DIGITAL SIGNAL PROCESSING

By

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Topics to be Covered:

1. OVERLAP SAVE METHOD
2. OVERLAP ADD METHOD
3. LINEAR CONVOLUTION

OVERLAP SAVE METHOD:

- Overlap–save is the traditional name for an efficient way to evaluate the discrete convolution between a very long signal x(*n*) and a finite impulse response *FIR* filter h(*n*)
- In this method the input sequence is divided into blocks of data of size N=L+M-1.**where** L= length of an input sequence and M=the length of an impulse response
- Each block consist of last (M-1) data points of previous block followed by L new data points to form a data sequence of length N=L+M-1.
- For first block, M-1 points are set to zero.

The input sequence can be divided into blocks as

$X_1(n) = \{0,0,x(0),x(1),x(2)\}$

M-1=2zeros

$$X2(n) = \{x(1),x(2),x(3),x(4),x(5)\}$$

Last two data points from previous block

$$X3(n) = \{x(4),x(5),x(6),x(7),x(8)\}$$

$$X4(n) = \{x(7),x(8),x(9),x(10),x(11)\}$$

$$X5(n) = \{x(10),x(11),x(12),x(13),x(14)\}$$

$$X6(n) = \{x(13),x(14),0,0,0\}$$

Thus, $Y1(n)=x1(n)\, N\, h(n)=\{y1(0),y1(1),y1(2),y1(3),y1(4)\}$

$$Y2(n)=x2(n)\, N\, h(n)=\{y2(0),y2(1),y2(2),y2(3),y2(4)\}$$

$$Y3(n)=x3(n) \textcircled{N} h(n)=\{y3(0),y3(1),y3(2),y3(3),y3(4)\}$$

$$Y4(n)=x4(n) \textcircled{N} h(n)=\{y4(0),y4(1),y4(2),y4(3),y4(4)\}$$

$$Y5(n)=x5(n) \textcircled{N} h(n)=\{y5(0),y5(1),y5(2),y5(3),y5(4)\}$$

$$Y6(n)=x6(n) \textcircled{N} h(n)=\{y6(0),y6(1),y6(2),y6(3),y6(4)\}$$

Therefore, the output blocks are abutted together to get

Y(n)={y1(2),y1(3),y1(4),y2(2),y2(3),y2(4),y3(2),y3(3), y3(4),y4(4),y5(2),y5(3),y5(4),y6(2),y6(3)}

EXAMPLE 1 : ***Determine the output of linear FIR filter whose impulse response is h(n)={1,2,3} and the input signal is x(n)= {1,2,3,4,5,6,7,8,9} using over lap save method.***

SOLUTION:

Given, $x(n)= \{1,2,3,4,5,6,7,8,9\}$ & $h(n)=\{1,2,3\}$

Therefore, $L=9; M=3;$

Adding M-1 zeros in x1(n),we get

$x1(n)=\{0,0,1,2,3\}$

$x2(n)=\{2,3,4,5,6\}$

$x3(n)=\{5,6,7,8,9\}$

$x4(n)=\{8,9,0,0,0\}$

$h(n)=\{1,2,3,0,0\}$

By circular convolution,

$$y1(n) = x1(n) \, (N) \, h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 12 \\ 9 \\ 1 \\ 4 \\ 10 \end{bmatrix}$$

