

# Homework 1

STAT 517 - Winter 2023

1. Consider an Ising model defined a regular  $L \times L$  lattice with a regular first order neighborhood, and such that the nodes can take discrete values on  $\{1, \dots, K\}$ .
  - (a) Show that the Ising model can be written as an  $L$ -th order Markov chain. (Hint: carefully consider how to order the pairs of nodes).
  - (b) How can you use this fact to generate an algorithm to evaluate the partition function without explicitly enumerating all  $2^{L^2}$  possible configurations.
  - (c) What is the complexity of the algorithm (as a function of  $L$ )? How useful do you think this approach would be in practice?
2. Problem 2 of GUTTORP (page 219).
3. Implement the perfect sampler for the Ising model introduced in Huber (2003) and use it to reproduce Figure 4.5 of GUTTORP.
4. Problem D4 of GUTTORP (page 226).
5. Consider problem 2 in Lab 1.
  - (a) Discuss how the algorithm derived in the lab can be extended to also estimate the parameter  $\phi$  of the latent Potts model.
  - (b) Derive and implement such an algorithm, and use it to fit the model to the same data used in the lab. Compare your results against those obtained in class.