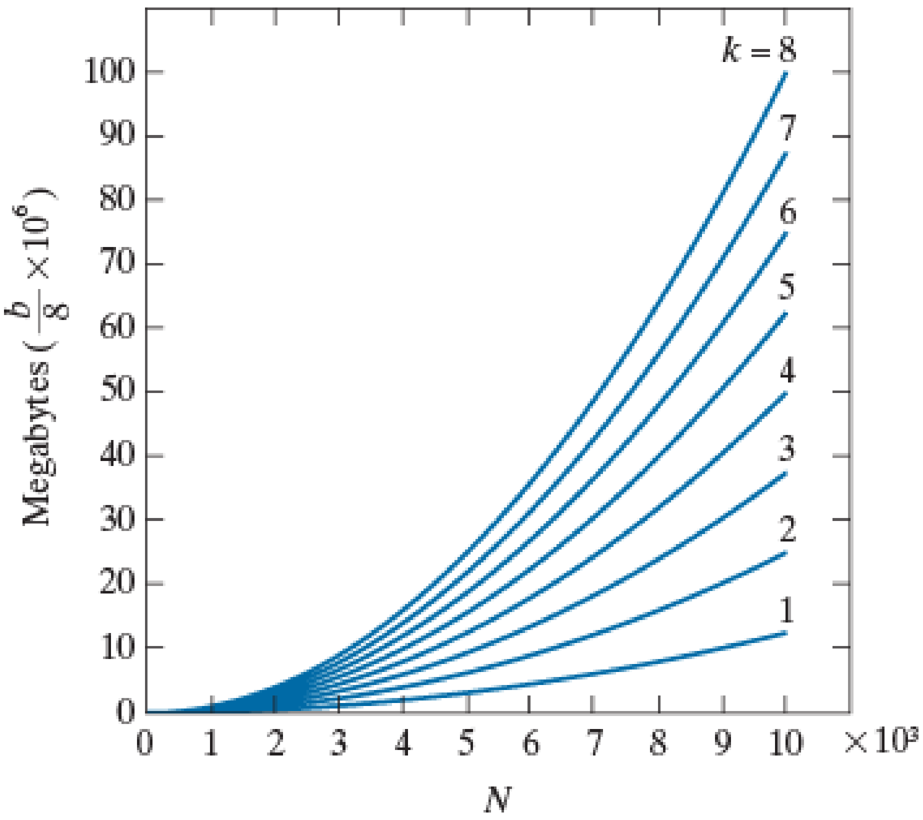
bits to store the image = N x M x k
gray level = L = 2^k

TABLE 2.1
Number of storage bits for various values of N and k.

N/k	1 (L = 2)	2 (L = 4)	3 (L = 8)	4 (L = 16)	5 (L = 32)	6 (L = 64)	7 (L = 128)	8 (L = 256)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Number of Storage bits for different choice of N and k (N=M)

FIGURE 2.21
Number of megabytes required to store images for various values of N and k.



Resolution

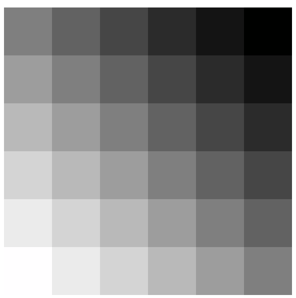
(Width × Height) = Col(N) × Row(M)

Resolution	Size (Width × Height)
SD	720 × 576
HD (720p)	1280 × 720
Full HD (1080p)	1920 × 1080
Ultra HD (4K)	3840 × 2160
UHD-2 (8K)	7680 × 4320

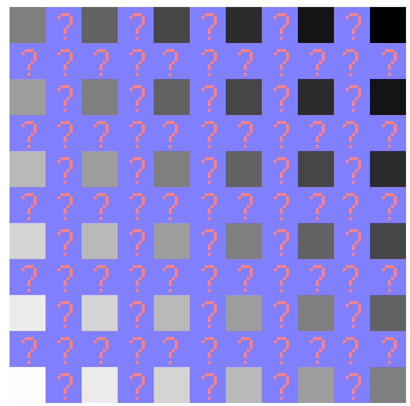
- The size of an image (W×H) is called its resolution.

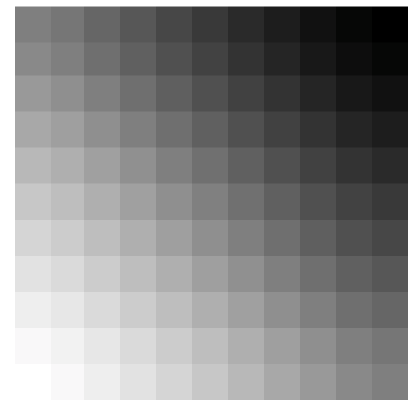
- **DPI (Dots per inch):** The number of dots per unit inch in linear direction in a printed picture (resolution for an image printed on page)
- **PPI (Pixel per inch):** The number of pixels per unit inch in linear direction in a image displayed on a screen (resolution for an image displayed on screen)
- **Spatial Resolution:** Spatial resolution is the smallest detectable detail in an image.
- **Grey level Resolution:** Gray-level resolution similarly refers to the smallest detectable change in gray level.

