Digital Image Fundamental

- Basic Relationships Between Pixels:
  - 4-Neighbors

$$N_4(p):\{(x+1,y),(x-1,y),(x,y+1),(x,y-1)\}$$

  - Diagonal Neighbors

$$N_D(p):\{(x+1,y+1),(x+1,y-1),(x-1,y+1),(x-1,y-1)\}$$

  - 8-Neighbors:

$$N_8(p):N_4(p)\cup N_D(p)$$

- Basic Relationships Between Pixels:

$$N_4 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad N_D = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad N_8 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Digital Image Fundamental

- Adjacency:
  - p and q are 4-adjacent: $q \in N_4(p)$
  - p and q are 8-adjacent: $q \in N_8(p)$
  - p and q are m-adjacent:

$$\{q \in N_4(p)\} or \{\{q \in N_D(p)\} and \{N_4(p) \cap N_4(q)\} = \varnothing\}$$

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8 **m**

Digital Image Fundamental

- Distance Measure:

  a. $D(p,q) \geq 0 \quad D(p,q) = 0 \quad \text{iff} \quad p = q$

  b. $D(p,q) = D(q,p)$

  b. $D(p,q) \leq D(p,r) + D(r,q)$

- Examples:
  - Euclidean: $D_e(p,q) = \sqrt{(x-s)^2 + (y-t)^2}$
  - D_4 (City Block or Manhattan): $D_4(p,q) = |x-s| + |y-t|$
  - D_8 (Chessboard): $D_8(p,q) = Max\{|x-s|, |y-t|\}$

Digital Image Fundamental

- Constant Distance Contour:
  - D4 (Left)
  - D8 (Right)

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Connectivity

Connectivity forms the basis for establishing the boundaries of an objects and also components of regions in an image. Two pixels are connected:

1. if the pixels are adjacent (E.g. are they 4-neighbours)
2. if their gray levels satisfy a specified criterion of similarity (E.g. equal or belongs to a set V – falls within a given range)

   In a binary image with values 0 and 1, two-pixels may be 4-neighbors, but they are said to be connected only if they have the same value.

Connected Set, Region, Boundary

- Connected Set
  - Two pixels are said to be connected in S (subset of pixels in an image) if there exists a path between them consisting entirely of pixels in S
  - For any pixel p in S, the set of pixels that are connected to it in S is called a connected component of S.
  - If it only has one connected component, then set S is called a connected set.
- Region R (subset of pixels)
  - R is a region of the image if R is a connected set.
  - The boundary (border or contour) of a region R is the set of pixels in the region that have one or more neighbors that are not in R.
- Difference between Boundary and Edge
  - The boundary of a finite region forms a closed path (a "global" concept)
  - Edges are formed from pixels with derivative values that exceed a preset threshold (a "local" concept).

Digital Image Fundamental

Mathematical Tools:

- Array and Matrix Operations
- Linear and nonlinear Operation
  - Fourier Filtering, Ordered Statistics Filtering, …
- Arithmetic Operation (+, -, *, /)
  - Averaging, Subtraction, …
- Set and Logical Operations
  - Fuzzy or Crisp Sets

Digital Image Fundamental

- Image Averaging:
  - Consider an additive noise condition:

$$g(x,y)=f(x,y)+\eta(x,y)$$

  - Conditions:
    - *Noise*, $\eta(x,y)$:
      - Uncorrelated
      - i.i.d
      - Zero Mean
    - Subject, $f(x,y)$:
      - Physical Stationary
      - Repeatable Experiments

