Integrale:

1. $\int dx = x + C$.
2. $\int x\,dx = \frac{x^2}{2} + C$
3. $\int x^{\alpha} dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \alpha \neq -1$.
4. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$
5. $\int \frac{1}{x} dx = \ln|x| + C$.
6. $\int a^x dx = \frac{a^x}{\ln a} + C, a>0, a\neq 1$.
7. $\int e^x dx = e^x + C$.
8. $\int \sin x\,dx = -\cos x + C$.
9. $\int \cos x\,dx = \sin x + C$.
10. $\int \frac{1}{\cos^2 x} dx = tg\,x + C$
11. $\int \frac{1}{\sin^2 x} dx = -ctg\,x + C$.
12. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C = -\arccos x + C$.
13. $\int \frac{1}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C$
14. $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$.
15. $\int \frac{1}{1+x^2} dx = arctg\,x + C = -arcctg\,x + C$.
16. $\int \frac{1}{a^2+x^2} dx = \frac{1}{a} arctg \frac{x}{a} + C$.
17. $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| + C$.

Derivate.

1. $C' = 0$, C - const.
2. $x' = 1$
3. $(x^{\alpha})' = \alpha \cdot x^{\alpha-1}$.
4. $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$
5. $(\sqrt[n]{x})' = \frac{1}{n\sqrt[n]{x^{n-1}}}$
6. $\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$
7. $(a^x)' = a^x \ln a, a>0, a\neq 1$
8. $(e^x)' = e^x$
9. $(\log_a x)' = \frac{1}{x \ln a}, a>0, a\neq 1$
10. $(\ln x)' = \frac{1}{x}$
11. $(\sin x)' = \cos x$
12. $(\cos x)' = -\sin x$.
13. $(tg\,x)' = \frac{1}{\cos^2 x}$
14. $(ctg\,x)' = -\frac{1}{\sin^2 x}$
15. $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
16. $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$
17. $(arctg\,x)' = \frac{1}{1+x^2}$
18. $(arcctg\,x)' = -\frac{1}{1+x^2}$
19. $(sh\,x)' = \left(\frac{e^x - e^{-x}}{2}\right)' = ch\,x$
20. $(ch\,x)' = \frac{e^x + e^{-x}}{2} = sh\,x$

$(f(x)+g(x))' = f'(x) + g'(x)$

$(f(x)\cdot g(x))' = f'(x)\cdot g(x) + f(x)\cdot g'(x)$.

$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) g(x) - f(x)\cdot g'(x)}{g^2(x)}$

$(C f(x))' = C f'(x)$

$(f(u(x)))' = f'(u(x)) \cdot u'(x)$

Suma pt. progresia geometrica:

$S = \frac{b}{1-q}$ b - 1 element al progresiei, q - ratia (puterea) $(q)^n$ pentru $0<q<1$ => conv.

$\frac{1}{f(x)\cdot g(x)} = \frac{A}{f(x)} + \frac{B}{g(x)}$

$e^{-\infty} = \frac{1}{e^{+\infty}} = \frac{1}{+\infty} = 0$

$e^{+\infty} = +\infty$

$\ln 0 = -\infty$

$\ln(+\infty) = +\infty$

$arctg(+\infty) = \frac{\pi}{2}$.

$arctg(-\infty) = -\frac{\pi}{2}$

$\sin \varphi(x) \sim \varphi(x)$.

$tg[\varphi(x)] \sim \varphi(x)$

$\arcsin \varphi(x) \sim \varphi(x)$

$arctg\,\varphi(x) \sim \varphi(x)$

$\cos \varphi(x) \sim 1 - \frac{\varphi^2(x)}{2}$

$e^{\varphi(x)} \sim 1 + \varphi(x)$.

$\ln(1+\varphi(x)) \sim \varphi(x)$

Seria Dirichlet.

$\sum_{n=1}^{\infty} \frac{1}{n^{\alpha}}, \alpha>0$

$\alpha > 1$ => convergentă

$\alpha \leq 1$ => divergentă.

Criteriul Leibnitz.

$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$

$a_1 \geq a_2 \geq \ldots \geq a_n \geq \ldots$, $\lim_{n\to\infty} a_n = 0$ => seria conv.

Criteriul de comp cu inegalitati.

Fie $\sum_{n=1}^{\infty} a_n$ și $\sum_{n=1}^{\infty} b_n$. $a_n \geq b_n > 0$

Dacă $\sum_{n=1}^{\infty} a_n$ este conv/div atunci și $\sum_{n=1}^{\infty} b_n$ este conv/div.