Enlargement

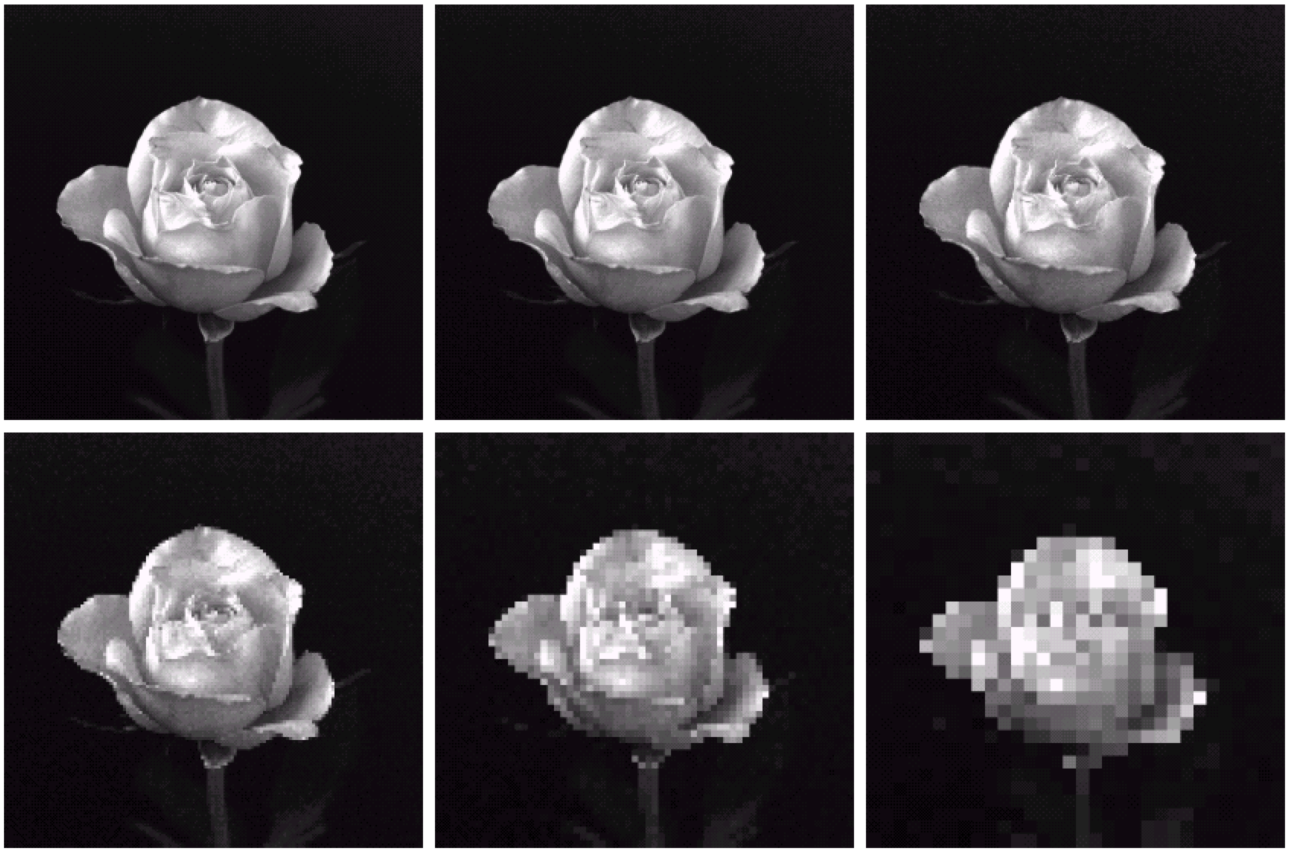


Enlarge 183%





Comparing spatial resolution



a b c
d e f

FIGURE 2.20 (a) 1024 × 1024, 8-bit image. (b) 512 × 512 image resampled into 1024 × 1024 pixels by row and column duplication. (c) through (f) 256 × 256, 128 × 128, 64 × 64, and 32 × 32 images resampled into 1024 × 1024 pixels.

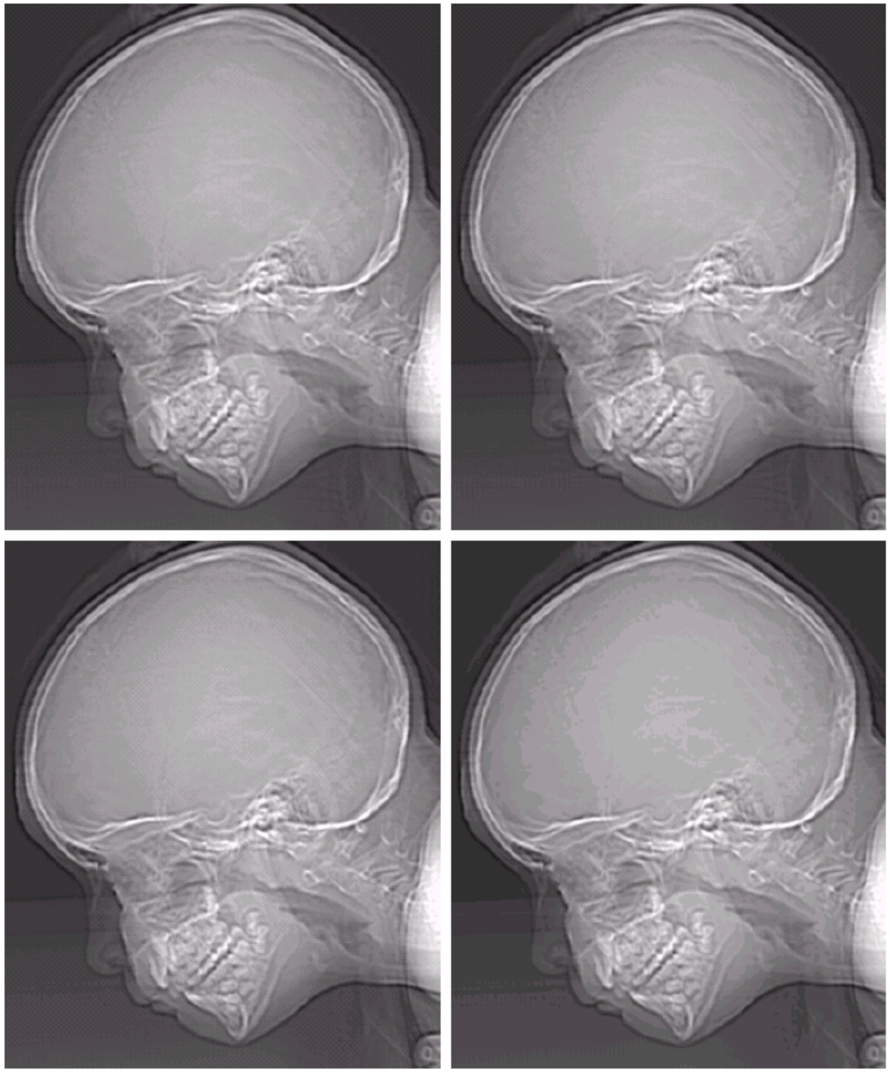
Comparing Spatial Resolution

1250 dpi | 300 dpi | 150 dpi | 75 dpi

Gray-Level Resolution

Number of samples are constant,
while gray-level L changes

k = 8 to 5



a b
c d

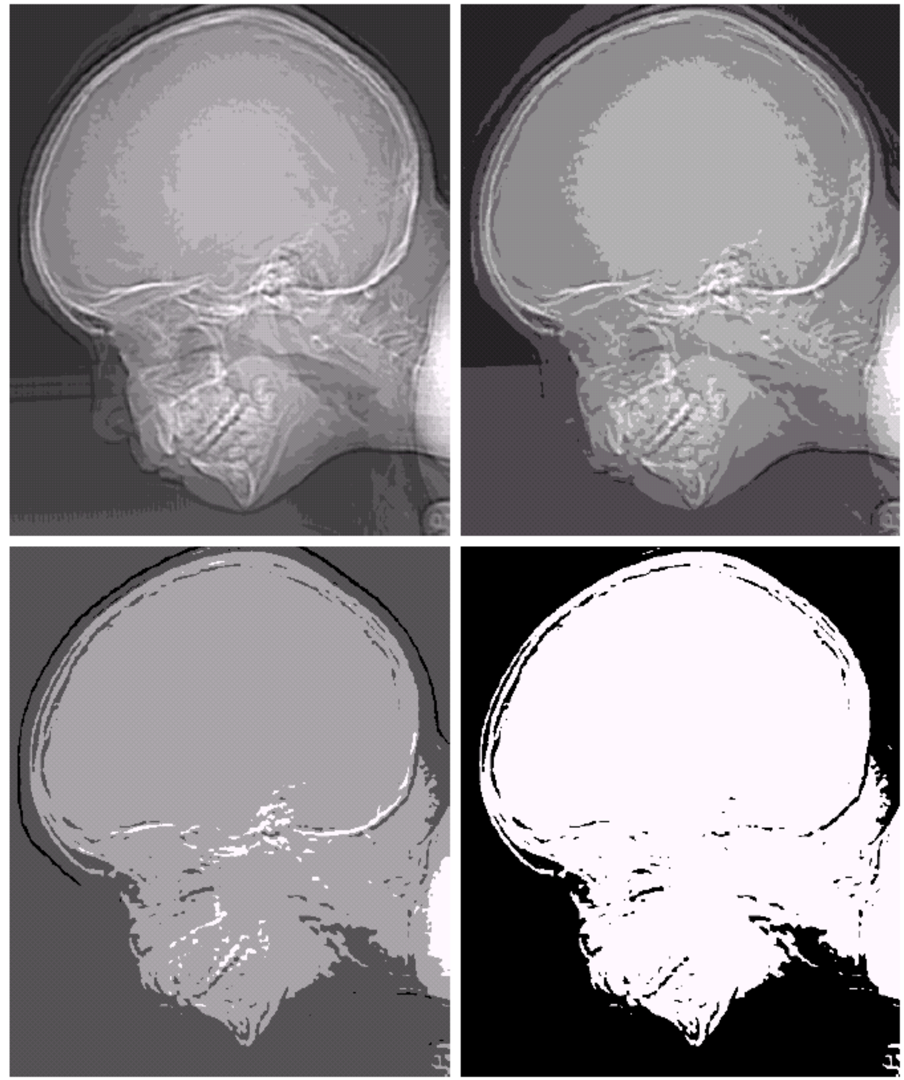
FIGURE 2.21
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

