

# A review of R neural network packages (with NNbenchmark): accuracy and ease of use

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## Abstract

In the last three decades, neural networks (NN) have evolved from an academic topic to a common scientific computing tool. CRAN currently hosts approximately 80 packages in May 2020 involving neural network modeling, some offering more than one algorithm. However, to our knowledge, there is no comprehensive study which check the accuracy, the reliability and the ease-of-use of those NN packages.

In this paper, we attempted to test this rather large number of packages against the common set of datasets with different levels of complexity, and to benchmark and rank them with certain metrics.

Restricting our evaluation to regression algorithms applied on the one-hidden layer perceptron and ignoring those for classification or other specialized purposes, there were approximately 60 package::algorithm pairs left to test. The criteria used in our benchmark were: (i) the accuracy, i.e. the ability to find the global minima on 13 datasets, measured by the Root Mean Square Error (RMSE) in a limited number of iterations; (ii) the speed of the training algorithm; (iii) the availability of helpful utilities; (iv) and the quality of the documentation.

We have attempted to give a score for each evaluation criterion and to rank each package::algorithm pair in a global table. Overall, 15 pairs are considered accurate and reliable and can be recommended for daily usage. Most others should be avoided as they are either less accurate, too slow, too difficult to handle, or have poor or no documentation.

To carry out this work, we developed various codes and templates, as well as the NNbenchmark package used for testing. This material is available at <https://akshajverma.com/NNbenchmarkWeb/index.html> and <https://github.com/pkR-pkR/NNbenchmark>, and can be used to verify our work and, we hope, improve both packages and their evaluation. Finally, we provide some hints and features to guide the development of an idealized neural network package for R.

## Introduction

(PK) For the last 30 years, neural networks have evolved from an academic topic to a common tool in scientific computing. As a convenience in the general conversation, the same term is used in a generic manner (for a shortcut to) for different model structures and applications: multilayer perceptron for regression, multilayer perceptron for classification, multilayer perceptron for specialized applications (mixture of models, conditional distribution, etc), recurrent neural network for autoregressive time series (NARMAX models), convolutional neural networks for dimension reduction and pattern recognition, deep neural networks for image or voice recognition.

Most of the above types of neural networks can be found in R packages hosted on CRAN but without any warranty about the quality or the accuracy of the calculation, since CRAN has no tools to check them. Package users have to believe package authors who are assumed to have selected the best algorithms and tuned them with the appropriate hyperparameters. This is rather problematic with neural networks as many poor algorithms have been mentioned in the literature, and are probably implemented in a few packages.

The goal of this paper is to verify the accuracy and the ease of use (including the quality of the documentation) of the packages hosted on CRAN that supply the most common neural network structure + algorithm, i.e. the multilayer perceptron for regression purpose.

The R Project for Statistical Computing ([www.r-project.org](http://www.r-project.org)), as any opensource platform, relies on its contributors to keep it up to date. Neural networks (NN), inspired on the brain's own connections system, are a class of models in the growing field of machine learning for which R has a number of tools. During the last 30 years, neural networks have evolved from an academic topic to a common tool in scientific computing. Previously, neural networks were considered more theory than practice, partly because the algorithms used were computationally demanding.

As a convenience in the general conversation, the same term is used in a generic manner for different model structures and applications: multilayer perceptron for regression, multilayer perceptron for classification, multilayer perceptron for specialized applications, recurrent neural network for autoregressive time series, convolutional neural networks for dimension reduction and pattern recognition, deep neural networks for image or voice recognition. Most of the above types of neural networks can be found in R packages hosted on CRAN but without any warranty about the accuracy or the speed of computation. This is an issue as many poor algorithms are available in the literature and hence poor packages implemented on CRAN.

