Large Sample ($n > 30$)

Level of Significance	1%	5%
One-tailed Test	$z_{0.01} = 2.33$	$z_{0.05} = 1.645$
Two-tailed Test	$z_{0.01} = 2.58$	$z_{0.05} = 1.96$

Single Mean:

$$z = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Difference of Mean:

$$z = \frac{\overline{x}_1 - \overline{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Problems on Single Mean:

Problem 1.13:

A sample of 900 members has a mean 3.4 cm and standard deviation 2.61 cm. Is the sample from a large population of mean 3.25 cms and standard deviation of 2.61cms? (Test at 5% level of significance. The value of z at 5% level is $|z_\alpha| < 1.96$).

(A/M 2010)

Solution:

Given: $n = 900,\ \overline{x} = 3.4,\ s = 2.61$ and $\mu = 3.25$.

Null Hypothesis $H_0:\ \mu = 3.25$

Alternative Hypothesis $H_1:\ \mu \neq 3.25$ [two-tailed Test]

Test Statistic:

$$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{3.4 - 3.25}{2.61/\sqrt{900}} = \frac{0.15}{0.087} = 1.7241$$

The table value of z at 5% leve is 1.96.

Calculated Value < Table Value

Hence we accept the null hypothesis.

Problems on Difference of Means:

Problem 1.14:

The mean height of two large samples of 1000 and 2000 members are 67.5 inches and 68.0 inches respectively. Can the samples be regarded as drawn from the same populations of standard deviation 2.5 inches at 5% level of Significance?

(M/J 2012),(N/D 2018)

Solution:

Given: $n_1 = 1000,\ n_2 = 2000,\ \bar{x}_1 = 67.5,\ \bar{x}_2 = 68$

and $\sigma_1 = \sigma_2 = 2.5$.

Null Hypothesis $H_0 : \mu_1 = \mu_2$

Alternative Hypothesis $H_1 : \mu_1 \neq \mu_2$ [two-tailed Test]

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{67.5 - 68}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{2000}}} = \frac{-0.5}{\sqrt{0.00938}} = -5.163$$

$\Rightarrow |z| = 5.163$ (Calculated Value)

The table value of z at 5% leve is 1.96.

Calculated Value > Table Value

Hence we reject the null hypothesis.

Problem 1.15:

A mathematics test was given to 50 girls and 75 boys. The girls made an average grade of 76 with a SD of 6, while boys made an average grade of 82 with a SD of 2. Test whether there is any significant difference between the performance of boys and girls.
(N/D 2012),(M/J 2016)

Solution:

Given: $n_1 = 50,\ n_2 = 75,\ \bar{x}_1 = 76,\ \bar{x}_2 = 82,\ \sigma_1 = 6$ and $\sigma_2 = 2$.

Null Hypothesis $H_0:\ \mu_1 = \mu_2$ (There is no significant difference between girls and boys.)

Alternative Hypothesis $H_1:\ \mu_1 \neq \mu_2$ [two-tailed Test]

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{76 - 82}{\sqrt{\frac{(6)^2}{50} + \frac{(2)^2}{75}}} = \frac{-6}{\sqrt{0.7733}} = -6.823$$

$\Rightarrow |z| = 6.823$ (Calculated Value)

The table value of z at 5% leve is 1.96.

Calculated Value > Table Value

Hence we reject the null hypothesis.

Problem 1.16:

A random sample of 100 bulbs from a company P shows a mean life 1300 hours and standard deviation of 82 hours. Another random sample of 100 bulbs from company Q showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company P superior to bulbs of company Q at 5% level of significance? (N/D 2017)

Solution:

Given: $n_1 = 100,\ n_2 = 100,\ \bar{x}_1 = 1300,\ \bar{x}_2 = 1248,\ \sigma_1 = 82$ and $\sigma_2 = 93$.

Null Hypothesis $H_0 : \mu_1 = \mu_2$ (There is no significant difference between company P and Q.)

Alternative Hypothesis $H_1 : \mu_1 > \mu_2$ [one-tailed Test]

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{1300 - 1248}{\sqrt{\frac{(82)^2}{100} + \frac{(93)^2}{100}}} = \frac{52}{\sqrt{153.73}} = 4.194$$

$\Rightarrow |z| = 4.194$ (Calculated Value)

The table value of z at 5% leve is 1.645.

Calculated Value > Table Value

Hence we reject the null hypothesis.

Problem 1.17:

A simple sample of heights of 6400 Englishmen has a mean of 170 cm and a S.D of 6.4 cm, while a simple sample of 1600 Americans has a mean of 172 cm and a S.D of 6.3 cm. Do the data indicate that Americans are, on the average, taller than Englishmen at 1% level of significance.

Solution:

Given: $n_1 = 6400$, $n_2 = 1600$, $\bar{x}_1 = 170$, $\bar{x}_2 = 172$, $\sigma_1 = 6.4$ and $\sigma_2 = 6.3$.

Null Hypothesis $H_0 : \mu_1 = \mu_2$ (There is no significant difference between the heights of Englishmen and Americans.)

Alternative Hypothesis $H_1 : \mu_1 < \mu_2$ [one-tailed Test]

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{170 - 172}{\sqrt{\frac{(6.4)^2}{6400} + \frac{(6.3)^2}{1600}}} = \frac{-2}{\sqrt{0.031}} = -11.359$$

$\Rightarrow |z| = 11.359$ (Calculated Value)

The table value of z at 1% leve is 2.33.

Calculated Value > Table Value

Hence we reject the null hypothesis.

Problem 1.18:

The sales manager of a large company conducted a sample survey in two places A and B taking 200 samples in each case. The results were the following table. Test whether the average sales is the same in the 2 areas at 5% level. (N/D 2013)

	Place A	Place B
Average Sales	Rs. 2,000	Rs. 1,700
S.D	Rs. 200	Rs. 450

Solution:

Given: $n_1 = 200,\ n_2 = 200,\ \bar{x}_1 = 2000,\ \bar{x}_2 = 1700,\ \sigma_1 = 200$ and $\sigma_2 = 450$.

Null Hypothesis $H_0: \mu_1 = \mu_2$

Alternative Hypothesis $H_1: \mu_1 \neq \mu_2$ [two-tailed Test]

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{2000 - 1700}{\sqrt{\frac{(200)^2}{200} + \frac{(450)^2}{200}}} = \frac{300}{\sqrt{1212.5}} = 8.616$$

$\Rightarrow |z| = 8.616$ (Calculated Value)

The table value of z at 5% leve is 1.96.

Calculated Value > Table Value

Hence we reject the null hypothesis.

Problem 1.19:

Examine whether the difference in the variability in yields is significant at 5% level of significance, for the following.

(N/D 2010)

	Set of 40 plots	Set of 60 plots
Mean yield per plot	1256	1243
S.D. per plot	34	28

Solution:

Given: $n_1 = 40,\ n_2 = 60,\ \bar{x}_1 = 1256,\ \bar{x}_2 = 1243,\ \sigma_1 = 34$ and $\sigma_2 = 28$.

Null Hypothesis $H_0 :\ \mu_1 = \mu_2$

Alternative Hypothesis $H_1 :\ \mu_1 \neq \mu_2$ [two-tailed Test]

Test Statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$$

$$z = \frac{1256 - 1243}{\sqrt{\dfrac{(34)^2}{40} + \dfrac{(28)^2}{60}}} = \frac{13}{\sqrt{41.97}} = 2.01$$

$\Rightarrow |z| = 2.01$ (Calculated Value)

The table value of z at 5% leve is 1.96.

Calculated Value > Table Value

Hence we reject the null hypothesis.