Gray-Level Resolution

k = 4 to 1

e f
g h

FIGURE 2.21 *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



Three types of image (Low/Medium/High Details):

a b c

FIGURE 2.22 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

Sampling-Quantization Tradeoff

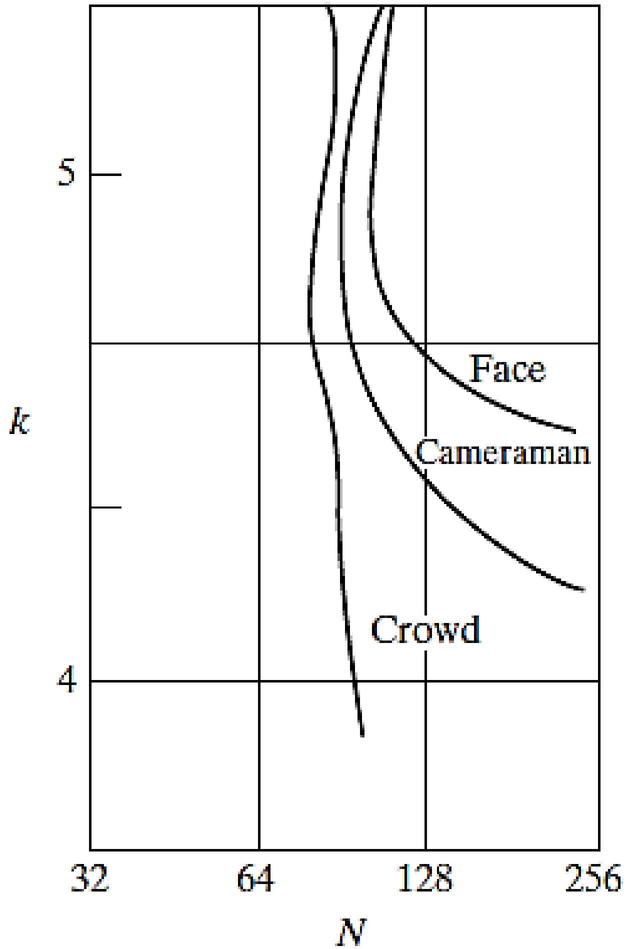


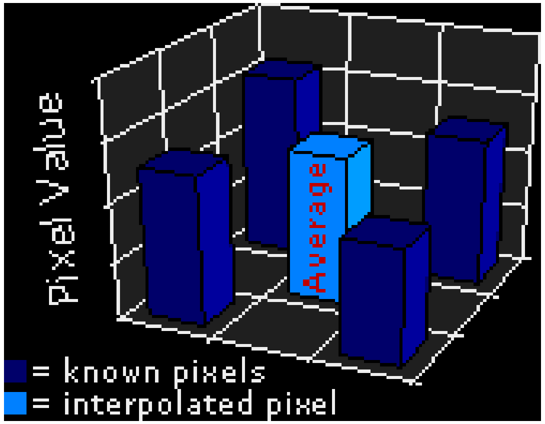
FIGURE 2.23 Typical isopreference curves for the three types of images in Fig. 2.22.

Zooming and Shrinking of Digital Images

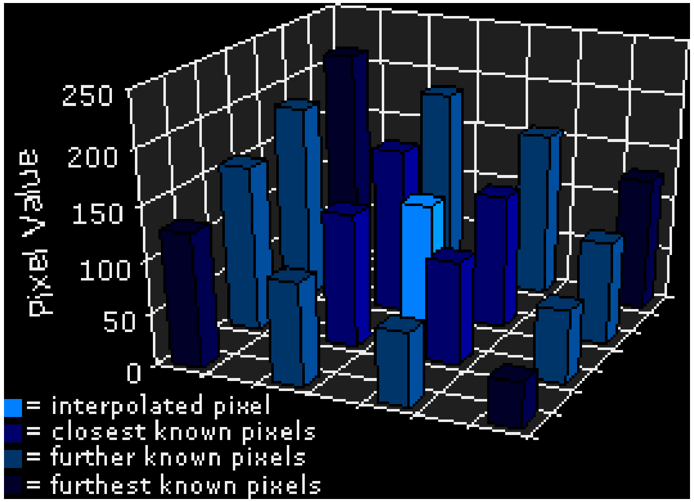
- **Zooming** : Oversampling
    - Creation of new pixel locations
    - The assignment of gray levels to those new locations
  - Nearest Neighbor Interpolation
    - Special case : Pixel Replication, if the size of new image is integer multiple of the original image size
    - NN is fast but produces checker-board effect
  - Bilinear Interpolation: uses four nearest neighbors of a point
  - Bi-cubic Interpolation: uses sixteen nearest neighbors of a point