$$y2(n) = x2(n) \, (N) \, h(n)$$

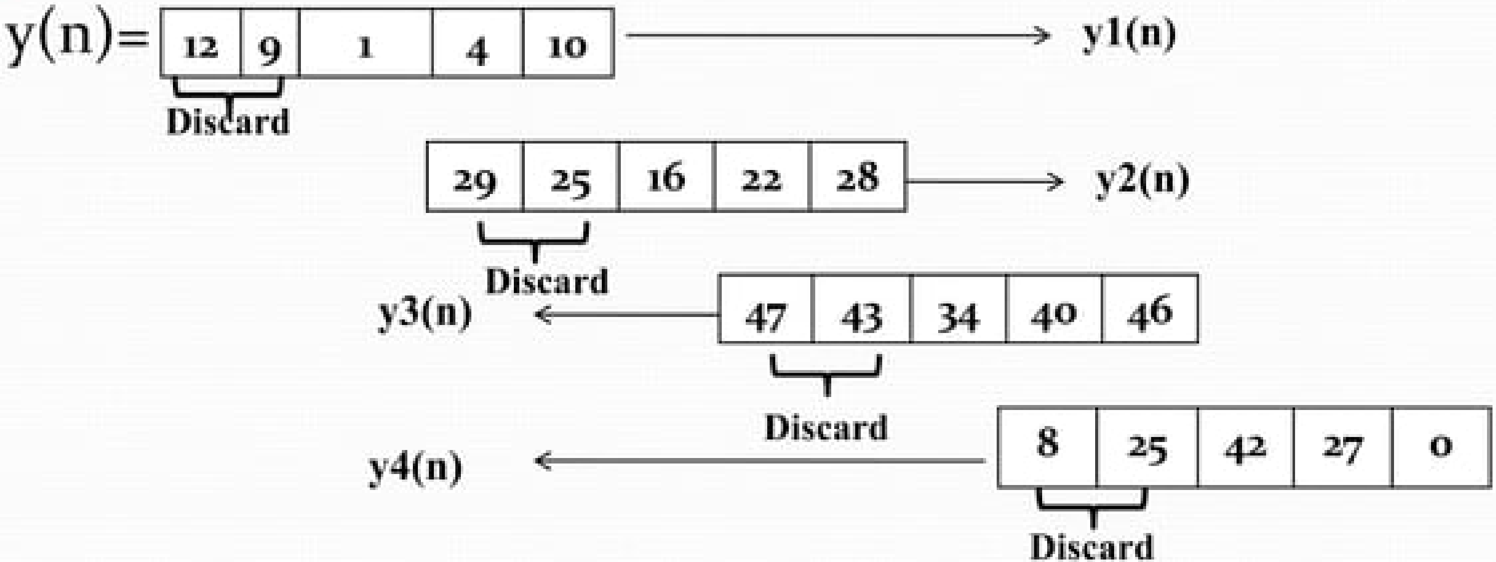
$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 29 \\ 25 \\ 16 \\ 22 \\ 28 \end{bmatrix}$$

$y3(n) = x3(n) \; (N) \; h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 47 \\ 43 \\ 34 \\ 40 \\ 46 \end{bmatrix}$$

$y4(n) = x4(n) \; (N) \; h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ 25 \\ 42 \\ 27 \\ 0 \end{bmatrix}$$



Therefore,

$$y(n)=\{1,4,10,16,22,28,34,40,46,42,27\}$$

<u>EXAMPLE 2:</u> Determine the output of linear FIR filter whose impulse response is h(n)={1,-3,5} and the input signal is x(n)= {-1,4,7,3,-2,9,10,12,-5,8} using over lap save method.

SOLUTION:

Given, x(n)= {-1,4,7,3,-2,9,10,12,-5,8} & h(n)={1,-3,5}

Therefore, L=9;M=3;

Adding M-1 zeros in x1(n),we get

x1(n)={0,0,-1,4,7}

x2(n)={4,7,3,-2,9}

x3(n)={-2,9,10,12,-5}

x4(n)={12,-5,8,0,0}

h(n)={1,-3,5,0,0}

By circular convolution,

$$y1(n)= x1(n) \; \textcircled{N} \; h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -1 \\ 4 \\ 7 \end{bmatrix} = \begin{bmatrix} -1 \\ 35 \\ -1 \\ 7 \\ -10 \end{bmatrix}$$

