Karnaugh Map

Sum of product form Notation for this page

Inputs to the function are binary variables. Output is a non-negative number.
Variables x, y and z are Boolean variables (i.e. they take value 0 or 1). a' is (1−a)
abc is ordinary product. (5)(7)(3) is 105.
(a+b+c) is maximum of a, b and c. (12+34+15+19) is 34.

The expression (1−y)z becomes 0 when y=1 or z=0. Hence only boxes 1 and 5 will have 1. All other boxes will be 0. Similarly expression xz will be 1 only in box 5 and 7. The expression (1−y)z+xz will become 2 in box 5, it will be 1 in boxes 1 and 7. In other boxes it will be zero. However, if '+' is treated as maximum then in box 5 it will be 1.

\ x yz	0	1
00	0	4
01	1	5
10	2	6
11	3	7

The function f (defined at right) can be written as 42y+39(1−x)(1−y). However if '+' means maximum then it can be also be written as 42y+39(1−x). It is because in boxes 2 and 3, 42 will suppress 39. Hence smallest size expression is 42y+39x'. Following method is used.

\ x yz	0	1
00	39	0
01	39	0
10	42	42
11	42	42

42y

\x yz	0	1
00	0	0
01	0	0
10	42	42
11	42	42

39(1−x)(1−y)

\x yz	0	1
00	39	0
01	39	0
10	0	0
11	0	0

39(1−x)=39x'

\x yz	0	1
00	39	0
01	39	0
10	39	0
11	39	0

max(42y,39x')

\x yz	0	1
00	39	0
01	39	0
10	42	42
11	42	42

The function f (defined below) can be written as 73xy'+42yz'+35x'.

The function f

\x yz	0	1
00	35	73
01	35	73
10	42	42
11	35	0

73x(1−y)=73xy'

\x yz	0	1
00	0	73
01	0	73
10	0	0
11	0	0

42y(1−z)=42yz'

\x yz	0	1
00	0	0
01	0	0
10	42	42
11	0	0

35(1−x)=35x'

\x yz	0	1
00	35	0
01	35	0
10	35	0
11	35	0

In the above function let us replace 42 by 22. Now above technique can not be applied. It is because 22 will not suppress 35 (rather 35 will suppress 22). Now the expression is 73xy'+35y'+35x'z +22z'.

The function g

\x yz	0	1
00	35	73
01	35	73
10	22	22
11	35	0

73x(1−y)=73xy'

\x yz	0	1
00	0	73
01	0	73
10	0	0
11	0	0

35y'+35x'z

\x yz	0	1
00	35	35
01	35	35
10	0	0
11	35	0

22(1−z)=22z'

\x yz	0	1
00	22	22
01	0	0
10	22	22
11	0	0

Product of sum form

Here those functions are defined, whose input is binary and output is between 0 and 1.
(a+b+c) is ordinary sum. 5+7+12 is 24.
abc is minimum of a, b and c. (2)(3)(0.4) is 0.4. a' is short form for (1−a)

The function h (defined below) can be written as h(x,y,z)=(x+y)(y'+0.2)(x'+y+0.6)

The function h

x \ yz	0	1
00	0	0.6
01	0	0.6
10	0.2	0.2
11	0.2	0.2

x+y

x \ yz	0	1
00	0	1
01	0	1
10	1	2
11	1	2

y'+0.2

x \ yz	0	1
00	1.2	1.2
01	1.2	1.2
10	0.2	0.2
11	0.2	0.2

x'+y+0.6

x \ yz	0	1
00	1.6	0.6
01	1.6	0.6
10	2.6	1.6
11	2.6	1.6

The function (table at right) can be written as
(x'+z)(y+z'+0.2)(x+y'+0.4)(x+y+z+0.4)(x'+y'+z'+0.7)
However we can also write it in more compressed form as
(x'+z)(y+z'+0.2)(x+0.4)(0.7)
Following method is used to reach at above. Here we have used the fact the minimum is taken in place of product.

