Question 1

A football coach is frustrated with his team's lack of speed. He measures each player's 40-yard dash speed and then sends all of them to a speed and agility camp. He then measures their times again after. The data is below. Is there sufficient evidence to say that the camp helped the players speed? Run a test.

Before	After
4.88	4.7
5.1	4.85
4.41	4.35
4.73	4.77
4.6	4.56
4.8	4.78
4.95	4.7
4.98	4.9
5.2	5.0
5.13	5.1
5.05	5.1
4.9	4.7
4.7	4.56
4.6	4.34
5.11	4.9

a) Write an appropriate hypothesis test for this situation and state the appropriate testing procedure

Hypothesis

Null Hypothesis H_0: The mean difference in time before the camp and after the camp is zero. i.e.; μBefore - μAfter =0

Alternate hypothesis H_1: The mean difference in time before the camp and after the camp is greater than zero. i.e., μBefore - μAfter >0

Correlated or Paired t-test

A paired or correlated t-test is used in the case of matched pairs of similar units or when there are cases of repeated measures.

Here the player's 40-yard dash speed before and after attending a speed and agility camp is measured. Hence paired t-test is used.

b) Compute the necessary summary statistics for the test in part (a)

1. Subtract each After(Y) from each Before(X)

Before(X)	After(Y)	X-Y
4.88	4.7	0.18
5.1	4.85	0.25
4.41	4.35	0.06
4.73	4.77	-0.04
4.6	4.56	0.04
4.8	4.78	0.02
4.95	4.7	0.25
4.98	4.9	0.08
5.2	5.0	0.2
5.13	5.1	0.03
5.05	5.1	-0.05
4.9	4.7	0.2
4.7	4.56	0.14
4.6	4.34	0.26
5.11	4.9	0.21

2. Add up all values from step 1

Before(X)	After(Y)	X-Y
4.88	4.7	0.18
5.1	4.85	0.25
4.41	4.35	0.06
4.73	4.77	-0.04
4.6	4.56	0.04
4.8	4.78	0.02
4.95	4.7	0.25
4.98	4.9	0.08
5.2	5.0	0.2
5.13	5.1	0.03
5.05	5.1	-0.05
4.9	4.7	0.2
4.7	4.56	0.14
4.6	4.34	0.26
5.11	4.9	0.21
	Sum:	1.83

3. Square the difference from step 1

Before(X)	After(Y)	X-Y	$(X-Y)^2$
4.88	4.7	0.18	0.0324
5.1	4.85	0.25	0.0625
4.41	4.35	0.06	0.0036
4.73	4.77	-0.04	0.0016
4.6	4.56	0.04	0.0016
4.8	4.78	0.02	0.0004
4.95	4.7	0.25	0.0625
4.98	4.9	0.08	0.0064
5.2	5.0	0.2	0.04
5.13	5.1	0.03	0.0009
5.05	5.1	-0.05	0.0025
4.9	4.7	0.2	0.04
4.7	4.56	0.14	0.0196
4.6	4.34	0.26	0.0676
5.11	4.9	0.21	0.0441
	Sum:	1.83	

4. Add up all of the squared differences from Step 3

Before(X)	After(Y)	X-Y	$(X-Y)^2$
4.88	4.7	0.18	0.0324
5.1	4.85	0.25	0.0625
4.41	4.35	0.06	0.0036
4.73	4.77	-0.04	0.0016
4.6	4.56	0.04	0.0016
4.8	4.78	0.02	0.0004
4.95	4.7	0.25	0.0625
4.98	4.9	0.08	0.0064
5.2	5.0	0.2	0.04
5.13	5.1	0.03	0.0009
5.05	5.1	-0.05	0.0025
4.9	4.7	0.2	0.04
4.7	4.56	0.14	0.0196
4.6	4.34	0.26	0.0676
5.11	4.9	0.21	0.0441
		Sum= 1.83	Sum = 0.3857

5. Calculate t- score using the formula:

$$t = \frac{\sum d}{\sqrt{\frac{n(\sum d^2)-(\sum d)^2}{n-1}}}$$

$\sum d = 1.83$, $n=15$, $\sum d2 = 0.3857$

on substituting values on the equation, we get t=

$$1.83/0.41719$$

$$t = 4.387$$

6. Degrees of freedom = df = n-1 = 15-1 =14

c) Perform the t-test and report the p-value.