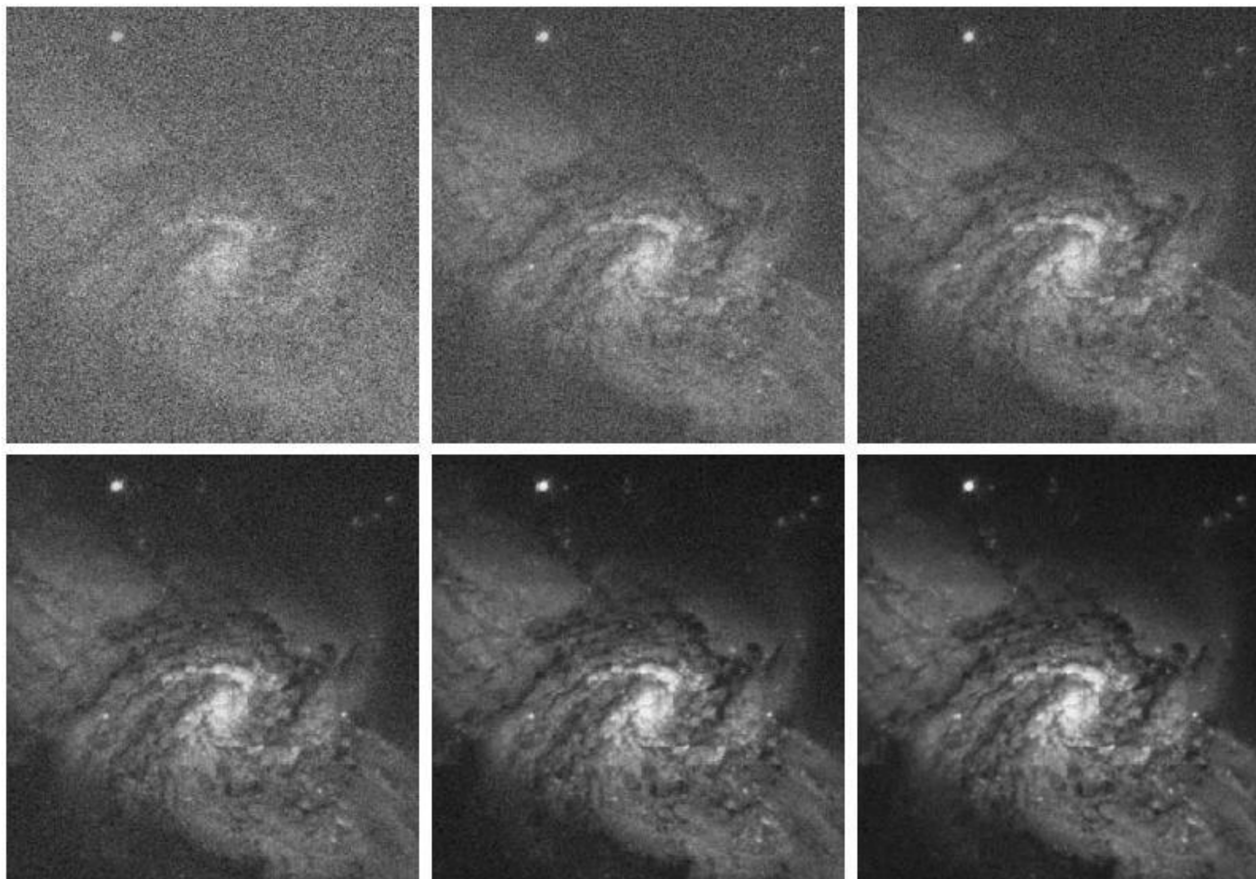
Digital Image Fundamental

- Image Averaging:

$$\bar{g}(x,y)=\frac{1}{N}\sum_{i=1}^{N} g_i(x,y) \Rightarrow \begin{cases} E\{\bar{g}(x,y)\}=f(x,y) \\ \sigma^2_{\bar{g}(x,y)}=\frac{1}{N}\sigma^2_{\eta(x,y)} \end{cases}$$

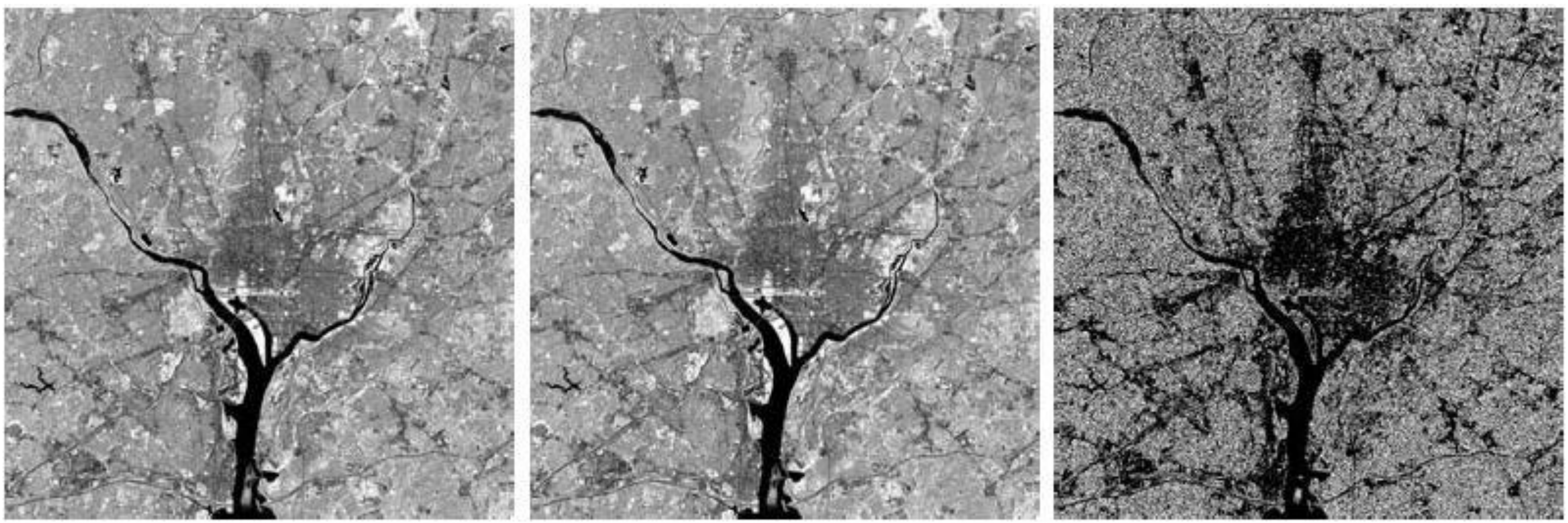
Digital Image Fundamental

- Example:
  - 5, 10,20, 50, and 100 averaging.



Digital Image Fundamental

- Image Subtraction:
  - Original (Left), LSB set to zero (Center), Difference (Right)



