

DISCRETE-TIME SIGNALS AND SYSTEMS, PART 1

1. Lecture 2 - 36 minutes

General System

$x(n) \rightarrow \boxed{T[\]} \rightarrow y(n)$

$x(n) \rightarrow y(n)$

Special Class:

Linear $\hat{=}$ Shift-invariant LSI

Linearity

If $x_1(n) \rightarrow y_1(n)$
 $x_2(n) \rightarrow y_2(n)$

then: $ax_1(n) + bx_2(n) \rightarrow ay_1(n) + by_2(n)$

$\sum a_k x_k(n) \rightarrow \sum a_k y_k(n)$

Shift-invariance

$x(n-n_0) \rightarrow y(n-n_0)$

$\delta(n) \rightarrow h(n)$
 (unit sample response)

$\delta(n-k) \rightarrow h(n-k)$

a.

$x(n) = \sum_{k=-\infty}^{\infty} x(k) \delta(n-k)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$

Convolution sum
 $n-k=r; k=n-r$

$y(n) = \sum_r x(n-r) h(r)$

$y(n) = x(n) * h(n)$
 $= h(n) * x(n)$

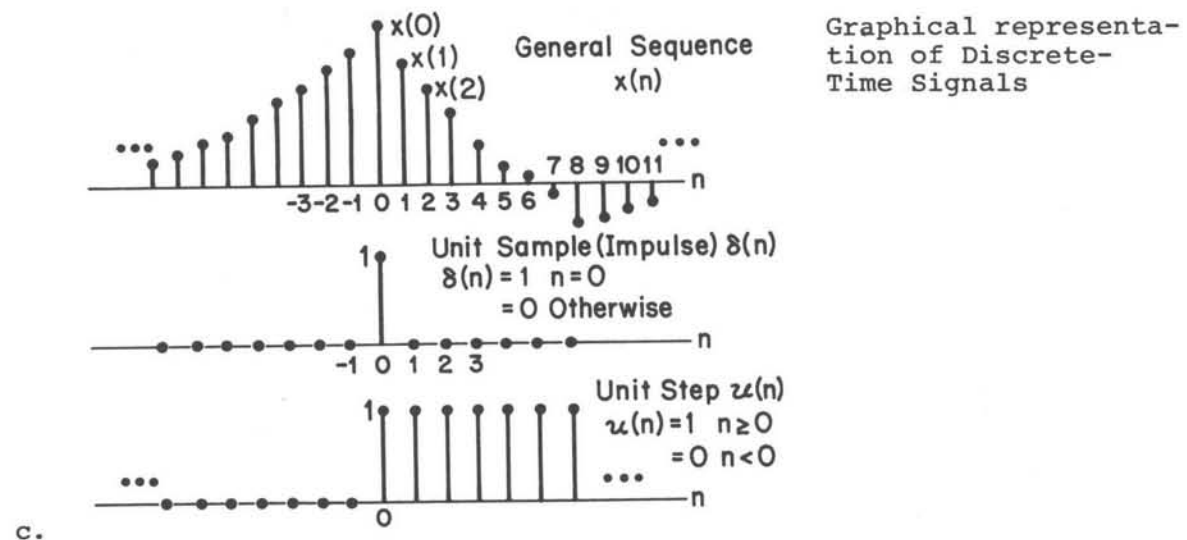
$x(n) \rightarrow \boxed{h(n)} \rightarrow y(n)$

$h(n) \rightarrow \boxed{x(n)} \rightarrow y(n)$

$x(n) \rightarrow \boxed{h_1(n)} \rightarrow \boxed{h_2(n)} \rightarrow y(n)$

$x(n) \rightarrow \boxed{h_2(n)} \rightarrow \boxed{h_1(n)} \rightarrow y(n)$

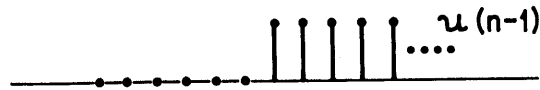
b.



c.