- **Shrinking**: Under sampling

Interpolation



Bilinear interpolation



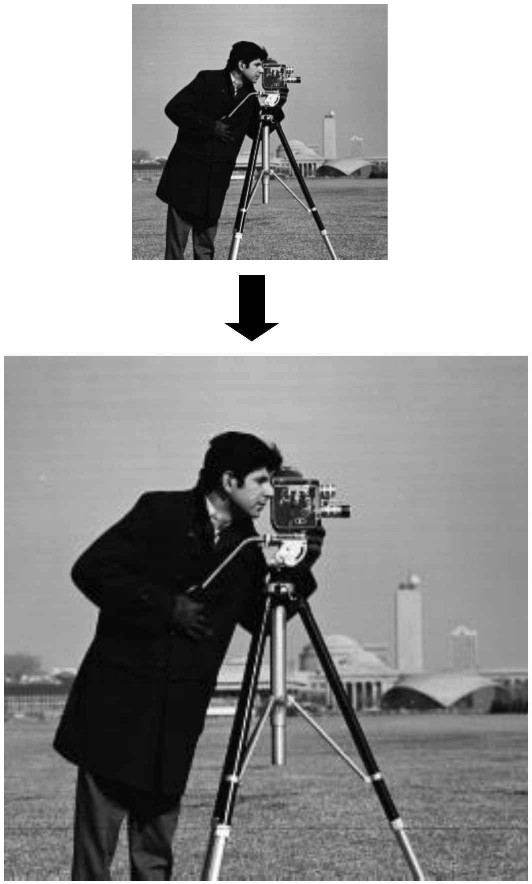
Bicubic interpolation

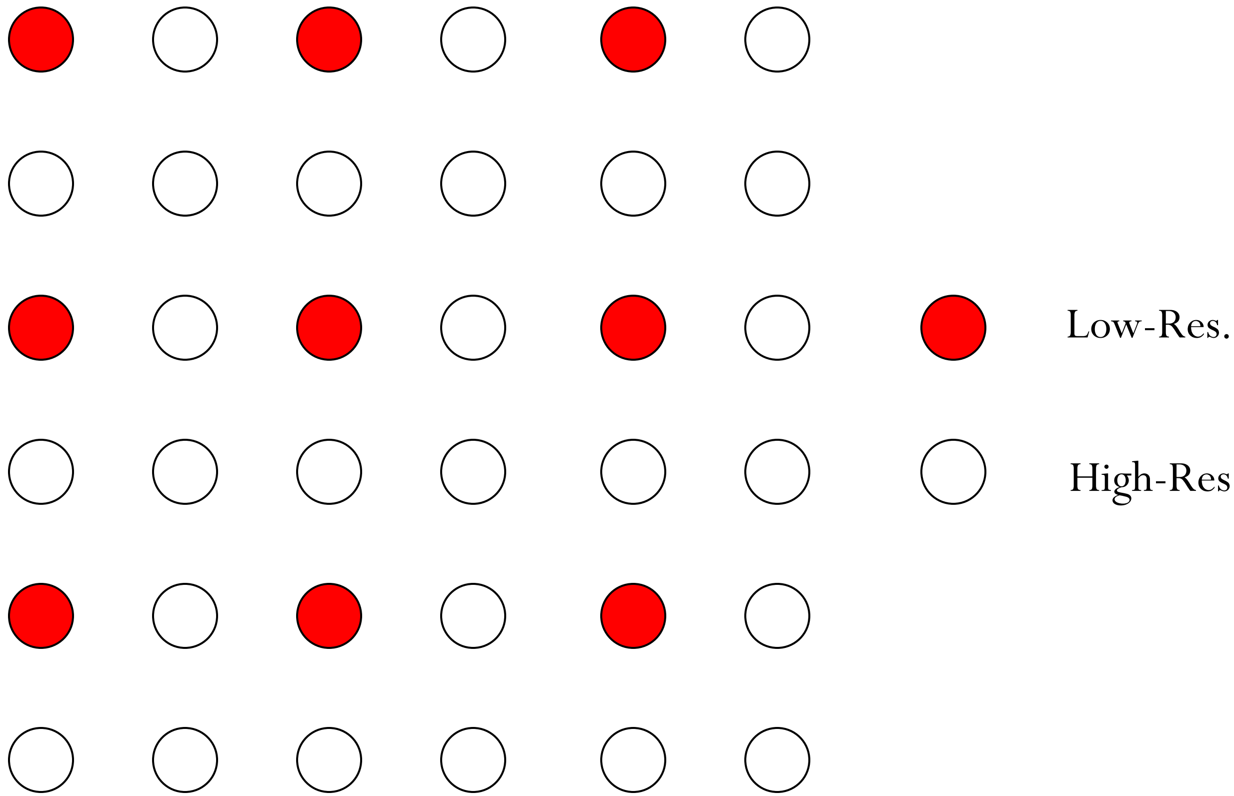
Zooming

10x Optical zoom

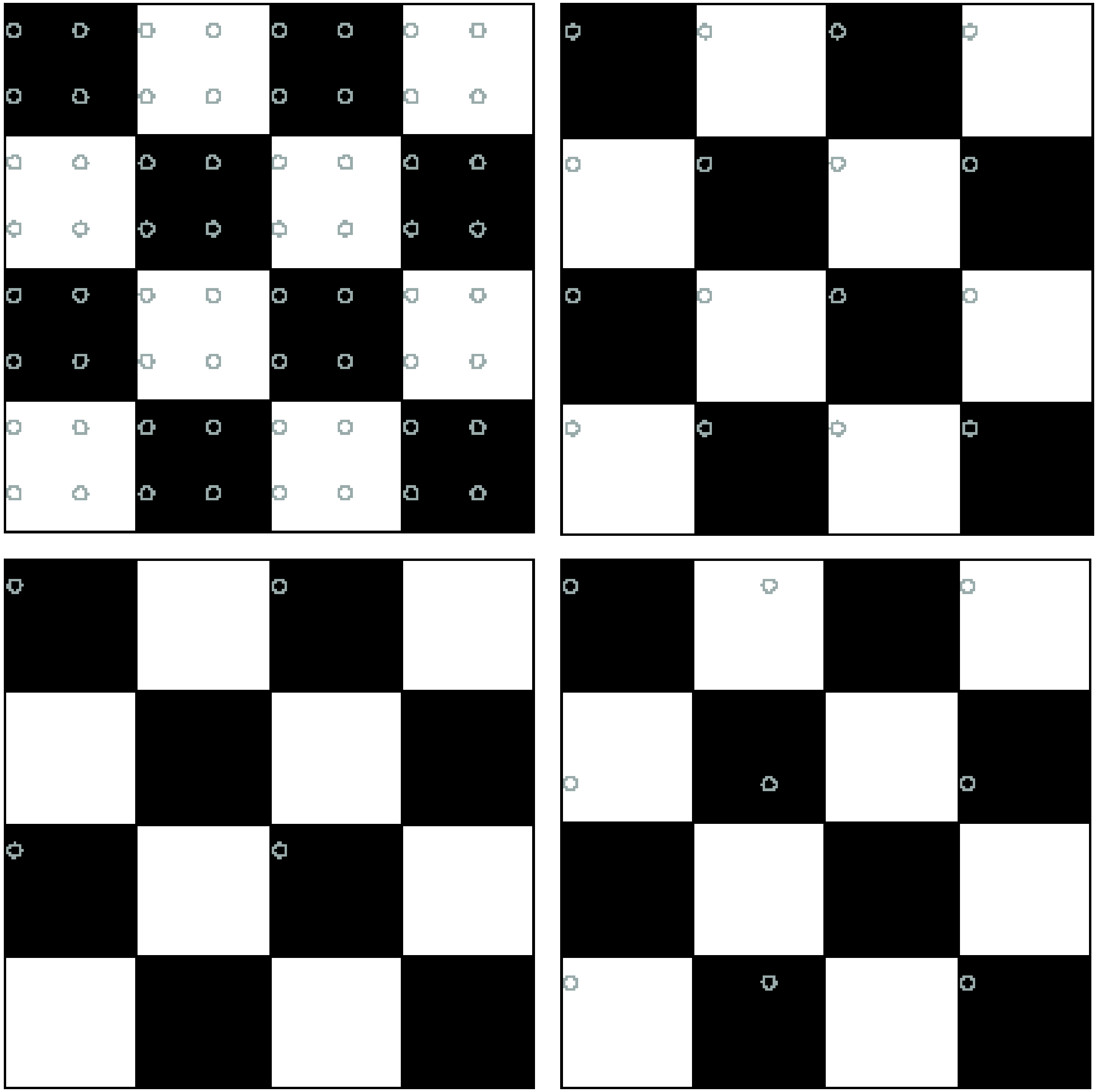
10x Digital zoom

Scenario I: Resolution Enhancement





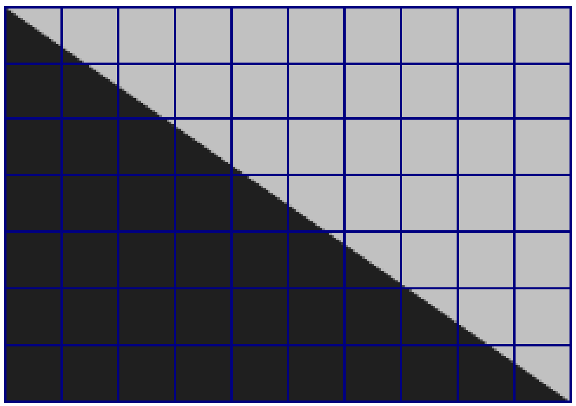
2D example



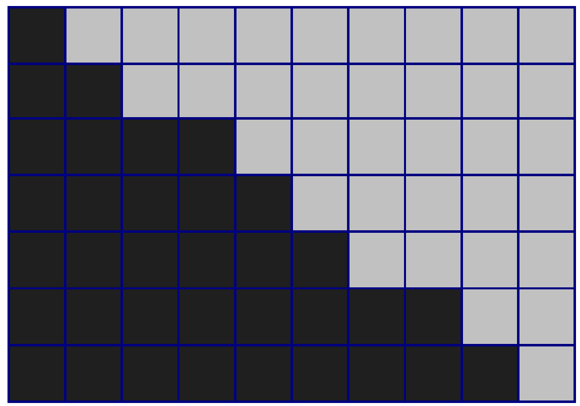
Good sampling
(Up sampling)

Bad sampling
(Down sampling)

