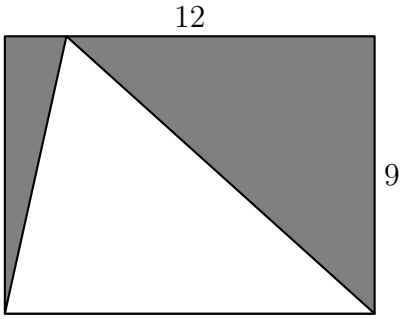
Art of Problem Solving

2017 Halloween AMC 8

Mock by techguy2, anser, popper224, md123, and SpinTop. Submit answers at http://math.samyok.us.

1. What is the area of the shaded region if the dimensions of the rectangle are 9 and 12?

(A) 48 **(B)** 54 **(C)** 60 **(D)** 66 **(E)** 72



2 The temperature of Spookyville is twice the temperature of Chillytown. If Chillytown is 32° Fahrenheit warmer than Spookyville, what is the temperature of Chillytown in Fahrenheit?

(A) -64 **(B)** -32 **(C)** 0 **(D)** 32 **(E)** 64

3 What is the value of $50 - 1 + 49 - 2 + ... + 27 - 24 + 26 - 25$?

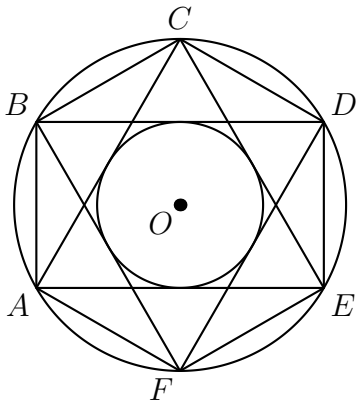
(A) 500 **(B)** 575 **(C)** 600 **(D)** 625 **(E)** 1225

4 On Halloween night, Avery trick-or-treated from 7:37 pm to 8:57 pm and received 96 pieces of candy. Joseph trick-or-treated for only half an hour, but his candy-receiving rate was equal to Avery's. How many pieces of candy did Joseph receive? **(A)** 18 **(B)** 24 **(C)** 36 **(D)** 48 **(E)** 72

5 Wendy the Witch spilled potion on her pumpkin pie, causing the whipped cream to expand. If the magical whipped cream expands at a rate of 3 millimeters per minute, how many hours will it take for the whipped cream to expand 3.6 meters?

(A) 12 **(B)** 15 **(C)** 18 **(D)** 20 **(E)** 24

6 Find the ratio of the area of the small circle to the area of the big circle such that:
The small circle is inscribed in the small hexagon
The big hexagon is formed by extending two sides of the small hexagon until they meet
The point where they meet is a vertex of the big hexagon
The big hexagon is inscribed in the big circle.



(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{6}$

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7 Define the operation $\diamond$ as summing the digits of the numbers on both sides and multiplying the sums. For example, $12 \diamond 14 = (1+2) \times (1+4) = 15$. What is the value of $(((((2 \diamond 81) \diamond 81) \diamond 81) \diamond 81) \diamond 81)$?

(A) 2 **(B)** 9 **(C)** 18 **(D)** 81 **(E)** 162

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8 A mummy costume originally cost \$25. When Halloween was approaching, the price increased by 20%. After Halloween, the price decreased by 40

(A) \$12 **(B)** \$15 **(C)** \$16 **(D)** \$18 **(E)** \$20

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9 Given that Halloween is always on October 31st and that Halloween in 2017 was on Tuesday, what day of the week was Halloween in 2000 on?

(A) Tuesday **(B)** Wednesday **(C)** Thursday **(D)** Friday **(E)** Saturday

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10 Raven cuts out vampire teeth from a piece of paper. All teeth are symmetrical. The innermost four teeth are congruent with all sides of length 1 cm and three

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right angles. The outermost two teeth are congruent with two right angles, three sides of length 1 cm, and the line of symmetry of length 3 cm. What is the area of the vampire teeth?

(A) 8cm^2 **(B)** 9cm^2 **(C)** 10cm^2 **(D)** 11cm^2 **(E)** 12cm^2

11 If

$$\begin{aligned} g+h&=o \\ o+u&=l \\ l+g&=s \\ h+u+s&=14 \end{aligned}$$

find $g+h+u$.

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

12 Rectangles that have integer side lengths and a perimeter of 12 inches are called *spine-tingling*. What is the sum of the areas of all distinct *spine-tingling* rectangles?

(A) 6 in^2 **(B)** 17 in^2 **(C)** 22 in^2 **(D)** 144 in^2 **(E)** 161 in^2

13 A spherical apple that has a diameter of 4 inches is covered in $\frac{93\pi}{2}$ cubic inches of caramel, spread evenly over the apple's exterior. When the caramel dries, a delicious treat is formed. What is the radius (in inches) of this new treat?