$$y2(n)= x2(n) \; \textcircled{N} \; h(n)$$

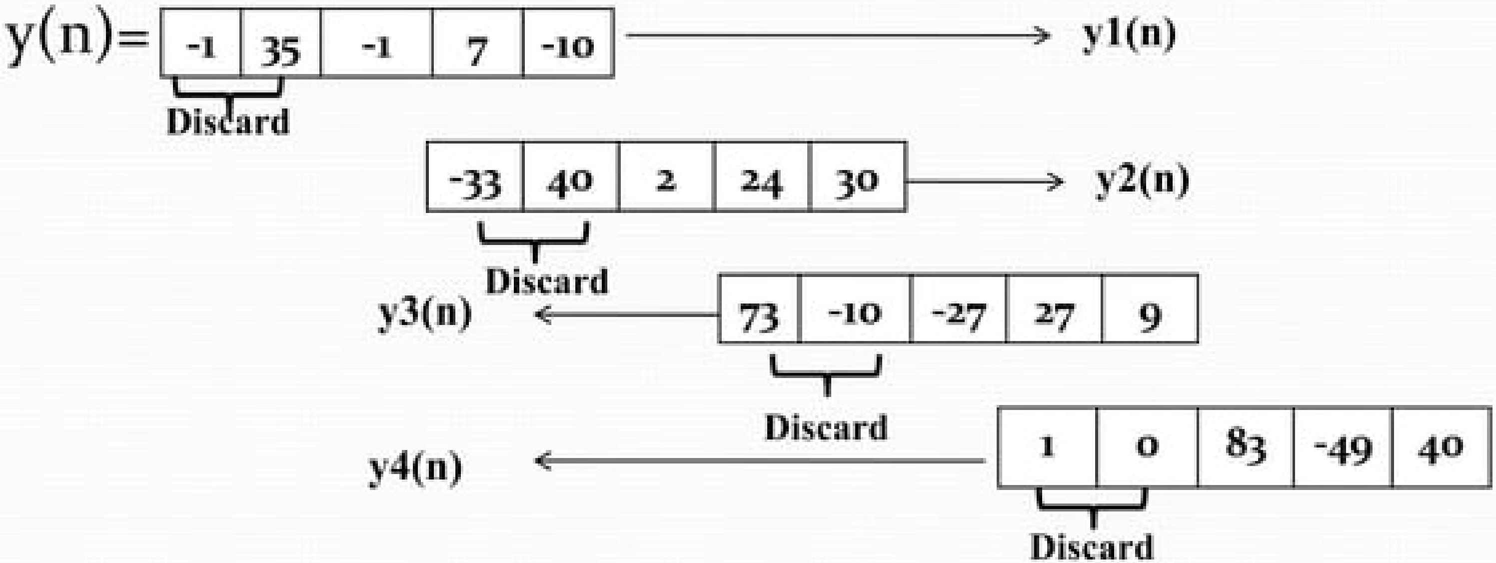
$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 7 \\ 3 \\ -2 \\ 9 \end{bmatrix} = \begin{bmatrix} -33 \\ 40 \\ 2 \\ 24 \\ 30 \end{bmatrix}$$

$y3(n) = x3(n) \text{ (N) } h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 9 \\ 10 \\ 12 \\ -5 \end{bmatrix} = \begin{bmatrix} 73 \\ -10 \\ -27 \\ 27 \\ 9 \end{bmatrix}$$

$y4(n) = x4(n) \text{ (N) } h(n)$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ -5 \\ 8 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 83 \\ -49 \\ 40 \end{bmatrix}$$



Therefore,

$$y(n)=\{1,7,-10,2,24,30,-27,27,9,83,-49,40\}$$

OVERLAP ADD METHOD:

- The length of the sequence is be Ls and the length of the impulse response is M.
- The sequence is divided into blocks of data size having length L and M-1.
- Zeros are appended to it make the data size of L+M-1.

Let the output blocks are of the form,

y1(n)={y1(0),y1(1),…….,y1(L-1),y1(L),….,y1(N-1)}

y2(n)={y2(0),y2(1),……,y2(L-1),y2(L),….,y2(N-1)}

y3(n)={y3(0),y3(1),…….,y3(L-1),y3(L),……,y3(N-1)}

the output sequence is

Y(n)={y1(0),y1(1),….,y1(L-1),y1(L)+y2(0),….,y1(N-1)+ y2(M-2),y2(M),……,y2(L)+y3(0),y2(L+1)+y3(1),….,y3(N-1)}

EXAMPLE 1: Determine the output of linear FIR filter whose impulse response is h(n)={1,2,3} and the input signal is x(n)= {1,2,3,4,5,6,7,8,9} using over lap add method.