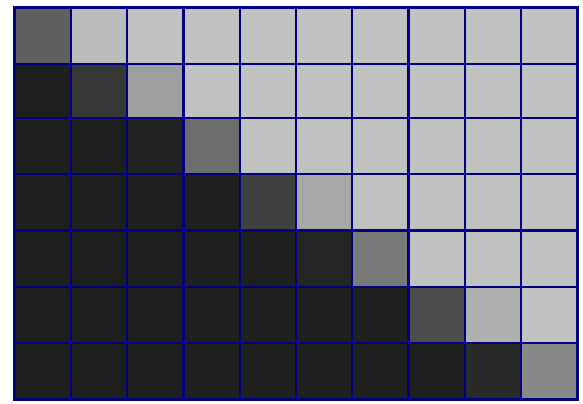
Aliasing



Perfect diagonal

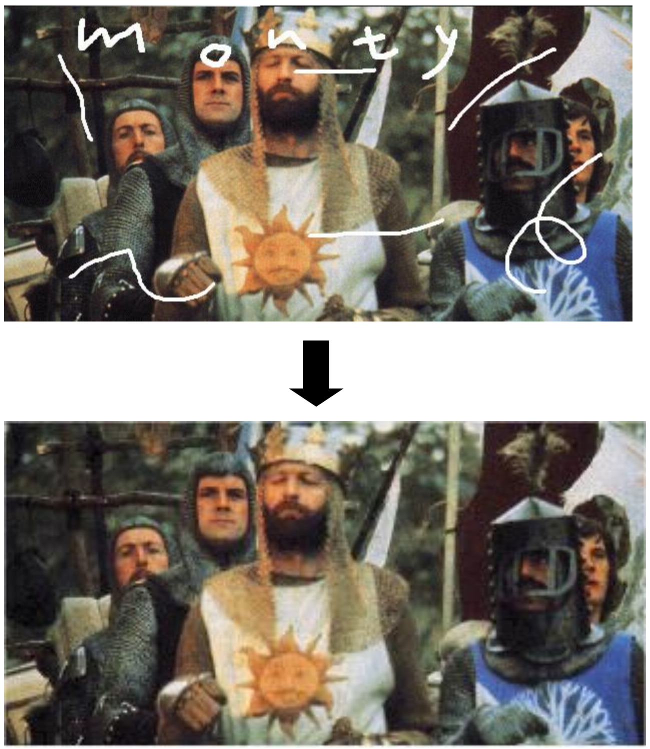


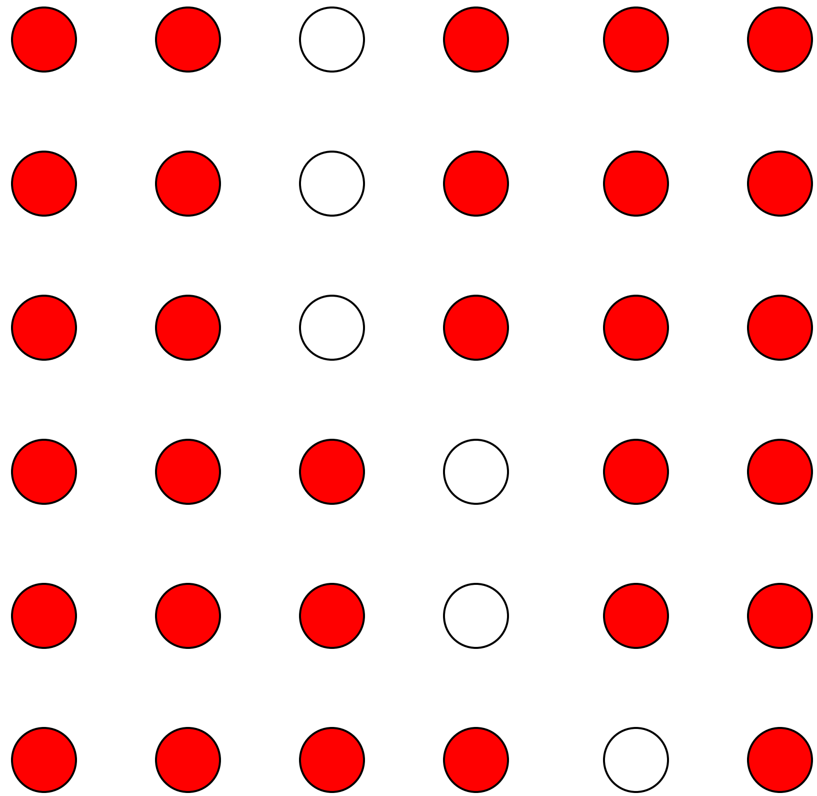
Resampled to Low Resolution
Aliased



Resampled to Low Resolution
Anti-Aliased

Scenario II: Image Inpainting

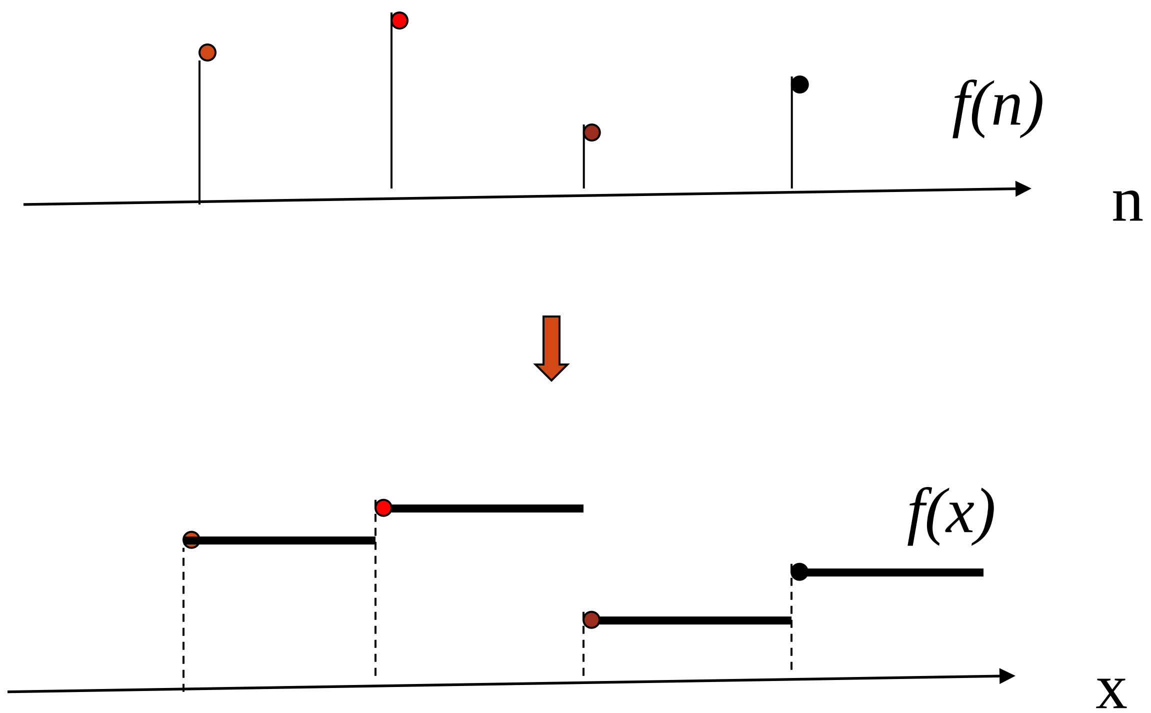




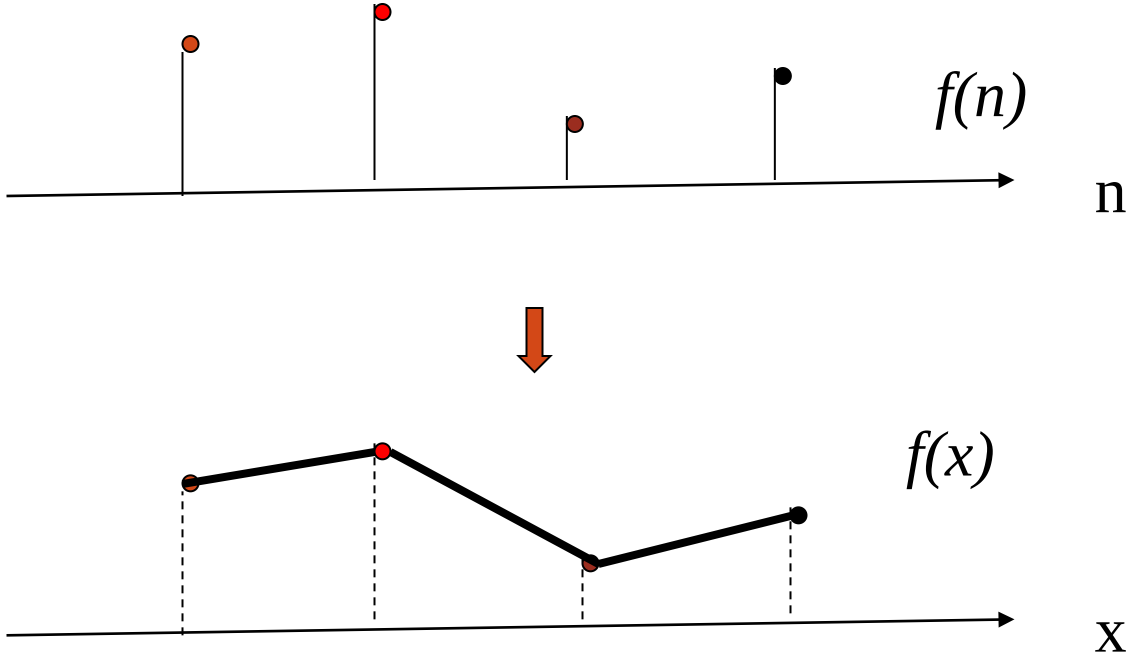
Non-damaged

Damaged

1D Zero-order (Replication)

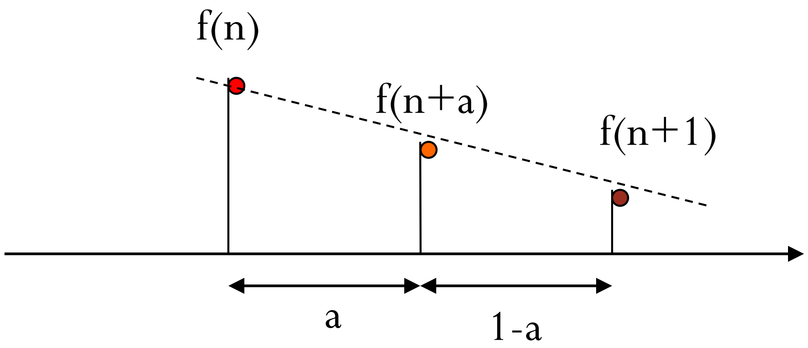


1D First-order Interpolation (Linear)



Linear Interpolation Formula

Heuristic: the closer to a pixel, the higher weight is assigned
Principle: line fitting to polynomial fitting (analytical formula)



$$\boxed{f(n+a)=(1-a)\times f(n)+a\times f(n+1),\ 0<a<1}$$

Note: when a=0.5, we simply have the average of two

Numerical Examples

f(n)=[0,120,180,120,0]

