APPM 4650 **EXAM #1** **Summer 2018**

Due Tuesday, July 10, at the start of class

Please show all of your work. This exam is to be taken under the honor system. The only resources that you may use are your text book, your class notes, and me. You may use a hand calculator, but any other device with an on–off switch is not allowed — the only exception to this requirement is the next sentence. If you have any questions, send me an email at *adam@colorado.edu* and be sure to include a phone number where I can reach you.

At the top of your submitted exam, please include the following statement, with your signature below the statement.

> *This take-home exam was worked entirely be me. With the exception of Adam Norris, I have had absolutely no communication of any sort with another person concerning this exam. In addition, I have followed the requirements of the honor system, as stated at the top of the exam, and the CU Honor Code statement.*

1. (25 points) The square root of c, for $c > 0$, can be found by iterating with the expression $x_{n+1} = \dfrac{1}{2}\left(x_n + \dfrac{c}{x_n}\right)$ where $x_0 > 0$.

   (a) Estimate $\sqrt{3}$ using this scheme.

   (b) Derive the above expression.

   (c) If $x_0 > 0$, will the approximations of $\sqrt{c}$ always be larger, or smaller, than the true value? Justify your answer.

   (d) Repeat part (c) but now assume $x_0 < 0$.

2. (25 points) Newton's method is generally used for "curved" functions with a simple root. In particular, if $g(r) = 0$, $g'(r) > 0$ and $g''(r) \neq 0$, then as demonstrated in class, we found that Newton's method yields second order convergence. However, suppose that we now have a function $f(x)$ that has the same root as $g(x)$, but it is "straighter" near the root r. In particular, $f(x)$ has the properties $f(r) = 0$, $f'(r) \neq 0$, but now $f''(r) = 0$, $f'''(r) = 0$, and $f''''(r) \neq 0$. If Newton's method is used to find the root r of $f(x)$, determine the order convergence. Hint: recall how we originally demonstrated second order convergence with Newton's method.

3. (25 points) Consider the problem of using the values of a function $f(x)$ at three non-uniformly spaced points (x_0, f_0), (x_1, f_1), (x_5, f_5), where $f_i = f(x_i)$, and $h = x_i - x_{i-1}$, to calculate an approximation to $f''(x_2)$.

   (a) One method involves writing out the Lagrange polynomial that passes through the points listed above. Write out this Lagrange polynomial $P(x)$.

   (b) Now, calculate the first and second derivatives, $P'(x)$ and $P''(x)$, and evaluate $P''(x)$ at x_2, to arrive at an approximation for $f''(x_2)$.

   (c) Use Taylor Series expansions about the point x_2 to determine the magnitude of the error associated with your approximation of $f''(x_2)$ from part (b).

4. (25 points) Determine the values of a and b for $\displaystyle\int_0^1 x^3 f(x)\, dx \approx af\left(\frac{1}{4}\right) + bf\left(\frac{3}{4}\right)$ using the "direct method." Be sure to note the weighting function x^3 that appears in the integrand. Also, determine the order of magnitude of the error.