SOLUTION:

Given, x(n)= {1,2,3,4,5,6,7,8,9} & h(n)={1,2,3}

Therefore, L=9;M=3;

Adding M-1 zero,we get

x1(n)={1,2,3,0,0}

x2(n)={4,5,6,0,0}

x3(n)={7,8,9,0,0}

h(n)={1,2,3,0,0}

By circular convolution,

$$y1(n) = x1(n) \, \text{Ⓝ} \, h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 10 \\ 12 \\ 9 \end{bmatrix}$$

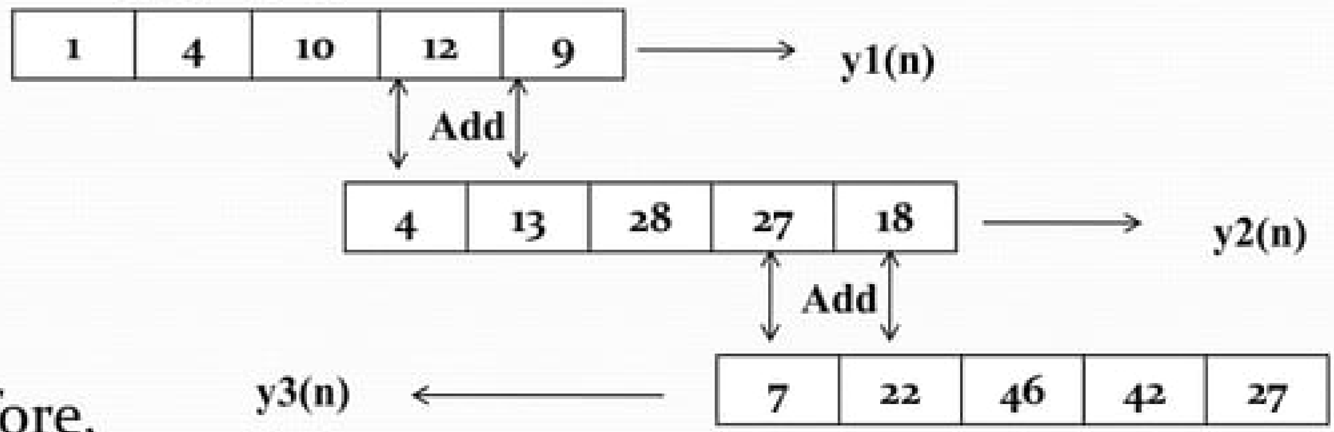
$$y2(n) = x2(n) \, \text{Ⓝ} \, h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \\ 28 \\ 27 \\ 18 \end{bmatrix}$$

$$y3(n) = x3(n) \, \textcircled{N} \, h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 & 2 \\ 2 & 1 & 0 & 0 & 3 \\ 3 & 2 & 1 & 0 & 0 \\ 0 & 3 & 2 & 1 & 0 \\ 0 & 0 & 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 9 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ 22 \\ 46 \\ 42 \\ 27 \end{bmatrix}$$

$y(n)=$



Therefore,

$$y(n)=\{1,4,10,16,22,28,34,40,46,42,27\}$$

EXAMPLE 2: Determine the output of linear FIR filter whose impulse response is h(n)={1,-3,5} and the input signal is x(n)= {-1,4,7,3,-2,9,10,12,-5,8} using over lap add method.

SOLUTION:

Given, x(n)= {-1,4,7,3,-2,9,10,12,-5,8} & h(n)={1,-3,5}

Therefore, L=9;M=3;

Adding M-1 zeros, we get

x1(n)={-1,4,7,0,0}

x2(n)={3,-2,9,0,0}

x3(n)={10,12,-5,0,0}

x4(n)={8,0,0,0,0}

h(n)={1,-3,5,0,0}

By circular convolution,

$$y1(n) = x1(n) \textcircled{N} h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ 7 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -10 \\ -1 \\ 35 \end{bmatrix}$$

