Probability.

1) A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles, If two marbles are drawn at random, what is the probability that at least one is green?

Total = 15 marbles

$$15C_2 = \frac{15!}{2!(15-2)!} = \frac{15\times14\times13\times12\times1}{2\times1\times13!}$$

$$= \frac{15\times14}{2\times1} = 105$$

at least one green = 1 − prob of no green

Non green = 3+4+2 = 9

$$9C_2 = \frac{9\times8}{2\times1} = 36$$

$$= \frac{36}{105} = \frac{12}{35}$$

$$1 - \frac{12}{35} = \frac{35-12}{35} = \frac{23}{35}$$

2) A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If two marbles are picked at random, what is the prob that they are either blue or yellow

Total marbles = 15

$$15C_2 = 105$$

blue or yellow = 3+2 = 5

$$5C_2 = \frac{5\times4}{2\times1} = 10 \qquad \frac{10}{105} = \frac{2}{21}$$

3) A box contains 3 blue marbles

Total number of marbles = 15

4 marbles = $15C_4 = \frac{15 \times 14 \times 13 \times 12}{4! \times 1}$

$= 15 \times 7 \times 13$

$= 1365$

No of non-blue = 4+6+2 = 12

$12C_4 = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2 \times 1} = 11 \times 5 \times 9 = 495$

$= \frac{495 \div 5}{1365 \div 5} = \frac{99}{273} = \frac{33}{91}$

4) 10 books are placed at random in a shelf. The probability that a pair of books will always be together is?

Total books = 10

Number of ways to arrange 9 units = 9!

the two books within the pair can be arranged in 2! ways

$= 9! \times 2!$

Total no. of ways to arrange 10 books = 10!

$\frac{9! \times 2!}{10!} = \frac{9! \times 2!}{10 \times 9!} = \frac{2}{10} = \frac{1}{5}$

5) What is the probability that a leap year has 53 sundays and 52 mondays

A leap yr has 366 days

366 days = 52 weeks and 2 extra days

2 extra days can be some pairs

(S, M) (M, T) (T, W) (W, T) (T, F) (F, S) (S, S)

There are 7 possible combination for 2 extra (Sunday, Monday) out of 7 possible outcomes

$= 1/7$

6. Out of 20 consecutive integers, two are chosen at random. The probability that their sum is odd is?

Two consecutive integers are $n, n+1$

Sum is $n+(n+1) = 2n+1$

$2n$ is even, $2n+1$ is odd

$\therefore$ sum of two integers is always odd

The probability is 1

7) A box contains 3 blue marbles, 4 red, 6 green marbles and 2 yellow marbles. If three marbles are drawn what is the probability the one is yellow and two are red

Total marbles = 15 $\quad {}^{15}C_3 = 455$

red marbles = 4

1 yellow marble from 2 $= {}^2C_1 = 2$

2 red marbles from 4 $= {}^4C_2 = \frac{4\times3}{2\times1} = 6$

1 yellow & 2 red $= {}^2C_1 \times {}^4C_2$

$= 2 \times 6 = 12$

$$\frac{{}^2C_1 \times {}^4C_2}{{}^{15}C_3} = \frac{12}{455}$$

8) Out of 10 persons working on a project, 4 are graduates, if 3 are selected, what is the prob that there are at least one graduate among them

Total persons = 10 $\quad {}^6C_3 = \frac{6\times5\times4}{3\times2\times1} = 20$

No. of graduates = 4 $\quad \frac{20}{120} = \frac{1}{6}$

non-graduates = $10-4 = 6$

${}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 10\times3\times4 = 120$

Probability of least one $= 1 - \frac{1}{6} = \frac{5}{6}$ //

9. In a party there are 5 couples. Out of them 5 people are chosen at random. find the probability that there are at the least two couples?

Total no of couples = 5

No of people chosen = 5

$$^{10}C_5 = \frac{10\times9\times8\times7\times6}{5\times4\times3\times2\times1} = 2\times3\times2\times7\times3 = 252$$

2 couples $^5C_2 = 10$ ways

10. The probability of a lottery ticket being a prized ticket is 0.2. When 4 tickets are purchased, the prob of winning a prize on atleast 1 ticket

$p = 0.2$

ticket not being a prized $1-p = 1-0.2$

$= 0.8$

∴ prob of no prize on 4 tickets

$= 0.8 \times 0.8 \times 0.8 \times 0.8 = (0.8)^4$

$= 0.4096$

prob of winning at least one ticket =

$1 - 0.4096$

$= 0.5904$

11. There are two boxes, one containing 39 red balls & the other containing 26 green balls, you are allowed to move the balls b/w the boxes so that when you choose a box random & a ball at random from the chosen box, the prob.

Box 1 : 39 red balls

Box 2 : 26 green balls

$Box1 = \frac{1}{2}$

$Box2 = \frac{1}{2}$

Total probability of red ball = P(Box 1) × P(Red / Box 1) + P(Box 2) × P(Red / Box 2)

$= \frac{1}{2} \times \frac{R_1}{N_1} + \frac{1}{2} \times \frac{R_2}{N_2}$

$\frac{R_1}{N_1}$ and $\frac{R_2}{N_2}$

We have 39 red balls & 26 green balls placing one red ball in Box 1, so $R_1 = 1, N_1 = 1$

$(39 - 1) = 38$.

So Box 2 = $R_2 = 38$, $N_2 = 38 + 26 = 64$

$P(\text{red}) = \frac{1}{2} \times \frac{1}{1} + \frac{1}{2} \times \frac{38}{64} = \frac{51}{64}$

12. Total no. of balls = 6 + 8 + 7 = 21 balls

Prob of red ball = $P = \frac{6}{21} = \frac{2}{7}$

Not a red ball = $q = 1 - \frac{2}{7} = \frac{5}{7}$

No. of trials = $n = 5$

binomial prob = $P(X = k) = {}^nC_k p^k q^{n-k}$

if $P(X = 3) = {}^5C_3 = 10$ $\quad 10 \times \left(\frac{2}{7}\right)^3 \times \left(\frac{5}{7}\right)^2 = \frac{2000}{16807}$

if $P(x = 4) = {}^5C_4 = 5$ $\quad = \frac{400}{16807}$

if $P(x = 5) = {}^5C_5 = 1$

$= \frac{32}{16807}$ $\quad = \frac{2432}{16807}$

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