Write a wise saying and your name will live forever.

— Anonymous

6 Discussion

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CORRECTION Discussion re-written to reflect new results in Chapter 5

This thesis had the aim of developing novel strategies for the statistical analysis of rare genetic variants in context of complex disease research. In the time since I began working towards this broadly defined goal, many major milestones have been achieved in human genetic research, in particular due to (still ongoing) advancements in high-throughput sequencing technologies. This facilitated research on a previously unseen scale and offered a wide range of opportunities to be explored.

One major achievement of human genetic research is seen in the success of genome-wide association (GWA) studies which, within less than a decade, have identified thousands of loci associated to a large number of disease phenotypes. At that time, not much was known about the role of rare variants in complex disease, wherein I saw an opportunity to contribute to current (and highly dynamic) research. I became interested in the development of novel approaches that would allow researchers to harness the growing amount of genomic data to find rare allele associations to disease risk.

The recent past has also seen a major change in the division between areas of applied medical research and the traditionally more theoretical field of population genetics. The line between those fields became more blurred, such that it could be argued that it was never there to begin with. The discoveries made in the human genome, in particular through the Human Genome Project (International Human Genome Sequencing

Consortium, 2001, 2004), have necessitated a deeper understanding of how genomic variation as a consequence of demographic and evolutionary forces contributes to variation of phenotypic traits. I became fascinated by the possibility to use genomic data to draw inferences about past historic events in populations, and to further use such insights to make predictions about the effects on disease traits. In that regard, I intended to focus on rare variants which, due to their presumed recent origin through mutation, could be seen as a source of information about the recent past. My interest grew into what now covers the major parts of this thesis.

6.1 Summary and main conclusions

The following main directions can be distinguished in this thesis. First, I attempted to develop a computational method to integrate different sources of data, so as to increase the number of rare variants that can be interrogated in association analysis. Second, a large part of this thesis is concerned with methods development to detect shared haplotype segments that are inherited from a common ancestor. Lastly, my primary goal was to develop the methodology and statistical theory required to computationally estimate the age of an allele observed at a single locus. In the following, I briefly review the methodology I developed and give a summary of the main ideas, findings, and conclusions.

6.1.1 Imputation and association analysis or low-frequency alleles

In Chapter 2, I attempted to develop a method to combine independent genomic datasets through genotype imputation as a way to improve statistical power to implicate rare variants in GWA studies. Rare variants are less likely to exert high risk effects on complex disease phenotypes, or otherwise could be picked up through conventional linkage analysis. Variants at lower frequencies may nonetheless play a role in disease aetiology, but are often highly population or cohort specific. Imputation from a single reference panel may therefore not produce genotype data where low-frequency alleles would be likely to be implicated in disease risk through association tests.

I developed a computational tool (*meta-imputation*) to combine genotype data obtained in separate imputations from independent reference panels. The intuition was that the information contained in different studies could be leveraged by bringing these datasets together through imputation into the same study sample. I thereby attempted to capture variants at lower frequencies or those specific to one or few source panels. The method produces a single, canonical genotype dataset that can be used in subsequent association analyses.

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The method was evaluated, first, in terms of the achieved genotype accuracy, which was compared to each of the separately imputed datasets that were previously combined through meta-imputation. An second evaluation was performed through a series of simulated GWA studies. This, however, turned out to be particularly challenging due to substantial computational demands. Thousands of simulated datasets had to be generated, which were used as sample for imputations from multiple reference panels, such that each dataset was treated as an independent GWA study. The main findings are summarised as follows.

- I showed that the method was able to improve overall genotype accuracy when data imputed from multiple studies were combined.
- The accuracy of meta-imputed genotypes improved notably in cases where data from more diverse population groups were available.
- Statistical power in association analysis was increased at low-frequency variants with intermediate or high penetrance, but where differences seen for common risk variants were negligibly small.
- Rare variants with low penetrance were unlikely to reach statistical significance.

Note that I used Phase I data from the 1000 Genomes Project (1000G). Shortly after my work on this project was completed, 1000G Phase III was released. I decided not to repeat the analysis conducted for the evaluation of meta-imputation with the newly released dataset, because the larger sample size and higher marker density would entail that the time and computational load needed to repeat the analysis would be too prohibitive.

In light of the imputation reference panel that became available through the Haplotype Reference Consortium (HRC), see McCarthy *et al.* (2016), potential applications of the meta-imputation method may be limited to specific niche problems. For example, for populations that are currently underrepresented in the HRC panel, or when novel datasets become available that are more suited for imputations into a given study cohort than the HRC alone.

6.1.2 Shared haplotype inference around rare variants

Recent advancements in high-throughput sequencing technologies have enabled the study of the rare variation present in the human genome. It is assumed that frequency is indicative for the time since an allele was created through a mutation event. Rare alleles are therefore likely to indicate recent relationships among haplotypes, which is informative about recent demographic history and population structure, within and among populations. Given deeper insight into the sharing structure of alleles, it would be possible, for example, to infer the timing of demographic events. Such inferences could be useful to make further statements with regard to selection or other forces that shaped the genetic variation observed in a population.