A neural network algorithm requires complicated calculations to improve the model control parameters. As with other optimization problems, the gradient of the chosen cost function that indicates the lack of suitability of the model is sought. This lets us improve the model by changing the parameters in the negative gradient direction. Parameters for the model are generally obtained using part of the available data (a training set) and tested on the remaining data. Modern software allows much of this work, including approximation of the gradient, to be carried out without a large effort by the user.

The training process can generally be made more efficient if we can also approximate second derivatives of the cost function, allowing us to use its curvature via the Hessian matrix. There are a large number of approaches, of which quasi-Newton algorithms are perhaps the most common and useful. Within this group, methods based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm for updating the (inverse) Hessian approximation provide several well-known examples. In conducting this study, we believed that these second-order algorithms would perform better than first-order methods for fit-in-memory datasets.

Regardless of our belief, we wished to be able to conduct a thorough examination of these training algorithms in R. There are many packages, but barely any information to allow comparison. Our work, reported here, aims to provide a framework for benchmarking neural network packages. We restrict our examination to packages for R, and in this report focus on those that provide neural networks of the perceptron type, that is, one input layer, one normalized layer, one hidden layer with a nonlinear activation function that is usually the hyperbolic tangent  $\tanh()$ , and one output layer. The criteria used in our benchmark were: (i) the accuracy, i.e. the ability to find the global minima on 13 datasets in a limited number of iterations; (ii) the speed of the training algorithm; (iii) the availability of helpful utilities; (iv) and the quality of the documentation. We restricted our evaluation to regression algorithms applied on the one-hidden layer perceptron and ignored those for classification or other specialized purposes.

## Methodology

??JN: \*\*\*\*\*

In working on material below, I think we need to provide some explanation of goals - What does "convergence" mean in our context? - What do we mean by RMSE, other measures? Should define here for later use. - What do we mean by "performance"? Other goals?

As a reminder, RMSE and other convergence metrics are defined in Appendix A.

Our research process was divided into 3 phases.

### Phase 1 - Preparation of benchmark datasets

#### Datasets => NEED TO BE FINISHED??

All the datasets we use cannot generally be modeled using a non-iterative calculation such as Ordinary Least Squares. Varying levels of difficulty in modeling the different data sets are intended to allow us to further classify different algorithms and the packages that implement them. Sonja Surjanovic and Derek Bingham of Simon Fraser University created a useful website from which three of the multivariate datasets were drawn. We note the link, name and difficulty level of the three datasets:

- <http://www.sfu.ca/~ssurjano/fried.html> (Friedman - average)
- <http://www.sfu.ca/~ssurjano/detpep10curv.html> (Dette - medium)
- <http://www.sfu.ca/~ssurjano/ishigami.html> (Ishigami - high)

The other multivariate dataset, Ref153, was taken from ...

Three of the univariate datasets we used were taken from a website of the US National Institute for Standards and Technology (NIST): [https://www.itl.nist.gov/div898/strd/nls/nls\\_main.shtml](https://www.itl.nist.gov/div898/strd/nls/nls_main.shtml). (Gauss1 - low; Gauss2 - low; Gauss3 - average)

Univariate datasets Dmod1, Dmod2 are from ...

Dreyfus1 is a pure neural network which has no error. This can make it difficult for algorithms that assume an error exists. Dreyfus2 is Dreyfus1 with errors. NeuroOne from ...

Finally, we also consider a Simon Wood test dataset, used in (Wood, 2011) for benchmarking generalized additive models. Precisely, we consider a generation of Gaussian random variates  $Y_i, i = 1, \dots, n$  with the mean  $\mu_i$  defined as

$$\mu_i = 1 + f_0(x_{i,0}) + f_1(x_{i,1}) + f_2(x_{i,2}) + f_3(x_{i,3}) + f_4(x_{i,4}) + f_0(x_{i,5})$$

and standard deviation  $\sigma = 1/4$  where  $f_j$  are Simon Wood's smooth functions defined in Appendix B,  $x_{i,j}$  are uniform variates and  $n = 20,000$ .