(A) 1.5 **(B)** 2 **(C)** 2.5 **(D)** 3 **(E)** 3.5

14 Allison and Bob each have 6 raffle tickets. There are 4 prizes, and each one has its own bucket. Alison wants to place all her tickets into the bucket for the pirate costume, but Bob wants to place two tickets into each one except for the pirate costume. There are already 6 tickets in each bucket, and Alison and Bob are the last ones to place tickets. If the probability that Alison gets exactly one prize is a, and the probability that Bob gets exactly one prize is b, what is $|a-b|$?

(A) $\frac{5}{64}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{9}{32}$ **(D)** $\frac{23}{64}$ **(E)** $\frac{27}{64}$

15 Elizabeth originally had a two-digit number of candies. When she split the candies into piles of 2, 1 candy was left over and when she split the candies into piles of 5, 2 candies were left over. Then, 9 children came and Elizabeth gave each child an equal number of candies. Afterwards, Elizabeth counted that she had 23 candies remaining. How many pieces did she give to each child?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

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16 Tootsie and Autumn are playing the classical game of phantom potion pumpkin. Phantom beats potion, potion beats pumpkin, and pumpkin beats phantom. If they throw the same object, then the result is a tie and they throw again. However, Tootsie considers pumpkins unlucky so she never throws pumpkin and Autumn never throws potion. They will both throw an object randomly. What is the probability that Tootsie will win the game?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{3}{4}$

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17 How many ordered pairs of integers (a, b) exist such that $a^2 \cdot b^2 = 144$?

(A) 4 **(B)** 6 **(C)** 12 **(D)** 18 **(E)** 24

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18 Jack and Jill are planning to walk up the steepest, scariest hill in town to the dark eerie mansion on top. Jack and Jill will arrive on the creaky front porch at a random time between 8:00 pm and 8:20 pm independently of the other. Both of them will wait for 10 minutes on the creaky front porch. If at any time they are both on the creaking front porch, they will ring the doorbell together. What is the probability that they ring the doorbell of the dark eerie mansion together?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

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19 In the library there are witches, stools, and black cats. Witches have 2 legs, stools have 3 legs, and black cats have 4 legs. If the sum of the number of witch legs and stool legs is 20, the sum of the number of stool legs and cat legs is 32, and the sum of the number of witch legs and cat legs is 28. What is the sum of the number of witches, stools, and black cats in the library?

(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

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20 Frank N. Stein and the other seven members of his Halloween Club are each going to contribute money to buy a new inflatable zombie. Frank doesn't know exactly how much his share is, but he is feeling very generous and gives a \$50 bill to the treasurer, Fan Tom, and tells him to keep the change, if there is any. The other seven members are going to pay an equal amount. After some calculations, Fan figures out that with Frank donating \$50, more than his required share, everyone else can pay \$5 less and the club will still have exactly the amount needed to buy the inflatable zombie. What is the total amount of money needed to buy the inflatable zombie?

(A) 10 **(B)** 15 **(C)** 20 **(D)** 120 **(E)** 140

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21 At S&S's Spooooky Sweets Shop, there are 2 types of candy corn, 4 flavors of taffy, 3 flavors of gummy bears, and 3 flavors of lollipops. The Special Halloween Combo Basket includes exactly one type of Candy Corn, zero, one, or two different flavors of Taffy, exactly two flavors of gummy bears, and any number of lollipop flavors (from 0 to 3). How many combo baskets can be made that satisfy the Special Halloween Combo Basket?

(A) 72 **(B)** 108 **(C)** 288 **(D)** 480 **(E)** 528

22 Reese looks at the ingredient list of the peanut butter cup she is eating. Every ingredient has a whole number percent next to it, there are no percents that are equal, and they are ranked from the largest percent to the smallest percent. She notices peanuts is listed as the second ingredient making up 20 percent of the peanut butter cup. Let a be the maximum possible number of ingredients in the chocolate and let b be the minimum possible number of ingredients in the chocolate. What is $a - b$?

(A) 10 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 14

23 Team Plants and Team Zombies are the last teams standing in a volleyball tournament. The championship match consists of at most 5 games and will end when a team wins 3 games. Team Plants and Team Zombies have an equal chance of winning each game. Given that Team Plants won the fourth game, what is the probability that Team Plants won the tournament?

(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

24 Three skeletons are walking up a hill in a search for gold locks. Baby Skeleton decides to "bounce" from Mama Skeleton to Papa Skeleton the whole time. Mama and Papa each walk at a rate of 5 feet per second. Baby can run downhill from Mama to Papa at a rate of 15 feet per second and can walk uphill at 7.5 feet per second. If Mama and Papa start 50 feet apart and Baby starts next to Papa, how many seconds will it take for Baby to travel 200 feet?

(A) $23\frac{1}{10}$ **(B)** $23\frac{2}{3}$ **(C)** $24\frac{1}{10}$ **(D)** $24\frac{2}{3}$ **(E)** $25\frac{1}{3}$

25 16 delectable candies are placed in a circle and numbered from 1 to 16 in a clockwise fashion. Your method of eating all 16 candies is to pick a candy to eat first, and then eat every third candy traveling counterclockwise. Your favorite candy is candy#3, so you will eat it last. What is the sum of the number on first candy you should eat and the number on the candy eaten second to last?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18