

REASONING SYSTEMS

DAY 3

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DAY 3 AGENDA

3.1 Reasoning using Optimization Techniques – Evolutionary Computation and Genetic Algorithms

3.2 Evolutionary Computation and Genetic Algorithm Applications

3.3 Workshop – Genetic Algorithms

3.1

REASONING USING OPTIMIZATION TECHNIQUES – EVOLUTIONARY COMPUTATION AND GENETIC ALGORITHMS

- **Problem Solving AI**

Plan; Optimize; Search for solutions

- **Reasoning systems need conduct search and optimization to get optimal (sub-optimal) solutions.**
- **There are different types of optimization techniques available.**

Optimization Problems



Businesses make pricing decisions to maximize profits



Consumers make investment decisions to maximize return

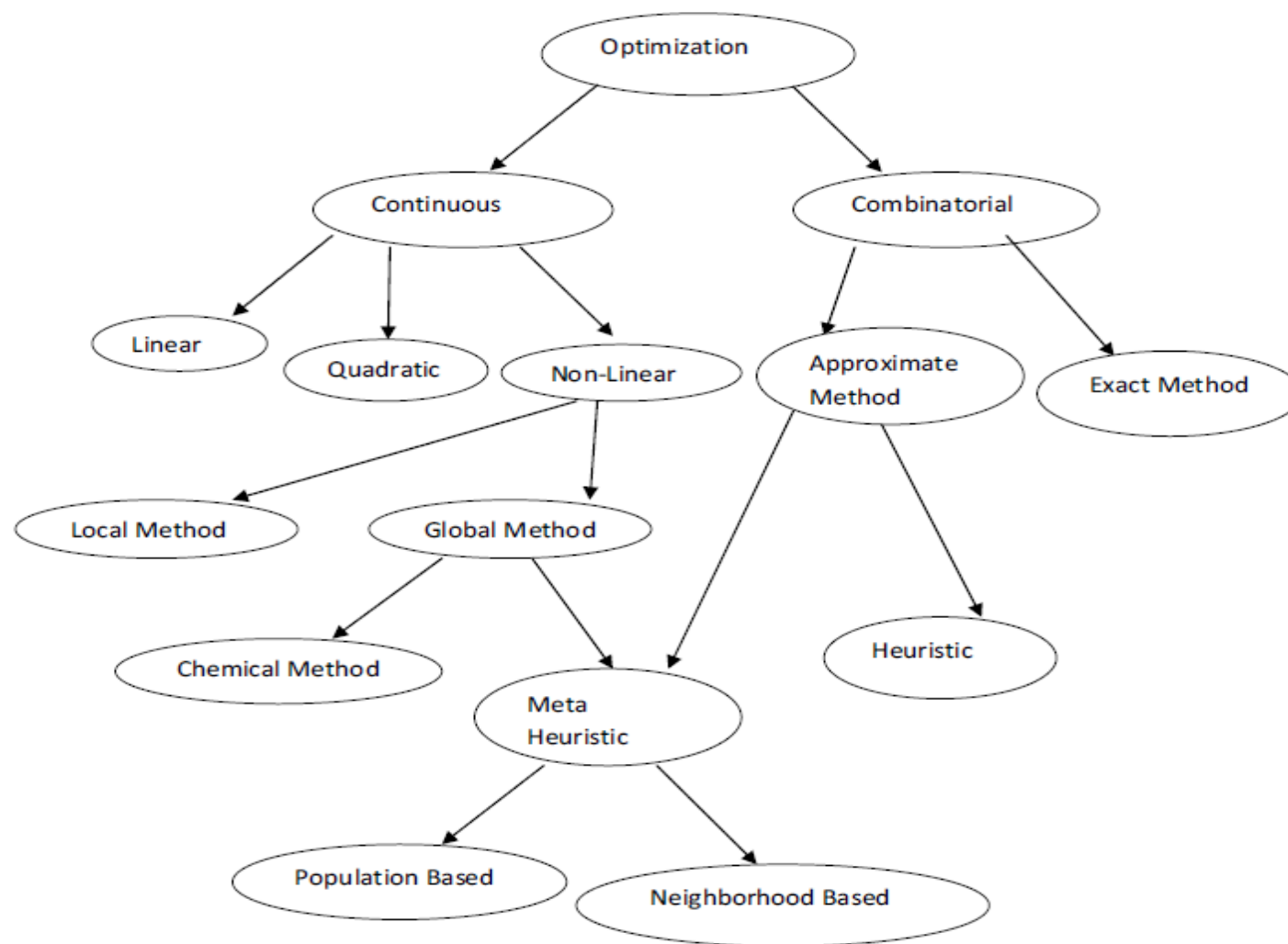


Figure credit: David Applegate, Robert Bixby, Vasek Chvatal and William Cook.

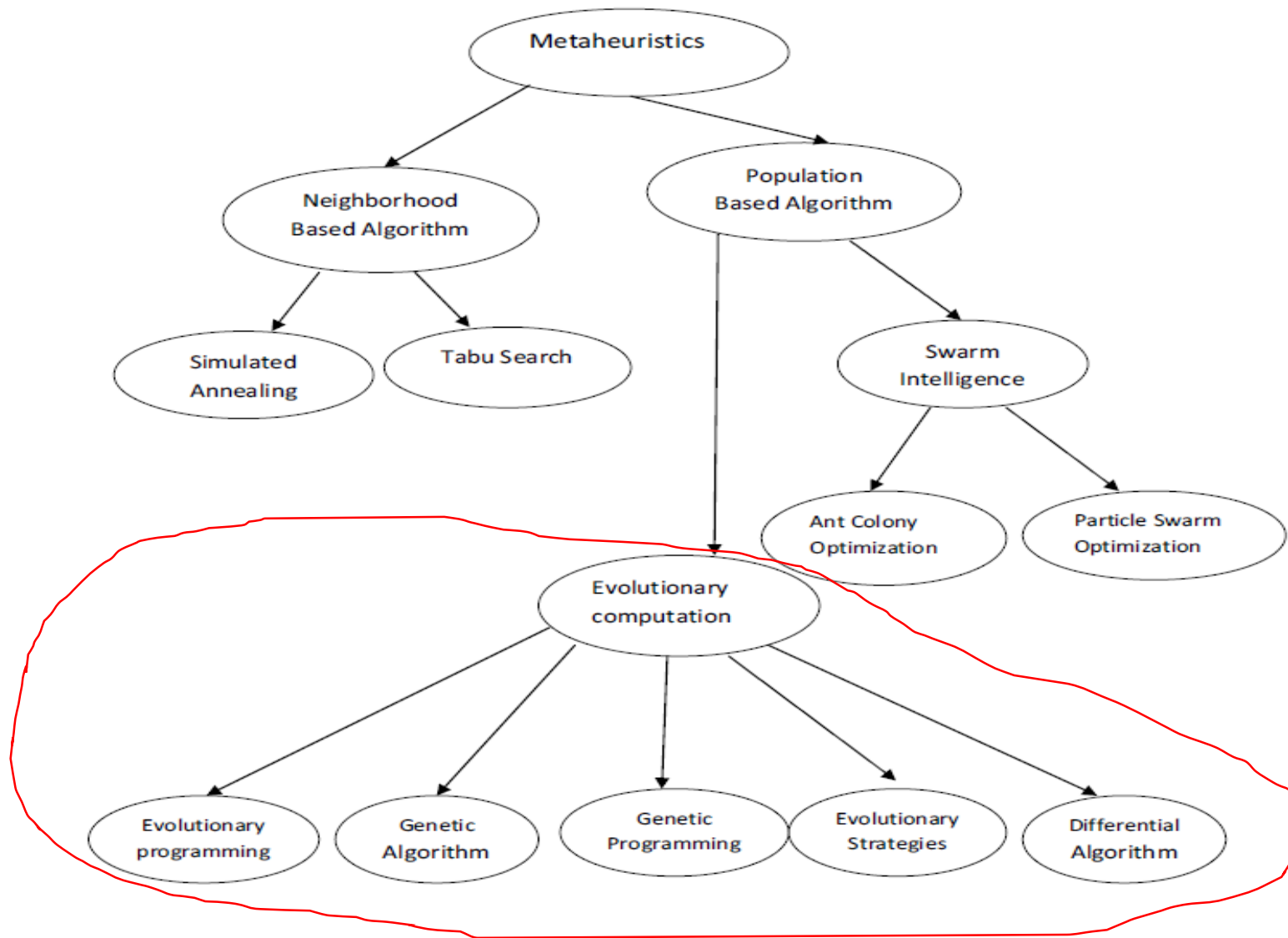


Travelling Salesman Problem (TSP) to minimize traveling distance

Optimization Techniques



Optimization Techniques

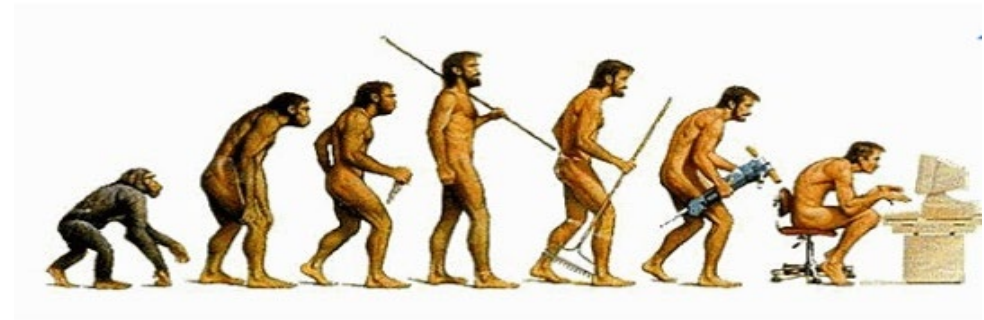


Introduction to Evolutionary Computation

“In computer science, **evolutionary computation** is a subfield of artificial intelligence (more particularly computational intelligence) that involves continuous optimization and combinatorial optimization problems. Its algorithms can be considered global optimization methods with a metaheuristic or stochastic optimization character and are mostly applied for black box problems (no derivatives known), often in the context of expensive optimization.”