### Packages

Using **RWsearch** (Kiener, 2020) we sought to automate the process of searching for neural network packages. All packages that have "neural network" as a keyword in the package title or in the package description were included. In May 2020, around 80 packages falls into this category. Packages **nlsr**, **minpack.lm**, **caret** were added because the former 2 are important implementations of second-order algorithms while the latter is the first cited meta package in the CRAN's task view for machine learning, <https://CRAN.R-project.org/view=MachineLearning>, as well as the dependency for some of the other packages tested. Restricting to regression analysis left us with 49 package::algorithm pairs in 2019 and 60 package::algorithm pairs in 2020.

### Phase 2 - Review of packages and development of a benchmarking template

From documentation and example code, we learned that not all packages selected by the automated search fit the scope of our research. Some have no function to generate neural networks. Others were not regression neural networks of the perceptron type or were only intended for very specific purposes.

#### Templates for Testing Accuracy and Speed

As we inspected the packages, we developed a template for benchmarking. The structure of this template (for each package) is as follows:

- (1) Set up the test environment - loading of packages, setting working directory and options;
- (2) Summary of datasets;

(3) Loop over datasets: (a) setting parameters for a specific dataset (b) selecting benchmark options (c) training a neural network with a tuned functions for each package (d) calculation of RMSE and MAE (??definition, reference) (e) plot each training over one initial graph, then plot the best result (f) add results to the appropriate existing record (\*.csv file) and (g) clear the environment for next loop; and

(4) Clearing up the environment for the next package. (5) It is optional to print warnings.

To simplify this process, we developed tools in the NNbenchmark package, of which the first version was created as part of GSoC 2019. In GSoC 2020, 3 functions encapsulating the template that had been made generic with `do.call` were added:

1. In `trainPredict_1mth1data` a neural network is trained on one dataset and then used for predictions, with several utilities. Then, the performance of the neural network is summarized.
2. `trainPredict_1data` serves as a wrapper function for `trainPredict_1mth1data` for multiple methods.
3. `trainPredict_1pkg` serves as a wrapper function for `trainPredict_1mth1data` for multiple datasets.

A function for the summary of accuracy and speed, `NNsummary`, was also added. The package repository is <https://github.com/pkR-pkR/NNbenchmark>, with package templates in <https://github.com/pkR-pkR/NNbenchmarkTemplates>.

### Ease of Use Scoring

We decided ease of use based on what we considered a user would need when using a neural network package for nonlinear regression, namely, utility functions and sufficient documentation.

#### (1) Utilities (1 star)

- (a) a predict function exists
- (b) scaling capabilities exist

#### (2) Sufficient documentation (2 stars)

- (a) the existence of useful example/vignette = (1 star)
  - clear, with regression = 2 points
  - unclear, examples use iris or are for classification only = 1 point
  - no examples = 0 points
- (b) input/output is clearly documented, e.g., what values are expected and returned by a function = (1 star)
  - clear input and output = 2 points
  - only one is clear = 1 point
  - both are not documented = 0 points

For a total of 0 to 3 stars.

## Phase 3 - Collection of and analysis of results

### Results collection

Looping over the datasets using each package template, we collected results in the relevant package directories in the templates repository.

### Analysis

To rank how well a package converged and its speed, we developed the following method:

1. The results datasets are loaded into the R environment as one large list. The dataset names, package:algorithm names and all 10 run numbers, durations, and RMSE are extracted from that list
2. For the duration score (DUR), the duration is averaged by dataset. 3 criteria for the RMSE score by dataset are calculated:
  - a. The minimum value of RMSE for each package:algorithm as a measure of their best performance
  - b. The median value of RMSE for each package:algorithm as a measure of their average performance, without the influence of outliers
  - c. The spread of the RMSE values for each package which is measured by the difference between the median and the minimum RMSE (d51)
3. Then, the ranks are calculated for every dataset and the results are merged into one wide dataframe.
  - a. The duration rank only depends on the duration.
  - b. For minimum RMSE values, ties are decided by duration mean, then the RMSE median
  - c. For median RMSE values, ties are decided by the RMSE minimum, then the duration mean
  - d. The d51 rank only depends on itself
4. A global score for all datasets is found by a sum of the ranks (of duration, minimum RMSE, median RMSE, d51 RMSE) of each package:algorithm for each dataset
5. The final table is the result of ranking by the global minimum RMSE scores for each package:algorithm
6. In addition to the previous metrics, two other convergence metrics have been considered: the Mean Absolute Error (MAE) and the Worst Absolute Error (WAE), see Appendix A. The ranking on those two metrics may help distinguish packages with close RMSE values. However, we do not choose the MAE for overall ranking as there is no consensus in the literature, see e.g. (Willmott and Matsuura, 2005; Chai and Draxler, 2014).

To rank how easy or not a package was to use (TO BE DISCUSSED FURTHER): -  
 Functionality (util): scaling, input, output, trace - Documentation (docs): examples, structure/functions, vignettes

## Results

**Tables** (NOTE: FINAL MEASURE FOR CONVERGENCE - RMSE RANKS? OR A COMBINATION OF OTHER MEASURES? As in Christophe's recent email: L1 MAE(), L2 RMSE(), Linfinity (WAE)) -> see Appendix

(ALSO: THE FOLLOWING IS SIMPLY ALPHABETIC LIST FOR ALL TESTED, I WILL DIVIDE THE TABLE INTO 4: 2nd ORDER always recommended, 1st ORDER recommended, 1st ORDER not recommended, untested packages)

**Table 1: Review of Tested Packages**

No	Package::Function	Algorithm	RMSE	DUR	UTIL	DOCS
1	<b>AMORE</b> ::train	1. ADAPTgd 2. ADAPTgdwm 3. BATCHgd 4. BATCHgdwm				
2	<b>ANN2</b> ::neuralnetwork	5. adam 6. rmsprop 7. sgd				
3	<b>automl</b> ::automl_train_manual	8. trainwgrad.adam 9. trainwgrad.RMSprop 10. trainwpso				
4	<b>brnn</b> ::brnn	11. Gauss-Newton				
5	<b>CaDENCE</b> ::cadence.fit	12. optim(BFGS) 13. pso::psoptim 14. Rprop				
6	<b>caret</b> ::avNNet	15. nnet::optim(BFGS)				
7	<b>deepdive</b> ::deepnet	16. adam 17. gradientDescent 18. momentum 19. rmsProp				
8	<b>deepnet</b> ::nn.train	20. BP				
9	<b>elmNNRcpp</b> ::elm_train	21. ELM				
10	<b>ELMR</b> ::OSelm_train.formula	22. ELM				
11	<b>EnsembleBase</b> ::Regression.Batch.Fit	23. nnet::optim(BFGS)				
12	<b>h2o</b> ::deeplearning	24.				
13	<b>keras</b> ::fit	25. adadelta 26. adagrad 27. adam 28. adamax 29. nadam 30. rmsprop 31. sgd				
14	<b>MachineShop</b> ::fit	32. nnet::optim(BFGS)				
15	<b>minpack.lm</b> ::nlsLM	33. LM				
16	<b>monmlp</b> ::fit <b>monmlp</b> ::fit	34. BFGS 35. Nelder-Mead				
17	<b>neuralnet</b> :: ::neuralnet	36. backprop 37. rprop- 38. rprop+ 39. sag 40. slr				
18	<b>nlsr</b> ::nlxb	41. NashLM				
19	<b>nnet</b> ::nnet	42. optim(BFGS)				
20	<b>qrnn</b> ::qrnn.fit	43. nlm()				
21	<b>radiant.model</b> ::radiant.model	45. nnet::optim(BFGS)				
22	<b>rminer</b> ::fit	46. nnet::optim(BFGS)				