7. Find the p-value in the t-table using the degree of freedom and alpha value

t-test table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

- t-value

Significance level (α) = 0.05
Degree of freedom (df) = n-1
= 15 - 1
= 14

From the t-test table,
t- value = 1.761
Calculated t-value = 4.387

The calculated t-value is greater than the table value at an alpha level of 0.05

- P-value

P-value = 0.0003103
= 3.103 x 10^4

P-value is less than the alpha level $p < 0.05$

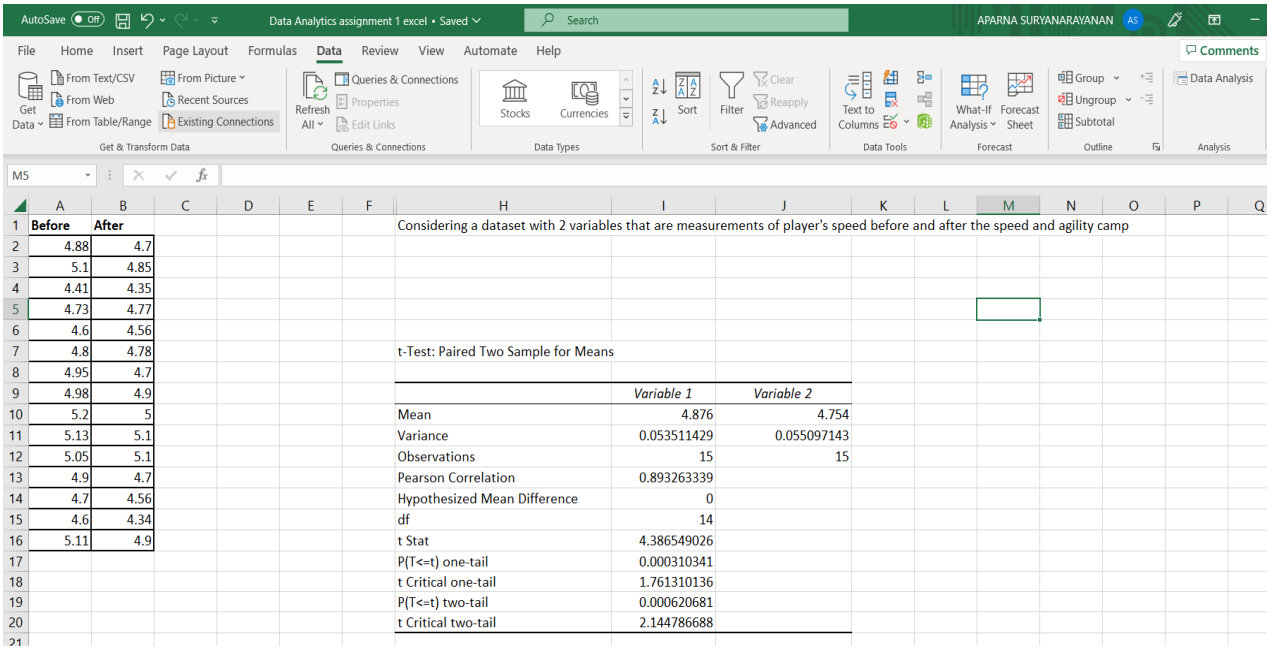
d) Interpret your results in the conclusion

Conclusion:

The calculated t-value is greater than the table value at the alpha level of 0.05
The p-value is less than the alpha; $p < 0.05$

