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Assignment: 03

Statistics

Discrete distributions:

1) Bi-nomial 2) Hypergeometric
3) poisson

① Bi-nomial:

Bernouli trial: single experiment having boolean value

Bernouli process: set of bernouli trials

Formula:

$$b(x, n, p) = \binom{n}{x} p^{x} q^{n-x}$$

$q = 1 - p$

Mean $= np$, Variance $= npq$

(5.4)

a) $P = 0.75$, $n = 5$

$P(x = 2) = ?$ $\because q = 1 - 0.75 = 0.25$

$= P(2, 5, 0.75)$

$\Rightarrow \binom{5}{2} (0.75)^2 (0.25)^3 = 0.087$

b) $P(x \leq 3)$

$= P(3) + P(2) + P(1)$ or $1 - P(x > 3)$

$= \binom{5}{3} (0.75)^3 (0.25)^2 + \binom{5}{2} (0.75)^2 (0.25)^3$

$+ \binom{5}{1} (0.75^1)(0.25)^4 =$

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$= 0.26 + 0.027 + 0.014$

$= 0.029$

(5.5)

a) Given that

$p = 30\% = 0.30$

$q = 1 - 0.30 = 0.70$

a) $n = 20$, $P(x \geq 10) = 1 - P(x \leq 10)$

$= P(10) + P(9) + P(8) + \cdots P(1)$

or

$P(10) + P(11) + P(12) + \cdots P(20)$

$= 0.03 + 0.012 + 0.0038 + 0.0010 + 0.00021$

$+ 0.000037 + 0.0000050 + 0.00000050 +$

$0.000000036 + 0.0000000016 + 0.000000000034$

$\Rightarrow 0.0480$ or 4%

b) $n = 20$, $P(x \leq 4)$

$= \binom{20}{4} p^4 q^{16} + \binom{20}{3} p^3 q^{17} + \binom{20}{2} p^2 q^{18}$

$+ \binom{20}{1} p^1 q^{19}$

$= 0.23 = 23\%$

c) $n=20$, $P(x=5)$

$= \binom{20}{5}(0.30)^5(0.70)^{15}$

$\Rightarrow 0.178 \Rightarrow 17\%$

(5.6)

$n=6$, $p=1/2$, $q=1/2$

a) $P(2 \leq x \leq 5)$

$= P(x=2) + P(x=3) + \cdots P(x=5)$

$= 0.23 + 0.312 + 0.23 + 0.093$

$= 0.8$

b) $P(x<3)$

$= P(2) + P(1)$

$= \binom{6}{2}(0.5)^2(0.5)^4 + \binom{6}{1}(0.5)^1(0.5)^5$

$= 0.23 + 0.093$

$= 0.3\cancel{2}37$

(5.56)

$\lambda = 3$

a) $p(X=5) = p(5,3)$

$$= \frac{(2.718^{-3})(3^5)}{5!} = 0.1008$$

b) $p(x<3)$ or $p(x \le 2)$

$= p(2) + p(1) + p(0)$

$= p(2,3) + p(1,3) , p(0,3)$

$$= \frac{(2.718^{-3})(3^2)}{2!} + \frac{(2.718^{-3})(3)}{1!} + \frac{(2.718)^{-3}\, 3^0}{} \quad \because 0! = 1$$

$= 0.224 + 0.149 + 0.049$

$= 0.423$

c) $P(x \geq 2) = 1 - P(x \leq 1)$

$P(x \leq 1) \Rightarrow P(1) + P(0) = 0.149 + 0.049$

$= 1 - 0.198 \Rightarrow 0.80$

(5.58)

$n = 6$

a) $P(x \leq 3)$

$= P(3) + P(2) + P(1) + P(0)$

$= P(3,6) + P(2,6) + P(1,6) + P(0,6)$

$= \dfrac{2.718^{-6}\, 6^3}{3!} + \dfrac{2.718^{-6}\, 6^2}{2!} + \dfrac{2.718^{-6}\, 6^1}{1!}$

$+ \dfrac{2.718^{-6}\, 6^0}{0!}$

$= 0.089 + 0.044 + 0.014 + 0.0024$

$= 0.15$ or 15%

b) $P(6 \leq x \leq 8)$

$= P(6,6) + P(7,6) + P(8,6)$

$= \dfrac{2.718^{-6}\, 6^6}{6!} + \dfrac{2.718^{-6}\, 6^7}{7!} + \dfrac{2.718^{-6}\, 6^8}{8!}$

$= 0.160 + 0.137 + 0.103$

$= 0.40$ or 40%.

(5.60)

$\mu = 12$

a) $P(x<7) = P(X \le 6)$

$= P(6) + P(5) + \dots + P(0)$

$$= \frac{2.718^{-12}\,12^{6}}{6!} + \frac{2.718^{-12}\,12^{5}}{5!} + \frac{2.718^{-12}\,12^{4}}{4!}$$

$$+ \frac{2.718^{-12}\,12^{3}}{3!} + \frac{2.718^{-12}\,12^{2}}{2!} + \frac{2.718^{-12}\,2^{1}}{1!}$$

$$+ \frac{2.718^{-12}\,12^{0}}{0!}$$

$= 0.025 + 0.012 + 0.0053 + 0.0017 + 0.00044$

$+ 0.000073 + 0.00000615$

$= 0.04$ or 4%

$q = 0.96$

b) $P(2, 3, 0.04) = ?$

$p = 0.04$

$$P(x, n, p) = \binom{n}{x} p^{x} q^{n-x}$$

$$P(2, 3, 0.04) = \binom{3}{2} (0.04)^{2} (0.95)$$

$= 0.0060$

(5.41)

$P = 0.7, \quad n = 18$

$\cancel{P(19 < x}$

$P(9 < x < 14) = ?$

$$1 - \sum_{x=10}^{13} b(x, 18, 0.7)$$

$$1 - \left[b(10, 18, 0.7) + b(11, 18, 0.7) + b(12, 18, 0.7) + b(13, 18, 0.7) \right]$$

0.60 or 60%

(15.40)

Sample is independent & probability of success is constant, so it's bi-nomial approxomation

$$p = \frac{n(E)}{n(SS)} = \frac{4000}{10{,}000} = 2/5 =$$

$1 - 2/5$ because 40,000 are against new sales tax

$= 0.6$

$n = 15$

$P(X \leq 7) = ?$

$$b(x, n, p) = \binom{n}{x} p^x q^{n-x}$$

$= b(0, 15, 0.6) + b(1, 15, 0.6) + b(2, 5, 0.6) + b(3, 5, 0.6) + b(4, 5, 0.6) + b(5, 5, 0.6) + b(6, 5, 0.6) + b(7, 5, 0.6)$

$$= \sum_{x=0}^{7} b(x, 15, 0.6) = 0.21 \text{ or } 21\%$$

(5.29)

$n=6$, $N=9$, $k=4$, $x=2$

$$h(x;N,n,k)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$h(2;9,6,4)=\frac{\binom{4}{2}\binom{5}{4}}{\binom{9}{6}}$$

$$=5/14 = 0.35$$

(5.34)

$N=9$, $n=5$, $k=4$, $x=2$

$$h(x;N,n,k)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$=\frac{\binom{4}{2}\binom{5}{3}}{\binom{9}{5}} = 10/21 = 0.47$$