Criteriul D'Alembert.

$\sum_{n=1}^{\infty} a_n$ există $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = l$.

$l<1$ => seria conv.

$l>1$ => seria div.

$l=1$ => nu putem face concluzii.

Criteriul Raabe - Duhamel.

pentru $\sum_{n=1}^{\infty} a_n$ $\exists \lim_{n\to\infty} \left(\frac{a_n}{a_{n+1}} - 1\right) n = l$.

$l>1$ => converge.

$l<1$ => diverge.

Criteriul radical Cauchy.

$\sum_{n=1}^{\infty} a_n$ există $\lim_{n\to\infty} \sqrt[n]{a_n} = l$.

$l<1$, seria converge.

$l>1$, seria diverge.

Criteriul de comparație la limită.

Fie $\sum_{n=1}^{\infty} a_n$ și $\sum_{n=1}^{\infty} b_n$ $\exists \lim_{n\to\infty} \frac{a_n}{b_n} = K$.

$0<K<\infty$ => au aceeași natură de conv.

$K=0$ și $\sum_{n=1}^{\infty} b_n$ conv => $\sum_{n=1}^{\infty} a_n$ => conv.

$K=\infty$ și $\sum_{n=1}^{\infty} b_n$ div => $\sum_{n=1}^{\infty} a_n$ => div.

Algoritmul de găsire a domeniului de conv.

1) Găsim raza de convergență.

$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|$ sau $R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|a_n|}}$

2) Pentru $x-x_0 \in (-R, R)$ - S. converge.

Pentru $x - x_0 \in (-\infty, -R) \cup (R, +\infty)$ - S. div.

3) pentru $x - x_0 = \pm R$ - cercetăm aparte (punem în loc de $x \Rightarrow \pm R$).

4) Concluzia

Serii Taylor:

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots, x \in (-\infty; +\infty)$.

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^n \frac{x^{2n}}{(2n)!} + \ldots, x \in (-\infty, +\infty)$

$(1+x)^m = 1 + mx + \frac{m(m-1)}{2!} x^2 + \ldots + \frac{m(m-1)\ldots(m-n+1)}{n!} x^n + \ldots, x \in (-1; 1)$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots + (-1)^n \frac{x^{n+1}}{n+1} + \ldots, x \in (-1; 1)$.

$arctg\,x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots + (-1)^n \frac{x^{2n+1}}{2n+1} + \ldots, x \in (-1; 1)$.

$\arcsin x = x + \frac{1}{2}\cdot\frac{x^3}{3} + \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} + \ldots + \frac{1\cdot 3 \cdot \ldots \cdot (2n-1)}{2\cdot 4 \cdot \ldots \cdot 2n}\cdot\frac{x^{2n+1}}{2n+1} + \ldots, x \in (-1; 1)$

$\arccos x = \frac{\pi}{2} - x - \frac{1}{2}\cdot\frac{x^3}{3} - \frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5} - \ldots - \frac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4 \cdot \ldots \cdot (2n)}\cdot\frac{x^{2n+1}}{2n+1}, x \in (-1; 1)$

$sh\,x = x + \frac{x^3}{3!} + \ldots + \frac{x^{2n+1}}{(2n+1)!} + \ldots, x \in (-\infty; +\infty)$.

$ch\,x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{2n}}{(2n)!} + \ldots, x \in (-\infty; +\infty)$.

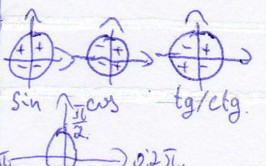
α	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
α°	0°	30°	45°	60°	90°	180°
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1
$tg\,\alpha$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-	0
$ctg\,\alpha$	-	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	-

$\sin^2\alpha + \cos^2\alpha = 1$.

$\sin(2\alpha) = 2\cos\alpha \sin\alpha$

$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$; $\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$



$f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + \frac{df}{dx}(x_0, y_0)\Delta x + \frac{df}{dy}(x_0, y_0)\Delta y$

puncte de extreme locale de 2 var.

(1) $\begin{cases} f'_x(x,y)=0 \\ f'_y(x,y)=0 \end{cases}$ $M(x_0,y_0)$

(2) derivatele parțiale de ordin II

$f''_{xx}(x,y)$; $f''_{xy}=f''_{yx}(x,y)$.
$f''_{yy}(x,y)$.

