INDIAN INSTITUTE OF TECHNOLOGY, DELHI

ASSIGNMENT DESIGN

Problem Statement

Write a MIPS Assembly Program for obtaining the area under a curve formed by joining successive points by a straight line.

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COL216 - Assignment 1

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1 Approach and Design

1.1 Input - Output

- Source Code: assignment.asm takes input from the keyword and prints the output on the console
- Tester Code: tester.asm takes input from the TestInput.txt and prints output on the console
- Use double precision as area calculations showed significant errors during single precision testing

1.2 Algorithm

- Initialize Counter to 1 and AreaTillNow to 0
- Read and Store User Input n = number of points
- first Input: Read User Input as (prevX, prevY) = (x_1, y_1)
- loop:
. - if Counter > n then Display Area
. - else increase Counter by 1 and Read User Input as (currX, currY) = (x_i, y_i)
. - calculate the area under the curve between prev and curr and add to AreaTillNow
. - (area calculation and derivation is attaached at the end)
. - update prevX and prevY to currX and currY. (prevX, prevY) <- (currX, currY)
. - jump back to loop
- Display Area prints the area on the console
- Loop Invariant : Before Iteration i. AreaTillNow = Area Enclosed by first i points. $1 \leq i \leq n$.

1.3 Design

1.3.1 Register

Integer Registers:

v0: used for making different syscalls, and also storing user inputs
a0: used in making syscalls (outputting strings and integers/double)
t0: Stores 'n', the total number of points
t1: Stores 'counter' ($1 \leq counter \leq n + 1$)

Floating Point Registers (Double Precision):

f0: used for storing user input when syscalls are made | also stores currY (y_2) while calculating area
f6: the constant HALF (0.5)
f4: value of area calculated till now
f8: stores prevX (x_1) while calculating area
f10: stores prevY (y_1) while calculating area
f12: stores currX (x_1) while calculating area | also used to make syscalls to display area calculated on console
f14: stores the product y_1 * y_2 while calculating area
f16: stores x_2 - x_1 while calculating area

f18: stores either $y_1 + y_2$ or just y_1^2, depending on location of points
f20: stores either | 0.5 * (x_2 - x_1) * ($y_1 + y_2$) | or just y_2^2 depending on location of points

(* used only when points are on opposite side of X-axis *)
f22: stores $y_1^2 + y_2^2$
f26: stores | 0.5 * (x_2 - x_1) * ($y_1^2 + y_2^2$) / (y_2 - y_1) | also used while comparing $y_1 * y_2$ with 0.0
f28: stores y_2 - y_1

1.3.2 Main Memory

ASCII:
msg_input: .asciiz "Enter the number of points (n): "
error_zero: .asciiz "Error: Number of points is zero. Area under the curve is not defined."
error_invalid: .asciiz "Error: Number of points cannot be negative."
error_notSorted: .asciiz "Error: Points not sorted w.r.t X-coordinate."
newline: .asciiz "\n"
msg_point: .asciiz "\nPoint "
msg_Xcod: .asciiz "Enter X coordinate: "
msg_Ycod: .asciiz "Enter Y coordinate: "
msg_separator: .asciiz "----------"
msg_total: .asciiz " \n Total area enclosed by the line plot and X-axis is: "

DOUBLE:
prevX: .double 0.0
prevY: .double 0.0
currX: .double 0.0
currY: .double 0.0

1.4 Raising Errors

- if n = 0 we raise Error: Number of points is zero. Area under the curve is not defined.
- if n < 0 we raise Error: Number of points cannot be negative.
- if input points are not sorted we raise Error: Points not sorted w.r.t X-coordinate.

2 Testing Strategy

- Total of **420** test cases were generated and tested against as a part our extensive testing strategy
- use TestCaseGenerator.py to generate randomized test case files with correct output. Testing via tester.asm
- tester code prints output on console. we copy it into a text file and run Checker.py to compare the outputs
- compare.py calculates the Difference upto 12 decimal places and stores in "Difference_12_decimal_places.txt"
- we store count of cases with 0 difference upto 12 decimals and total number of cases to calculate accuracy
- Types of Test Cases Used:
. - Test Cases with 10, 50, 1000 and 10000 randomized points with varying coordinate ranges
. covering all possible cases (positive Y, negative Y, mixed Y and stacked X)
. - Corner Cases for n ≤ 0 or unsorted input or single point input
. - Manually Generated Cases includes only positive Y, only negative Y, mixed positive-negative Y
. and input stacked wrt X (example - [(1,2), (1,3), (1,4), (5,-1)])

3 Result

We achieved a 100% accuracy across all our test cases, with 0 difference in all outputs upto 12 decimal places.

