Lab 2

STAT 517 - Winter 2023

This lab focuses on a discussion of the continuous-time Markov chain model for a capacity-constrained queuing system with s servers (sometimes called an "$M/M/1$" queue), where the line capacity of the queue is c customers and customers are served on a "first come, first serve" basis.

1. Let the arrival rate be λ and the service rate for each server be μ. Derive and solve the Kolmogorov equations for this system. Can you solve the equations in closed form?

2. No matter what the answer to the previous question, write a numerical algorithm to evaluate the transition probabilities.

3. Compute the stationary distribution of the system.

4. Graph the % of time a customer can be serviced without waiting as function of the ration μ/λ for $s \in \{1, 2, 3, 4\}$ and $c = 6$.

5. Is the chain reversible?

6. Write code to simulate the queue for a length of period T, and use it to generate a sample for $\lambda = 4$, $\mu = 1$, $s = 3$, $c = 5$ and $T = 300$.

7. Now, assume that λ and μ are unknown. Write the likelihood function for the sample you just generated and use it to obtain the maximum likelihood estimators for λ and μ. Obtain 95% credible intervals for the estimates as well.