$$\delta(n) = u(n) - u(n-1)$$

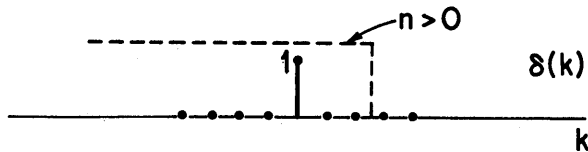
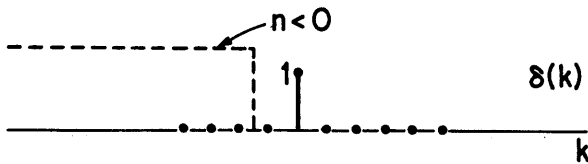
The unit-sample sequence in terms of the unit-step sequence.



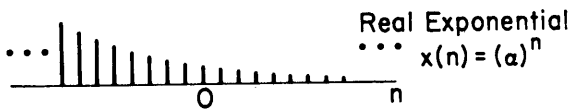
d.

$$u(n) = \sum_{k=-\infty}^n \delta(k)$$

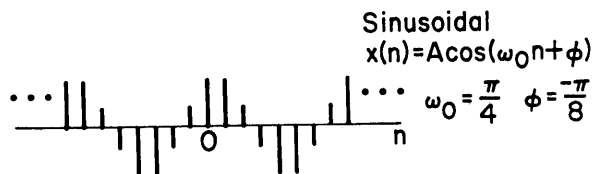
The unit-step sequence in terms of the unit-sample sequence.



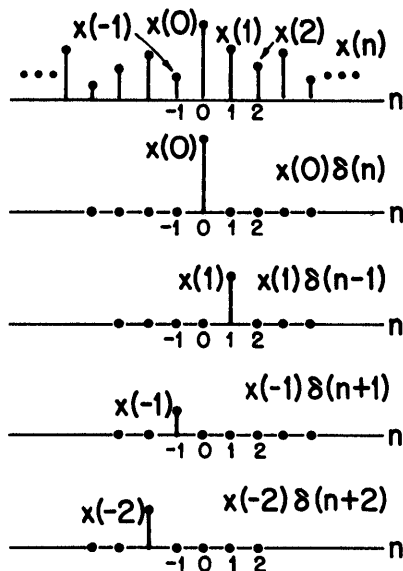
e.



Exponential and Sinusoidal sequences.



f.



Representation of an arbitrary sequence as a linear combination of delayed unit samples.

$$\begin{aligned}
 x(n) &= x(0)\delta(n) + x(1)\delta(n-1) \\
 &\quad + x(-1)\delta(n+1) + \dots \\
 &= \sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)
 \end{aligned}$$

Find convolution:

1. reverse $h(k)$ sequence
2. slide $h(k)$ sequence like window

For example,
 $x(k) = \{2, 3, 1, 2\}$
 $h(k) = \{1, 2\}$

Solution:

1. Reverse $h(k)$ sequence:

$h(k) = \{2, 1\}$

i) $\begin{array}{cccc} 2 & 3 & 1 & 2 \\ & 2 & 1 & \end{array}$ Illustration of folding and shifting for linear convolution.