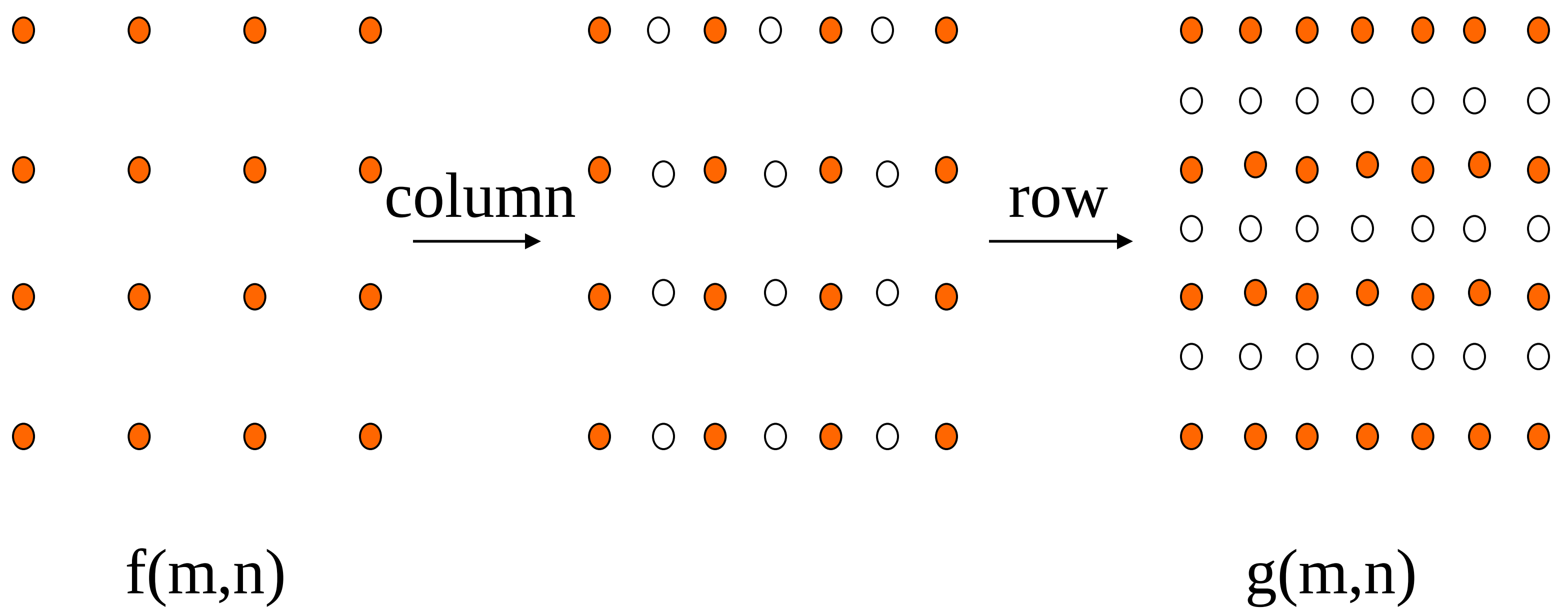
$\downarrow$ Interpolate at 1/2-pixel

f(x)=[0,60,120,150,180,150,120,60,0], x=n/2

$\downarrow$ Interpolate at 1/3-pixel

f(x)=[0,20,40,60,80,100,120,130,140,150,160,170,180,…], x=n/6

Graphical Interpretation of Interpolation at Half-pel



Numerical Examples

a	b
c	d

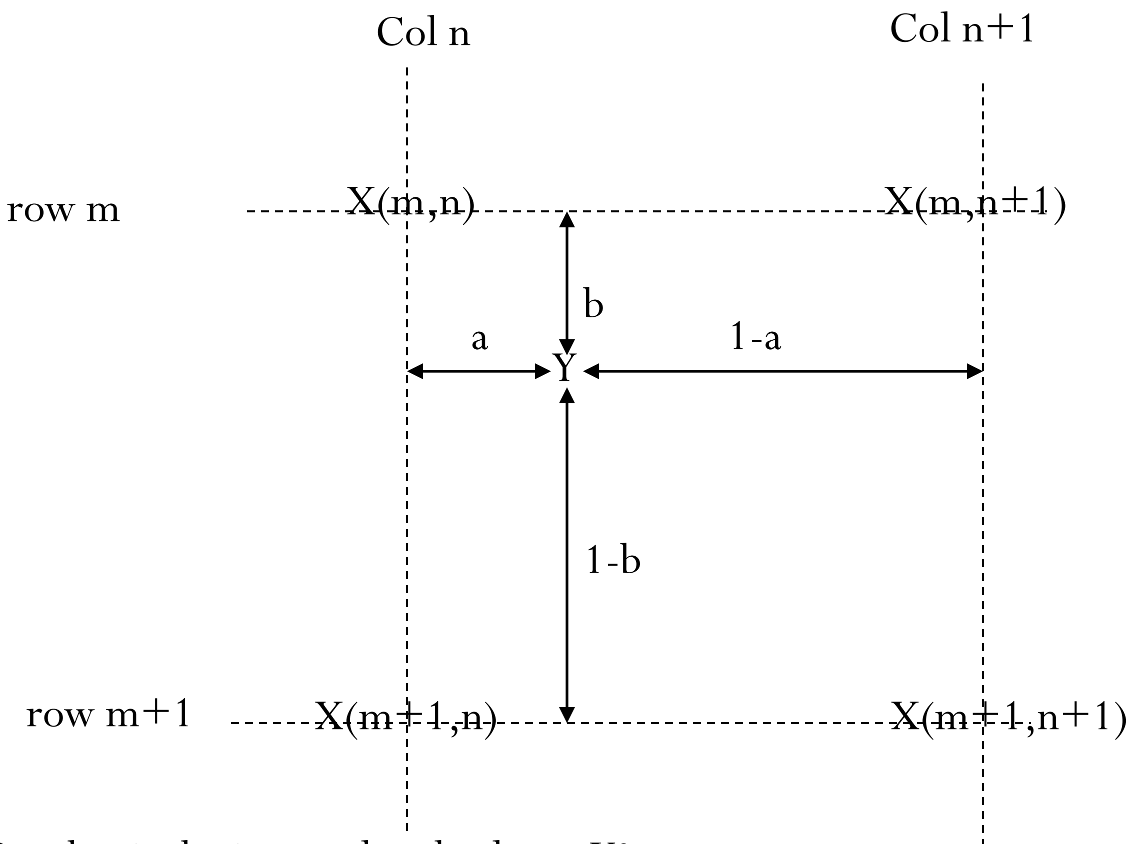
zero-order

a	a	b	b
a	a	b	b
c	c	d	d
c	c	d	d

first-order

a	(a+b)/2	b
(a+c)/2	(a+b+c+d)/4	(b+d)/2
c	(c+d)/2	d

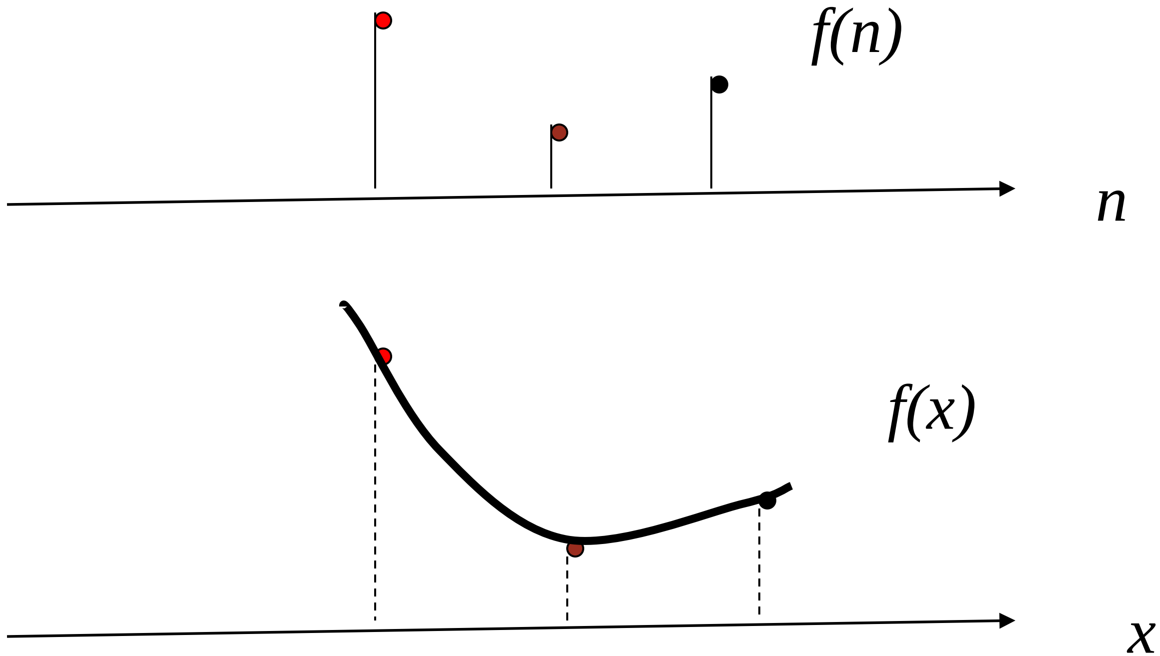
Numerical Examples (Con't)



Q: what is the interpolated value at Y?