---Wikipedia

- **Inspired by Darwinian natural evolution:**
 - Survival of the fittest
 - Selection on phenotype through environment
 - Genotypic inheritance
 - Reproduction
 - Blind variation



Evolutionary Computation Metaphor

EVOLUTION

Environment

Individual

Fitness



PROBLEM SOLVING

Problem

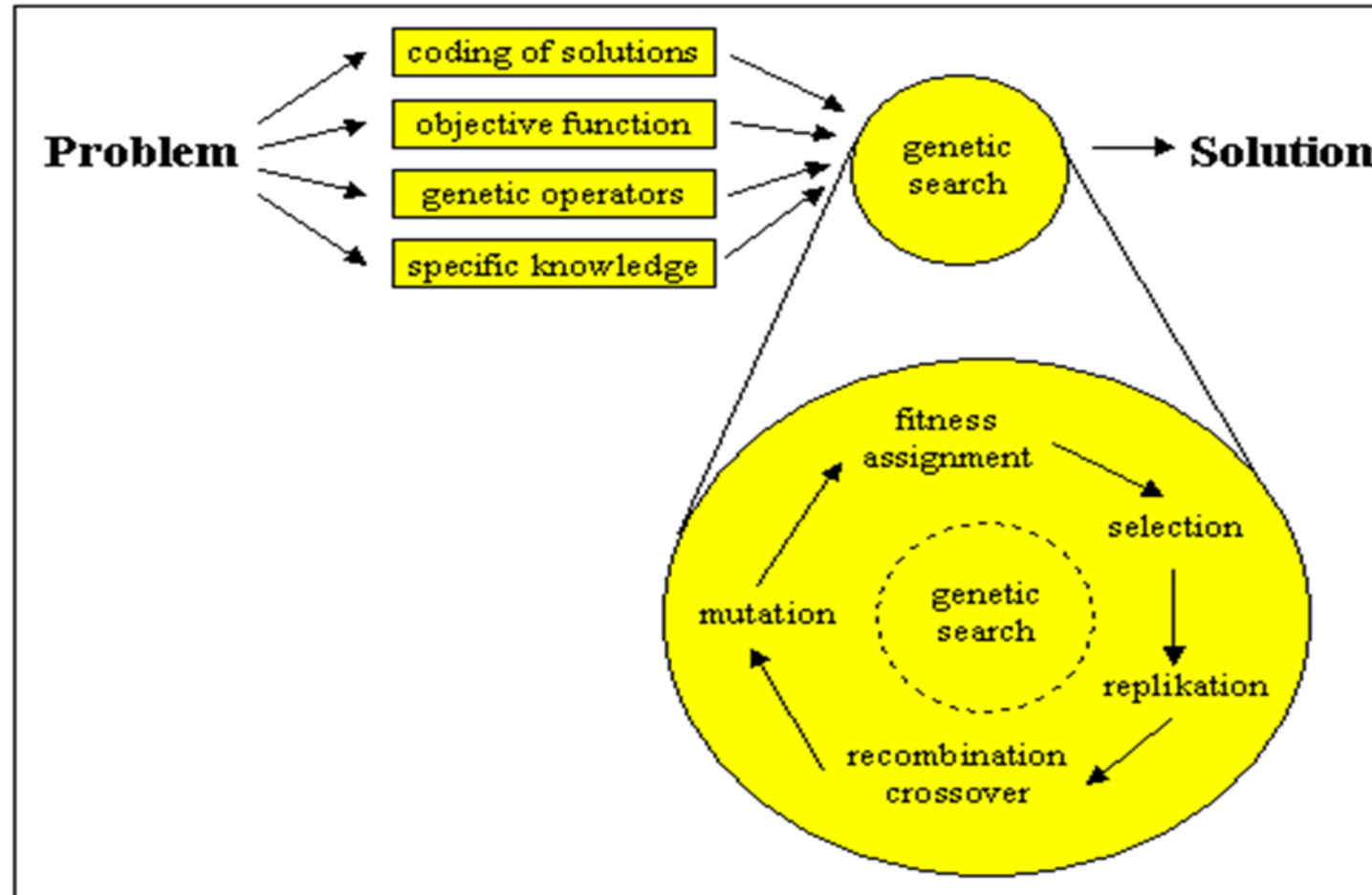
Candidate Solution

Quality

Evolutionary Algorithms History

- Evolutionary Programming
 - L. Fogel 1962 (San Diego, CA)
- Genetic Algorithms
 - J. Holland 1962 (Ann Arbor, MI)
- Evolution Strategies
 - I. Rechenberg & H.-P. Schwefel 1965 (Berlin, Germany)
- Genetic Programming
 - J. Koza 1989 (Palo Alto, CA)

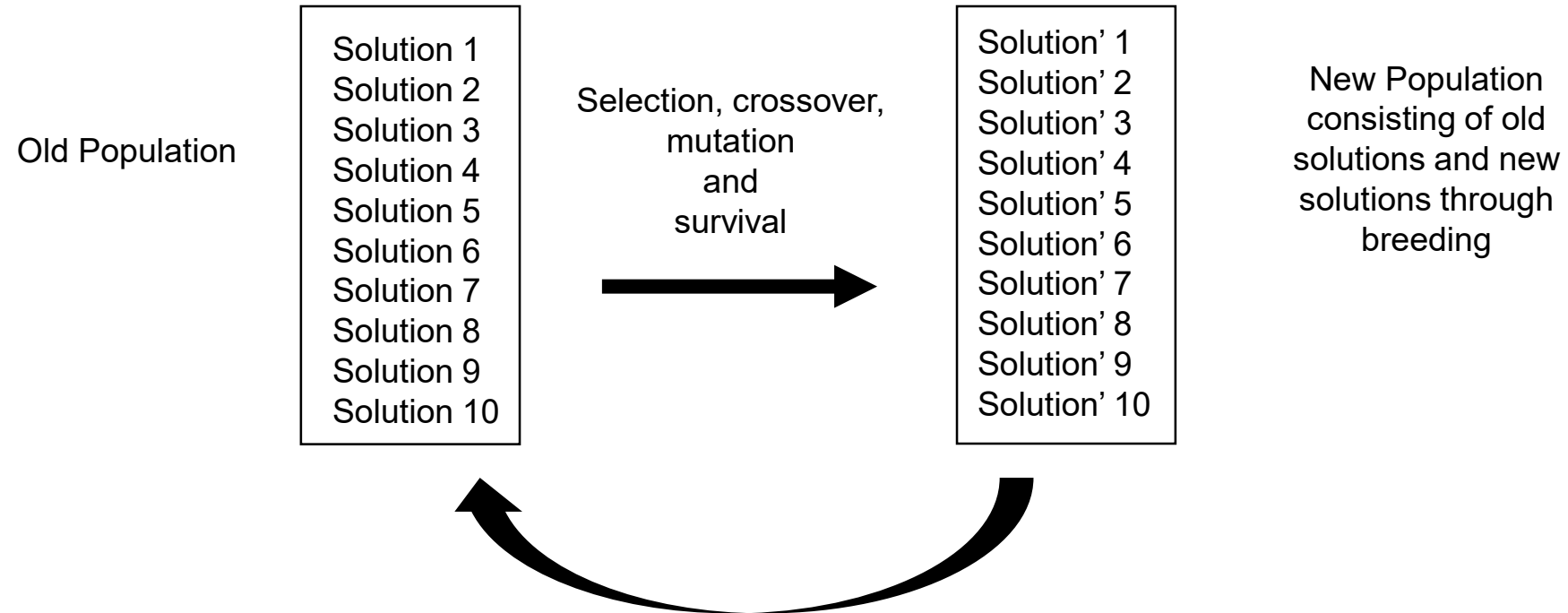
Problem Solution Using Evolutionary Algorithms



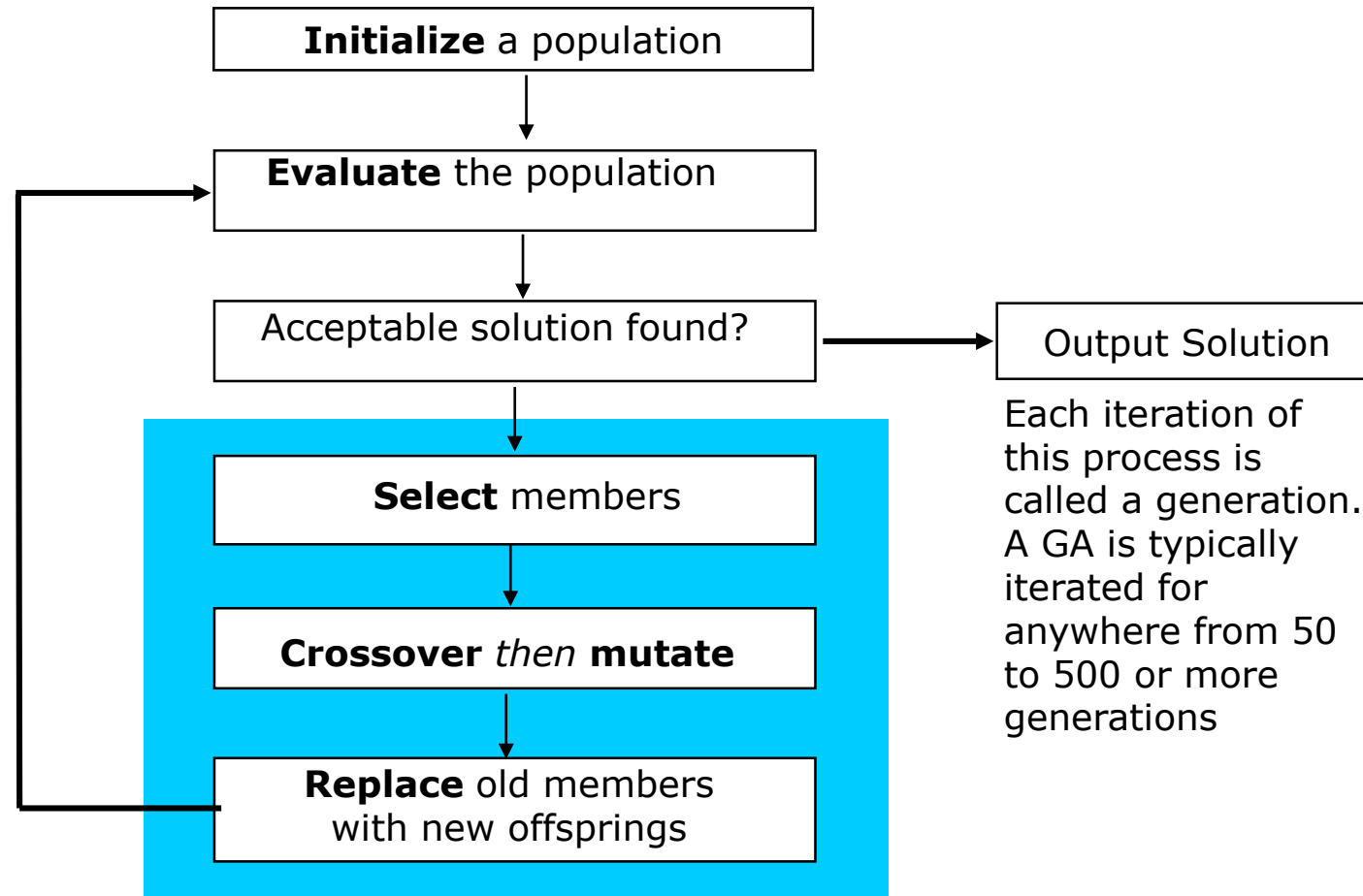
Genetic Algorithms – As typical EC algorithms

- Genetic algorithms are general-purpose search and optimization algorithms that use principles inspired by natural population genetics to evolve solutions to problems
- GAs operate on a population of individuals representing potential solutions to a given problem.
- GAs seek to produce better (fitter) individuals (solutions) by combining the better of the existing ones (through breeding - crossover and mutation).

