Differential Equations Formulas



1. Constant:
$$\frac{d}{dx}c = 0$$

3. Sum:
$$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$$

5. Quotient:
$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

7. Power:
$$\frac{d}{dx}x^n = nx^{n-1}$$

2. Constant Multiple:
$$\frac{d}{dx}cf(x) = cf'(x)$$

4. Product:
$$\frac{d}{dx}f(x)g(x) = f(x)g'(x) + g(x)f'(x)$$

6. Chain:
$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

8. Power:
$$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}g'(x)$$

Trigonometric:

9.
$$\frac{d}{dx}\sin x = \cos x$$

12.
$$\frac{d}{dx}\cot x = -\csc^2 x$$

10.
$$\frac{d}{dx}\cos x = -\sin x$$

13.
$$\frac{d}{dx} \sec x = \sec x \tan x$$

11.
$$\frac{d}{dx}\tan x = \sec^2 x$$

14.
$$\frac{d}{dx}\csc x = -\csc x \cot x$$

Inverse trigonometric:

15.
$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

18.
$$\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$$

16.
$$\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$$

19.
$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x| \sqrt{x^2 - 1}}$$

17.
$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

20.
$$\frac{d}{dx}\csc^{-1}x = -\frac{1}{|x|\sqrt{x^2-1}}$$

Hyperbolic:

$$24. \frac{d}{dx}\coth x = -\operatorname{csch}^2 x$$

$$22. \ \frac{d}{dx}\cosh x = \sinh x$$

25.
$$\frac{d}{dx}$$
 sech $x = -\operatorname{sech} x \tanh x$

23.
$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

26.
$$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \operatorname{coth} x$$

Inverse hyperbolic:

27.
$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{x^2 + 1}}$$

30.
$$\frac{d}{dx} \coth^{-1} x = \frac{1}{1 - x^2}$$

28.
$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2 - 1}}$$

31.
$$\frac{d}{dx}$$
 sech⁻¹ $x = -\frac{1}{x\sqrt{1-x^2}}$

29.
$$\frac{d}{dx} \tanh^{-1} x = \frac{1}{1 - x^2}$$

32.
$$\frac{d}{dx} \operatorname{csch}^{-1} x = -\frac{1}{|x|\sqrt{x^2+1}}$$

Exponential:

$$33. \frac{d}{dx}e^x = e^x$$

$$35. \frac{d}{dx} \ln |x| = \frac{1}{x}$$

$$34. \ \frac{d}{dx}b^x = b^x(\ln b)$$

$$36. \frac{d}{dx}\log_b x = \frac{1}{x(\ln b)}$$

1.
$$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

$$3. \quad \int e^u \, du = e^u + C$$

$$5. \int \sin u \, du = -\cos u + C$$

$$7. \quad \int \sec^2 u \, du = \tan u + C$$

9.
$$\int \sec u \tan u \, du = \sec u + C$$

$$11. \int \tan u \, du = -\ln|\cos u| + C$$

$$13. \quad \int \sec u \, du = \ln \left| \sec u + \tan u \right| + C$$

$$15. \quad \int u \sin u \, du = \sin u - u \cos u + C$$

17.
$$\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$$

$$19. \quad \int \tan^2 u \, du = \tan u - u + C$$

21.
$$\int \sin^3 u \, du = -\frac{1}{3} \left(2 + \sin^2 u \right) \cos u + C$$

23.
$$\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln \left| \cos u \right| + C$$

25.
$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln \left| \sec u + \tan u \right| + C$$

27.
$$\int \sin au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$$

29.
$$\int e^{au} \sin bu \, du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$$

$$\mathbf{31.} \quad \int \sinh u \, du = \cosh u + C$$

$$33. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$35. \int \tanh u \, du = \ln(\cosh u) + C$$

$$37. \quad \int \ln u \, du = u \ln u - u + C$$

$$39. \int \frac{1}{\sqrt{a^2 - u^2}} du = \sin^{-1} \frac{u}{a} + C$$

41.
$$\int \sqrt{a^2 - u^2} \ du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$$

43.
$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$$

$$2. \int \frac{1}{u} du = \ln |u| + C$$

$$4. \quad \int a^u \ du = \frac{1}{\ln a} a^u + C$$

$$6. \quad \int \cos u \, du = \sin u + C$$

$$8. \quad \int \csc^2 u \, du = -\cot u + C$$

$$10. \quad \int \csc u \cot u \, du = -\csc u + C$$

$$12. \int \cot u \, du = \ln \left| \sin u \right| + C$$

14.
$$\int \csc u \, du = \ln \left| \csc u - \cot u \right| + C$$

$$16. \quad \int u \cos u \, du = \cos u + u \sin u + C$$

18.
$$\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$$

$$20. \int \cot^2 u \, du = -\cot u - u + C$$

22.
$$\int \cos^3 u \, du = \frac{1}{3} (2 + \cos^2 u) \sin u + C$$

24.
$$\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$$

26.
$$\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln \left| \csc u - \cot u \right| + C$$

28.
$$\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$$

30.
$$\int e^{au} \cos bu \, du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$$

$$32. \quad \int \cosh u \, du = \sinh u + C$$

$$34. \quad \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$36. \int \coth u \, du = \ln \left| \sinh u \right| + C$$

38.
$$\int u \ln u \, du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$$

40.
$$\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln \left| u + \sqrt{a^2 + u^2} \right| + C$$

42.
$$\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln \left| u + \sqrt{a^2 + u^2} \right| + C$$

44.
$$\int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{a + u}{a - u} \right| + C$$