Ans.: (1-a)(1-b)X(m,n) + (1-a)bX(m+1,n) + a(1-b)X(m,n+1) + abX(m+1,n+1)

1D Third-order Interpolation (Cubic)*



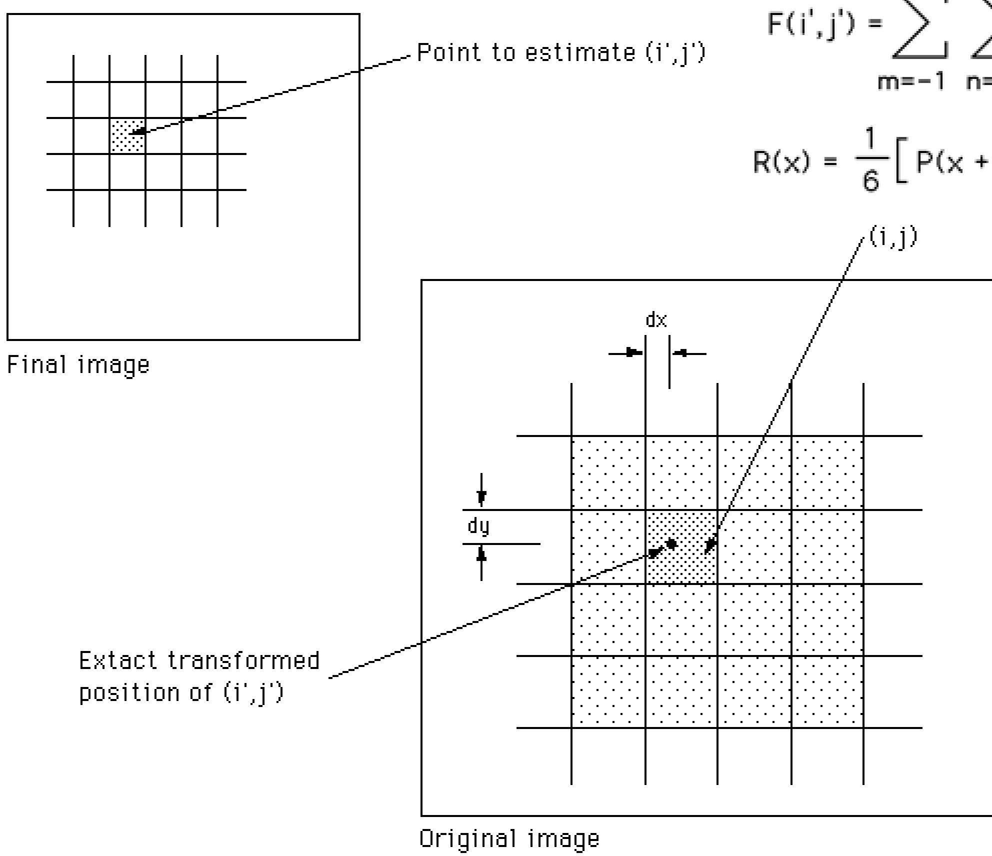
Cubic spline fitting

Bicubic Interpolation*

$$F(i',j') = \sum_{m=-1}^{2} \sum_{n=-1}^{2} F(i + m, j + n)\, R(m - dx)\, R(dy - n)$$

$$R(x) = \frac{1}{6}\left[P(x + 2)^3 - 4\, P(x + 1)^3 + 6\, P(x)^3 - 4\, P(x - 1)^3 \right]$$

$$P(x) = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$



Pixel Replication (NN)

low-resolution image (100×100)

high-resolution image (400×400)