Genetic Algorithms



GA Search Process



GA Pseudocode

Generate the initial population $P(0)$;

$t=0$;

repeat

Evaluate the fitness of each individual in $P(t)$;

Select parents from $P(t)$ based on their fitness;

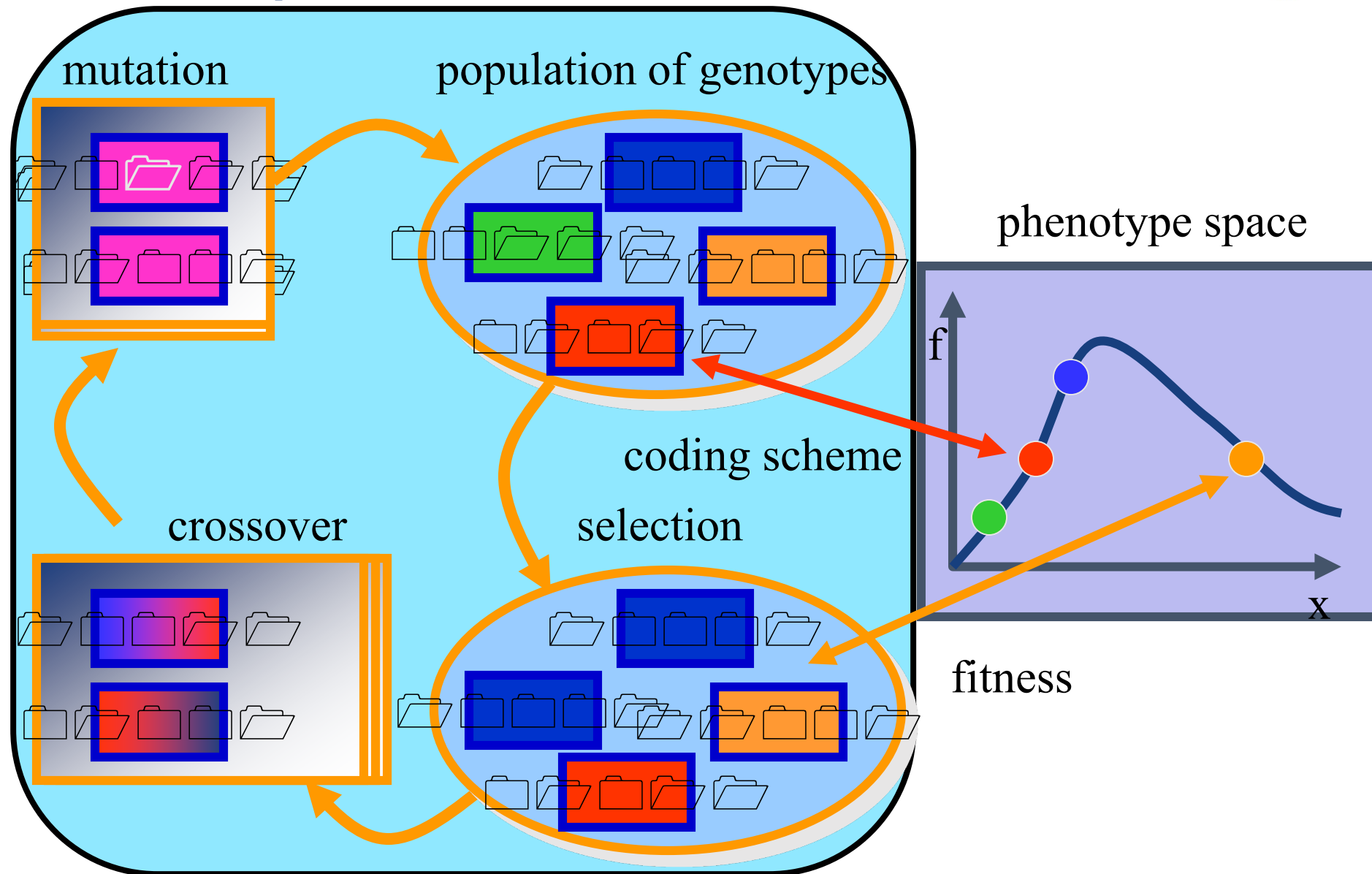
Applying **crossover** and **mutation** to parents to create $O(t)$;

Obtain population $P(t + 1)$ by combining $P(t)$ and $O(t)$;

$t = t + 1$;

until termination criterion satisfied;

Evolution as Optimisation -GA search process



Representing Solutions in GA

- **GAs encode a problem's solution in a form that is analogous to nature's chromosomes or sequences of DNA.**
 - Solutions are simply strings of values
 - Traditional primitive type is a single bit.
 - Integers, floating points or even higher level entities can be used.
- **GAs are inherently independent of the application domain.**

Encoded forms of problem solutions

010100100001011110000110010001110001

5 6 1 9 8 3 7 4 2 10

14.5 79.0 22.8 9.3

Core GA Concepts

- **Initial population**

- Can be initialized using whatever knowledge is available about the possible solutions
- Should represent a random sample of the search space
- Each member of the population is evaluated and assigned a measure of its fitness as a solution



- **New population**

- Structures in the current population are selected for replication based on their relative fitness
- Genetic operators (crossover, mutation) are performed to generate the new population

Core GA Concepts

- **Selection**

- Reproduction focuses attention on **high fitness individuals**, thus exploiting on the available fitness information.
- High-performing structures might be chosen several times for replication, while poor-performing structures might not be chosen at all.

- **Crossover**

- Combines the features of two parent structures (new combinations of genes are generated from previous ones) to form two similar offspring → **conserves** genetic information.

- **Mutation**

- Alters one or more components of a selected structure randomly (a direct analogy from nature and provides the way to introduce new information into the population) → population **diversity**

Core GA Concepts

- **The resulting offspring are then evaluated and inserted back into the population, replacing older members.**
 - Specific decisions about how many members are replaced during each iteration, and how members are selected for replacement, define a range of alternative implementations
 - Generational GA
 - GAs that replace the entire population
 - Steady-state GA
 - GAs that replace only a small fraction of chromosomes. Typically new chromosomes replace the worst chromosomes



Selection and Reproduction

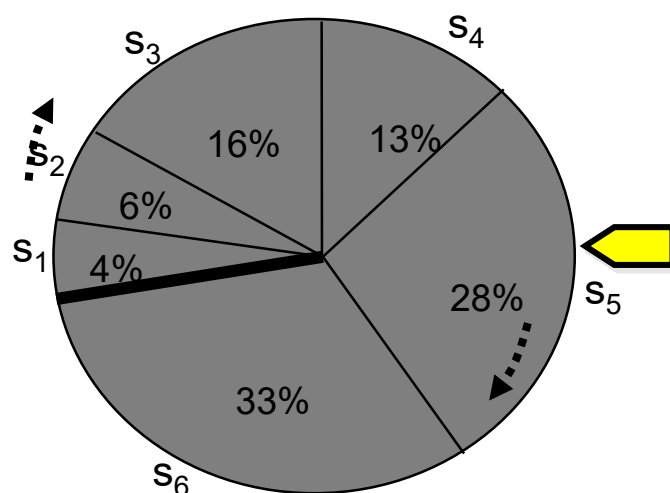
- **Selection and reproduction in GA always in the given sequence**
 1. **Select** fitter solutions (or individuals) to contribute genetic material to the next generation
→ selection method
 2. **Crossover** or recombine genetic materials from two parents to create new offsprings
→ crossover operator and crossover rate
 3. **Mutate** randomly selected components of new offsprings
→ mutation operator and mutation rate
 4. **Replace** some existing solutions in the current generation with the new offsprings to form the next generation
→ replacement strategy

- **Fitness-Proportionate Selection with “Roulette Wheel” aka Wheel of Fortune**

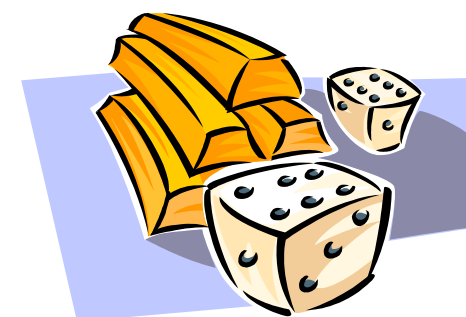
- Select new population with respect to the probability distribution based on fitness values
- A roulette wheel with slots sized according to the fitness is used
 - Calculate the fitness value $f(s_i)$ for each chromosome s_i
 - Find the total fitness of the population
 - $F = \sum_i f(s_i)$
 - Calculate the probability of selection p_i for each chromosome s_i
 - $p_i = f(s_i)/F$
 - Calculate a cumulative probability q_i for each chromosome s_i
 - $q_i = \sum_j p_j$

Selection Methods - Roulette Wheel

- Spin the roulette wheel, each time a single chromosome for a new population is selected in the following way
 - Generate a random (float) number r from the range $[0..1]$
 - If $r < q_1$ then select the first chromosome; otherwise select the **i -th chromosome s_i** such that $q_{i-1} < r < q_i$



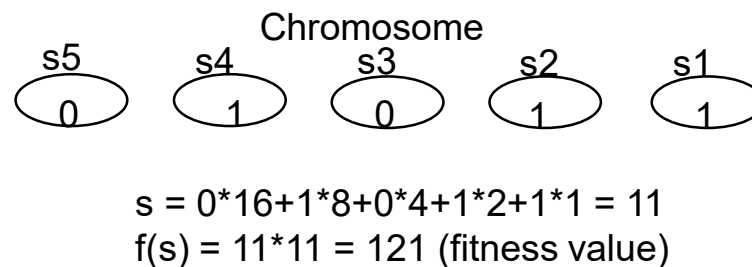
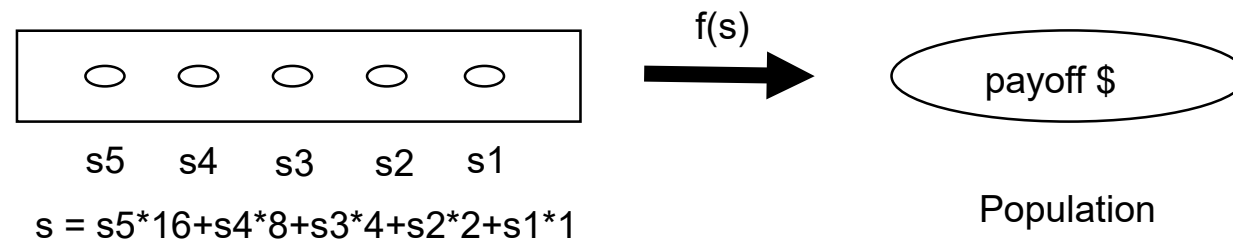
A “roulette-wheel sampling” giving each individual a slice of a circular roulette wheel equal in area to the individual’s fitness