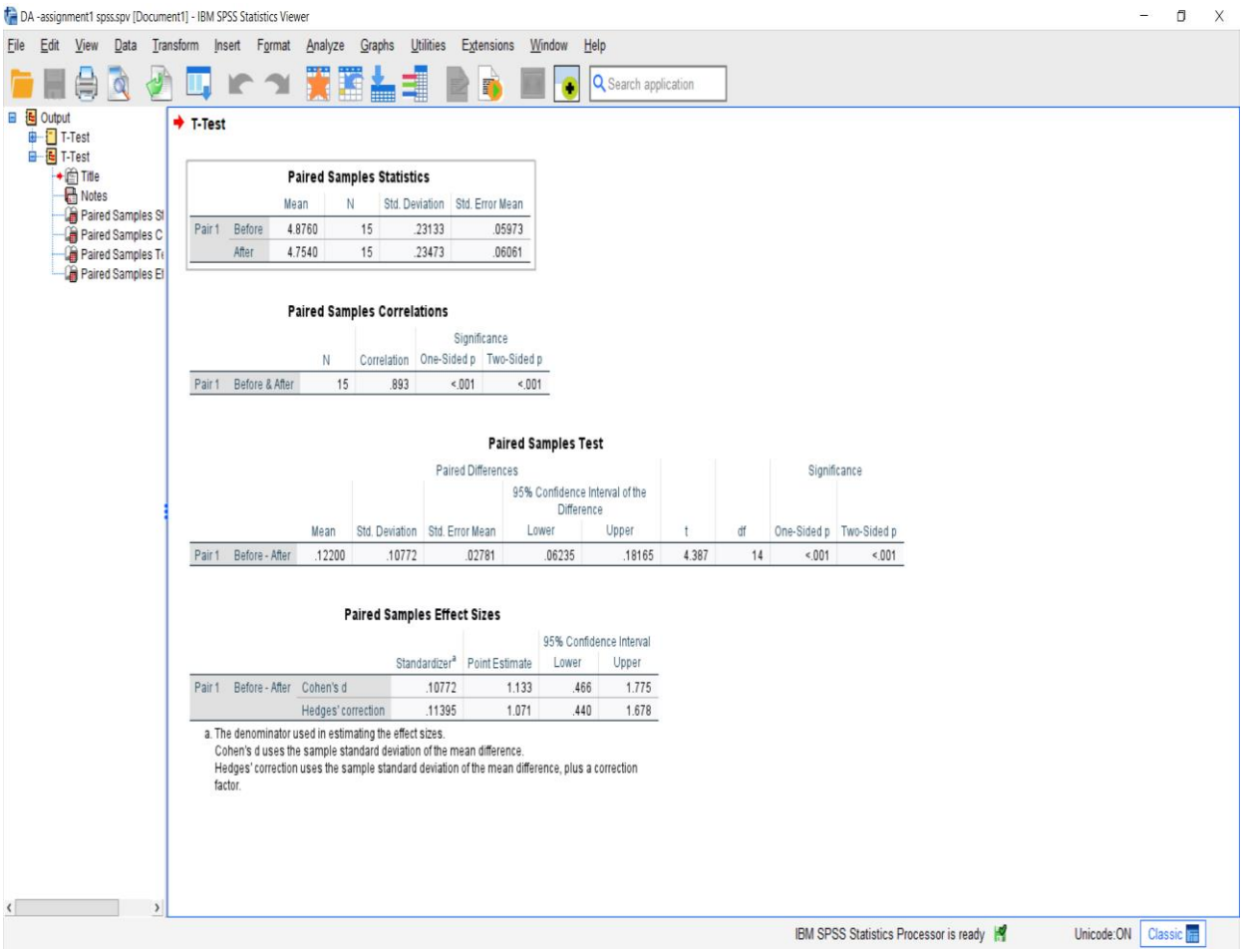
Since the p-value is less than alpha ($3.103 \times 10^4 < 0.05$) we reject the null hypothesis. In conclusion, we found that the mean difference in times from before the camp to after the camp has decreased and that the camp has helped the player's speed.

Calculation using Excel



	A	B	H	I	J
1	**Before**	**After**	Considering a dataset with 2 variables that are measurements of player's speed before and after the speed and agility camp		
2	4.88	4.7			
3	5.1	4.85			
4	4.41	4.35			
5	4.73	4.77			
6	4.6	4.56			
7	4.8	4.78	t-Test: Paired Two Sample for Means		
8	4.95	4.7			
9	4.98	4.9		*Variable 1*	*Variable 2*
10	5.2	5	Mean	4.876	4.754
11	5.13	5.1	Variance	0.053511429	0.055097143
12	5.05	5.1	Observations	15	15
13	4.9	4.7	Pearson Correlation	0.893263339	
14	4.7	4.56	Hypothesized Mean Difference	0	
15	4.6	4.34	df	14	
16	5.11	4.9	t Stat	4.386549026	
17			P(T<=t) one-tail	0.000310341	
18			t Critical one-tail	1.761310136	
19			P(T<=t) two-tail	0.000620681	
20			t Critical two-tail	2.144786688	

Calculation Using SPSS



T-Test

Paired Samples Statistics

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before	4.8760	15	.23133	.05973
	After	4.7540	15	.23473	.06061

Paired Samples Correlations

		N	Correlation	Significance One-Sided p	Significance Two-Sided p
Pair 1	Before & After	15	.893	<.001	<.001

