Due Wednesday, June 20, 2018

1. Show that repeated application of $P(x) = (x-\alpha)Q(x) + R$ will eventually produce the Taylor series expansion of the polynomial function $f(x)$. That is, show that $R_k = \dfrac{P^{(k)}(\alpha)}{k!} = \dfrac{f^{(k)}(\alpha)}{k!}$. What is the center of this expansion?

2. Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate $f(8.4)$ if $f(8.1) = 16.94410$, $f(8.3) = 17.56492$, $f(8.6) = 18.50515$, and $f(8.7) = 18.82091$.

3. Use Neville's method for the previous problem.

4. Use cubic splines to approximate the function $f(x) = x^4$ on the interval $-3 \le x \le 3$. Use the values of the function $f(x)$ at $x = -3, -1, 1, 3$. Match the slope of the spline to the slope of the function at the locations $x = -3$ and $x = 3$. You should get the following results:
   (a) $P_0 = -8x^3 - 22x^2 - 24x - 9$ valid on $-3 \le x \le -1$
   (b) $P_1 = 2x^2 - 1$ valid on $-1 \le x \le 1$
   (c) $P_2 = 8x^3 - 22x^2 + 24x - 9$ valid on $1 \le x \le 3$

5. Use cubic splines to approximate the function $f(x) = x^4$ on the interval $-1 \le x \le 1$ by using the values of the function $f(x)$ at $x = -1, 0, 1$. Match the slope of the spline to the forward and backward approximations of $f'(x)$,
$$f'(x) \approx \frac{f_1 - f_0}{h} \qquad \text{and} \qquad f'(x) \approx \frac{f_2 - f_1}{h}$$
at locations $x = -1$ and $x = 1$. Let P_0 be valid on $-1 \le x \le 0$ and let P_1 be valid on $0 \le x \le 1$.

6. Use cubic splines to approximate the function $f(x) = x^4$ on the interval $-1 \le x \le 1$ by using the values of the function $f(x)$ at $x = -1, 0, 1$. Use "free boundary conditions" at locations $x = -1$ and $x = 1$. Let P_0 be valid on $-1 \le x \le 0$ and let P_1 be valid on $0 \le x \le 1$.