Example: Problem Encoding

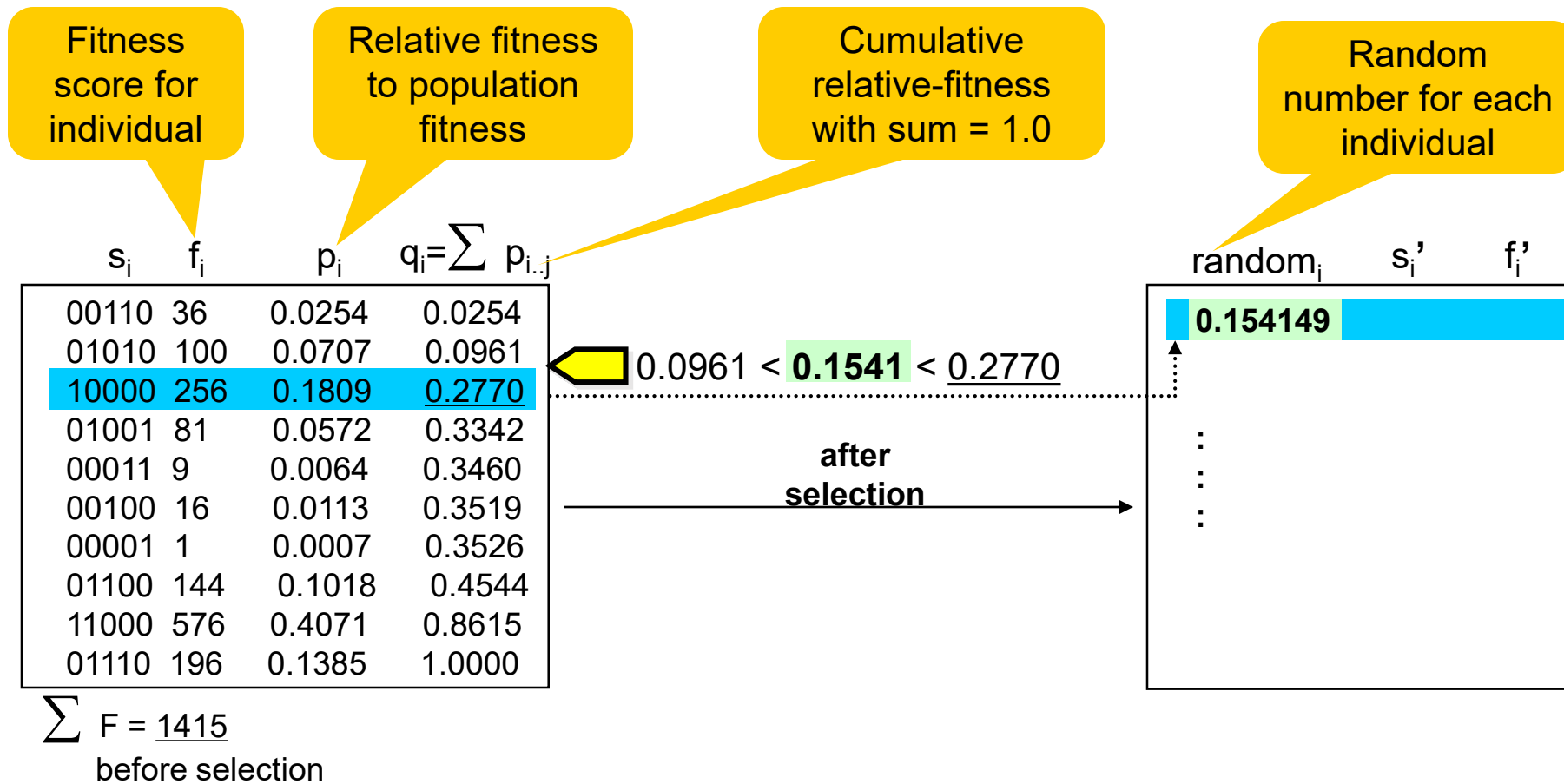
- Consider a black box switching problem which concerns a black box device with a bank of five input switches. For every setting of the five switches, there is an output signal f ; mathematically $f = f(s)$ where s is a particular setting of the five switches. The objective of the problem is to set the switches to obtain the maximum possible f value.

$$f(s) = s^2 \text{ on the integer interval } [0,31]$$



00110	36
01010	100
10000	256
01001	81
00011	9
00100	16
00001	1
01100	144
11000	576
01110	196

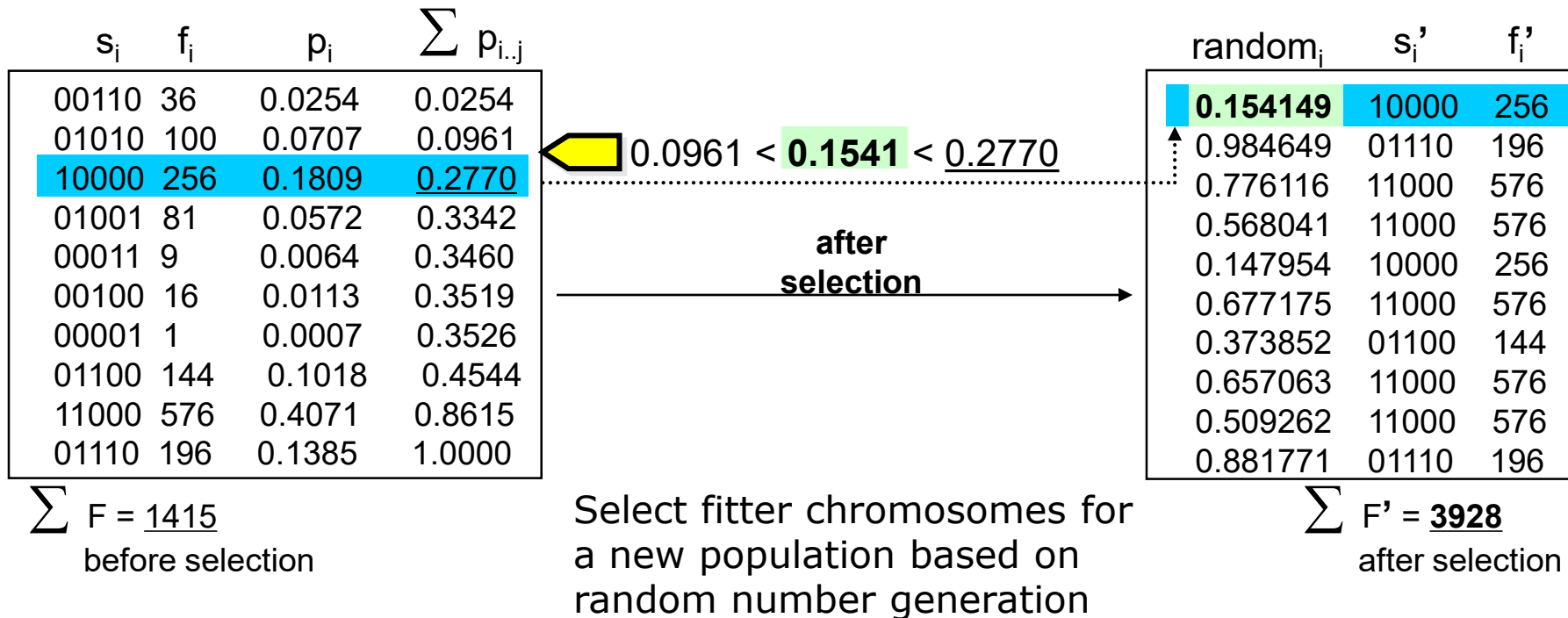
Example: Selection (Roulette Wheel)



Example: Selection (Roulette Wheel)

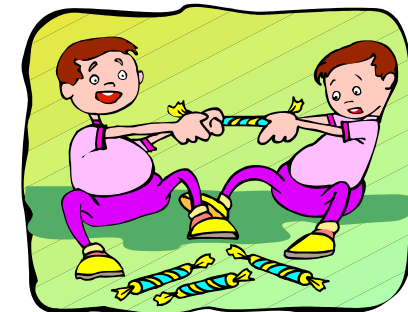
Generation 0

Probability distribution calculation based on fitness values



This represents an improvement of +2513

- **Tournament selection method**
 - Choosing individuals for reproduction randomly.
 - **Binary tournament**
 - Two individuals are chosen at random and the better of the two individuals is selected.
 - **Larger or N-tournament**
 - N individuals are chosen at random from the population, with the best being selected for reproduction.



- **Rank selection method**

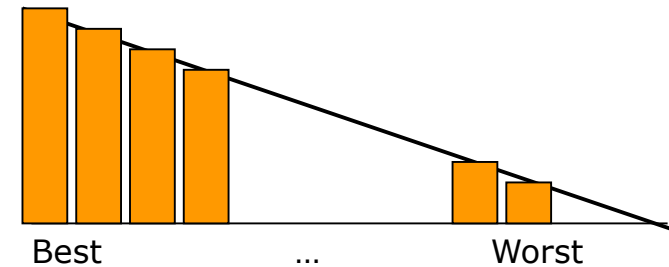
- The population is ordered according to the measured fitness values. A new fitness value is then ascribed, inversely related to their rank.

- **Linear ranking**

- Each chromosome in the population is ranked in increasing order of fitness, from 1 to N, and assigned a new subjective fitness using a linear ranking function.

- **Exponential ranking**

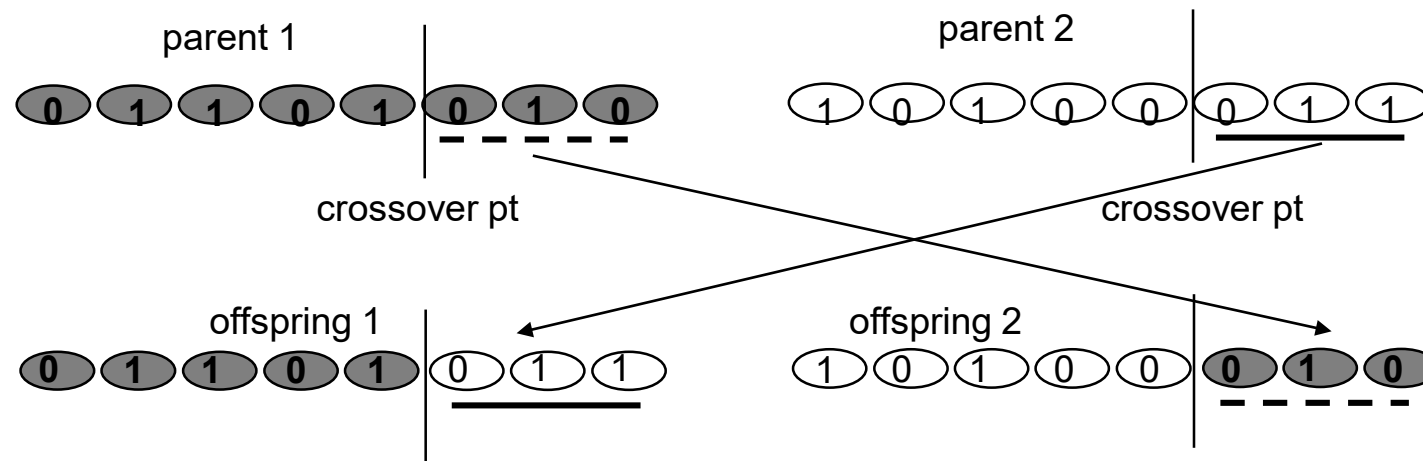
- First chromosome assigned fitness of 1;
the second fitness of s (typically 0.99);
the third fitness of s^2 and so on.



Crossover Operators

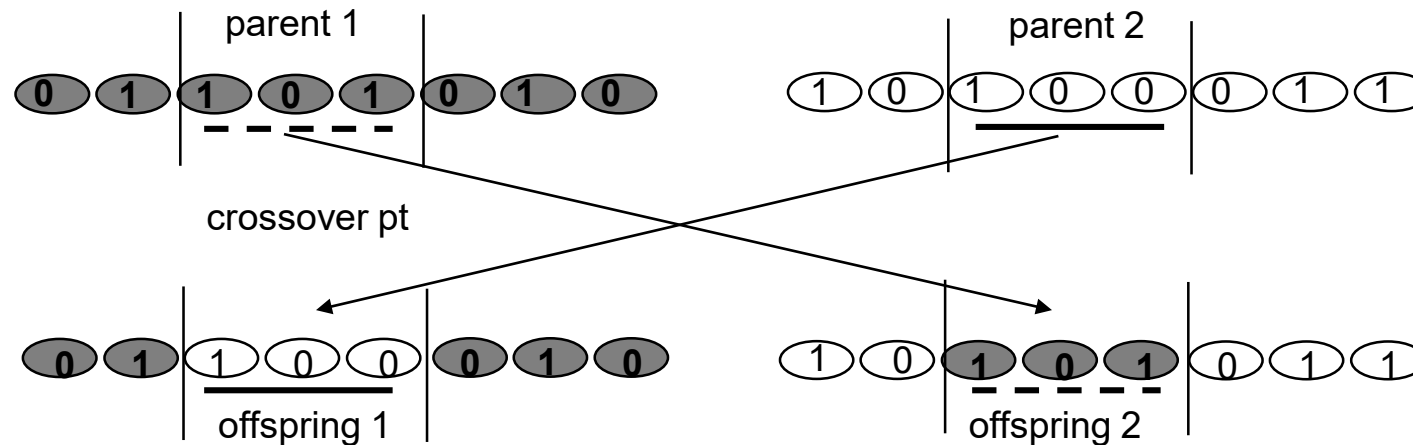


- **Segments are cut-and-spliced between the chromosomes (strings)**
 - Segments of two parent chromosomes are swapped producing two offsprings
 - Crossover site chosen randomly
- **Single-point crossover**



