Sorting Algorithms Analysis and Comparison

Purpose

The aim of this experiment is to analyze and compare three different sorting algorithms based on their efficiency (or complexity). The complexity is measured using the runtime of each of the algorithms on the same unsorted array. The number of swaps and iterations made is also computed to give an idea of the total number of operations each algorithm makes before it can sort the array.

Procedure

The three different sorting algorithms that were compared are Insertion sort, Quicksort, and Counting sort. Insertion sort was chosen because it is an improved version of the naïve Bubble sort and it would be interesting to see how it compares against the mighty Quicksort that employs the divide-and-conquer paradigm and uses recursion. Counting sort is a non-comparison based algorithm. Hence it always runs in linear time.

In Insertion sort, we loop over the entire array n times. When we loop over the array the i^{th} time, the first $i-1$ elements are sorted. And, at the end of the i^{th} iteration, the first i elements of the array are sorted. Thus, when we finish looping over the array n times, all the n elements are sorted in the array. Clearly, this outperforms the naïve Bubble sort but is still not as powerful as other algorithms.

Quicksort is a divide-and-conquer algorithm that partitions the array into two smaller sub-arrays. The middle element is chosen as a pivot and after comparing it to other elements, a swap is made based on the result of the comparisons. Then Quicksort is recursively called on the two sub-arrays until all the sub-arrays are sorted.

Counting sort is a non-comparison based algorithm that works only when it is known that the input array consists only of positive numbers with an upper bound. First a histogram is made of the keys that occur in the input array. In the second loop, the prefix-sum is computed. Finally, the correct position is assigned to each of the input key into a new array. We can then either copy the values from the temporary array into the original array or simple use the new sorted array.

Now that we've seen a brief overview of the algorithms we've implemented, note that Insertion sort is the slowest of the three. Quicksort uses an algorithmic approach called divide-and-conquer. Counting does not do any comparison and thus loops over the array at least 3 times. Since it takes this method, a pure in-place implementation of Counting sort is difficult to achieve. Hence Quicksort has an edge over Counting sort in terms of memory requirement; however, they must perform at almost the same levels.

To evaluate and compare the performance of these three algorithms, we will need a measure of the performance. For this metric, I will be primarily using the runtime of the algorithms. Along with this, counting the number of operations made (iterations and swaps) will also give an interesting perspective to the problem. Since, Counting sort has no direct swapping of two elements, I will count a copying operation from the temporary array to the original array as a swap operation.

Data

The runtime of each of the three algorithms will be tested against arrays of the following sizes:

$$\{10, 100, 500, 1000, 5000, 10000, 50000, 100000,$$

$$500000, 1000000, 5000000, 10000000, 50000000\}$$

The arrays were generated using the `random_device` from `<random>` library. It is a uniformly-distributed integer random number generator that produces non-deterministic random numbers. *Mersenne Twister* pseudo-random number generator was used in conjuncture with this. The `rand()` from `<cstdlib>` library creates a bias towards the lower end of the integers when generating numbers within a confined range. This limitation was overcome by using `uniform_int_distribution<int>` that guarantees unbiased numbers given a range.

Once an array was generated using this process, it was copied into two separate arrays. Thus, we have three identical, randomly generated arrays to be tested against three different algorithms.

Result

Table 1 Number of Swaps

Array Size	Insertion Sort	Quicksort	Counting sort
10	16	9	10
100	2498	181	100
500	60493	1191	500
1000	249053	2560	1000
5000	6309548	15376	5000
10000	25246025	33210	10000
50000	624496303	195973	50000
100000	2497811749	415558	100000
500000	62375174049	2365823	500000
1000000	-	4987030	1000000
5000000	-	28804556	5000000
10000000	-	61548675	10000000
50000000	-	360546020	50000000