In Chapter 3, an initial attempt was made to develop deterministic (rule-based) ways to utilise rare variants as "bookmarks" in the underlying genealogy, to identify haplotypes that recently derived from a common ancestor. I presented two approaches that are based on the four-gamete test (FGT) after Hudson and Kaplan (1985), through which the "breakpoint" of a recombination event along the sequence can be detected. While the FGT is based on observing specific allelic configurations in haplotype data, its simplified form, the discordant genotype test (DGT), requires only genotype data. I developed a computational tool (tidy) to detect breakpoint intervals around a given focal site in pairs of diploid individuals that carry the focal allele. The resulting interval is assumed to indicate the underlying shared haplotype segment that was inherited "identical by descent" (IBD) in each pair sharing the focal allele.

In Chapter 4, this idea was extended using a Hidden Markov Model (HMM) that operates on genotype data; referred to as HMM_g below. I determined genotype error rates in different sequencing and genotyping datasets, which I used to modify simulated data to perform subsequent evaluations of the developed detection methods in realistic settings. Error rates were also used to construct an empirical emission model to make the HMM robust towards error in applications to real data. I employed a similar approach in a novel haplotype-based HMM, presented in Chapter 5 and referred to as HMM_h below.

I assessed the performance of the detection methodology developed in this chapter in comparison to an existing, probabilistic method for IBD detection (*Refined IBD*), in simulated data with and without empirically measured distributions of data error, as well as in real data. In summary, the main findings are as follows.

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• The FGT generally achieved high levels of accuracy when used to detect breakpoints around rare variants (allele frequency ≤ 0.5%) in simulated data. The DGT performed similarly well in simulated settings, but resulted in an overall reduced precision of the detected breakpoints. Importantly, both showed a tendency to overestimate the actual shared haplotype interval.

- I showed that the accuracy of the FGT decreased when haplotypes were computationally phased (inferred from genotype data), but where the impact of phasing error was low on average. Note that genotype-based approaches (DGT and HMM_g) are insensitive towards phasing error.
- When data error was considered in simulations, segment detection using either the FGT or DGT resulted in vastly underestimated breakpoint intervals. I showed that segment truncation was due to the detection of false positive breakpoints, caused by a relatively small proportion of misclassified alleles or genotypes at single loci. I further showed that existing (probabilistic) methods may also be adversely affected by data error. Similar patterns were seen in applications to real data. Thus, I concluded that the deterministic approaches developed in this thesis (FGT and DGT) were unreliable to indicate the extent of actual shared haplotype regions.
- In simulated settings without considering data error, I showed that HMM_g outperformed the DGT, but performed less well compared to the FGT in terms of the precision to detect breakpoint locations. But both HMM_g and HMM_h were robust in presence of realistic distributions of data error, where segment length obtained in analyses of real data followed expected patterns.
- A main finding was that HMM_h was able to outperform all previous methods for segment breakpoint detection developed in this thesis. This was seen in simulated settings with and without consideration of data or phasing error, and was likewise suggested in applications to real data.

In conclusion, my work on shared haplotype inference has led to the development of HMM_h as the result of a steady progression to improve the methodology as a consequence of the faults and caveats seen through extensive evaluation of different approaches. However, the method employs an empirical emission model that was constructed on the basis of genotype error measured in the 1000 Genomes Phase III dataset, and was further constrained to consider relatively recent relationships. It is therefore implicitly assumed that the actual shared haplotype around a given focal site is "younger" than the immediately next segment along the sequence. While this may apply to the majority of rare variants, this assumption is less likely to be satisfied for "older" relationships, e.g. variants observed at higher frequencies.

6.1.3 Allele age estimation

Knowing the age of an allele would allow us to observe human evolutionary history back in time, to infer past demographic events and processes that resulted in the observed genetic variability in a population. The method for allele age estimation was presented in Chapter 5, which concluded this thesis.

Age is estimated for a given allele at a single locus as observed in the sample. This is done in a Bayesian setting in which a composite posterior distribution is computed from posterior probabilities of the time of coalescent events between pairs of haplotypes. According to observed allele sharing in the sample, a number of "concordant" and "discordant" haplotype pairs are formed and analysed in turn to obtain pairwise posteriors of the time to the most recent common ancestor (T_{MRCA}). These are used to indicate the time of coalescent events that delimit the branch on which the mutation event occurred, from which the focal allele derived. The age of an allele can be estimated directly from the resulting composite posterior distribution.

I provided the definitions for several models to obtain posterior densities from pairwise observations of mutational differences or recombinational length, where prior expectations were based on coalescent theory. As a result, the following "clock" models were defined; mutation clock (\mathcal{T}_M), recombination clock (\mathcal{T}_R), and combined clock (\mathcal{T}_M).