Crossover Operators

- Multi-point crossover (k-point crossover)

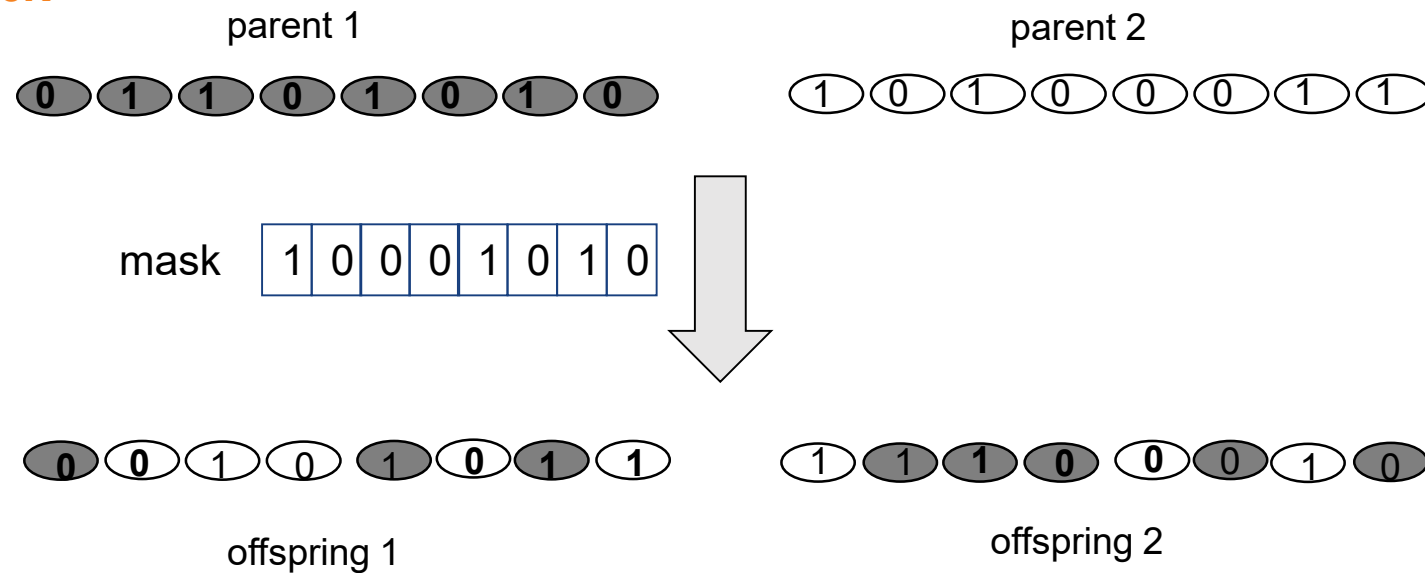


- Traditionally, the number of crossover points has been fixed at a very low constant value of 2.

Crossover Operators

- **Uniform crossover**

- Genes are copied from the first parent or from the second parent based on a randomly created mask

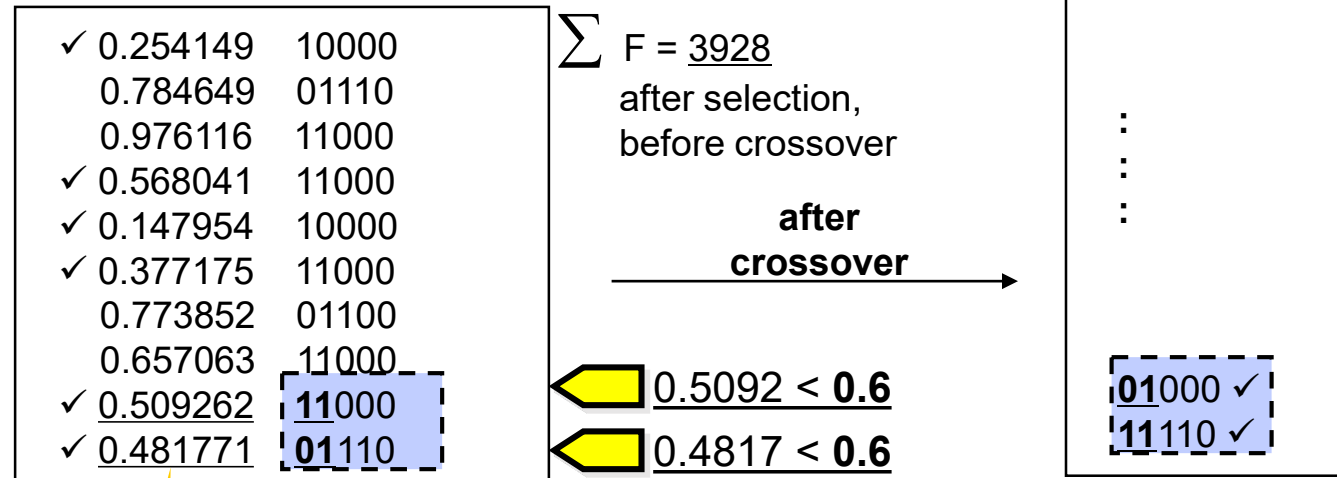


- **Bear in mind that there are other kinds of crossover operators besides point-based ones**

- The probability of crossover (crossover rate) p_c gives the expected number $p_c \times$ population size of chromosomes to undergo the crossover operation.
 - Generate a random number r from the range $[0..1]$
 - If $r < p_c$, select given chromosome for crossover.
 - For each pair of coupled chromosomes we generate a random integer number (the position of the crossover point) from the range $[1..m-1]$ where m is the number of bits in a chromosome
 - The parent chromosomes are replaced by the offspring chromosomes.

Example: Crossover Operators

crossover rate= 0.6

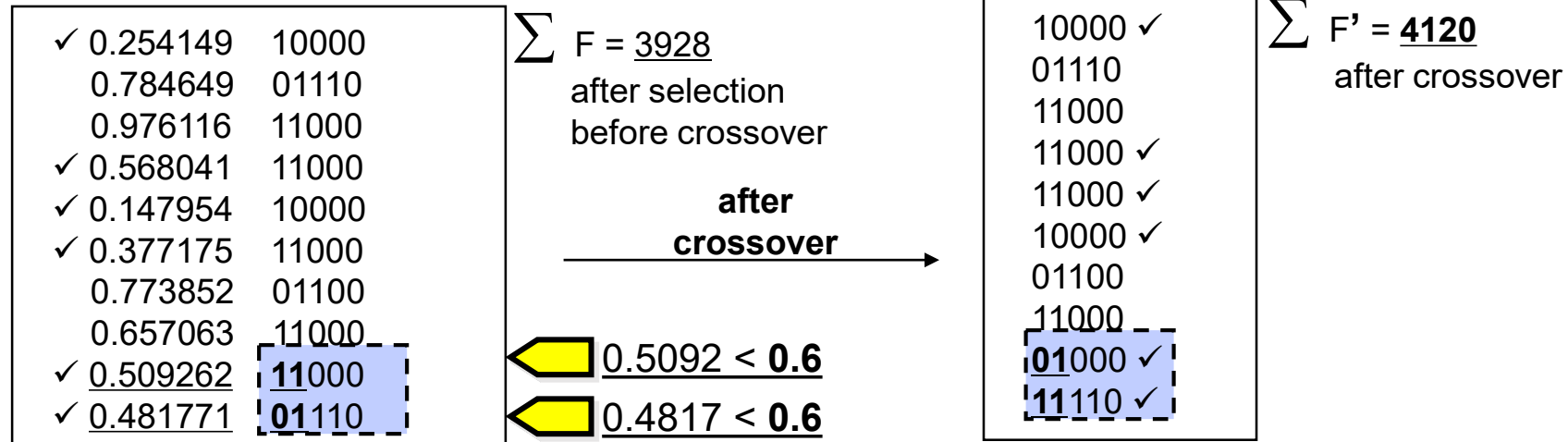


Random
number for each
individual

Individual selected if
random number < crossover rate

Example: Crossover Operators

crossover rate= 0.6



Parent	Offspring
1 0000	<u>1</u> 0000
1 1000	<u>1</u> 1000
10 000	11 000
11 000	<u>10</u> 000
11 000	01 000
01 110	11 110

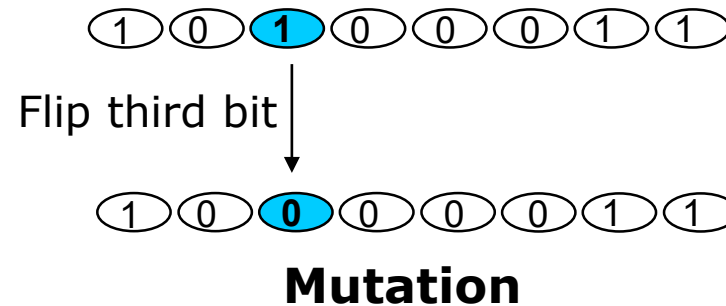
Crossover point
randomly selected

New population
with parent chromosomes
replaced by offspring
chromosomes

This represents a further
improvement of +192

Mutation Operators

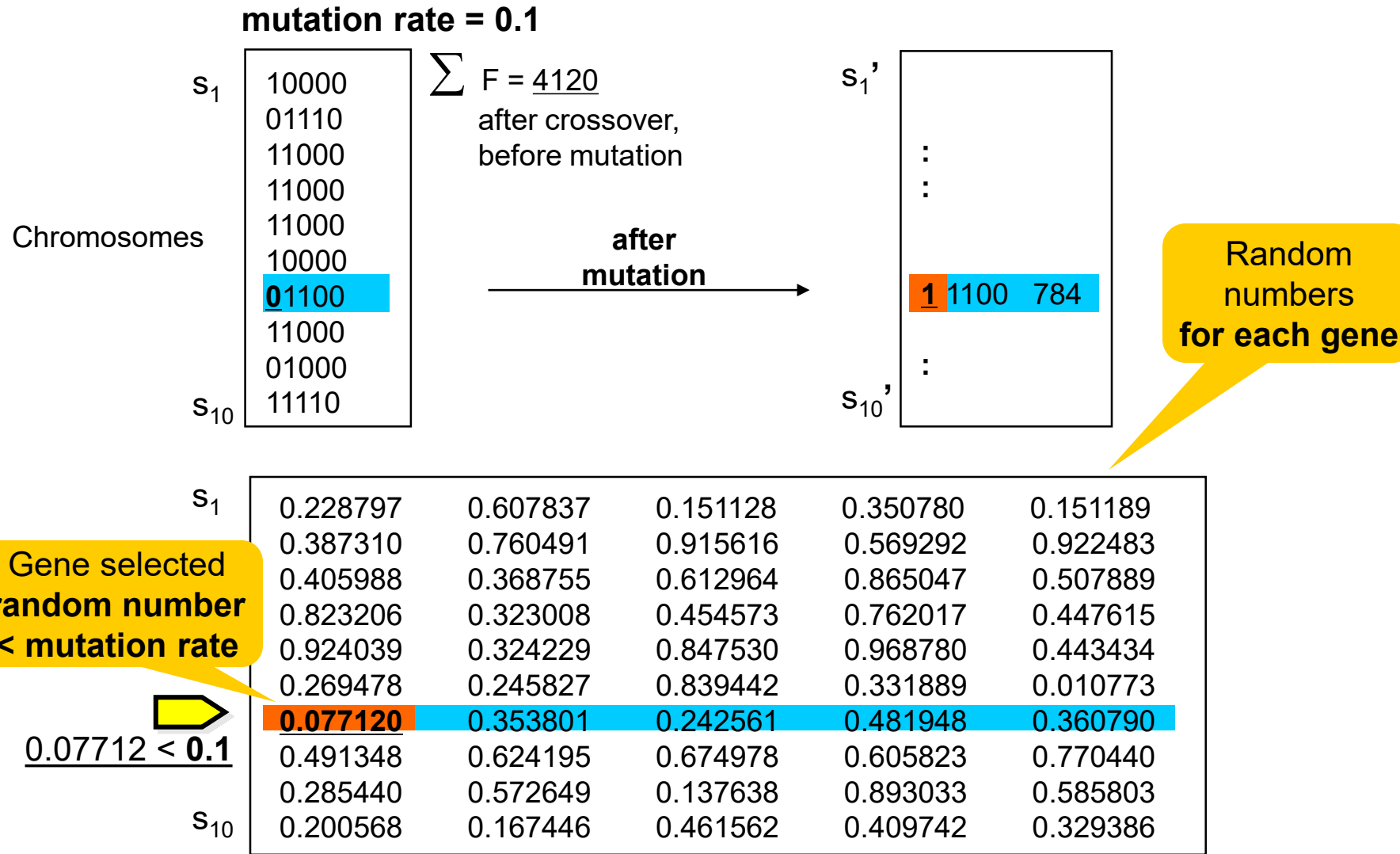
- **Replace the value at some randomly-chosen position by a new arbitrary value**
 - The role of mutation is to maintain genetic diversity
 - A range of mutation operators have been proposed, ranging from completely random alterations to more heuristically motivated local search operators