$$y2(n) = x2(n) \textcircled{N} h(n)$$

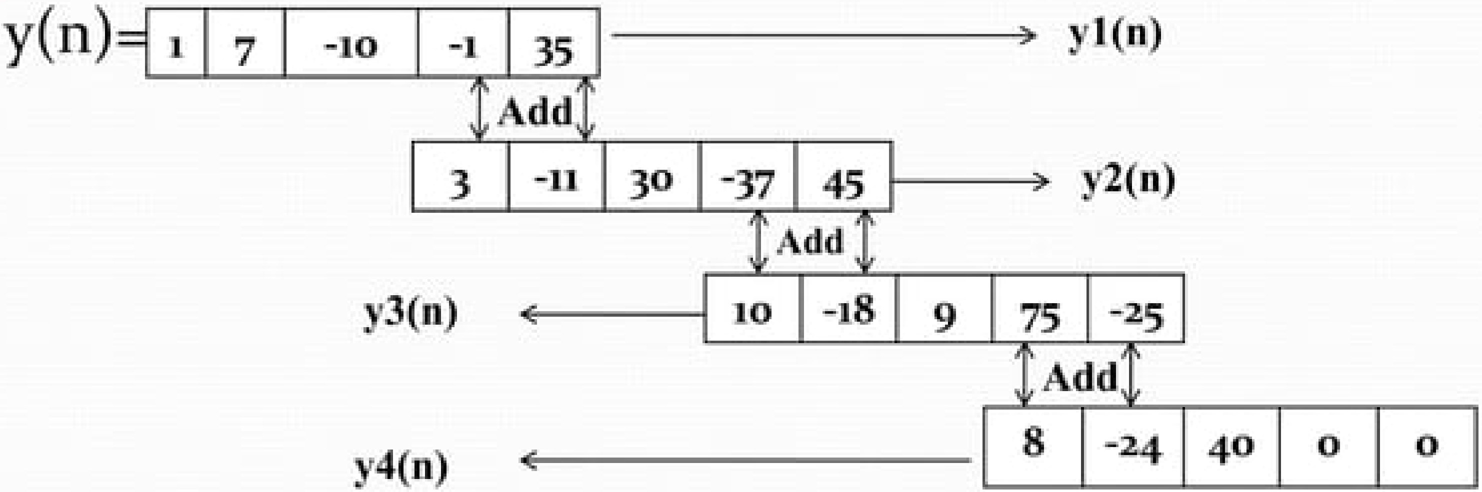
$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 9 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ -11 \\ 30 \\ -37 \\ 45 \end{bmatrix}$$

$$y3(n) = x3(n) \textcircled{N}\ h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 12 \\ -5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -18 \\ 9 \\ 75 \\ -25 \end{bmatrix}$$

$$y4(n) = x4(n) \textcircled{N}\ h(n)$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & -3 \\ -3 & 1 & 0 & 0 & 5 \\ 5 & -3 & 1 & 0 & 0 \\ 0 & 5 & -3 & 1 & 0 \\ 0 & 0 & 5 & -3 & 1 \end{bmatrix} \begin{bmatrix} 8 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8 \\ -24 \\ 40 \\ 0 \\ 0 \end{bmatrix}$$



Therefore,

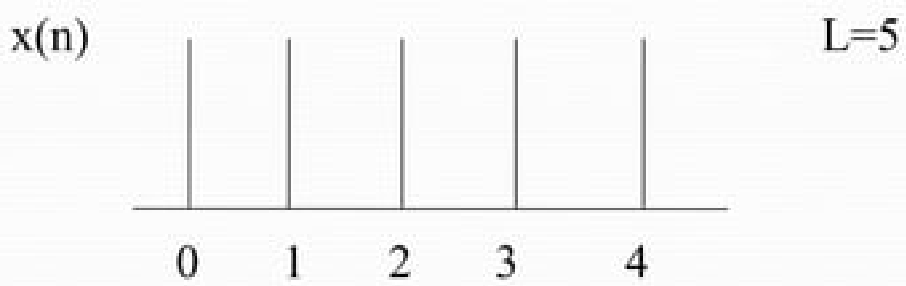
$y(n)=\{1,7,-10,2,24,30,-27,27,9,83,-49,40\}$

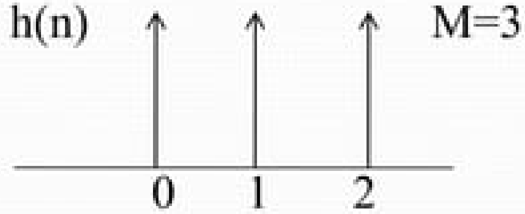
LINEAR CONVOLUTION FROM CIRCULAR CONVOLUTION:

- Let us consider two finite duration sequences x(n) and h(n). The duration of x(n) is L and y(n) is M samples. The linear convolution of x(n) and h(n) is given by the formula