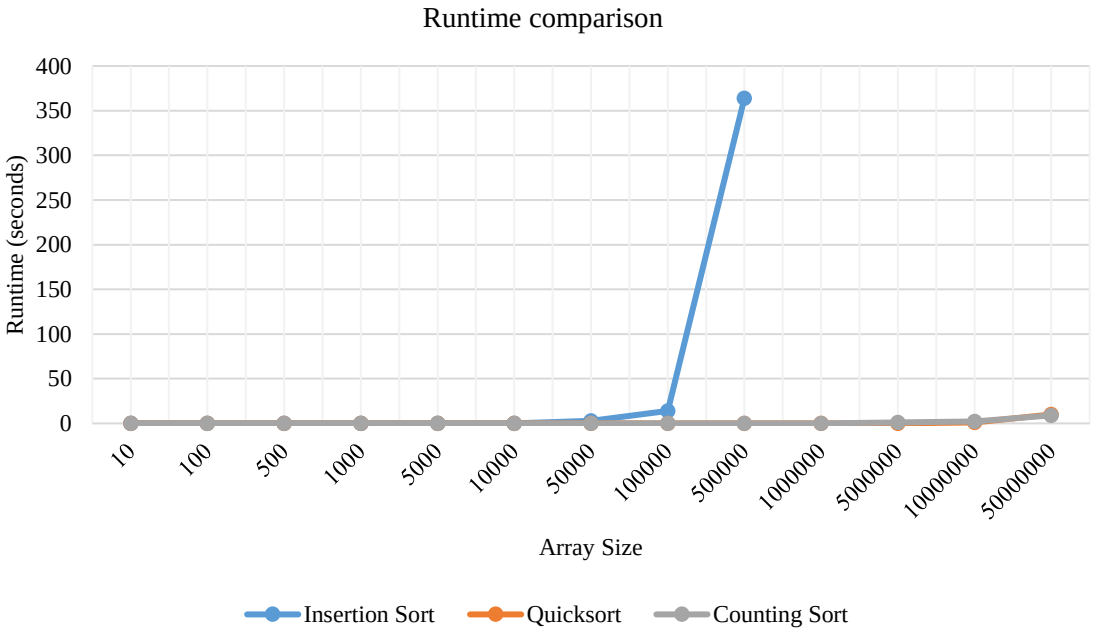
Table 2 Number of Iterations

Array Size	Insertion Sort	Quicksort	Counting sort
10	26	30	100000030
100	2598	685	100000300
500	60993	4515	100001500
1000	250053	10422	100003000
5000	6314548	72839	100015000
10000	25256025	153932	100030000
50000	624546303	819547	100150000
100000	2497911749	1744521	100300000

500000	62375674049	10086047	101500000
1000000	-	21577722	103000000
5000000	-	115492919	115000000
10000000	-	252160734	130000000
50000000	-	1256715538	250000000

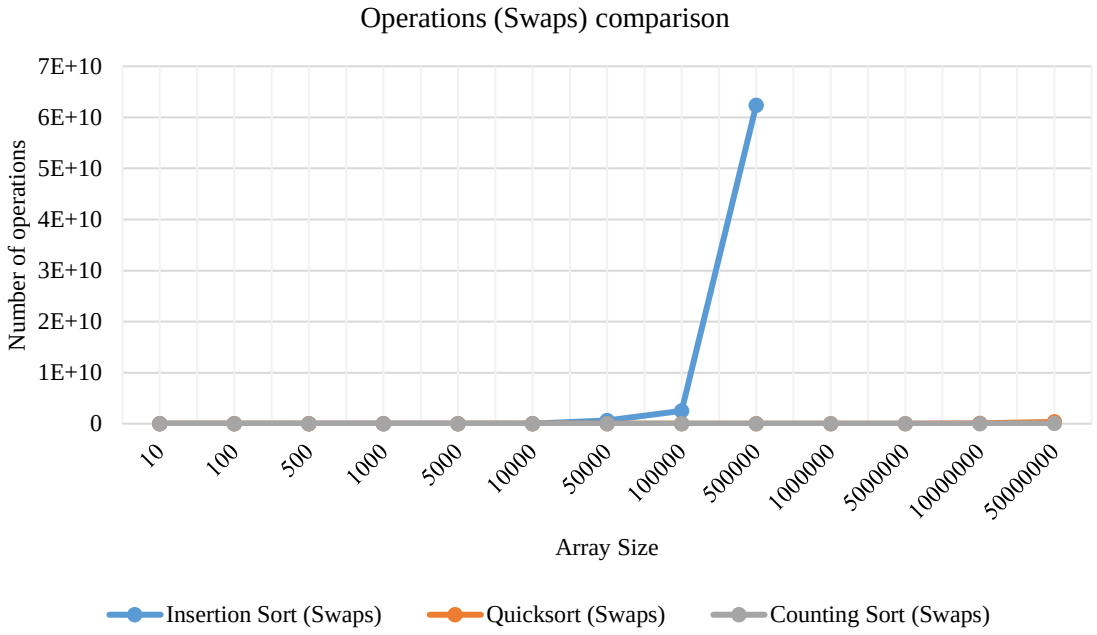
Table 3 Runtime in milliseconds

Array Size	Insertion Sort	Quicksort	Counting sort
10	0	0	0
100	0	0	0
500	0	0	0
1000	0	0	0
5000	0	0	0
10000	0	0	0
50000	3000	0	0
100000	14000	0	0
500000	364000	0	0
1000000	-	0	0
5000000	-	0	1000
10000000	-	1000	2000
50000000	-	10000	9000



As expected from the discussion, Insertion sort performed very poorly for arrays larger than 100,000 (around 13 seconds). At 500,000, it took more than 6 minutes to sort the array. And at 1,000,000, it could not complete the sorting process even after 30 minutes.

Counting sort and Quicksort were much better. However, Counting sort has an edge over quick sort when the input range is not much bigger than the input array size. At the size of 5 million, Counting sort took 1 second to complete, while Quicksort took 0 milliseconds to complete. At the size of 50 million, Counting sort took 9 seconds while Quicksort completed the operation in 10 seconds.



Comparing the number of operations made by each algorithm also leads to a similar conclusion. From the data, we observe that the number of operations made by Counting sort is great when the input array size is much smaller than the input range. However, for large arrays, Counting sort takes fewer number of operations to sort the array than Quicksort. We can therefore conclude and say that Counting sort is the best out of the three algorithms when we are dealing with large databases. The one drawback in Counting sort is that the keys should be confined to a range. Quicksort performs well and can be used when the database is very small or we do not know that the array keys are integers confined to a range. And Insertion Sort is the poorest of the three algorithms and took more than 6 minutes to sort a 100,000 array. Therefore, on large scale datasets, Quicksort or Counting sort is preferred, with Counting sort performing better having fewer operations made.

Sources

- Counting Sort pseudocode - http://www.albany.edu/~csi503/pdfs/handout_9.1.pdf Retrieved on March 18 2017, 2:20 PM (GMT-0600)
- Quicksort and Insertion Sort - Recitation 9 starter code