Language and Notation

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This is a compendium of things which we would normally explain in maths competitions for teenaged schoolchildren.

1. Established mathematical notation will be used.
2. We might use a colon or a vertical line as a separator in set notation so $\{x \mid x \in Z, x > 0\} = \{y\colon y \in Z, y > 0\}$.
3. Floor and ceiling notation, so for x a real number we let $\lfloor x \rfloor = \max\{z \mid x \in \mathbb{Z}, z \leq x\}$. Similarly $\lfloor x \rfloor = \min\{z \mid x \in \mathbb{Z}, z \geq x\}$.
4. Fractional part notation. If x is a real number, we define $\{x\}$ to mean $x - \lfloor x \rfloor$.
5. We write a line over a non-negative integer written in base 10 notation to indicate that it is being viewed as a string of digits rather than a number. Thus the second digit of $\overline{1729}$ is 7 but 1729 does not have a second digit because it is an integer.
6. We allow a phrase such as x is a 3-digit positive integer to mean that if written in Arabic notation as $x = a_m a_{m-1} \cdots a_1$ with a_i all digits and $a_m \neq 0$, then $n = m$. We allow 'the sum of the digits of n' to mean: write n in Arabic base 10 notation and then sum the digits.
7. We allow informal probability language such as: a point is chosen uniformly at random in the interval $[0, 1]$.
8. We use $\binom{n}{r}$ to denote the number of ways of choosing r things from n things.
9. The sum over the empty set is 0 and the product over the empty set is 1.
10. We use an ellipsis to denote an obvious pattern, either on the line of print of midline (as appropriate) so the set of the first n positive integers can be written $\{1, 2, \ldots, n\}$ and their sum is $1 + 2 + \cdots + n$.
11. For integers l, m, n then l^{m^n} denotes $l^{(m^n)}$.
12. $m^0 = 1$ for all integers m (including 0) if doing combinatorial enumeration. If x is real then x^0 needs to be clarified if $x = 0$.
13. British or American versions of English can be used. Thus "highest common factor" means the same as "greatest common divisor".
14. A prefix or subscript may be used to indicate features of a triangle associated with vertices. Thus triangle ABC has three altitudes, and the one dropped from A could be denoted the altitude through A, the A-altitude or the altitude h_a. Similarly for median lines.
15. If the term *natural number* is used, then it will be made clear if 0 is a natural number.
16. := means 'is defined to be equal to'.