$2 \times 1 = 2$

ii) $\begin{array}{cccc} 2 & 3 & 1 & 2 \\ & 2 & 1 & \end{array}$

$2 \times 2 + 3 \times 1 = 7$

iii) $\begin{array}{cccc} 2 & 3 & 1 & 2 \\ & 2 & 1 & \end{array}$

$2 \times 3 + 1 \times 1 = 7$

iv) $\begin{array}{cccc} 2 & 3 & 1 & 2 \\ & 2 & 1 & \end{array}$

$1 \times 2 + 2 \times 1 = 4$

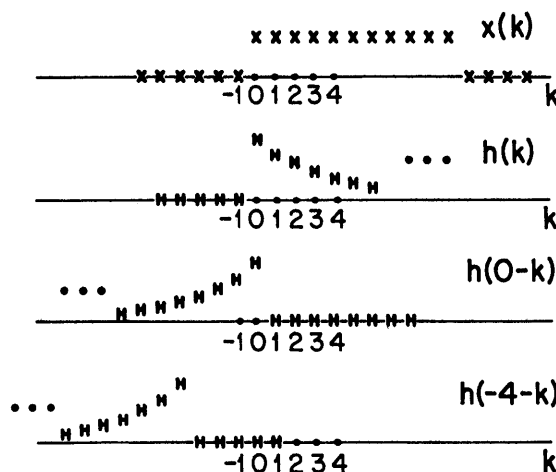
v) $\begin{array}{cccc} 2 & 3 & 1 & 2 \\ & 2 & 1 & \end{array}$

$2 \times 2 = 4$

So, convolution is $\{2, 7, 7, 4, 4\}$

g.

$$y(n) = \sum_{k=-\infty}^{+\infty} x(k) h(n-k)$$



h.

2. Correction

In the lecture I indicate that the sinusoidal sequence $A \cos(\omega_0 n + \phi)$ with $\omega_0 = 3\pi/7$ and $\phi = -\pi/8$ is not periodic. In fact it is periodic although not with a period of $2\pi/\omega_0$. (See problem 2.1(a)). For $\omega_0 = 3/7$ the sinusoidal sequence will not be periodic.

3. Comments

In this lecture we introduce the class of discrete-time signals and systems. The unit sample, unit step, exponential and sinusoidal sequences are basic sequences which play an important role in the analysis and representation of more complex sequences. The class of discrete-time systems that we focus on is the class of linear shift-invariant systems. The representation of this class of systems through the convolution sum and some properties of convolution are developed.

4. Reading

Text: Section 2.0 (page 8) through eq. (2.51) page 28 section 2.4.

5. Problems

Problem 2.1

Determine whether or not each of the following sequences is periodic. If your answer is yes, determine the period.

(a) $x(n) = A \cos\left(\frac{3\pi}{7}n - \frac{\pi}{8}\right)$ $2\pi / (3\pi/7) = 14/3 \Rightarrow \text{Rational number Periodic signal} \Rightarrow \text{period} = 14/3$

(b) $x(n) = e^{j(n/8 - \pi)}$ $2\pi / (1/8) = 16\pi \Rightarrow \text{Irrational number} \Rightarrow \text{non-periodic}$

Problem 2.2

A sequence $x(n]$ is shown below. Express $x(n]$ as a linear combination of weighted and delayed unit samples.

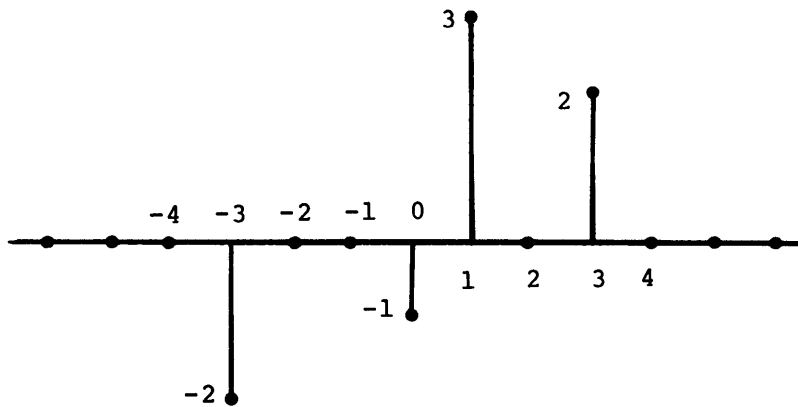


Figure P2.2-1

Problem 2.3

For each of the following systems, $y(n]$ denotes the output and $x(n]$ the input. Determine for each whether the specified input-output relationship is linear and/or shift-invariant.

(a) $y(n) = 2x(n) + 3$

(b) $y(n) = x(n) \sin\left(\frac{2\pi}{7}n + \frac{\pi}{6}\right)$

(c) $y(n) = [x(n)]^2$

(d) $y(n) = \sum_{m=-\infty}^n x(m)$

Problem 2.4

For each of the following pairs of sequences, $x(n]$ represents the input to an LSI system with unit-sample response $h(n]$. Determine each output $y(n]$. Sketch your results.