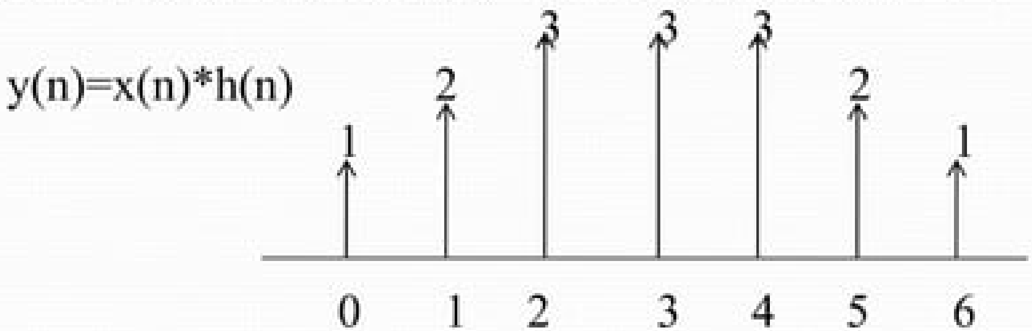
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

where y(n) is a finite duration of sequence of L+M-1 samples.
The circular convolution of x(n) and h(n) give N samples where N=Max(L,M).
Consider two sequences x(n) and h(n) having sequence length L=5 and M=3 respectively as shown in the figure.

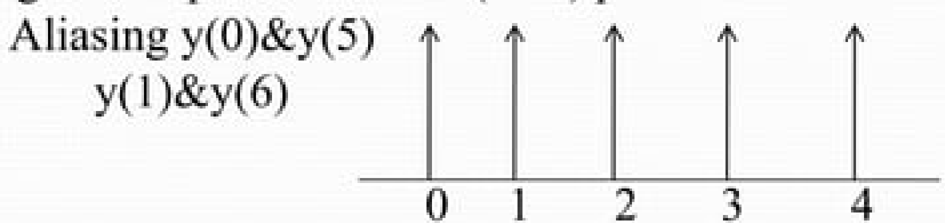




The linear convolution of x(n) and h(n) consist of 5+3-1=7 points.



The circular convolution of x(n) and h(n) consist of 5 points short of M-1=2 points. Therefore the circular convolution will contain corrupted points due to time domain aliasing. These points are first (M-1) points as shown in the figure.



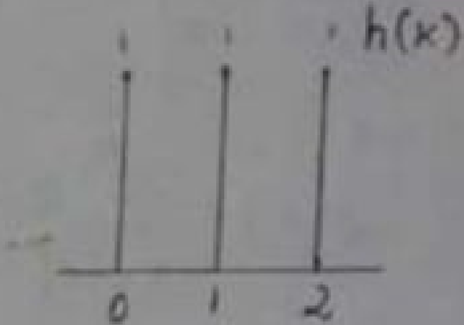
EXAMPLE:

Determine the output response $y(n)$ if $h(n) = \{1, 1, 1\}$; $x(n) = \{1, 2, 3, 1\}$ by using linear convolution method.

Solution:-

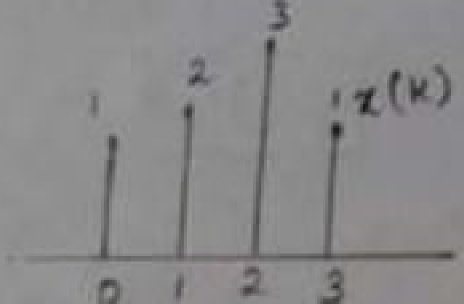
$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)$$

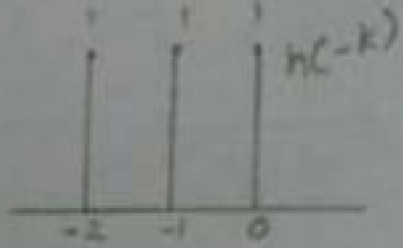
$$y(n) = h(n) * x(n)$$



When $n=0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k)$$



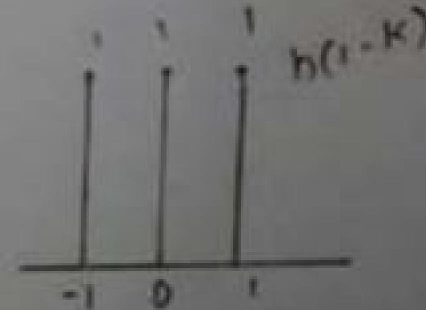


$$y(0) = (1)(1)$$

$$\boxed{y(0) = 1}$$

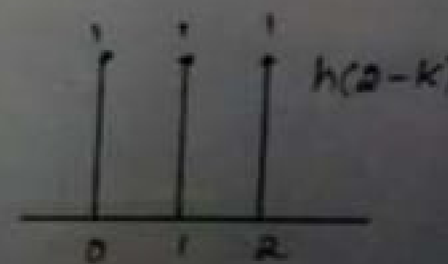
when $n = 1$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\,h(1-k)$$



