More commonly used form

Suppose X is a RV then $(X-\mu)^2$ is also a RV

st $(X-\mu)^2 \geq 0$ ~~for~~ $\forall X \in S$ [Here μ represents the mean)

$\therefore$ apply the previous result

$$Pr((X-\mu)^2 \geq c) < \frac{E[(X-\mu)^2]}{c}$$

$$Pr(|X-\mu| \geq \sqrt{c}) < \frac{\sigma^2}{c}$$

Let $\sqrt{c} = k\sigma$

$$c = k^2\sigma^2$$

$$\Rightarrow \boxed{Pr(|X-\mu| \geq k\sigma) < \frac{1}{k^2}}$$

This can be equivalent

$$Pr(|X-\mu| < k\sigma) > 1 - 1/k^2$$

$\Rightarrow$ Use $1 - Pr(|X-\mu| \geq k\sigma) = Pr(|X-\mu| < k\sigma)$

Chebyshev's Inequality

Let X be a non-negative RV if $E[X]$ eisfist

then

$$Pr[X \geq c] \leq \frac{E[X]}{c}$$

$\Rightarrow$ we are going to prove for discrete case [continous can be handel in a similar way]

$$E[X] = \sum_{x_i \in S} x_i f(x_i) \qquad S \subseteq R_+ \text{ only positive Real number}$$

choose $c \in S$

$$E[X] = \underbrace{\sum_{\substack{x_i < c \\ x_i \in S}} x_i f(x_i)}_{\pm ve} + \sum_{\substack{x_i \geq c \\ x_i \in S}} x_i f(x_i) \geq \sum_{\substack{x_i \geq c \\ x_i \in S}} x_i f(x_i)$$

$$\geq c \sum_{x_i \geq c} f(x_i) \quad [\text{As } x_i \geq c]$$

$$E[X] \geq c \, Pr(X \geq c)$$

$$\boxed{Pr(X \geq c) \leq \frac{E[X]}{c}}$$

$$\frac{d \ln M(t)}{dt} = \left.\frac{M'(t)}{M(t)}\right|_0 = \frac{E[x]}{1} =$$

$$\frac{d^2 \ln M(t)}{dt^2} = \frac{d}{dt}\left[\frac{M'(t)}{M(t)}\right] = \cancel{\frac{M''(t) M'(t)}{M(t)^2}}$$

$$= \left.\frac{M''(t) M(t) - M'(t) M'(t)}{M(t)^2}\right|_0 = \frac{E[x^2] - E[x]}{1}$$

$$= \sigma^2$$

4) Characteristic function

~~Unlike Moment generating function Characterist functio~~

It is defined as :-

$$E[e^{ixt}]$$

- Unlike moment generation functi it exist for all t

$$\left| \int_{-\infty}^{\infty} e^{ixt} f(x)\, dx \right| \leq \int_{-\infty}^{\infty} f(x)\, dx$$

$$\boxed{|e^{ixt}| \leq 1}$$

Note :- Moment generation functions are unique for a distribution if 2 distributions have same MGF then they are same.

Let σ^2 denote the variance

$$\sigma^2 = E[(x^2 - u)^2]$$

$$= E[x^2 - 2xu + u^2]$$

Now E is a linear function

$$\Rightarrow \sigma^2 = E[x^2] - 2E[x]u + u^2$$

$$E[x] = u$$

$$\boxed{\sigma^2 = E[x^2] - u^2}$$

Standard deviation $= \sqrt{\sigma^2}$

3) Moment generating function :-

Suppose $\exists\, h > 0$ st $E[e^{tx}]$ exist for $t \in (-h, h)$

Then

$$E[e^{tx}] = \int_{-\infty}^{\infty} e^{tx} f(x)\, dx = M(t)$$

$$M(0) = 1$$

$$M'(0) = u$$

$$M''(0) = E[x^2]$$

and so on..

Some Special Expectation

Note

Expectation are linear functions.

Linear function : Suppose $f : U \to V$ st and $\alpha \in$ Scalars

$$f(U+V) = f(U) + f(V)$$

$$f(\alpha U) = \alpha f(U)$$

Orsimply

$$f(\alpha U + \beta V) = \alpha f(U) + \beta f(V)$$

1) Let X be a discrete RV then [f be the pmf]

$$E[X] = \sum_{x} x f(x)$$

This is called the mean of the distribution X usually represeneted as μ [Similar result can be obtained for continous distribution.]

$$E[X] = \int_{S} x f(x)\, dx$$ S represets the support

2) Another special expectation is obtained by $E[(X-\mu)^2]$ here μ is the mean of the distribution. This expected value is called Variance.