Console Interface -

Console

```
Enter the number of points (n): 5

Point 1
Enter X coordinate: 1
Enter Y coordinate: 1

Point 2
Enter X coordinate: 3
Enter Y coordinate: 4

Point 3
Enter X coordinate: 5
Enter Y coordinate: 3

Point 4
Enter X coordinate: 6
Enter Y coordinate: 7

Point 5
Enter X coordinate: 9
Enter Y coordinate: 5

Total area enclosed by the line plot and X-axis is: 35

_____________________________________________________________

Enter the number of points (n): 0
Error: Number of points is zero. Area under the curve is not defined.

_____________________________________________________________

Enter the number of points (n): -1
Error: Number of points cannot be negative.

_____________________________________________________________

Enter the number of points (n): 1

Point 1
Enter X coordinate: 2
Enter Y coordinate: 3

Total area enclosed by the line plot and X-axis is: 0

_____________________________________________________________

Enter the number of points (n): 3

Point 1
Enter X coordinate: 1
Enter Y coordinate: 2

Point 2
Enter X coordinate: 3
Enter Y coordinate: 4

Point 3
Enter X coordinate: 2
Enter Y coordinate: 5
Error: Points not sorted w.r.t X-coordinate.

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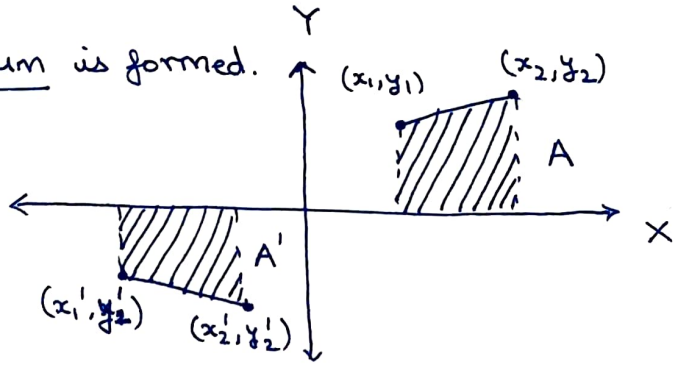
AREA FORMED BETWEEN A LINE SEGMENT (joining (x_1, y_1) and (x_2, y_2)) AND X-AXIS (Formula)

Case I: $y_1 y_2 \geq 0$

In this case, both ends of the line segment lie on the same side of the x-axis

Hence, a trapezium is formed.



From area of trapezium, I get

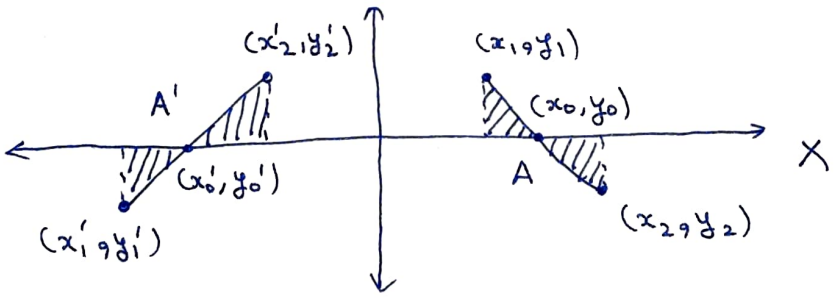
$$A = \frac{1}{2}(y_1 + y_2)(x_2 - x_1) \quad \text{and} \quad A' = \frac{1}{2}(-y_1' - y_2')(x_2 - x_1)$$

So, in general, area formed = $\underbrace{|0.5 * (y_1 + y_2)(x_2 - x_1)|}$

Formula used in algorithm (code)

Case II: $y_1 y_2 < 0$

In this case, ends of the line segment lie on opposite sides of the x-axis. Hence, two triangles are formed.



Finding (x_0, y_0):

(x_0, y_0) lies in the interior of the segment $\Rightarrow$

$$x_0 = \frac{\lambda x_2 + x_1}{\lambda + 1} \quad \text{and} \quad y_0 = \frac{\lambda y_2 + y_1}{\lambda + 1}, \quad \lambda > 0$$

and y_0 lies on X-axis $\Rightarrow y_0 = 0 \Rightarrow \boxed{\lambda = -y_1 / y_2}$

So, $x_0 = \dfrac{y_2 x_1 - y_1 x_2}{y_2 - y_1}$ and $y_0 = 0$

Now, area A = sum of two shaded triangles

$$= \frac{1}{2}(x_0 - x_1) \cdot y_1 + \frac{1}{2}(x_2 - x_0)(-y_2)$$

$$= \frac{1}{2} y_1 \frac{y_1 (x_1 - x_2)}{(y_2 - y_1)} + \frac{1}{2} \frac{(-y_2) y_2 (x_2 - x_1)}{(y_2 - y_1)}$$

$$= -\frac{1}{2} \frac{(y_1^2 + y_2^2)(x_1 - x_2)}{(y_1 - y_2)} = \frac{1}{2} \frac{(y_1^2 + y_2^2)(x_2 - x_1)}{y_1 - y_2}$$

Similarly, A' = sum of two shaded triangles

$$= \frac{1}{2} \frac{(y_1'^2 + y_2'^2)(x_2' - x_1')}{y_2' - y_1'}$$

In either case, area formed = $\left| \dfrac{0.5 * (y_1^2 + y_2^2)(x_2 - x_1)}{(y_2 - y_1)} \right|$

Formula used in algorithm (code)