(3) Matricea Hesiană.

$$H(M_0)=\begin{pmatrix} f''_{xx}(M_0) & f''_{xy}(M_0) \\ f''_{xx}(M_0) & f''_{yy}(M_0) \end{pmatrix}=\Delta_2.$$

$\Delta_1=f''_{xx}(M_0)$.

Pentru:

$\Delta_1>0$; $\Delta_2>0$ → minim local.

$\Delta_1<0$; $\Delta_2>0$ → maxim local.

$\Delta_2<0$ → punct de tip „șa"

$F(x,y,\lambda)=f(x,y)+\lambda\,\varphi(x,y)$.

(1) $\begin{cases} F'_x=0 \\ F'_y=0 \\ F'_\lambda=0 \end{cases} \Rightarrow \tilde{M}_0$

(2) F''_{xx} F''_{xy} F''_{yy}.
φ'_x φ'_y
în $\tilde{M}_0$.

(3)
$$\Delta=-\begin{vmatrix} 0 & \varphi'_x(M_0) & \varphi'_y(M_0) \\ \varphi'_x(M_0) & F''_{xx}(\tilde{M}_0) & F''_{xy}(\tilde{M}_0) \\ \varphi'_y(M_0) & F''_{yx}(\tilde{M}_0) & F''_{yy}(\tilde{M}_0) \end{vmatrix}$$

Pentru.

$\Delta<0 \Rightarrow M_0(x_0;y_0)$ → max. loc. cond.

$\Delta>0 \Rightarrow M_0(x_0;y_0)$ → min loc. cond.

Distanța de la un punct la o dreaptă

pentru. $d_1: ax+by+c=0$.

$$d=\frac{|ax_M+by_M+c|}{\sqrt{a^2+b^2}}$$

pentru: $d_2: y=mx+n$.

$$d=\frac{|mx_M-y_M+n|}{\sqrt{1+m^2}}$$

distanța minimă.

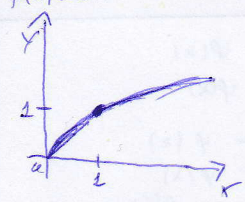
$f=d^2$.

$f(x,y)$ – dreapta. $d(x,y)$.

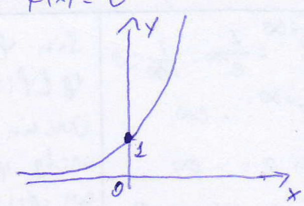
$\varphi(x,y)$ – punct.

$F(x,y,\lambda)=d+\lambda\cdot$punct.

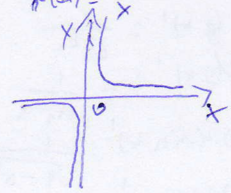
$f(x)=\sqrt{x}$



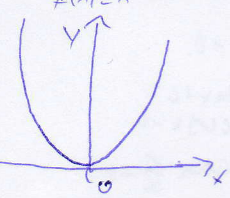
$f(x)=e^x$



$f(x)=\frac{1}{x}$



$f(x)=x^2$



Integrale folosind schimb de variabile

$\begin{cases} x=\rho\cos\varphi \\ y=\rho\sin\varphi. \end{cases}$ | ⇒ schimbă și calc. în D.
ex. $2\cos\varphi\le\rho\le4\cos\varphi$.

φ ⇒ cadranele.
ex. $0\le\varphi\le\frac{\pi}{4}$.

$\sqrt{3}=60°$

$\frac{1}{\sqrt{3}}=30°$

$y=x=45°$.

$A_D=\iint_D dx\,dy$.

le egalăm.

Ecuația dreptei ce trece prin 2 puncte.

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$$

$V=\iint_D f(x,y)\,dx\,dy$.

densit.: $m=\iint_D \rho(x,y)\,dx,dy$

$\int_a^{+\infty}\frac{1}{x^\alpha}dx$ — $\alpha>1$ → converge; $\alpha\le1$ → diverge (scrim f asemănătoare).

$\int_a^b\frac{1}{(b-x)^\alpha}dx$ — b-punct singular. înlocuim b în $f(x)\ne0$
— $0<\alpha<1$ → converge
— $\alpha\ge1$ → diverge. $=\varphi(x)$

$-1\le\sin/\cos\le1$.

$\int_a^{+\infty}f(x)dx$; $\int_a^{+\infty}g(x)\,dx$. a.î. $l=\lim_{x\to\infty}\frac{f(x)}{g(x)}\ne0$

$\int_a^{+\infty}g(x)\,dx$ – conv/div $\Rightarrow\int_a^{+\infty}f(x)$ – conv/div.