Table 1: Review of Tested Packages

No	Package::Function	Algorithm	RMSE	DUR	UTIL	DOCS
23	<b>RSNNS::</b>	47. BackpropBatch				
	<b>RSNNS::</b>	48. BackpropChunk				
	<b>RSNNS::</b>	49. BackpropMomentum				
	<b>RSNNS::</b>	50. BackpropWeightDecay				
	<b>RSNNS::</b>	51. Quickprop				
	<b>RSNNS::</b>	52. Rprop				
	<b>RSNNS::</b>	53. SCG				
	<b>RSNNS::</b>	54. Std-Backpropagation				
24	<b>snnR</b>	55.				
25	<b>traineR</b>	55.				
26	<b>validann::</b>	56. BFGS				
	<b>validann::</b>	57. CG				
	<b>validann::</b>	58. L-BFGS-B				
	<b>validann::</b>	59. Nelder-Mead				
	<b>validann::</b>	60. SANN				

(THE FOLLOWING IS JUST AN ALPHABETICALLY ORDERED LIST OF CURRENTLY UNTESTED PACKAGES)

Table 2: Review of Ommitted Packages

No	Name (package)	Category	Comment
1	<b>appnn</b>	AP	This package provides a feed forward neural network to predict the amyloidogenicity propensity of polypeptide sequences
2	<b>autoencoder</b>	AP	This package provides a sparse autoencoder, an unsupervised algorithm that learns useful features from the data its given
3	<b>BNN</b>	RE*	This package uses a feed forward neural network to perform regression as provided in the examples, however, it is unclear whether it fits the form of perceptron that is the scope of our research. Moreover, it states that it is intended for variable selection. Although how exactly the package would be used to do so isn't accessible in the package, especially considering the source code is based on .c code that users of R might not understand. It's performance is slow, which may have to do with the 100.000 iterations it needs, although quite accurate for simple datasets.
4	<b>Buddle</b>	RE**	(errors)
5	<b>cld2</b>	00	
6	<b>cld3</b>	AP	
7	<b>condmixt</b>	AP	
8	<b>deep</b>	CL	
9	<b>DALEX2</b>	00	removed keyword, included in 2019
10	<b>DamiaNN</b>	RE**	(errors) exported functions, still doesn't work
11	<b>DChaos</b>	??	removed keyword for some reason, need to check out!
12	<b>deepNN</b>	RE**	(errors) I/O weird, ragged vector array
13	<b>DNMF</b>	AP	
14	<b>evclass</b>	CL	
15	<b>gamlss.add</b>	RE	there is some code but dist not appropriate
16	<b>gcForest</b>	00	
17	<b>GMDH</b>	TS	
18	<b>GMDH2</b>	CL	
19	<b>GMDHreg</b>	RE*	
20	<b>grnn</b>	RE**	
21	<b>hybridEnsemble</b>	??	
22	<b>isingLenzMC</b>	AP	
23	<b>leabRa</b>	??	
24	<b>learNN</b>	??	
25	<b>LilRhino</b>	AP	
26	<b>neural</b>	CL	
27	<b>NeuralNetTools</b>	UT	tools for neural networks
28	<b>NeuralSens</b>	UT	tools for neural networks
29	<b>NlinTS</b>	TS	Time Series
30	<b>nnetpredint</b>	UT	confidence intervals for NN
31	<b>nnfor</b>	TS	Times Series, uses neuralnet
32	<b>nntrf</b>	UT	
33	<b>onnx</b>		provides an open source format
34	<b>OptimClassifier</b>		choose classifier parameters, nnet
35	<b>OSTSC</b>		solving oversampling classification
36	<b>passt</b>		
36	<b>pnn</b>		Probabilistic
37	<b>polyreg</b>		polyregression as alternative to NN
38	<b>predictoR</b>		shiny interface, neuralnet
39	<b>ProcData</b>		
40	<b>QuantumOps</b>		classifies MNIST, Schuld (2018), removed keyword, in 2019
41	<b>quarrint</b>		specified classifier for quarry data
42	<b>rasclass</b>		classifier for raster images, nnet?
43	<b>rcane</b>		
44	<b>regressoR</b>		a manual rich version of predictoR
45	<b>rnn</b>		Recurrent
46	<b>RTextTools</b>		
47	<b>ruta</b>		
48	<b>simpleNeural</b>		



## Discussion and Recommendations

The following is a list of packages we included in this study, with brief descriptions.