$$y(1) = 1(1) + 1(2)$$

$$\boxed{y(1) = 3}$$



when $n = 2$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\,h(2-k)$$

$$y(2) = 1(1) + 1(2) + 1(3)$$

$$\boxed{y(2) = 6}$$

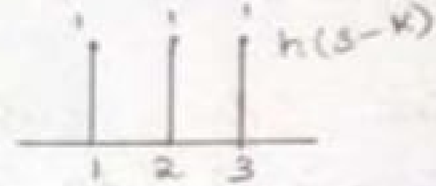
When $n = 3$.

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k)$$

$$= 1(2) + 1(3) + 1(1)$$

$$= 2 + 3 + 1$$

$$\boxed{y(3) = 6}$$

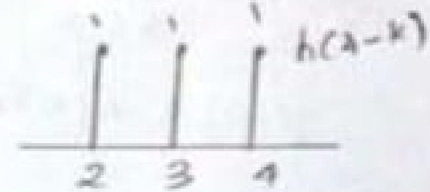


When $n = 4$

$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k)$$

$$= 1(3) + 1(1)$$

$$\boxed{y(4) = 4}$$

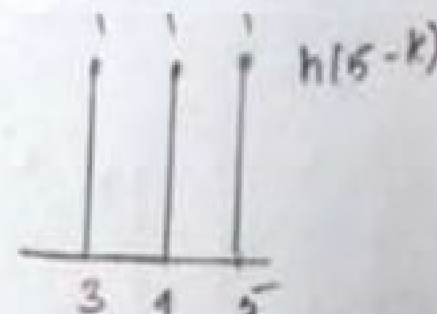


When $n = 5$

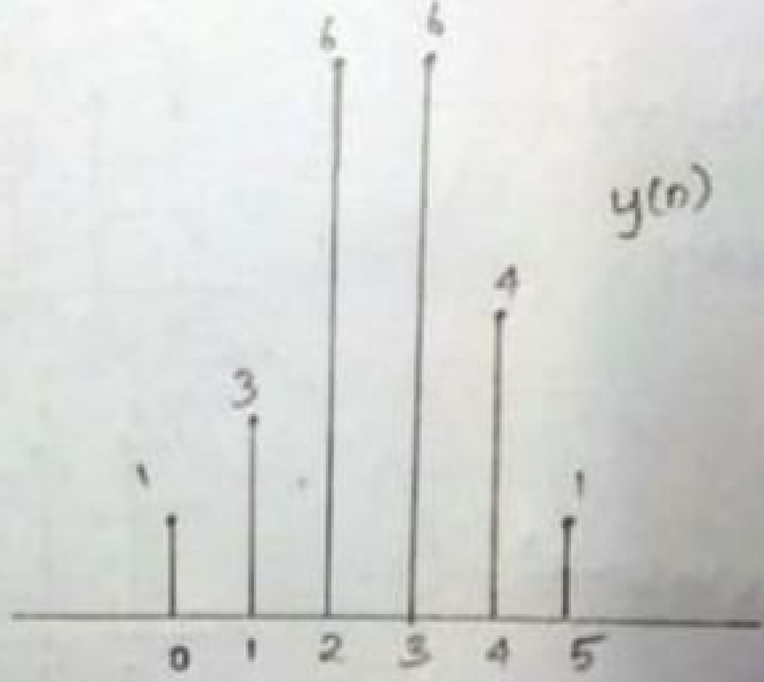
$$y(5) = \sum_{k=-\infty}^{\infty} x(k)h(5-k)$$

$$= 1(1)$$

$$\boxed{y(5) = 1}$$



$$y(n) = \{1, 3, 6, 6, 4, 1\}$$



Thus, linear convolution is done.

THANK YOU!!!