The methodology to detect shared haplotype segments around a given focal site, as developed in this thesis, was essential for the application of the age estimation method. This is because the observed values of the parameters specified by the clock models are determined from the data with respect to the shared haplotype structure inferred at a focal site in the sample. Note that this thesis had a strong emphasis on identity by descent (IBD) and shared haplotype inference for this reason. I implemented the age estimation method with the different segment detection methods in a computational tool (rvage) written in C++. In summary, the main findings are listed below.

- In a simulated setting when the actual shared haplotype structure at a focal site is known, I showed that allele age can be estimated with high accuracy, confirming the theoretical validity of the method.
- When shared haplotypes were inferred to subsequently estimate allele age based on observed parameters, haplotype-based detection methods generally outperformed genotype-based approaches in ideal simulated settings. The impact of phasing error was again seen as being low on average.

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• In simulations where data error was considered to facilitate realistic evaluations, I showed that using the deterministic detection approaches (FGT and DGT) resulted in very high estimation bias, such that allele age could not be estimated reliably, *e.g.* when applied to real data.

- Although both HMM_g and HMM_h were previously seen as being robust when applied in realistic simulated settings, I showed that HMM_g resulted in an increased bias to underestimate allele age. In contrast, I found that estimation bias was low when HMM_h was used, and that correlations with measures of the actual time of underlying mutation events were high overall. This suggested that allele age can be estimated reliably when HMM_h is applied to real data.
- Using HMM_h, I found that the detection of shared haplotype segments was far less biased when concordant pairs (carrier haplotypes) were analysed compared to discordant pairs (carrier and non-carrier haplotypes), where the inference of T_{MRCA} was similarly affected. However, I found that the reduced accuracy for discordant pairs had a relatively low impact on the subsequent estimation of allele age.
- I additionally assessed the accuracy of the inference of pairwise T_{MRCA} using HMM_h in comparison to posterior probabilities obtained with the Pairwise Sequentially Markovian Coalescent (PSMC) model. I showed that HMM_h performed similarly well to infer T_{MRCA} , and that estimation bias of allele age was low for both. However, I found that PSCM achieved higher levels of accuracy when analysing discordant pairs compared to HMM_h under each clock model, but where bias was nonetheless increased compared to inferences at concordant pairs. Note that this was seen in a simulated setting with known demographic parameters, and without considering data error.
- Although HMM_h is implicitly biased due to its use of an empirical emission model
 constrained to relatively recent relationships between haplotype pairs, I found that
 it nonetheless performed well in age estimation of alleles that derived from "old"
 mutation events or that occurred at higher frequency in the sample.
- Lastly, I showed that each clock model performed well to estimate allele age, but where \mathcal{T}_{MR} generally achieved lower estimation bias and showed higher correlation with measures of the actual mutation time.

228 6.2. Future directions

While the initial focus of this work was on rare variants, it would be useful to better assess the possibility of applying HMM_h to age estimation of alleles deriving from relatively old mutation events. The inference of shared haplotype intervals and T_{MRCA} was shown to be problematic for discordant haplotype pairs. Although this was found to have less impact on the estimation of allele age, compared to bias due to incorrect inferences at concordant pairs, it is a main caveat of the current implementation. I discuss future directions arising from the allele age estimation method in the section below.

6.2 Future directions

The allele age estimation method presented in Chapter 5 can be regarded as the main result of this thesis. As the application of this method is dependent on approximations to the underlying genealogy at a given site in the genome, the presented methodology for shared haplotype inference is of equal importance. Further development of such methods could therefore be seen as a goal for future work. For example, I used the PSMC model to achieve similar results, but where HMM $_h$ has the advantage of being relatively fast such that an analysis of hundreds of thousands of haplotype pairs remains computationally tractable.

A possible enhancement would be to substitute the empirical emission model with a theoretical one, in which the probability of observing allelic combinations along the sequence are modelled by their expectations, given an (approximate) expectation of the T_{MRCA}. However, the division between *ibd* and *non* as hidden states remains arbitrary and leads to the implicit assumption that genealogies outside the genealogy at the focal site indicate a far older relationship between the haplotype pair. More complex models could be devised, for example based on the PSMC model, but where computation time should be considered to maintain practical scalability.

In the introduction to this thesis, I advocated a variant-centric view of genetic variation. By looking at the genealogy at a particular site in the genome, it would be possible to arrive at conclusions about the evolutionary and demographic influences that have resulted in the observed allele distribution and shape of the genealogy. Since the genealogy can only be known through statistical inference, the allele age estimation method along with the methods to infer the shared haplotype structure at a particular site may provide the necessary toolset to make significant discoveries. Future research will show to what extent the methodology developed in this thesis is useful.

The key test for an acronym is to ask whether it helps or hurts communication.

— Elon Musk

Abbreviations

1000G 1000 Genomes ProjectDGT Discordant genotype test

FGT Four-gamete test

GWA Genome-wide associationHMM Hidden Markov Model

HRC Haplotype Reference Consortium

IBD Identity by descent

PSMC Pairwise Sequentially Markovian Coalescent T_{MRCA} Time to the most recent common ancestor

My definition of a scientist is that you can complete the following sentence: 'he or she has shown that ...'

- E. O. Wilson

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