1. **AMORE** (Limas et al., 2020),
2. **ANN2** (Lammers, 2020),
3. **appnn** (Família et al., 2015),
4. **autoencoder** (Dubossarsky and Tyshetskiy, 2015),
5. **automl** (Boulangé, 2020),
6. **BNN** (Jia, 2018),
7. **brnn** (Rodriguez and Gianola, 2020),
8. **Buddle** (Kim, 2020),
9. **CaDENCE** (Cannon, 2017a),
10. **cld2** (Ooms, 2018),
11. **cld3** (Ooms, 2020),
12. **condmixt** (Carreau, 2020),
13. **DamiaNN** (Siniakowicz, 2016),
14. **deep** (Mayer, 2019),
15. **deepdive** (Balakrishnan, 2020),
16. **deepnet** (Rong, 2014),
17. **deepNN** (Taylor, 2020),
18. **DNMF** (Jia and Zhang, 2015),
19. **elmNNRcpp** (Mouselimis and Gosso, 2020),
20. **ELMR** (Petrozziello, 2015),
21. **EnsembleBase** (Mahani and Sharabiani, 2016),
22. **evclass** (Denoeux, 2017),
23. **gamlss.add** (Stasinopoulos et al., 2020),
24. **gcForest** (Jing, 2018),
25. **GMDH** (Dag and Yozgatligil, 2016),
26. **GMDH2** (Dag et al., 2019),
27. **GMDHreg** (Tilve, 2020),
28. **gnn** (Hofert and Prasad, 2020),
29. **grnn** (Chasset, 2013a),
30. **h2o** (LeDell et al., 2020),
31. **hybridEnsemble** (Ballings et al., 2015),
32. **isingLenzMC** (Suzen, 2016),
33. **keras** (Allaire and Chollet, 2020),
34. **kerasR** (Arnold, 2017),
35. **leabRa** (Titz, 2017),
36. **learNN** (Quast, 2015),
37. **LilRhino** (Barton, 2019),
38. **minpack.lm** (Elzhov et al., 2016),
39. **MachineShop** (Smith, 2020),
40. **monmlp** (Cannon, 2017b),
41. **neural** (Nagy, 2014),
42. **neuralnet** (Fritsch et al., 2019),
43. **NeuralNetTools** (Beck, 2018),
44. **NeuralSens** (Portela González et al., 2020),
45. **NlinTS** (Hmamouche, 2020),
46. **nlsr** (Nash and Murdoch, 2019),
47. **nnet** (Ripley, 2020),
48. **nnetpredint** (Ding, 2015),
49. **nnfor** (Kourentzes, 2019),
50. **nntrf** (Aler and Valls, 2020),
51. **nnli2bRcpp** (Nikolaidis, 2020),
52. **onnx** (Tang and ONNX Authors, 2018),
53. **OptimClassifier** (Perez-Martin et al., 2020),