(a)

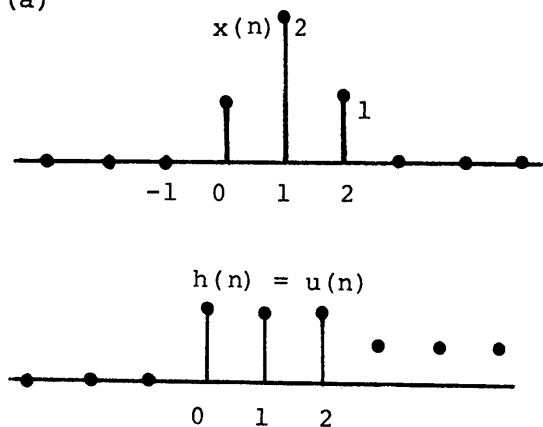


Figure P2.4-1

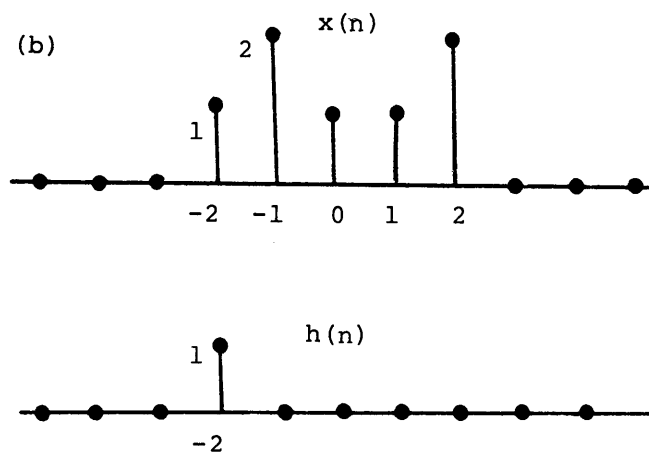


Figure P2.4-2

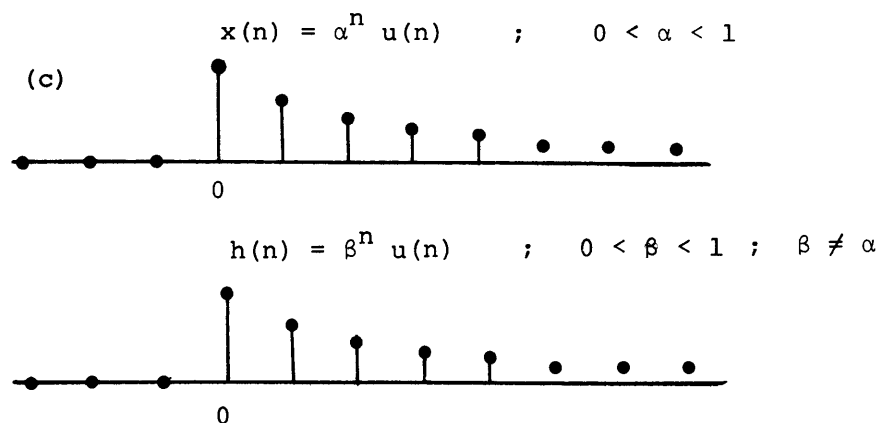


Figure P2.4-3

(d)

