Classification of Handwritten Digits with Support Vector Machines

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Abstract

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1. Introduction

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2. Mathematical Formulation

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Text requiring further explanation[1].

We will now apply this idea of Support Vector Machines to classifying handwritten digits from the MNIST dataset. The MNIST database consists of $60,000$ images of handwritten digits, with correct labels, that are size-normalized and centered. Each of these images is a 28×28 image of a single digit. So, our classification problem becomes that of learning the recognize the correct digit.

With classifying digits, we have 10 different classes i.e., $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Since this is a multi-class classification problem (there are more than 2 classes), we will employ the one-vs-all classification technique. In this technique, we construct 10 different hyperplanes - each hyperplane distinguishes the input image between one class and the rest. For example, one such hyperplane would classify the input image as either 0, or not. Another hyperplane would classify the input image as either 9, or not. Thus, we end up with 10 different hyperplanes. Note that we need 10, and not 9, hyperplanes to take into consideration the possibility that the image we are classifying might not be any of the 10 digits.

With this method, we run into the problem of the same image being assigned to two different classes. When that happens, we will choose the hyperplane that gives a higher output value for the image.

Let us introduce some old notation to our new data. Let the number of images we have be m. Our $\vec{X} \in \mathbb{R}^m$, here, becomes a vector of all the images from the dataset. Let us denote each element of $\vec{X}$ as x_i, which is, in turn, a single image from the dataset that is unrolled into a one dimensional vector. Let the length of the unrolled vectore be n. Therefore, $x_i \in \mathbb{R}^n$. Each pixel value of the image becomes a feature in our feature space. Since we are working with grayscale images, we do not have to worry about RGB channels. Hence, it might make sense to think of X as a matrix, $X \in \mathbb{R}^{m \times n}$.

Our y_i associated with each of the x_i depends on the hyperplane we are constructing. Assume we are constructing the k^{th} hyperplane i.e., the hyperplane that classifies the image as either digit k, or not digit k. Let us define the normal to this hyperplane as $\vec{w}^{(k)}$. Since our y_i depends on the value k, let us re-define our notation as $y_i^{(k)}$, where

$$y_i^{(k)} = \begin{cases} 1, & \text{if } x_i \text{ represents } k \\ -1, & \text{otherwise} \end{cases} \tag{1}$$

3. Results

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[1] Example footnote

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4. Conclusion

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