Lab 1

STAT 517 - Winter 2023

1. Discuss the Plant Disease problem in page s195 and 202 of GUTTORP, and the motivation for the use of an Ising model.

   (a) For the data coming from the first count, implement the MLE algorithm based on exponential tilting to estimate the two parameters of the Ising model. Provide a confidence interval for the estimates.

   (b) For the same data, implement the pseudo-likelihood method and provide alternative estimates for the two parameters. Provide a confidence interval for the estimates. Compare the results against the MLEs.

2. Image segmentation: consider the problem of segmenting an image into K sections (K is assumed known). The image can be seen $\mathbf{Y}$, where y_i representing the luminosity of pixel i, with . A common approach to modeling such data is to assume that the luminosity is a function of the section to which the pixel belongs, and that it can be modeled using a Gaussian distribution:

$$Y_i \mid \theta_1, \ldots, \theta_K, \sigma_1, \ldots, \sigma_K, Z_i \sim N\left(\theta_{Z_i}, \sigma^2_{Z_i}\right),$$

   where $Z_i \in \{1, \ldots, K\}$. To capture the idea that sections of the image should be made mostly of adjacent points, a two-dimensional Potts model with on a regular lattice can be used to model $\mathbf{Z}$:

$$P(\mathbf{Z} \mid \phi) \propto \exp\left\{\phi \sum_{i \sim j} \delta_{(Z_i, Z_j)}\right\}.$$

   where $\delta_{(Z_i, Z_j)}$ is 1 if $Z_i = Z_j$ and 0 otherwise. Typically, ϕ is unknown, but for the purpose of this exercise, we will consider it fixed.

   (a) How do you interpret $\theta_1, \ldots, \theta_K$ and $\sigma_1, \ldots, \sigma_K$ in this setting?

   (b) How do you interpret ϕ in this setting?

   (c) Derive and implement an algorithm to fit this model. Run the algorithm for the data in `imageseg.txt` for $K = 3$ and $\phi = 3$. (Note that this data lies on a 50×50 regular lattice.)

(d) Fit the model again but now assuming $\phi = 0$. How do the results differ qualitatively from those obtained using $\phi = 3$?

(e) What do you think the effect would be of using an extended regular lattice instead of a regular lattice to define the latent Potts model? You do not need to fit that model, just discuss the intuition.