Digital Image Fundamental

- Digital Subtraction Radiography (DSA):
  - Pre and Post Imaging

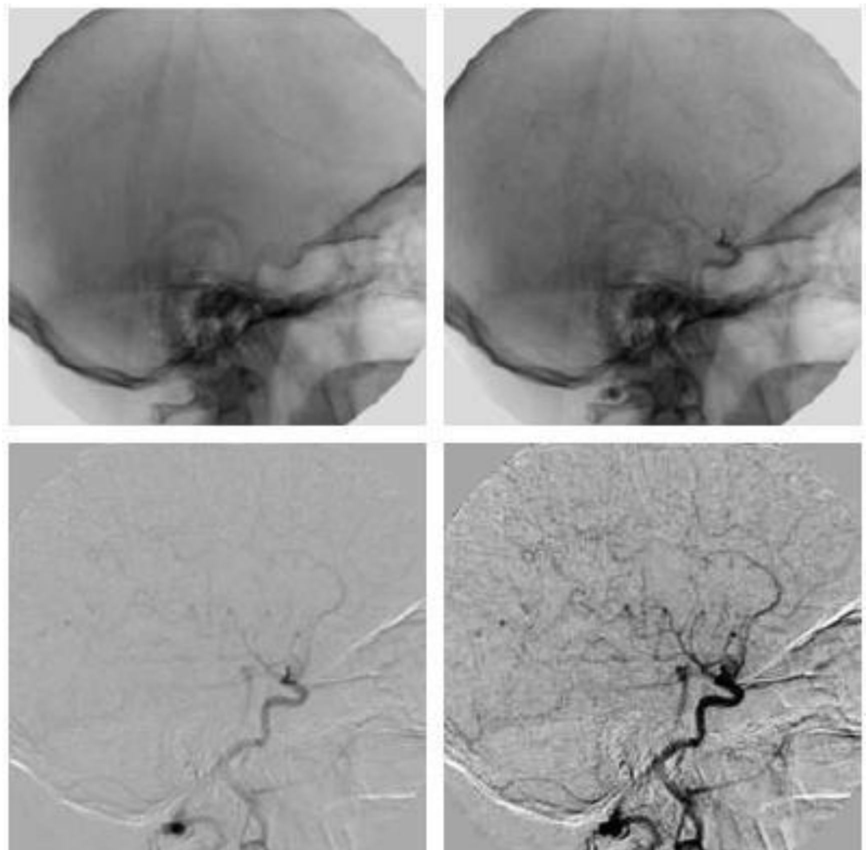
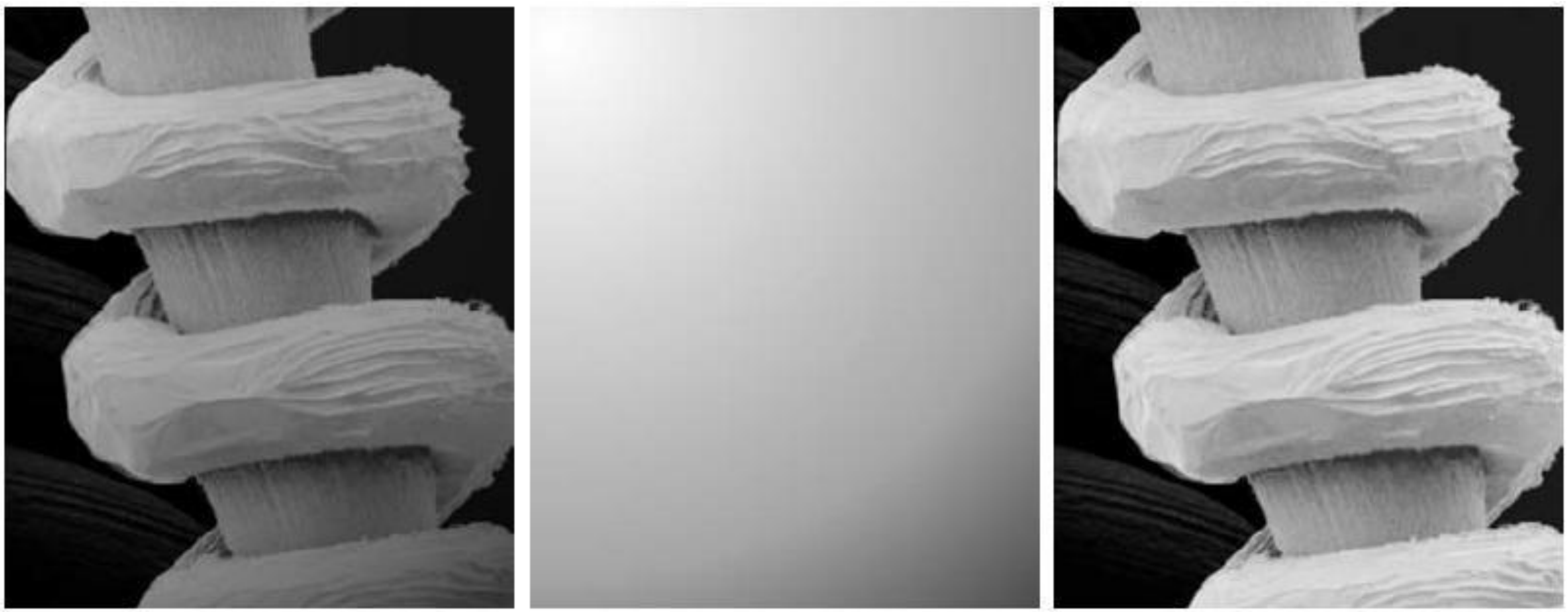


FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

Digital Image Fundamental

- Image Multiplication/Division:
  - Shading Correction

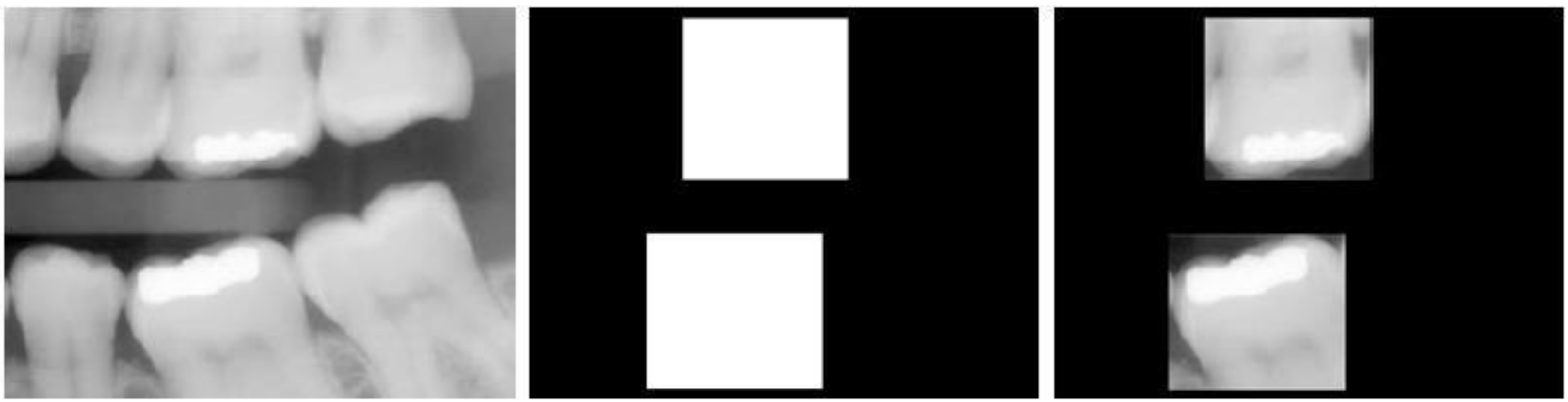


a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Digital Image Fundamental

- Image Multiplication/Division:
  - ROI Masking



a b c

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Digital Image Fundamental

- Set and Logical Operation:

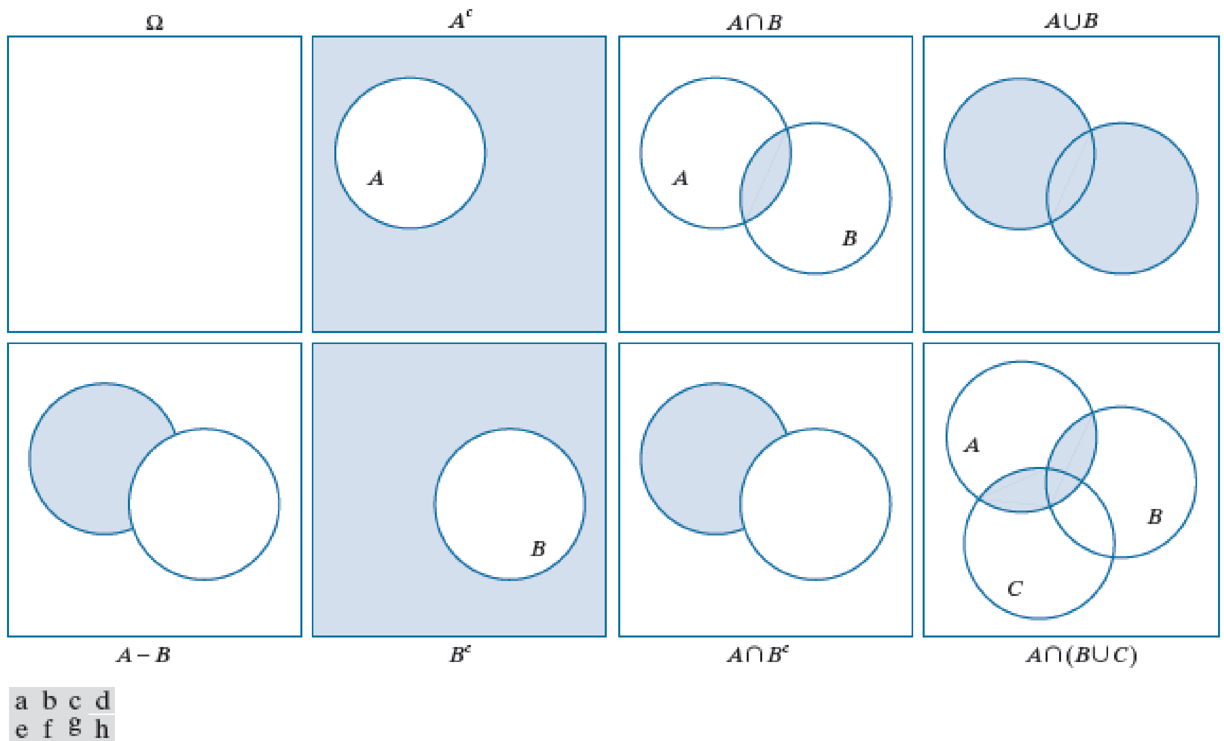


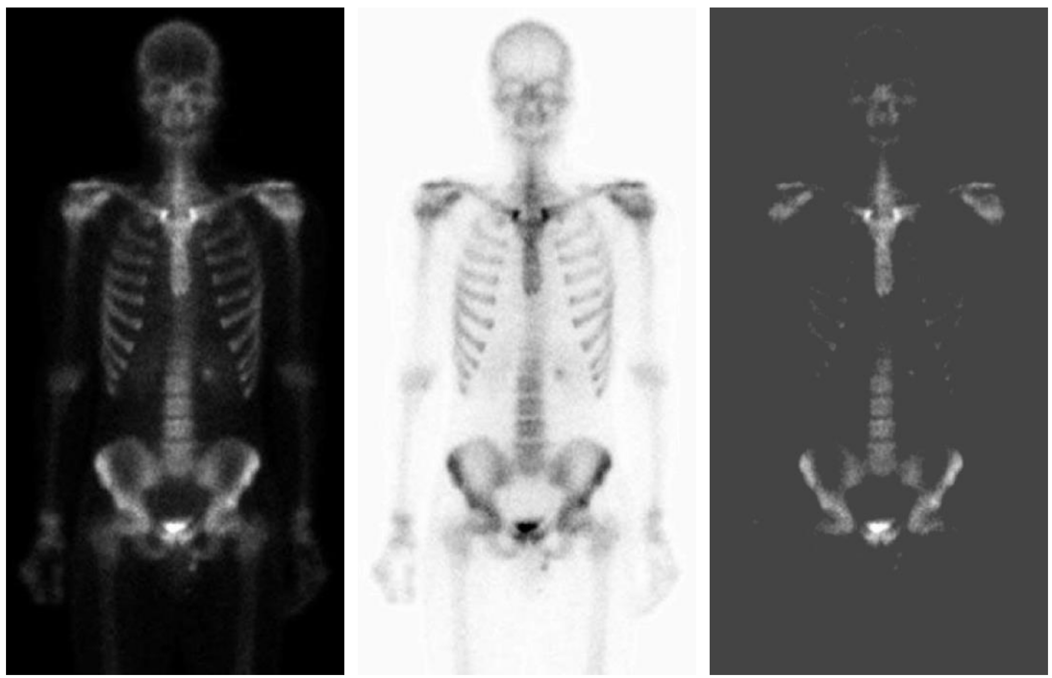
FIGURE 2.35 Venn diagrams corresponding to some of the set operations in Table 2.1. The results of the operations, such as A^c, are shown shaded. Figures (e) and (g) are the same, proving via Venn diagrams that $A - B = A \cap B^c$ [see Eq. (2-40)].

Digital Image Fundamental

- Example
  - Original (Left), Negative (Center), Right (union with Constant)

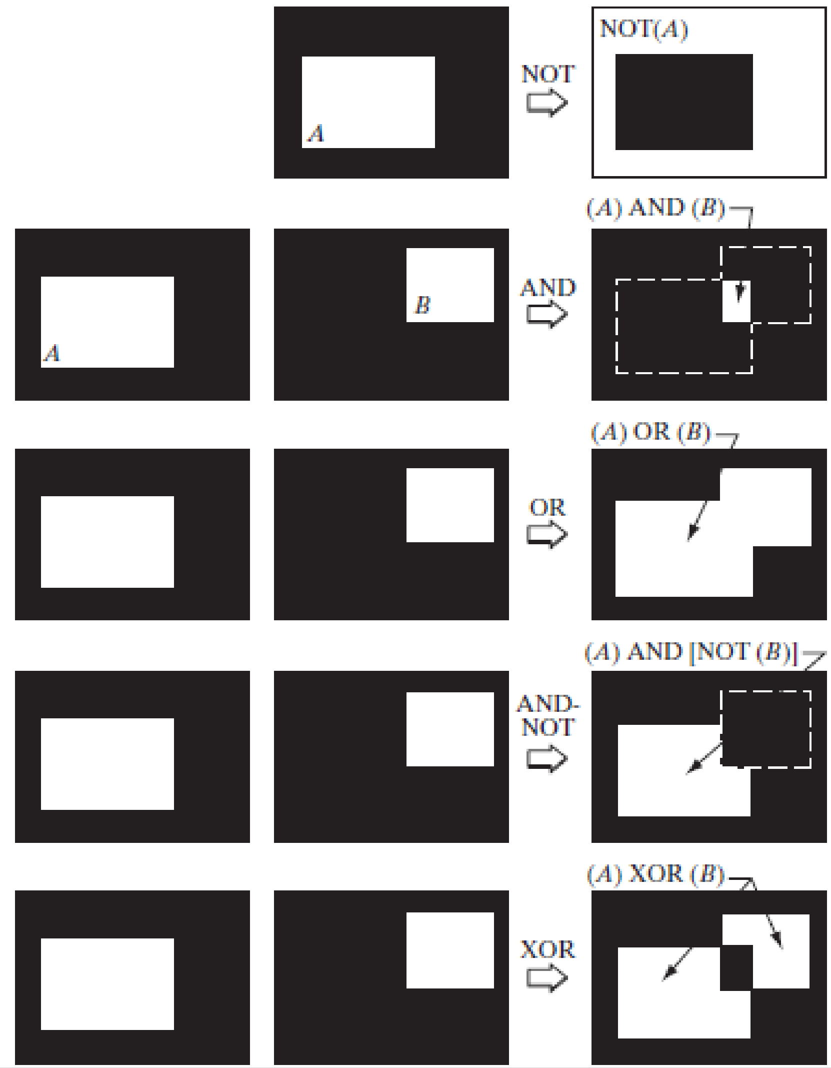
a b c

FIGURE 2.36 Set operations involving grayscale images. (a) Original image. (b) Image negative obtained using grayscale set complementation. (c) The union of image (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)



Digital Image Fundamental

- Logical Operation:



Digital Image Fundamental

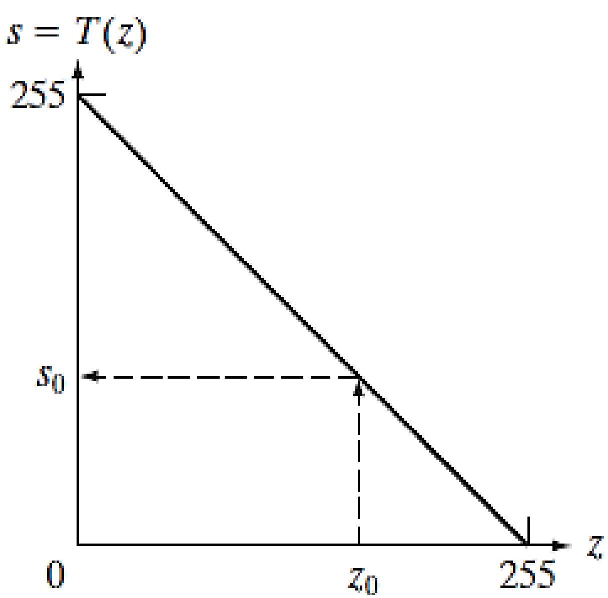
Mathematical Tools:

- Spatial Operations
  - Single pixel
  - Neighborhood

Digital Image Fundamental

- Single Pixel:

$$s = \mathrm{T}(z)$$



Digital Image Fundamental

- Neighborhood Operation:

$$g(x,y)=\frac{1}{mn}\sum_{(r,c)\in S_{xy}} f(r,c)$$

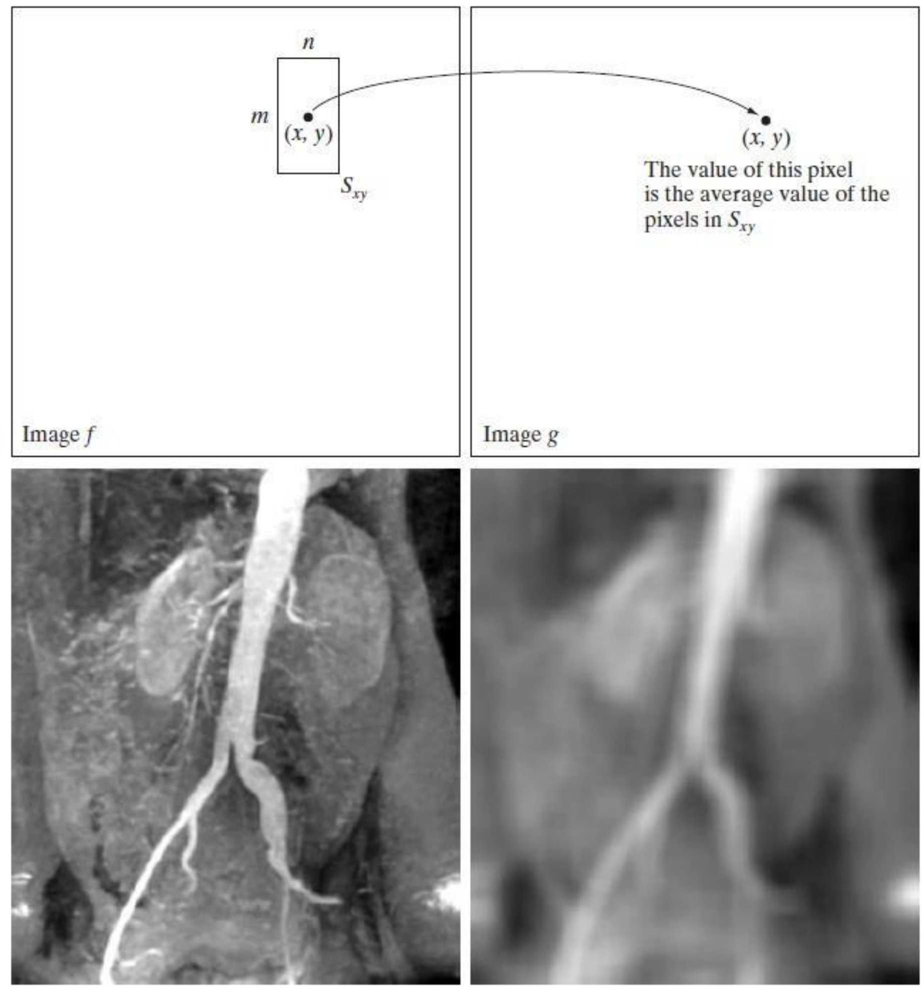


FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

Digital Image Fundamental

- Geometric Spatial Transform
  - General Formulation:

$$(x, y) = T\{(u, v)\}$$

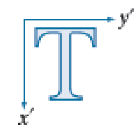
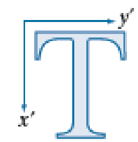
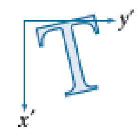
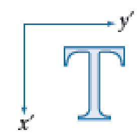
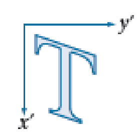
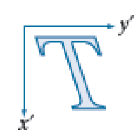
  - Example (Affine Transform)

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} u \\ v \\ 1 \end{pmatrix}$$

Digital Image Fundamental

- Affine Transformation

TABLE 2.3 Affine transformations based on Eq. (2-45).

Transformation Name	Affine Matrix, A	Coordinate Equations	Example
Identity	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = y$	
Scaling/Reflection (For reflection, set one scaling factor to −1 and the other to 0)	$\begin{bmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = c_x x$ $y' = c_y y$	
Rotation (about the origin)	$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x\cos\theta - y\sin\theta$ $y' = x\sin\theta + y\cos\theta$	
Translation	$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + t_x$ $y' = y + t_y$	
Shear (vertical)	$\begin{bmatrix} 1 & s_v & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x + s_v y$ $y' = y$	
Shear (horizontal)	$\begin{bmatrix} 1 & 0 & 0 \\ s_h & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$x' = x$ $y' = s_h x + y$	