54. [OSTSC](#) (Dixon et al., 2017),
55. [pnn](#) (Chasset, 2013b),
56. [polyreg](#) (Matloff et al., 2020),
57. [predictoR](#) (with contributions from Diego Jimenez A. and D., 2020),
58. [qrnn](#) (Cannon, 2019),
59. [QuantumOps](#) (Resch, 2020),
60. [quarrint](#) (Barthelemy et al., 2016),
61. [radiant.model](#) (Nijs, 2020),
62. [rasclass](#) (Wiesmann and Quinn, 2016),
63. [rcane](#) (Suresh et al., 2018),
64. [regressoR](#) (Rodriguez R., 2019),
65. [rminer](#) (Cortez, 2020),
66. [rnn](#) (Quast and Fichou, 2020),
67. [RSNNS](#) (Bergmeir, 2019),
68. [ruta](#) (Charte et al., 2019),
69. [simpleNeural](#) (Dernoncourt, 2020),
70. [snnR](#) (Wang et al., 2017),
71. [softmaxreg](#) (Ding, 2016),
72. [Sojourn.Data](#) (Hibbing and Lyden, 2019),
73. [spnn](#) (Ebrahimi, 2020),
74. [TeachNet](#) (Steinbuss, 2018),
75. [tensorflow](#) (Allaire and Tang, 2020),
76. [tfestimators](#) (Allaire et al., 2018),
77. [trackdem](#) (Bruijning et al., 2020),
78. [TrafficBDE](#) (Chatzopoulou et al., 2018),
79. [tsensembler](#) (Cerqueira et al., 2017),
80. [validann](#) (Humphrey, 2017),
81. [zFactor](#) (Reyes, 2019).
- 82.
- 83.
- 84.
- 85.
- 86.
- 87.
- 88.

A. Recommended: 2nd order algorithms Out of all the algorithms, these second algorithms generally performed better in terms of convergence despite being set to a much lower number of iterations, 200, than the first-order algorithms. Moreover, they performed better in terms of speed. The best in this class were. [minpack.lm](#) and. [nlslr](#), tied at rank number 1. The Levenberg-Marquardt (LM) algorithm used is fast and converges well. `stats::nls()` is used. However, these packages require a handwritten formula that may not be ideal for certain situations. A more popular package for neural networks is `nnet`. This might be because it is part of base R. It implements the BFGS algorithm with `stats::optim()`.

Ranked directly after are some packages that depend on `nnet` or use the same functions. They differ in how well they decide initial parameters. `rminer` (rank 4), `MachineShop` (rank 5), and `radiant.model` (rank 7) use `nnet`. Note, `radiant.model` has its iterations set to 10000, which originally made it slower yet converge better. We used a modified version of the package. At rank 6 is `validann`'s BFGS algorithm using `stats::optim()`. Its use of `optim`'s L-BFGS-B ranked at number 9 with `CaDENCE`'s use of `optim`'s BFGS.. [monmlp](#), from the same author as `CaDENCE` (Alex Cannon), uses the package. [optimx](#)'s BFGS ([Nash and Varadhan, 2020](#)).

Alex Cannon also implemented a quantile regression neural network in `qrnn` with `stats::nlm()`. It requires more iterations and is not as fast compared to the other second-order algorithms. However, it is a valuable implementation of quantile regression. Last but not least is [brnn](#)'s Gauss Newton algorithm which ranks at number 8. `brnn` is easy to use but

does not converge as well due to a hidden constraint: a missing first parameter. Furthermore, `brnn`'s algorithm minimizes the sum of squared errors and a penalty on parameters instead of just the sum of squared errors. This may prevent parameters to get highly correlated, especially with an almost degenerated Jacobian matrix.

B. Recommended: 1st order algorithms `validann` `optim` `CG` -slow `RSNNS` `SCG` `h2o` back-propagation `RSNNS` `Rprop` `ANN2` `adam` `CaDENCE` `Rprop` -SLOW `deepnet` `BP` `AMORE` `ADAPTgdwm` `AMORE` `ADAPTgd` `ANN2` `sgd` `automl` `trainwgrad` `ANN2` `rmsprop` `RSNNS` `BackpropChunk` `RSNNS` `BackWeightDecay` `RSNNS` `Std_Backpropagation` `RSNNS` `Backprop-Momentum` `automl` `trainwpso` `validann` `optim` `NelderMead` `snnR` `Semi` `Smooth` `Newton` `RSNNS` `BackpropBatch` `validann` `optim` `SANN` `monmlp` `optimx` `Nelder` `Mead`