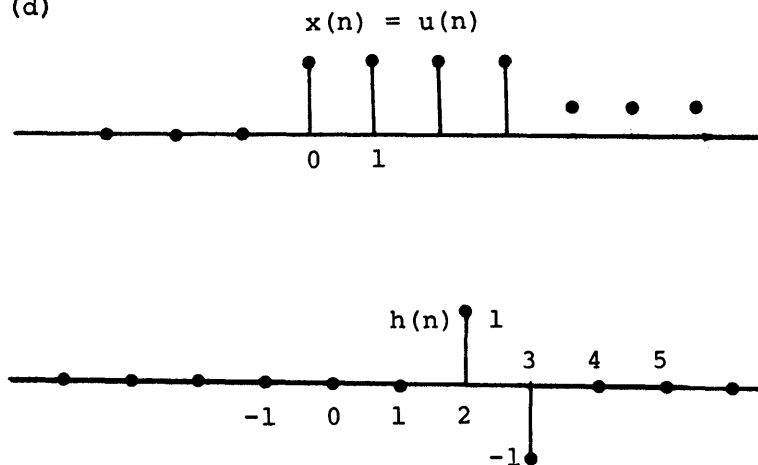


Figure P2.4-4

The following formulas may be useful:

$$\sum_{r=0}^{\infty} a^r = \frac{1}{1-a}, \quad |a| < 1$$

$$\sum_{r=0}^{N-1} a^r = \frac{1-a^N}{1-a}, \quad \text{all } a$$

Problem 2.5

The system shown below contains two linear shift-invariant subsystems with unit sample responses $h_1(n)$ and $h_2(n)$, in cascade.

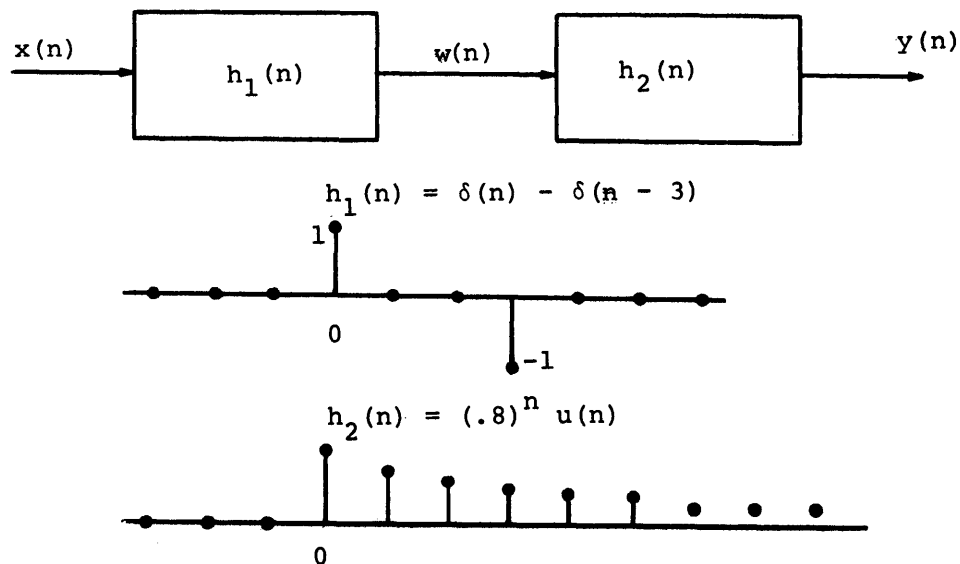


Figure P2.5-1

(a) Let $x(n) = u(n)$. Find $y_a(n)$ by first convolving $x(n)$ with $h_1(n)$ and then convolving the result with $h_2(n)$ i.e.

$$y_a(n) = [x(n) * h_1(n)] * h_2(n)$$

(b) Again let $x(n) = u(n)$. Find $y_b(n)$ by convolving $x(n)$ with the result of the convolution of $h_1(n)$ and $h_2(n)$ i.e.

$$y_b(n) = x(n) * [h_1(n) * h_2(n)]$$

Your results for parts (a) and (b) should be identical, illustrating the associative property of convolution.

Problem 2.6*

If the output of a system is the input multiplied by a complex constant then that input function is called an eigenfunction of the system.

(a) Show that the function $x(n) = z^n$, where z is a complex constant, is an eigenfunction of a linear shift-invariant discrete-time system.

(b) By constructing a counterexample, show that $z^n u(n)$ is not an eigenfunction of a linear shift-invariant discrete-time system.

* Asterisk indicates optional problem.

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