Q 1. Coca Cola and Pepsi are fighting to becomes America's most favourite beverage hence to settle their dispute they hire you. Your job is to conduct a survey and report which is the favourite beverage .They survey will determine that if you pick a random guy up what are the chances he prefer Coke or Pepsi.(Notice this is a Bernoulli Distribution). The company have told you that they are fine with an error of .05 in your estimation of probability but need a confidence of more than 95%.

(a) Determine the number of people you need to conduct the survey on to satisfy the companies demands? (You should produce the proof of why you are giving this number.)

(b) Can you some how decrease the number of people on which you have to conduct the survey. Try using CLT (Central Limit Theorem).

Q 2. More about MGF
We know that MGF of a distribution is unique . Derive the following qualities.

(a) Derive the MGF of Normal distribution X $\sim N(\mu, \sigma^2)$
First assume μ, σ^2 to $\in R$ then prove the general case $\mu \in R^d$ and $\sigma^2 \in M_d(R)$

(b) Suppose MGF of X is g(t). What is MGF of k*X+d , k,d$\in R$

(c) Note :- Suppose (X,Y) be bi-variety distribution. In this case MGF $M(t_1, t_2) = \iint e^{t_1 x + t_2 y} f(x, y)\, dx\, dy$
To prove that when two independent normal distribution are added the resultant distribution is also normal.

(d) what happens when we subtract two independent normal distribution.

(e) Given MGF find the distribution.

- $M(t) = \frac{1}{6}e^{3t} + \frac{1}{2}e^{5t} + \frac{1}{3}e^{7t}$
- $M(t) = \frac{5}{5-t}$

Q 3. More about Moments:- $E[X^n]$ is called the nth moment of RV X $E[(X-t)^n]$ is called the nth moment of RV X about the point t $E[(X-\mu)^n]$ is called the nth central moment of RV X

(a) Show that if $E[X^a]$ exist then $\forall b < a; E[X^b]$ exist.

(b) Show that if $E[(X-t)^a]$ exist then $\forall t' \in R; E[(X-t')^a]$ exist.

(c) Find the minimum value of $E[(X-t)^2]$ where t is arbitrary.

(d) Derive an expression for nth central moment of RV X in terms of kth moment for$k \leq n; n, k \in R$

Q 4. How can you tell by looking at DF of a RV that its continuous or discrete, Can a RV be neither ?

Q 5. Find the DF,mean,variance of following distribution(Also check if they are valid pdf):-

- $f(x) = \lambda e^{-\lambda x}; \forall x \geq 0$
- $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}; \forall x \in N_0$
- $f(x) = \frac{6}{\pi^2 n^2}$ and $n \in \{...., -5, -3, -1, 2, 4, 6, 8, 10, ...\}$
- $f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$ and $x \in (-\infty, \infty)$

Q 6. Let X be a random variable with moment-generating function M(t), $-h \leq t \leq h$. Prove that
$Pr(X \geq a) \leq e^{-at}M(t), 0 \leq t \leq h$
and that
$Pr(X \leq a) \leq e^{-at}M(t), -h \leq t \leq 0$

Q 7. Note:- For Multivariate Normal Distribution read about it on your own from Wikipedia or another source. We will discuss about this in class if time permits.
Given $X \sim N(u, \Sigma)$
$X = (x_1, x_2, ..., x_n)$.
Find $f(x_1, x_2, ..x_m|x_{m+1}, x_{m+2}, ..x_n)$

Q 8. X be iid U(0,1) then find let f(x) be some pdf with F(x) DF then find the distribution of $F^{-1}(X)$.

Q 9. Find U_1 and U_2 iid with U(0,1). let X=$\sqrt{-2ln(U_1)}cos(2\pi U_2)$ Y=$\sqrt{-2ln(U_1)}sin(2\pi U_2)$ Find the distribution of X and Y.

Q 10. Let X_1, X_2, X_3 be iid U(0,1).Find distribution of X_1, X_2, X_3.