Bilinear Interpolation

low-resolution
image (100×100)

high-resolution
image (400×400)

Bicubic Interpolation

low-resolution image (100×100)

high-resolution image (400×400)

Zooming Using NN & Bilinear Interpolation

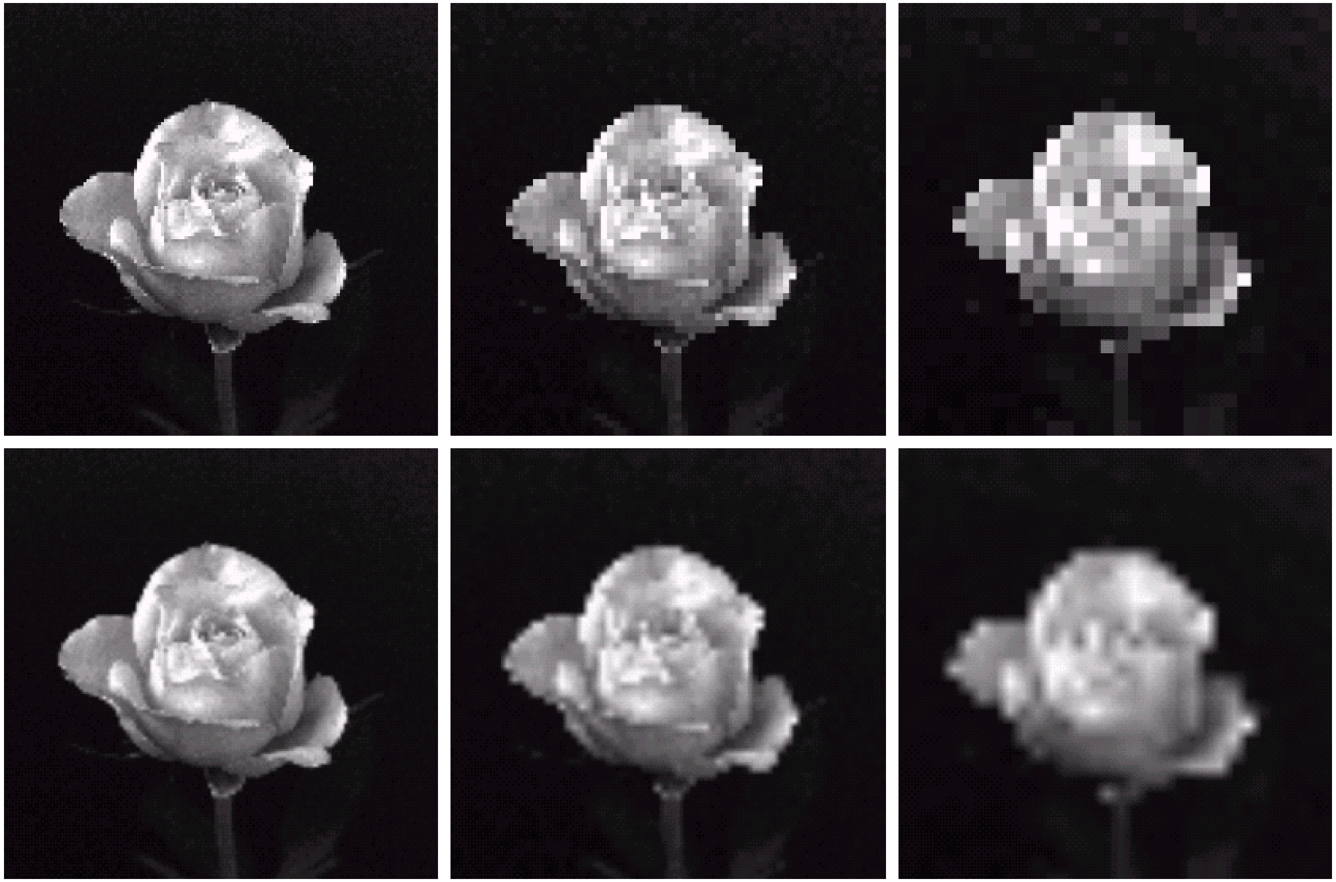
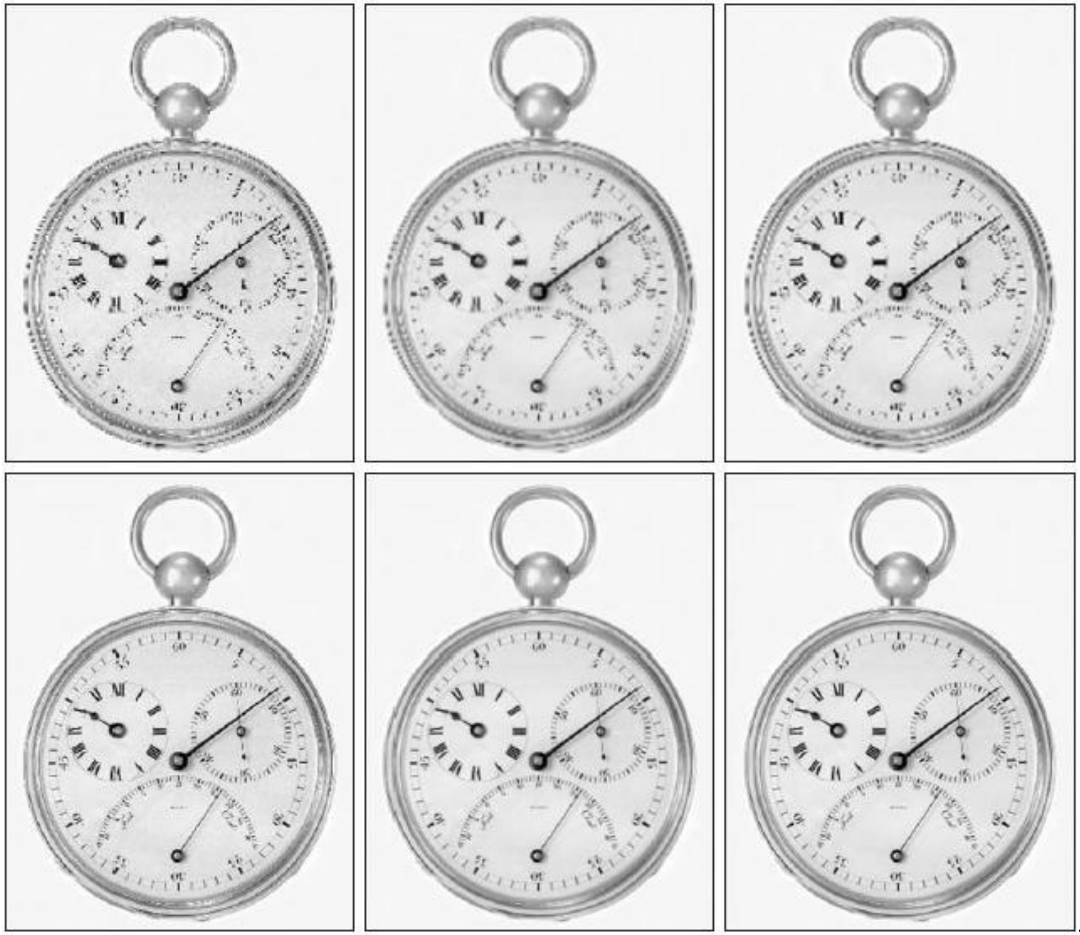


FIGURE 2.25 Top row: images zoomed from 128 × 128, 64 × 64, and 32 × 32 pixels to 1024 × 1024 pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

Image Interpolation (Example):

- Original size – 3692×2812 (930 dpi)

- Reduced to smaller size, and zoomed back to Original Size



- From 72 dpi,

  NN, BL, BC

- From 150 dpi,

  NN, BL, BC