x \ yz	0	1
00	0.4	0
01	0.2	0.2
10	0.4	0
11	0.4	0.7

x'+z

x \ yz	0	1
00	1	0
01	2	1
10	1	0
11	2	1

y+z'+0.2

x \ yz	0	1
00	1.2	1.2
01	0.2	0.2
10	2.2	2.2
11	1.2	1.2

x+0.4

x \ yz	0	1
00	0.4	1.4
01	0.4	1.4
10	0.4	1.4
11	0.4	1.4

0.7

x \ yz	0	1
00	0.7	0.7
01	0.7	0.7
10	0.7	0.7
11	0.7	0.7

The function (defined below) can be written as (y+z'+0.3)(x'+z'+0.3)(0.8). Those entries, which are bigger than 1 are not shown (because they will be suppressed)

The function

x \ yz	0	1
00	0.8	0.8
01	0.3	0.3
10	0.8	0.8
11	0.8	0.3

y+z'+0.3

x \ yz	0	1
00		
01	0.3	0.3
10		
11		

x'+z'+0.3

x \ yz	0	1
00		
01		0.3
10		
11		0.3

0.8

x \ yz	0	1
00	0.8	0.8
01	0.8	0.8
10	0.8	0.8
11	0.8	0.8

1. Let f(x,y,z)=67xy'z+75yz+16
   Fill the table corresponding to this function.

   ab is product of a and b. (6)(1)(3)=18
   a+b is max(a,b). 12+17+13=17.

x \ yz	0	1
00		
01		
10		
11		

2. Let f(x,y,z)=(x+y')(x+y+z'+0.6)
   Fill the table corresponding to this function.

   ab is minimum of a and b. (2.7)(1.1)(1.6)=1.1
   a+b is max(a,b). 12+17+13=17.

x \ yz	0	1
00		
01		
10		
11		

3. Let f(x,y,z)=(x'+0.7)(y+0.6)
   Fill the table corresponding to this function.

   ab is product of a and b. (2.7)(1.1)=2.97
   a+b is max(a,b). 12+17+13=17.

x \ yz	0	1
00		
01		
10		
11		

4. Let f(x,y,z)=(x'+0.7)(y+0.6)
   Fill the table corresponding to this function.

   ab is product of a and b (2.7)(1.1)=2.97
   a+b is sum of a and b 2.2+1.3=3.5

x \ yz	0	1
00		
01		
10		
11		

5. Let f(x,y,z)=0.66xy+8.3y'z
   Fill the table corresponding to this function.

   ab is minimum of a and b (2.7)(1.1)(1.8)=1.1
   a+b is maximum of a and b 1.8+2.2+2.3=2.3

x \ yz	0	1
00		
01		
10		
11		

6. Let f(x,y,z)=(0.96x+0.73y'z)(0.86)+0.51x'y
Fill the table corresponding to this function.

ab is minimum of a and b (2.7)(1.1)(1.8)=1.1
a+b is maximum of a and b 1.8+2.2+2.3=2.3

x \ yz	0	1
00		
01		
10		
11		

For following questions the answer of smallest size should be given. If there are more than one possible answers of same size then all of them should be given. Size of an expression is the number of multiplications (or min) and additions (or max) used.

7. Let the following function realizes the given table
87x'+75z+missing

What is missing (smallest size)?

ab is product of a and b. (27)(11)=297
a+b is maximum of a and b 22+13+19+11=22

x \ yz	0	1
00	87	61
01	87	75
10	87	76
11	87	75

8. Let the following function realizes the given table
(x'+y'+0.1)(w+0.11)(z+0.8)(0.9)(missing)

What is missing (smallest size)?

ab is minimum of a and b (7)(8)(5)=5
a+b is normal sum 22+13=35

xy \ zw	00	01	10	11
00	0.11	0.11	0.11	0.1
01	0.8	0.8	0.6	0.1
10	0.11	0.11	0.11	0.1
11	0.9	0.9	0.9	0.1

9. Let the following function realizes the given table
67x + 42y(1−w) + 37w + 19(1−x)(1−z) + missing

What is missing (smallest size)?

ab is normal product (7)(8)(5)=280
a+b is normal sum 22+13=35

xy \ zw	00	01	10	11
00	19	61	67	109
01	56	56	104	104
10	38	80	88	130
11	58	58	125	125

10. In the function, which realizes the given table,

What is coefficient of 55?(smallest size)

ab is normal product (7)(8)(5)=280
a+b is maximum of a and b 22+13+39+4=39
[Hint: The coefficient of 85 is x'yz']

xy \ zw	00	01	10	11
00	78	85	55	75
01	33	97	31	83
10	39	33	78	93
11	79	89	79	39