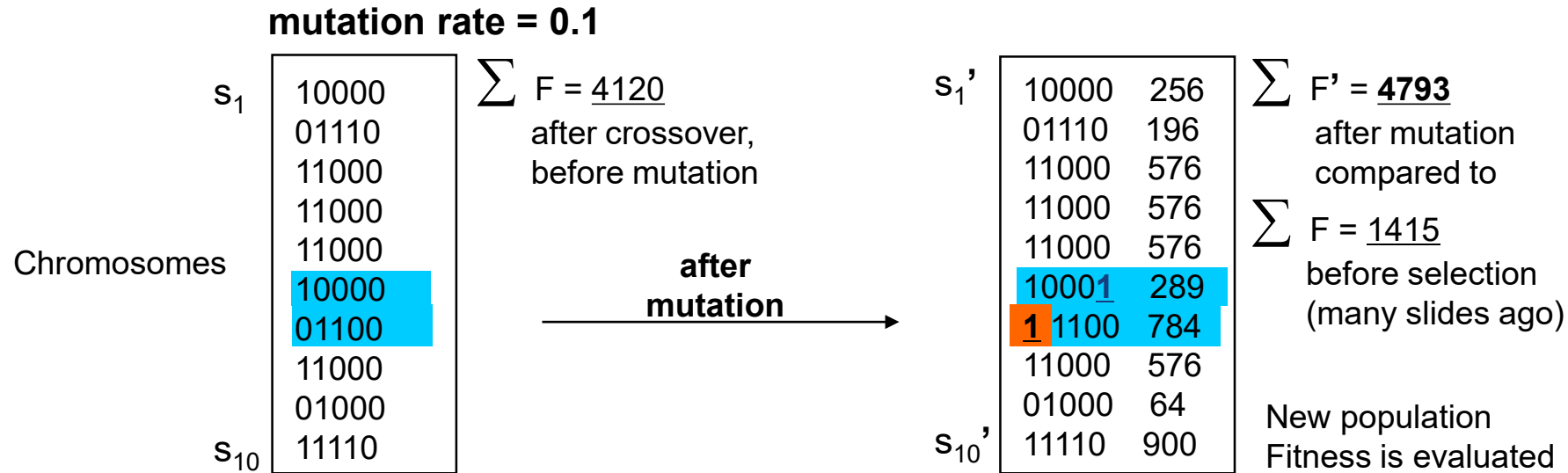
- Bear in mind that there are **other kinds of mutation operators** besides bit-flipping

- **Applying mutation operator to the individuals in the new population**
 - The probability of mutation (mutation rate) p_m gives the expected number of mutated bits $p_m \times \text{population size} \times \text{number of bits in a chromosome}$
 - Every bit (in all chromosomes in the whole population) has an equal chance to undergo mutation
 - Generate a random number r from the range $[0,1]$
 - If $r < p_m \rightarrow$ mutate the bit

Example: Mutation Operators



Example: Mutation Operators



s_1	0.228797	0.607837	0.151128	0.350780	0.151189
	0.387310	0.760491	0.915616	0.569292	0.922483
	0.405988	0.368755	0.612964	0.865047	0.507889
	0.823206	0.323008	0.454573	0.762017	0.447615
	0.924039	0.324229	0.847530	0.968780	0.443434
	0.269478	0.245827	0.839442	0.331889	0.010773
	0.077120	0.353801	0.242561	0.481948	0.360790
	0.491348	0.624195	0.674978	0.605823	0.770440
	0.285440	0.572649	0.137638	0.893033	0.585803
s_{10}	0.200568	0.167446	0.461562	0.409742	0.329386

0.07712 < 0.1

Gene selected
random number
< mutation rate

0.010773 < 0.1

This represents a further improvement of +673

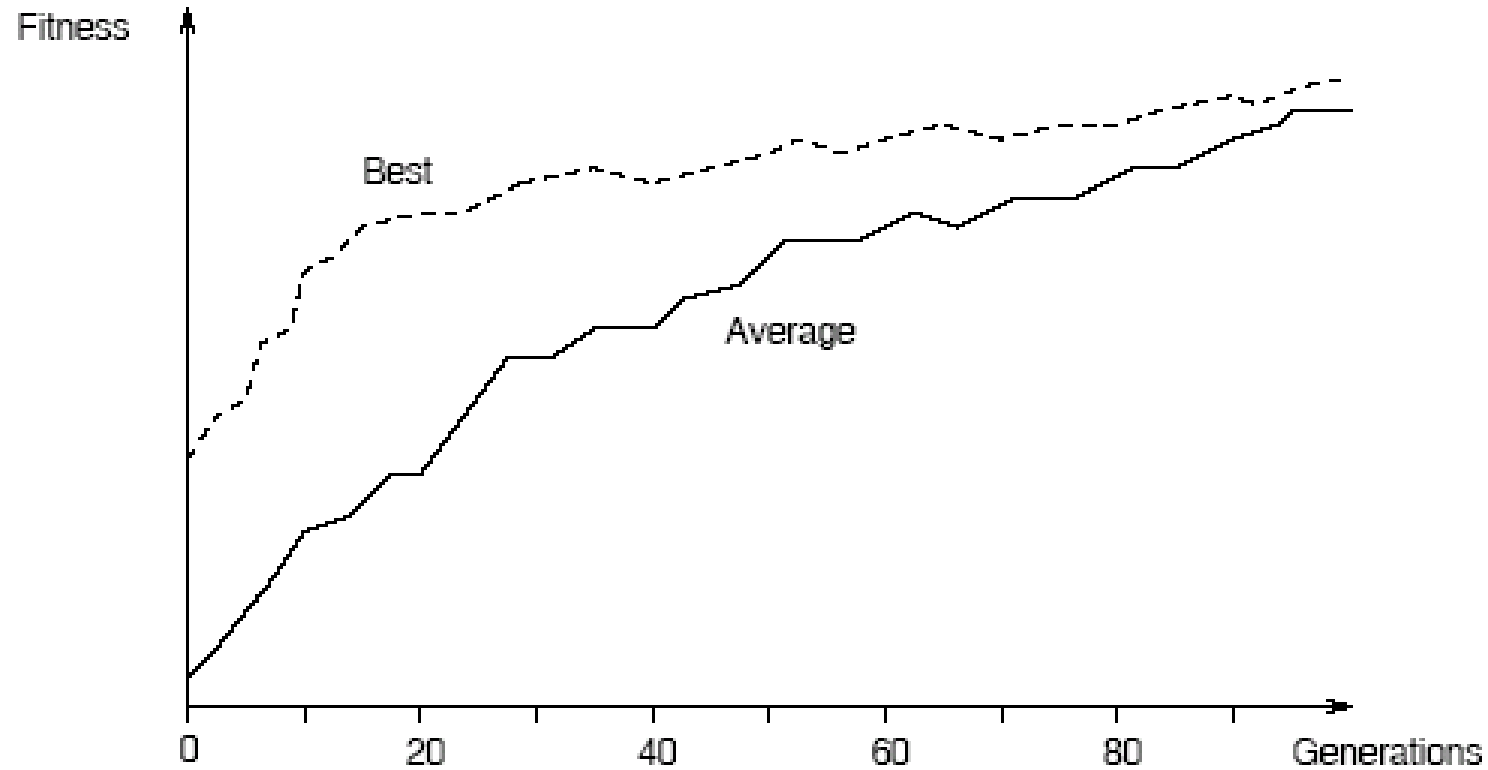
- Following selection, crossover and then mutation, the new population is ready for its next evaluation.
- The rest of evolution is just cyclic repetition of these steps.
- The total fitness of the new population is higher than total fitness of the previous generation – or so, one would hope 😊
- A stopping criterion must be specified:
 - After a fixed number of generations
 - After a chromosome with a certain high fitness value is located
 - After all the chromosomes in the population have attained a certain degree of homogeneity

Parameters for GAs

- **GA's performance in problem solving will depend on**
 - The method for encoding candidate solutions
 - The method for selecting the individuals in the population to reproduce for the next generation
 - Choices of genetic operators
 - The parameter settings
 - Population size
 - Probability of crossover (crossover rate) gives the probability that a pair of chromosomes will be combined
 - Probability of mutation (mutation rate) gives the probability that a bit will be flipped.

- Increasing the crossover probability increases recombination of schemata, but it also increases the disruption of good chromosomes.
- Increasing the mutation probability tends to transform the genetic search into a random search, but it also helps re-introduce lost genetic material.
- Increasing the population size increases its diversity and reduces the probability that the GA will prematurely converge to a local optimum, but it also increases the time required for the population to converge to the optimal regions in the search space.

A Sample GA Run



Taken from: Beasley, D. R. Bull, R. R. Martin. An Overview of Genetic Algorithms: Part 1, Fundamentals. University Computing 15:4 (1993) 170-181.

When & How to Use GAs?

- **When would you want to apply GA on a problem?**
 - No idea how to reasonably solve the problem
 - You cannot enumerate all possible solutions
 - You know how to evaluate how good or bad a solution is
- **What do we need to use Genetic Algorithms?**
 - A **goodness measure**
 - this is eventually translated into the fitness function
 - A **representation** of the solution
 - the more natural, the better
 - A **problem model** which defines the behaviour

3.2

Evolutionary Computation and Genetic Algorithm Applications

- **Evolutionary Computation and Genetic Algorithms have been successfully used in various areas such as optimisation, learning and design.**
 - Numerical & Combinatorial Optimization
 - Engineering Design
 - Interactive Creative Design
 - Machine Learning
 - Scheduling and Control
 - Etc...

- **GAs can be applied to a wide range of optimization and learning problems**
 - Routing and scheduling, machine vision, engineering design optimization, gas pipeline control systems, machine learning
 - Hundreds of applications have now been discovered and a variety of commercial software tools have been introduced.
- **Numerical function optimisation**
 - GA has been shown to outperform conventional optimisation techniques on difficult, discontinuous, multimodal, noisy functions.