Digital Image Fundamental

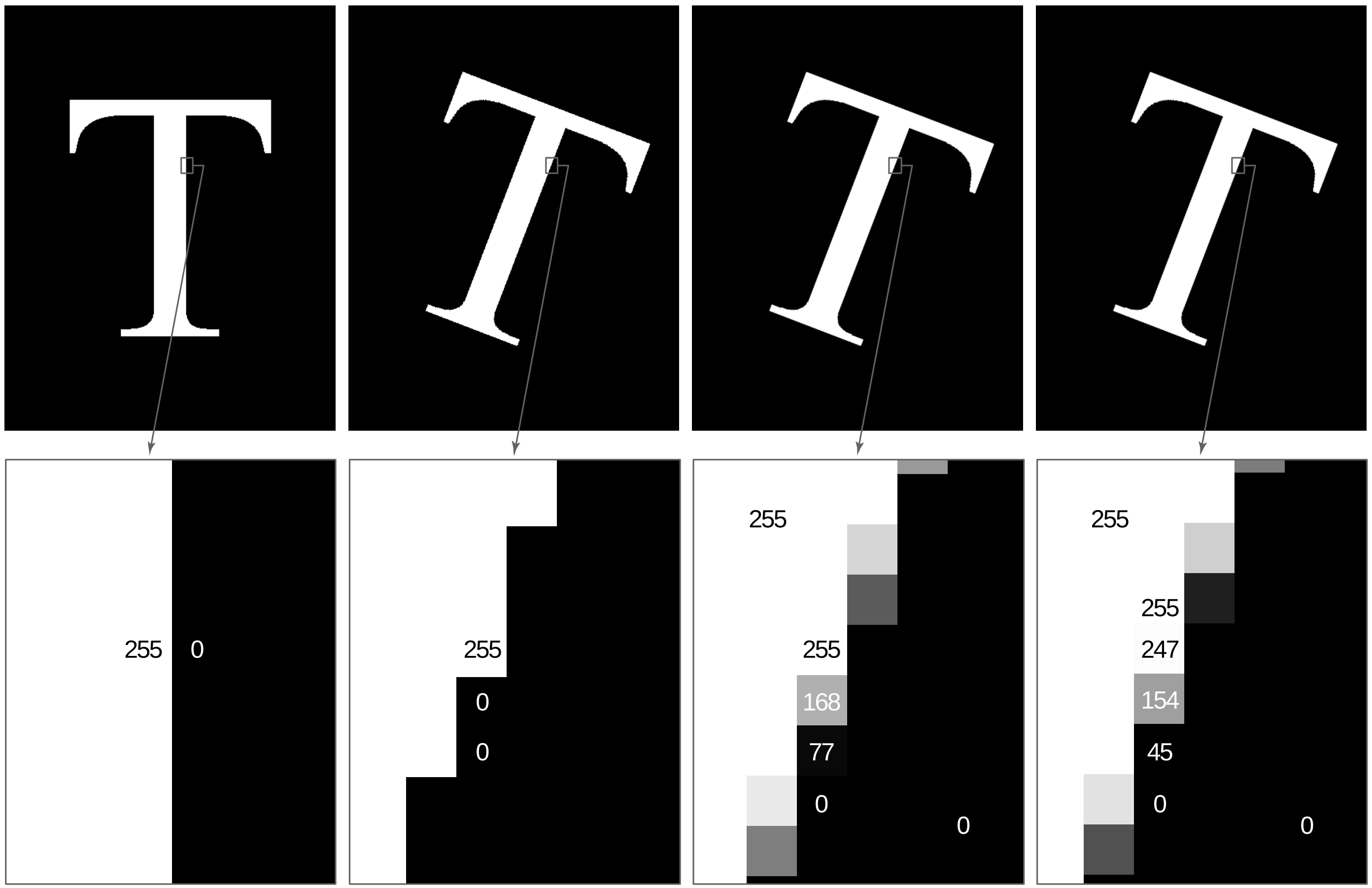
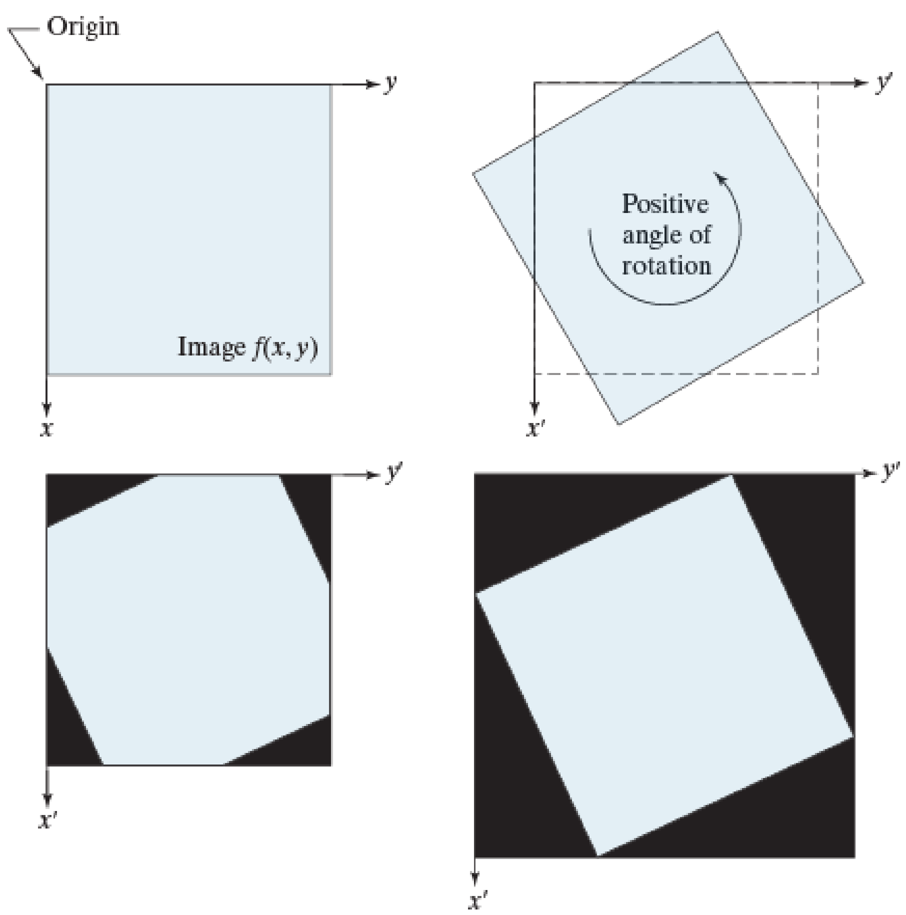


