Q1

mean = $\frac{15+22+18+25+20+17+23+19+21+16}{10}$

$\bar{x} = \frac{196}{10} = 19.6$ $\therefore N = 10$

in median first sort it =

Median =>

$15+16+17+18+19+20+21+22+23+25$

Median = $\frac{19+20}{2} = 19.5$

Mode => There is not repeating value of most frequent elements.

Range => L - S $\therefore$ largest smallest

$R = 25 - 15$

$= 10$

Variance => $s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{N}$

$s^2 = \frac{(15-19.6)^2 + (16-19.6)^2 + (17-19.6)^2 + (18-19.6)^2 - \ldots + (25-19.6)^2}{10-1}$

$s^2 = 10.49$

Standard Deviation (s)

$s = \sqrt{10.49} = 3.234$

Q2

Data 85, 92, 78, 88, 95, 82, 90, 87

$$\text{Mean} = \frac{85+92+78+88+95+82+90+87}{8}$$

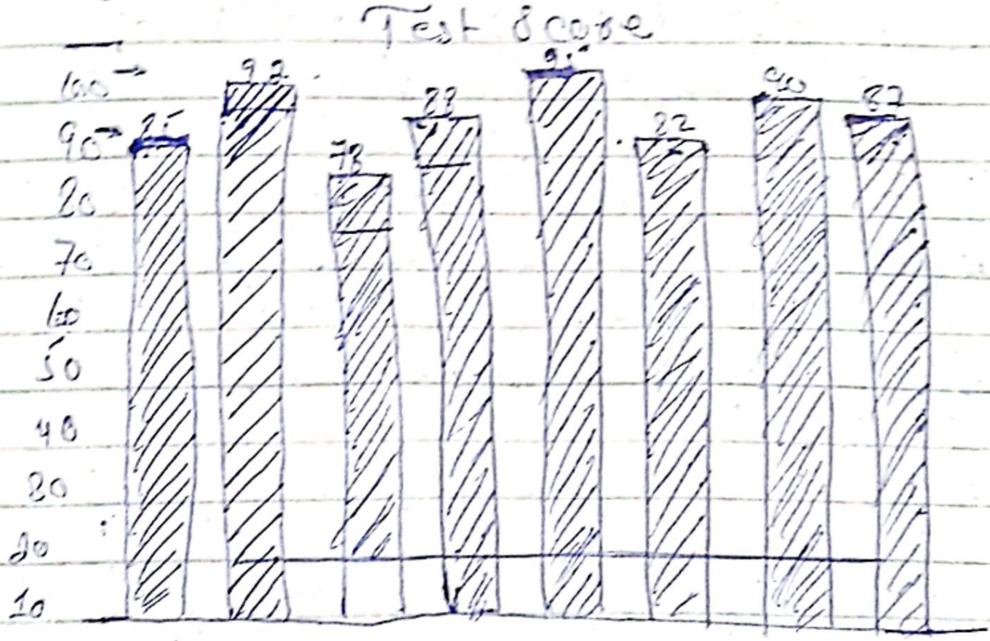
$N = 8$

$$= \frac{697}{8} = 87.125$$

$$\text{Variance } (s^2) = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

$$= \frac{(85-87.12)^2 + (92-87.12)^2 + (78-87.12)^2 + (95-87.12)^2 \ldots}{8-1}$$

