APPM 4650 Project #2 Summer 2018

Flow over a flat plate Due Monday, July 23, 2018

Consider the flow of a fluid, initially at temperature T_∞ and horizontal velocity U_∞, as it passes over a stationary flat plate uniformly held at temperature T_w. We are interested in calculating the temperature distribution, $T(x, y)$, as well as the x and y components of the velocity distribution, $u(x, y)$ and $v(x, y)$, respectively.

Conservation of mass, x-momentum, and energy, lead to the following three coupled partial differential equations known as the "boundary layer equations,"

$$\begin{aligned} u_x + v_y &= 0 && (1)\\ uu_x + vu_y &= \nu\, u_{yy} && (2)\\ uT_x + vT_y &= \alpha\, T_{yy} && (3) \end{aligned}$$

where ν is the kinematic viscosity and α is the thermal diffusivity. We require that at the leading edge of the plate, the fluid should have the temperature T_∞ and horizontal velocity U_∞. Thus,

$$T(0, y) = T_\infty \qquad u(0, y) = U_\infty \qquad v(0, y) = 0\,. \tag{4}$$

Along the plate itself, the fluid should "stick" to the plate and have the same temperature as the plate. In this case,

$$T(x, 0) = T_w \qquad u(x, 0) = 0 \qquad v(x, 0) = 0\,. \tag{5}$$

Finally, far from the plate, the fluid temperature and velocity should reach free stream values,

$$T(x, \infty) = T_\infty \qquad u(x, \infty) = U_\infty\,. \tag{6}$$

Note that far from the plate, we expect an induced vertical velocity, $v(x, y)$, the value of which should be determined as part of the solution.

To solve the problem, we introduce the following similarity transformation

$$\begin{aligned} u &= U_\infty F'(\eta) && (7)\\ v &= \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}\,[\eta F'(\eta) - F(\eta)] && (8)\\ G(\eta, Pr) &= \frac{T - T_\infty}{T_w - T_\infty} && (9) \end{aligned}$$

where the similarity variable η is defined as

$$\eta = \frac{y}{\sqrt{\frac{\nu x}{U_\infty}}} \tag{10}$$

and the Prandtl number is defined as $Pr = \nu/\alpha$. After the above transformations are performed, the conservations equations (1) through (3) reduce to the following set of coupled ordinary differential equations,

$$\begin{aligned} F''' + \frac{1}{2}FF'' &= 0 && (11)\\ G'' + \frac{Pr}{2}FG' &= 0 && (12) \end{aligned}$$

subject to the following boundary conditions,

$$\begin{aligned} F(0) &= 0 && (13) \\ F'(0) &= 0 && (14) \\ F'(\infty) &= 1 && (15) \\ G(0) &= 1 && (16) \\ G(\infty) &= 0\,. && (17) \end{aligned}$$

Notice that the F problem can be solved without a solution for G. Once F is determined, then G may be solved. Also, the coupled system of higher order equations can be rewritten as a larger system of coupled first order equations.

Initially, you will be using the RK-4 method to solve the system of first order equations resulting from (11). However, since this method is really used for initial value problems, you will need to employ the "shooting" method. In particular, you will replace the boundary condition (15) with a fictitious initial condition $F''(0) = guess_1$ and adjust the value of $guess_1$ until the final value $F'(\infty) = 1$ is obtained. This will give you the solution to the F problem.

Now, you need to solve the coupled problem in F and G. To do this you need to solve the system of first order equations resulting from (11) and (12). Final condition (17) will need to be replaced by a fictitious initial condition $G'(0) = guess_2$. The value of $guess_2$ will be adjusted until $G(\infty) = 0$ is satisfied. You now have the numerical solution to the entire F and G problems.

Here are some issues to consider as you write your report for this project.

1. Using a step size of $\Delta\eta = 0.1$, integrate from $\eta = 0$ to values of η that are large enough to assure that (15) and (17) are satisfied in an asymptotic sense. Make a table of values for F, F', F'', G, and G' as a function of η, for $Pr = 5$.

2. On a single graph, plot the dimensionless velocities $\frac{v}{U_\infty}\sqrt{\frac{x}{L}}\sqrt{Re} = \frac{1}{2}\left[\eta F'(\eta) - F(\eta)\right]$ and $\frac{u}{U_\infty} = F'(\eta)$ as a function of η. On this graph, clearly indicate the value of η corresponding to the thickness of the momentum boundary layer. Call this value η_m. Recall that $Re = \frac{U_\infty L}{\nu}$ is the Reynold's number.

3. Next, plot the dimensionless temperature $G(\eta)$ as a function of η. Use the same scales on this graph as for the velocity graphs. Clearly indicate the value of η that corresponds to the thickness of the thermal boundary layer. Call this value η_t. (Also add the plots of $G(\eta)$ for the different Pr values from part 7.)

4. From your tabulated values of F' and G versus η, interpolate to determine more accurate values of η_m and η_t. (Use more than two points for your interpolation!) You may assume that $G = 0.02$ and $U/U_\infty = 0.98$ represent the edge of the boundary layers.

5. Make a third figure in which you plot the growth of the momentum and thermal boundary layer thickness as a function of position along the plate. That is, plot the dimensionless groupings $\frac{\delta_m}{L}\sqrt{Re}$ and $\frac{\delta_t}{L}\sqrt{Re}$ as a function of the dimensionless distance along the plate $\frac{x}{L}$. Include a curve for each Pr value.

6. Make a fourth figure in which you plot η_t as a function of Prandtl number, specifically let $Pr = 0.1,\ 0.2,\ 0.5,\ 1,\ 2,\ 5$, and 10. Be sure to clearly state the values of η_t for each Pr number.

7. And of course, you must work in groups of size two or three.