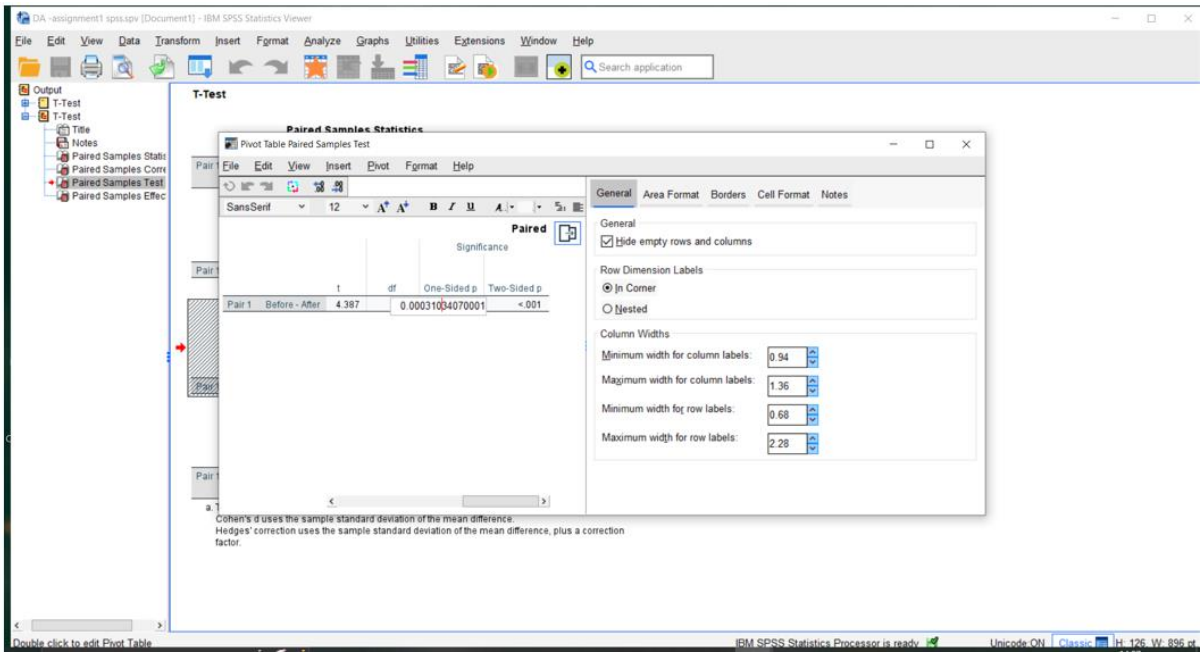
Paired Samples Test

		Paired Differences Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Significance One-Sided p	Two-Sided p
Pair 1	Before - After	.12200	.10772	.02781	.06235	.18165	4.387	14	<.001	<.001

Paired Samples Effect Sizes

			Standardizer[a]	Point Estimate	95% Confidence Interval Lower	Upper
Pair 1	Before - After	Cohen's d	.10772	1.133	.466	1.775
		Hedges' correction	.11395	1.071	.440	1.678

a. The denominator used in estimating the effect sizes.
Cohen's d uses the sample standard deviation of the mean difference.
Hedges' correction uses the sample standard deviation of the mean difference, plus a correction factor.



Question 2

The distribution of scores of students taking the LSATs is claimed to have a mean of 521. Sample 25 incoming Harvard Law School freshman LSAT scores and find a mean of 589 and a standard deviation of 37. Since Harvard is an Ivy League school, they think their freshmen are smarter than average law students. Test this theory by applying a suitable hypothesis test (that Harvard students score higher than average on the LSATs) at the 0.05 significance level.

Use one sample t test

1. The null hypothesis $H_0: \mu = 521$ The mean LSATS for Harvard freshman is 521.
2. Alternate hypothesis $H_1: \mu > 521$ The mean LSATS for Harvard freshman is greater than 521
3. Identify Statistical values

Summary statistics for the test

$$\text{mean } \mu = 521$$
$$\bar{x} = 589$$

Standard deviation s = 37
Significance level α = 0.05
n = 25
Degree of freedom df = n-1
= 25-1
=24

4. Calculate t-score

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$= (589 - 521) / (37 / \sqrt{25})$

$= 68/7.4$

$= 9.1891$

Calculated t-value $= 9.1891$

5. Find t- value from the table

t-value from table = 1.711

t-test table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										

Conclusion:

Calculated t -value > table t-value at significance level 0.05.
Therefore, we reject the null hypothesis. We can conclude that the mean LSATS for Harvard freshman is higher than the average LSATS.