$s^2 = 30.13$



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$$s^2 = 30.13$$

Q3

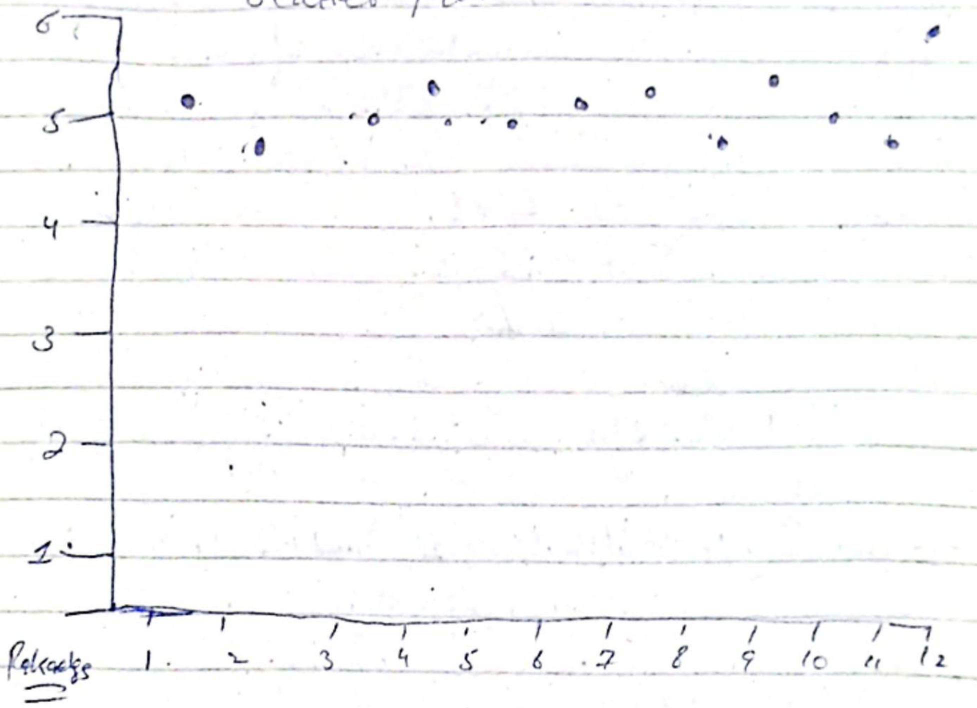
(iv) Mean $= \dfrac{5.2+4.8+5.0+4.9+5.1+5.3+4.7+5.4+5+4.6+5.6 \; (+5.5)}{12}$

$= \dfrac{61.1}{12} = 5.1$

Median $= \dfrac{5.1+5.3}{2} = \dfrac{10.4}{2} = 5.2$

mode $= 5.0$

Scatter Plot.



Q3

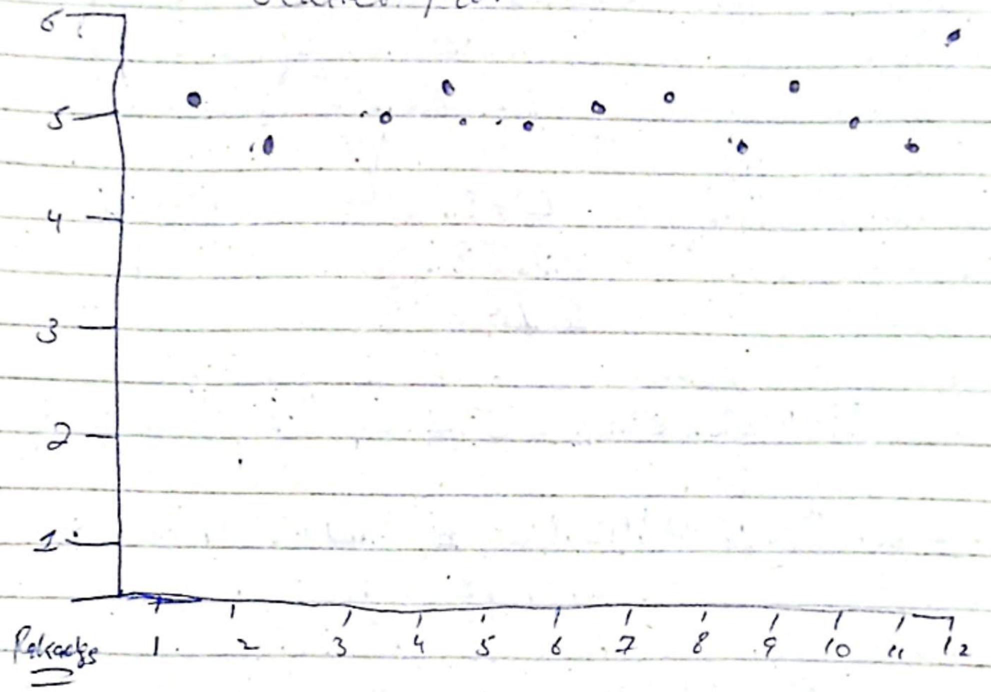
(ii) Mean $= \dfrac{5.2 + 4.8 + 5.0 + 5.5 + 4.9 + 5.1 + 5.3 + 4.7 + 5.4 + 5 + 4.6 + 5.6}{12}$

