

Given the coupled system

$$\begin{aligned} y_1' &= f_1(x, y_1, y_2, y_3) & \text{subject to} & & y_1(x_0) &= y_{1,0} \\ y_2' &= f_2(&) & \text{subject to} & y_2(x_0) &= y_{2,0} \\ y_3' &= f_3(&) & \text{subject to} & y_3(x_0) &= y_{3,0} \end{aligned}$$

we first calculate

$$\begin{aligned} k_{1,1} &= h \cdot f_1(x_j, y_{1,j}, y_{2,j}, y_{3,j}) & \text{where} & & y_{1,j} &= y_1(x_j) \\ k_{1,2} &= h \cdot f_2(&) & \text{where} & y_{2,j} &= y_2(x_j) \\ k_{1,3} &= h \cdot f_3(&) & \text{where} & y_{3,j} &= y_3(x_j) \end{aligned}$$

then calculate

$$\begin{aligned} k_{2,1} &= h \cdot f_1\left(x_j + \frac{h}{2}, y_{1,j} + \frac{k_{1,1}}{2}, y_{2,j} + \frac{k_{1,2}}{2}, y_{3,j} + \frac{k_{1,3}}{2}\right) \\ k_{2,2} &= h \cdot f_2\left(& \right) \\ k_{2,3} &= h \cdot f_3\left(& \right) \end{aligned}$$

then

$$\begin{aligned} k_{3,1} &= h \cdot f_1\left(x_j + \frac{h}{2}, y_{1,j} + \frac{k_{2,1}}{2}, y_{2,j} + \frac{k_{2,2}}{2}, y_{3,j} + \frac{k_{2,3}}{2}\right) \\ k_{3,2} &= h \cdot f_2\left(& \right) \\ k_{3,3} &= h \cdot f_3\left(& \right) \end{aligned}$$

then

$$\begin{aligned} k_{4,1} &= h \cdot f_1(x_j + h, y_{1,j} + k_{3,1}, y_{2,j} + k_{3,2}, y_{3,j} + k_{3,3}) \\ k_{4,2} &= h \cdot f_2(&) \\ k_{4,3} &= h \cdot f_3(&) \end{aligned}$$

and finally, we calculate

$$\begin{aligned} y_{1,j+1} &= y_{1,j} + \frac{1}{6} \cdot (k_{1,1} + 2k_{2,1} + 2k_{3,1} + k_{4,1}) \\ y_{2,j+1} &= y_{2,j} + \frac{1}{6} \cdot (k_{1,2} + 2k_{2,2} + 2k_{3,2} + k_{4,2}) \\ y_{3,j+1} &= y_{3,j} + \frac{1}{6} \cdot (k_{1,3} + 2k_{2,3} + 2k_{3,3} + k_{4,3}) \end{aligned}$$

to give us the new values for y_1 , y_2 and y_3 corresponding to x_{i+1} .