APPM 4650 Homework #4 Summer 2018

Due Monday, July 9, 2018

1. Find the values of a and b in the expression $\int_0^1 f(x)\,dx \approx af\left(\frac{1}{3}\right) + bf\left(\frac{2}{3}\right)$ by using the "direct method."

2. Use the 4^{th} order Runge–Kutta method for systems to approximate the solution of the following system of first–order differential equations for $0 \leq t \leq 1$, and $h = 0.2$, and compare the results to the actual solution

$$
\begin{aligned}
u_1' &= 3u_1 + 2u_2 - (2t^2+1)e^{2t} \\
u_2' &= 4u_1 + u_2 + (t^2 + 2t - 4)e^{2t}\,.
\end{aligned}
$$

The initial conditions are $u_1(0) = 1$ and $u_2(0) = 1$. For comparison, the exact solutions are $u_1(t) = \frac{1}{3}e^{5t} - \frac{1}{3}e^{-t} + e^{2t}$ and $u_2(t) = \frac{1}{3}e^{5t} + \frac{2}{3}e^{-t} + t^2e^{2t}$.

Looking to your immediate future, you will need to solve four coupled first–order ODEs. When you design and write your code for this problem, consider how you will extend it to four equations.

3. Consider the initial value, ordinary differential equation, $y' = x + y$ with $y(0) = 0$. Find $y(x)$ on $0 \leq x \leq 0.5$ with a step size of $h = 0.1$ using the following methods:

   (a) Euler's

   (b) 4^{th} order Runge–Kutta

   (c) Adams–Bashforth 2–point

   (d) Adams–Moulton 2–point

   (e) Improved Euler

   (f) Predictor–corrector using the Adams–Bashforth 4–point for the predictor and the Adams–Moulton 3–point as the corrector

To determine any necessary seed values for this problem, use the approximation $y \approx \dfrac{x^2}{2}$ which is only valid for $x << 1$.