FIGURE 2.40 (a) A 541 × 421 image of the letter T. (b) Image rotated −21° using nearest-neighbor interpolation for intensity assignments. (c) Image rotated −21° using bilinear interpolation. (d) Image rotated −21° using bicubic interpolation. (e)-(h) Zoomed sections (each square is one pixel, and the numbers shown are intensity values).

Digital Image Fundamental

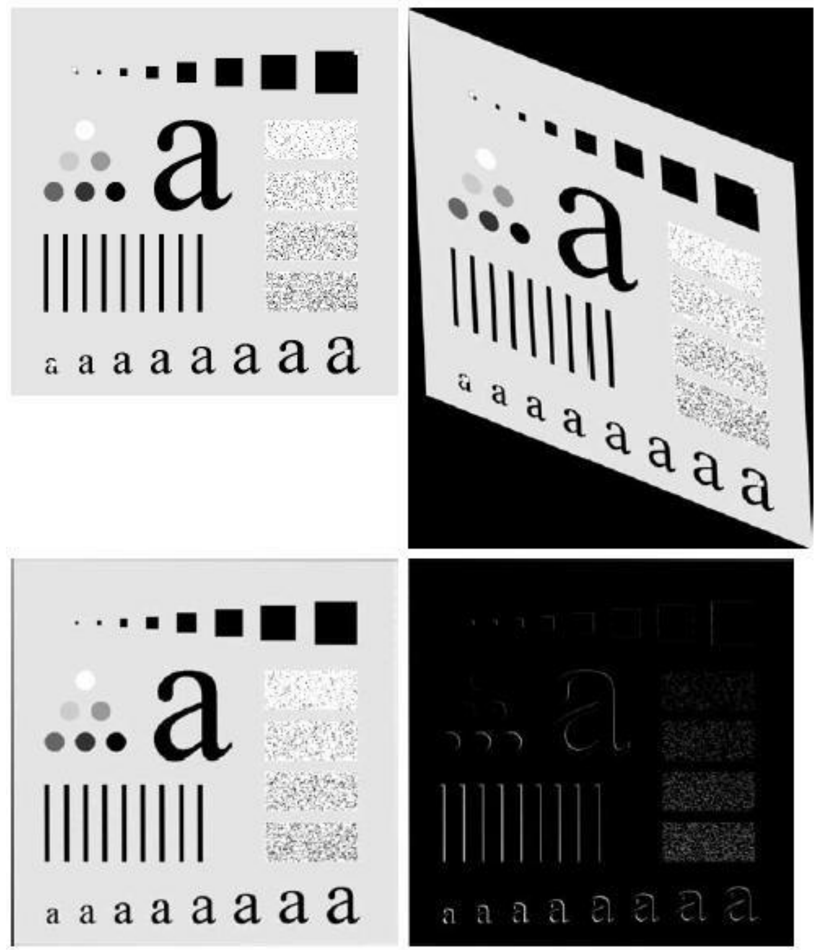
a	b
c	d

FIGURE 2.41
(a) A digital image.
(b) Rotated image (note the counterclockwise direction for a positive angle of rotation).
(c) Rotated image cropped to fit the same area as the original image.
(d) Image enlarged to accommodate the entire rotated image.



Digital Image Fundamental

- Image Registration (Example):
  - Using tie points



a b
c d

FIGURE 2.37 Image registration. (a) Reference image. (b) Input (geometrically distorted image). Corresponding tie points are shown as small white squares near the corners. (c) Registered image (note the errors in the border). (d) Difference between (a) and (c), showing more registration errors.

Digital Image Fundamental

- Paradigm of image processing:
  - Low-level processing
    - Inputs and outputs are images
    - Primitive operations: de-noise, enhancement, sharpening, …
  - Mid-level processing
    - Inputs are images, outputs are attributes extracted from images
    - Segmentation, classification,…
  - High-level processing
    - "Make sense" of an ensemble of recognized objects by machines

Assignment # 1

End Problems of Ch # 2

All except Reading Assignment

Odd Qns. > Odd Roll #s

Even Qns. > Even Roll #s

Must be:

- hand written
- on A4 size page

Reading Assignment:

Topics: 2.6.5 – 2.6.8

Problems: 2.7, 2.8, 2.14, 2.20, 2.21, 2.23-2.27

Assignment Title page Sample

Assignment # 1

Course: Digital Image Processing (EEE-415)
Instructor: Dr Ikramullah Khosa
Submitted by: Your complete name
(Complete Registration No)
Submission Date:

Assignment Topic: Digital Image Fundamentals

Assignment Statement:
Apply the concepts of digital image fundamentals to understand and solve exercise problems related to imaging sensors, sampling and quantization, and connectivity.