- **Combinatorial optimisation**

- Resource allocation problems
- Classical problem of travelling salesperson / bin-packing / job shop scheduling
- For example:
 - Assign jobs to machines over time such that the machines' idle time and the jobs' throughput time are minimized
 - Timetabling of examinations or classes in the universities, colleges

- **Financial forecasting**

- Searching for a set of rules or equations that will predict the ups and downs of a financial market, such as that for foreign currency and stocks.
- Hypothesis: linear combination of technical indicators can be used to forecast periods of rising price movement in the stock market.
- The contribution of each indicator in a forecasting model is indicated by a weighting coefficient.
- The coefficients are derived by a genetic algorithm.
- The resulting models are evaluated against historical price movement of the stocks.

- **Design optimisation**

- network routing
 - To maximise throughput for given bandwidth (first objective) while minimizing cost (second objective).
 - Use of network simulation tools to determine bandwidth utilisation and throughput.
- Satellite orbit selection
 - Avoid collision and reduce blackout window

- **Machine learning**

- Used to evolve aspects of particular machine learning systems, such as weights for neural networks, rules for learning symbolic production systems, and sensors of robots.
- Classifier systems which evolve if-then rules for a specific problem domain
 - For example: rule induction for financial decision making

- **Designing satellite antenna (NASA Space Technology 5)**
 - Encode antenna structure into a genome and use a GA to evolve an antenna that best meets the desired antenna performance as defined in a fitness function.
- **Generating self-animating characters**
 - Dynamic Motion Synthesis
 - Breeding computer characters which walk ‘naturally’ without any supervision
 - Used in conjunction with neural networks – hybrid system

- **GAlib, A C++ Library of Genetic Algorithm Components developed at MIT Technology Center**
 - The library includes tools for using genetic algorithms to do optimization in any C++ program using any representation and genetic operators.
 - <http://lancet.mit.edu/ga/>
- **JGAP, Java Genetic Algorithms Package**
 - a Genetic Algorithms and Genetic Programming component provided as a Java framework.
 - Easy to use, highly modular
 - Can plug in custom genetic operators
 - .Net version available
 - <http://jgap.sourceforge.net/>

- **Excel-based GA tools**

- Excel Solver
 - <http://www.solver.com/>
- Evolver
 - <http://www.palisade.com/evolver/>
- SolveXL
 - <http://www.solvexl.com/>

- **R packages for GA:**
 - Genalg
 - GA
 - RGP
 -
- **Python packages for GA:**
 - DEAP
 - Pyevolve
 - Pyvolution
 -

3.3 WORKSHOPS

GA Workshop: Solving Load Distribution Problem Using Excel Solver

Problem Description

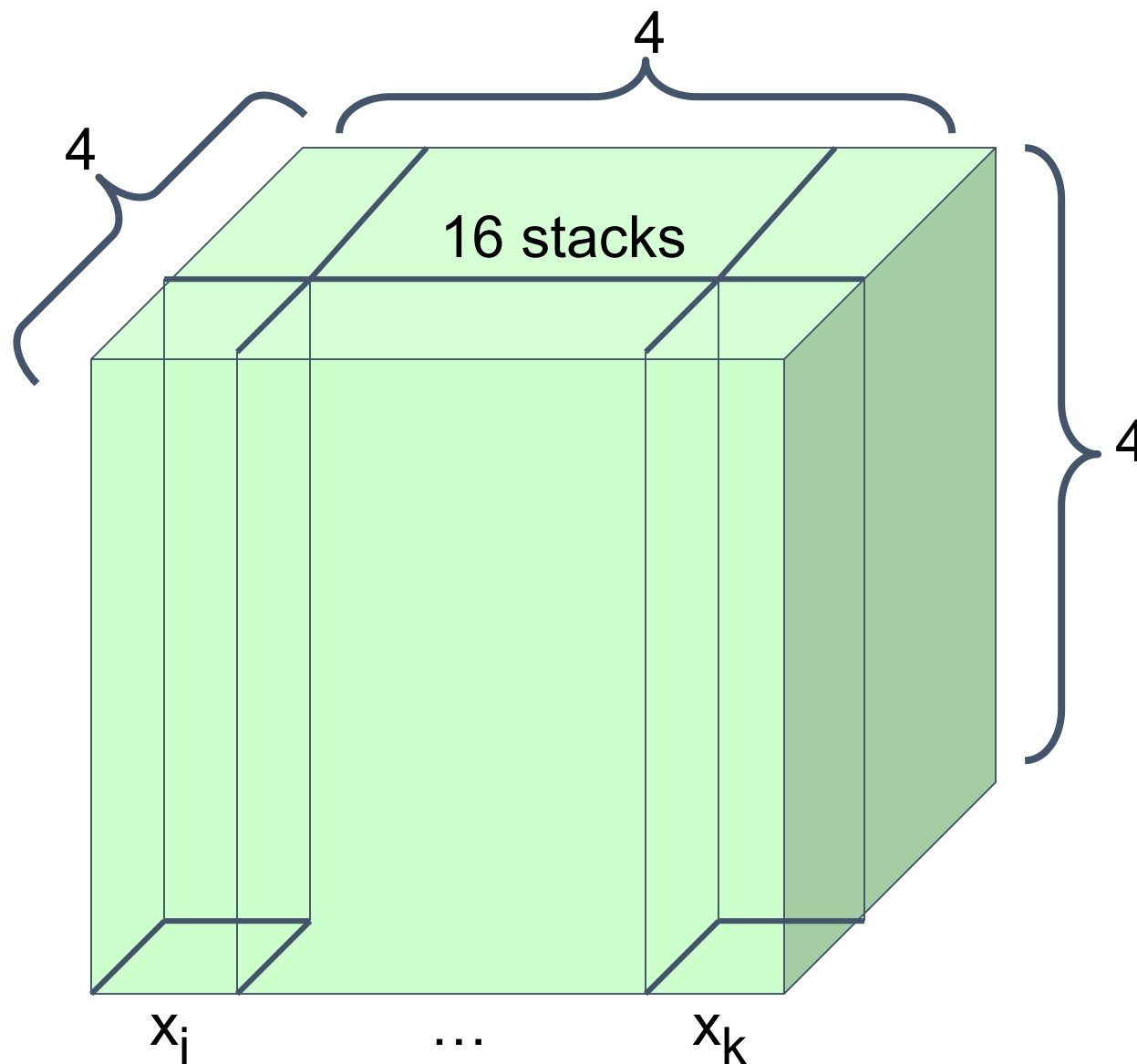
- In loading containers into a vessel or loading packages into an aircraft, the distribution of the weights should be more or less uniform.
- It is assumed the holding space for the containers or packages can be divided into 64 rectangular spaces.
- It is also necessary to ensure that the lighter containers or packages are on top of the heavier ones.
- The solution needs to determine how the packages are to be assigned to the rectangular spaces to ensure uniform distribution.

Problem Description

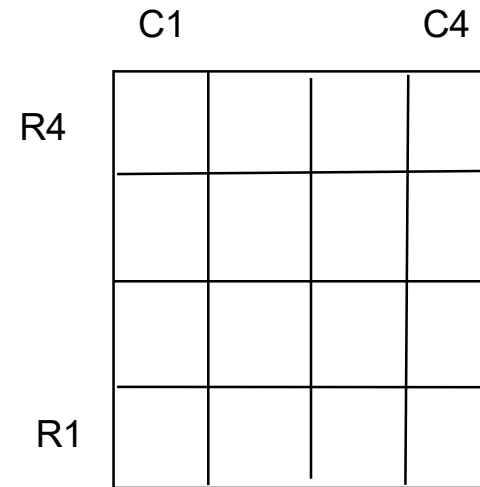
PackageID	Weight
1	27
2	57
3	33
4	55
5	65
6	26
7	21
8	28
9	43
10	40
11	27
12	62
13	25
14	33
15	67
16	30
17	46
18	31
19	58
20	59
21	37
22	55
23	61
24	25
25	38
26	65
27	51
28	30
29	40
30	69
31	33
32	61

PackageID	Weight
33	66
34	63
35	53
36	37
37	53
38	67
39	32
40	44
41	53
42	47
43	39
44	38
45	46
46	67
47	36
48	30
49	37
50	43
51	68
52	50
53	64
54	32
55	32
56	59
57	24
58	67
59	54
60	23
61	49
62	60
63	65
64	35

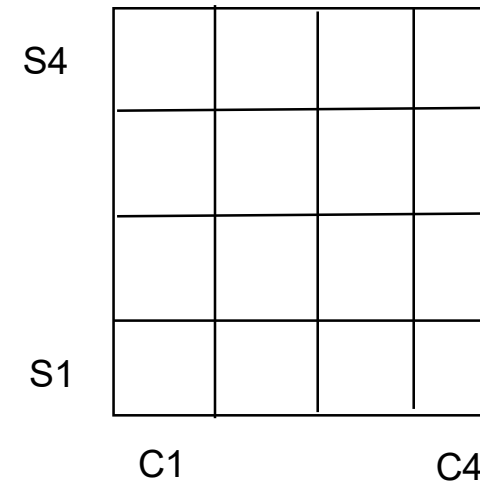
Graphical Representation



Graphical Representation



Looking from
the side of a
vessel



Looking from the
aeroplane view
of a vessel

1. Required information

- Supplied by client
 - Package ID
 - Package weight
 - Container/space location ID
- Derived by our model
 - Averages
 - Standard deviation

2. Representation of solution – we have some options

- Assigning packages to locations
 - The location remain static but the assigned packages change
 - In this model, the chromosome consists of Package IDs
- Assigning locations to packages
 - The packages remain static but the assigned locations change
 - In this model, the chromosome consists of location IDs

3. Fitness function

- Two different averages (of weights)
 - Overall across all packages (static)
 - For each stack (changes with assigned packages)
- Minimise the spread between the average stack weights and the overall average weight

$$\text{Minimise } \sum_i (x_i - AV)^2$$

where x_i is the average package weight in each stack and AV is the average of all the packages.

4. Constraints

- Exactly sixteen stacks, each with four levels
- Lighter packages on top of heavier ones – we don't want to have crushed packages upon arrival
- Explain how each constraint can be addressed based on the solution representation

Handling Constraints

- We are asked to propose only those solutions with lighter packages above heavier ones
BUT
we don't need to use hard constraints
- Hard constraints have an adverse impact on performance due to the generate-and-test strategy GA uses

- **Options for handling weight-ordering constraint**
 - Using soft constraints
 - Factor in the degree of the violation
 - Include this as penalty to fitness function,
BUT may need to scale penalty appropriately
 - Not using GA constraints at all!
 - Don't feel trapped into using GA constraints for everything
 - Simply sort the packages in each stack after finding a good solution – fitness is not affected!