C. Not recommended: 1st order algorithms `<- DISCUSS` `CUTOFF` By package `ELMR`, `elmNNRcpp` - fast ELM algorithms. Unfortunately, can't finetune, does not converge well. `neuralnet`: a large ammount of iterations, slow, erratic failures `tensorflow`: NOT EASY TO USE, subsequently `keras`, `tfestimators`, `ruta` ... user needs to understand the language However, advanced users might be able to highly specify a neural network to their needs (customization?)

By algorithm: `neuralnet` `rprop+` `neuralnet` `rprop-` `neuralnet` `slr` - once ranked well with 100000 iterations `AMORE` `BATCHgd` `CaDENCE` `pso` `psoptim` - need to reconfigure? `elmNNRcpp` - fast, no iterations `RSNNS` `Quickprop` (?) `AMORE` `BATCHgdwm` `tensorflow` `MomentumOptimizer` `tensorflow` `AdamOptimizer` `ELMR` - fast, no iterations `tensorflow` `GradientDescentOptimizer` `keras` `rmsprop` `keras` `adagrad` `keras` `sgd` `keras` `adadelta` `tensorflow` `AdagradOptimizer` `keras` `adam` `tensorflow` `FtrlOptimizer` `neuralnetwork` `sag` `tensorflow` `AdadeltaOptimizer` `neuralnet` `backprop` - note, might not actually reflect standings, somehow from template to template the learning rate disappeared. Will fix this in future runs

D. Untested => TO DO - LIST

## Conclusion and perspective

??JN: Can we start to put in some major findings? i.e., important positive findings, big negatives?

### Positives (no particular order)

1. the existence of algorithms that converge well
2. `nnet`, which uses `optim`'s BFGS, is already often chosen to represent neural networks for packages that are either a collection of independent machine learning algorithms, ensembles, or even applications in a field such as ...
3. the wide variety of neural networks available to users of R, from libraries of other programming languages to many different types of algorithms, hyperparameters, and uses

### Negatives (no particular order)

1. bad documentation
2. the lack of packages that expand the number of unique second order algorithms. (Perhaps even the existence of what can be considered as repetitive packages?)
3. the lack of clear default values, or bad default values

### Future work

As the field of neural networks continue to grow, there will always be more algorithms to validate. For current algorithms in R, our research should be extended to encompass more types of neural networks and their data formats (classifier neural networks, recurrent

neural networks, and so on). Different rating schemes and different parameters for package functions can also be tried out.

- The dreamed NN package: Recommendation to package authors
- Conclusion

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## Appendix

### Appendix A

Consider a set of observations  $y_i$  and its corresponding predictions  $\hat{y}_i$  for  $i = 1, \dots, n$ . The three metrics used were:

$$MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|, RMSE = \frac{1}{n} \sqrt{\sum_{i=1}^n (y_i - \hat{y}_i)^2}, WAE = \frac{1}{n} \max_{i=1, \dots, n} |y_i - \hat{y}_i|.$$

These values represent the absolute, the squared and the maximum norm of residual vectors.

### Appendix B

We define three smooth functions for Simon Wood's test dataset

$$f_0 = 5 * \sin(2\pi x), f_1 = \exp(3 * x) - 7, f_2 = 0.5x^{11} * (10(1 - x))^6 - 10(10 * x)^3 * (1 - x)^{10},$$

$$f_3 = 15 \exp(-5|x - 1/2|) - 6, f_4 = 2 - 1_{(x <= 1/3)}(6 * x)^3 - 1_{(x >= 2/3)}(6 - 6 * x)^3 - 1_{(2/3 > x > 1/3)}(8 + 2 \sin(9 * (x - 1/2)))$$

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