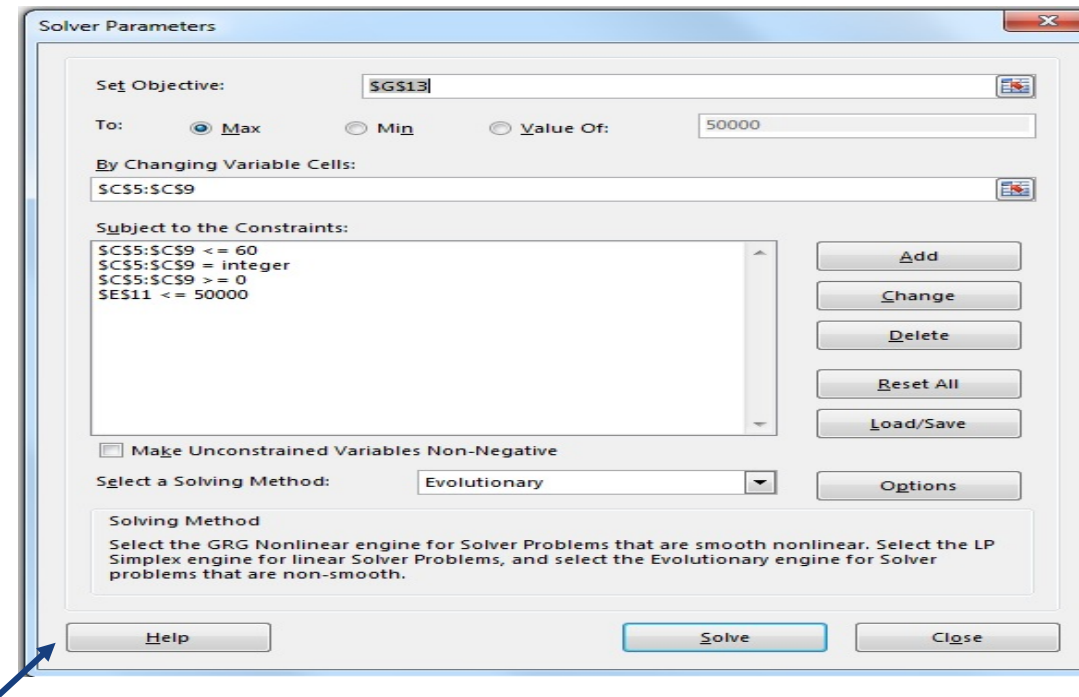
Appendix: Excel Solver

How to Load the Excel Solver Add-in

- **Open Excel**
- Click the **File** tab, and then click **Options**.
- Click **Add-Ins**, and then in the **Manage** box, select **Excel Add-ins**.
- Click **Go**.
- In the **Add-Ins available** box, select the **Solver Add-in** check box, and then click **OK**.
- If **Solver Add-in** is not listed in the **Add-Ins available** box, click **Browse** to locate the add-in.
- If you get prompted that the Solver add-in is not currently installed on your computer, click **Yes** to install it.
- After you load the Solver add-in, the **Solver** command is available in the **Analysis** group on the **Data** tab.

Excel Solver

- Excel Solver Help
 - <http://www.solver.com/excel-solver-help>



#Click HELP to get the help file.

GA Modelling – Excel Solver

Step 1:

- Decide what you are optimising for (e.g. maximise profit). This ‘goodness measure’ or objective will eventually translate into a fitness function

Step 2:

- Represent each solution chromosome as an array of real numbers or integers
- E.g. to find the best split of funds that will allow a company to maximise profits

advertising	marketing	production	salary		profit
8%	12%	50%	30%		\$2780

One solution

Fitness function

GA Modelling – Excel Solver

Step 3:

- Write a **fitness function** that evaluates the goodness of a solution chromosome.
- This function takes as input a candidate solution and returns a number that indicates how good the solution is (e.g. the amount of expected profit)

Step 4:

- Define any constraints on the values of your solution chromosome.

Exercise 1

The following jobs can be processed on any of the 5 machines. How can these jobs be assigned to the machine so that the total processing time for the jobs is minimum. The time taken to process each job on each machine is known.

Set the Excel spreadsheet as shown.

	A	B	C	D	E	F
1		Machine ID	Process Time			
2	Job1	1	=INDEX(\$B\$11:\$F\$15,1, B2)			
3	Job2	2	=INDEX(\$B\$11:\$F\$15, 2, B3)			
4	Job3	3	=INDEX(\$B\$11:\$F\$15, 3, B4)			
5	Job4	4	=INDEX(\$B\$11:\$F\$15, 4, B5)			
6	Job5	5	=INDEX(\$B\$11:\$F\$15, 5, B6)			
7		Total	=SUM(C2:C6)			
8						
9						
10	Machine ID	1	2	3	4	5
11	Job1	12	45	23	33	12
12	Job2	34	13	8	14	25
13	Job3	22	13	33	15	24
14	Job4	14	56	23	12	26
15	Job5	4	13	23	34	27

Exercise 2

You have a group of students whose average overall performance are known. You are required to divide them into 3 groups so that the members of each group can interact well with each other. To ensure that they can interact well with each other the deviations of their performance should be minimum.

Set the Excel spreadsheet as shown

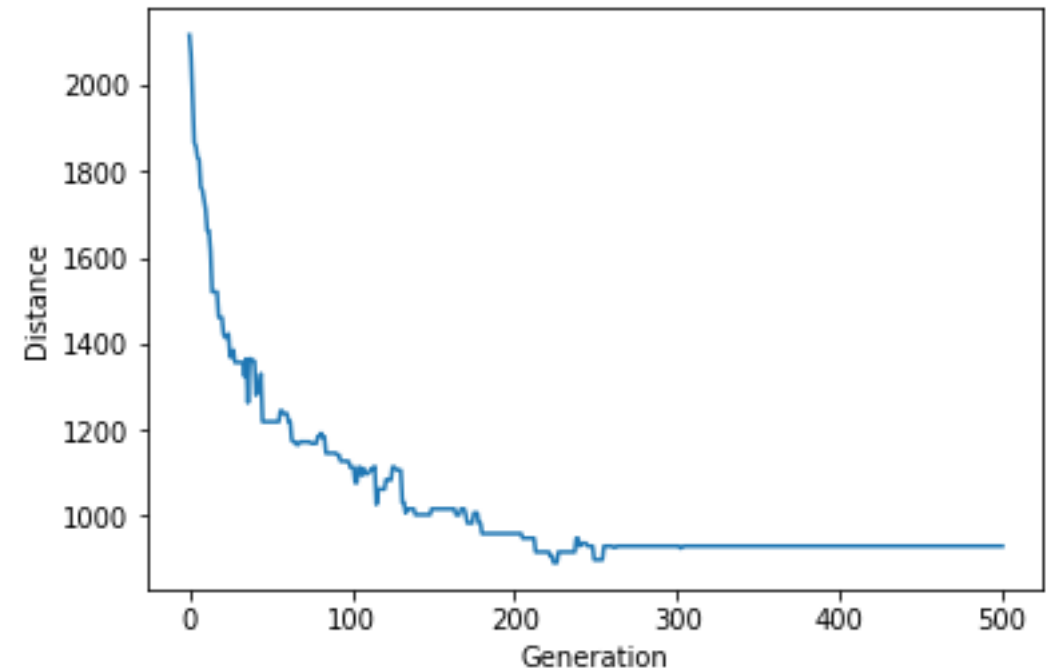
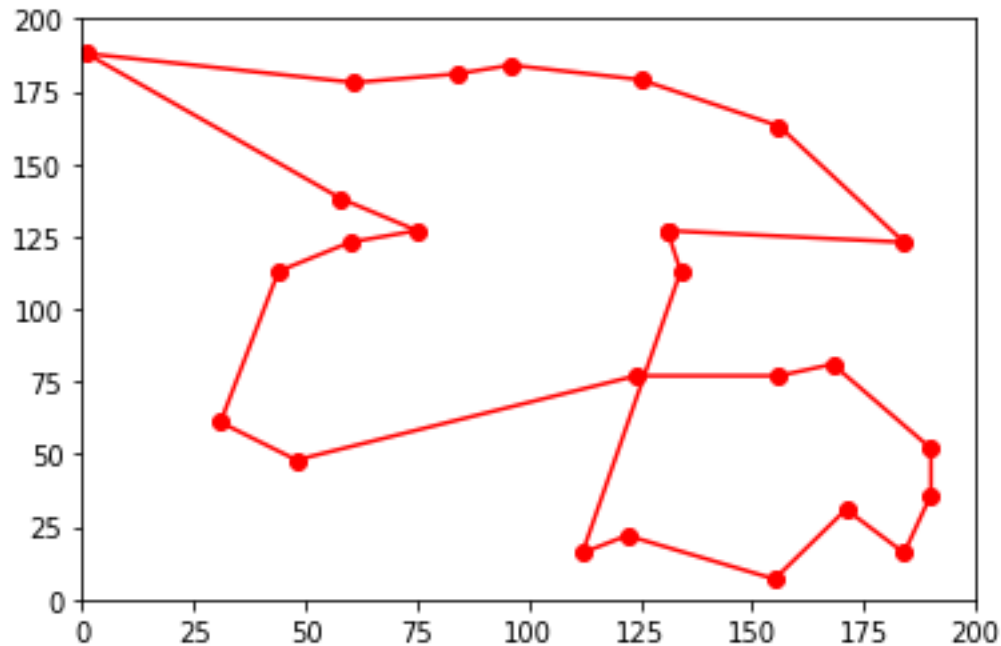
	A	B	C	D	E	F
1	Student	Performance	Group	Group1	Group2	Group3
2	Tom	50	1	=IF(C2=1, B2, "0")	=IF(C2=2, B2, "0")	=IF(C2=3, B2, "0")
3	Jerry	67	2	=IF(C3=1, B3, "0")	=IF(C3=2, B3, "0")	=IF(C3=3, B3, "0")
4	Ann	34	3	=IF(C4=1, B4, "0")	=IF(C4=2, B4, "0")	=IF(C4=3, B4, "0")
5	Bob	55	1	=IF(C5=1, B5, "0")	=IF(C5=2, B5, "0")	=IF(C5=3, B5, "0")
6	June	80	2	=IF(C6=1, B6, "0")	=IF(C6=2, B6, "0")	=IF(C6=3, B6, "0")
7	Nancy	90	3	=IF(C7=1, B7, "0")	=IF(C7=2, B7, "0")	=IF(C7=3, B7, "0")
8	Mike	77	1	=IF(C8=1, B8, "0")	=IF(C8=2, B8, "0")	=IF(C8=3, B8, "0")
9	Dolly	55	2	=IF(C9=1, B9, "0")	=IF(C9=2, B9, "0")	=IF(C9=3, B9, "0")
10	Paul	66	3	=IF(C10=1, B10, "0")	=IF(C10=2, B10, "0")	=IF(C10=3, B10, "0")
11	Lucy	83	1	=IF(C11=1, B11, "0")	=IF(C11=2, B11, "0")	=IF(C11=3, B11, "0")
12	John	44	2	=IF(C12=1, B12, "0")	=IF(C12=2, B12, "0")	=IF(C12=3, B12, "0")
13	Mary	73	3	=IF(C13=1, B13, "0")	=IF(C13=2, B13, "0")	=IF(C13=3, B13, "0")
14	Number			=COUNTIF(D2:D13, ">0")	=COUNTIF(E2:E13, ">0")	=COUNTIF(F2:F13, ">0")
15	Average			=AVERAGE(D2:D13)	=AVERAGE(E2:E13)	=AVERAGE(F2:F13)
16	Std Dev			=STDEV(D2:D13)	=STDEV(E2:E13)	=STDEV(F2:F13)
17						
18				=SUM(D16:F16)		

GA Workshop: Solving Travelling Salesman Problem Using Python

Problem Statement: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the origin city?"

TSP - Python

- Run the Python code provided
- Understand the GA concepts through the code
- Experiment with different parameter settings on popSize, mutationRate, etc.



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- 11) **Eiben, A.E., Smith, J.E., Introduction to Evolutionary Computing, Springer, 2010.**
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- 15) **Clinton Sheppard, Genetic Algorithms with Python, CreateSpace Independent Publishing Platform, 2016.**