$= \dfrac{61.1}{12} = 5.1$

Median $= \dfrac{5.1 + 5.3}{2} = \dfrac{10.4}{2} = 5.2$

mode $= 5.0$

Scatter Plot.



Q3

(i) Mean $= \dfrac{5.2+4.8+5.0+5.5+4.9+5.1+5.3+4.7+5.4+5+4.6+5.6}{12}$

$= \dfrac{61.1}{12} = 5.1$

Median $= \dfrac{5.1+5.3}{2} = \dfrac{10.4}{2} = 5.2$

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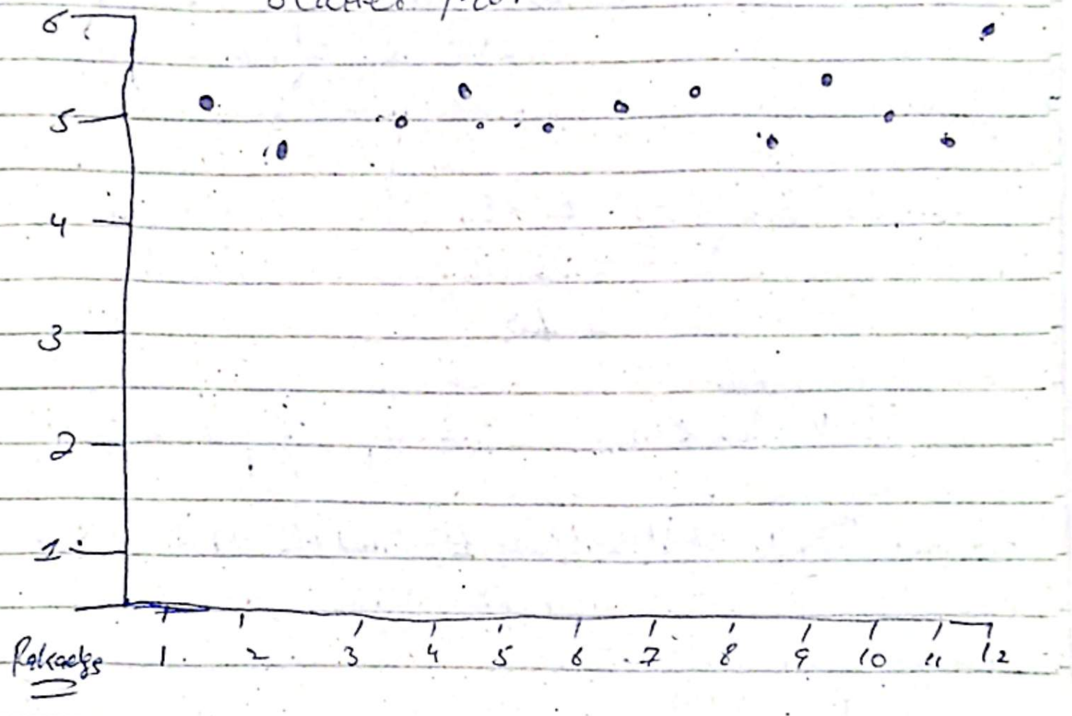
Scatter Plot:



Q4 Password of 4 char (A-Z 2 letter 0-9 2 digi

=> Without Repitation
1st letter options = 26
2nd letter options = 25
= 26 × 25 = 650 way

1st digits options from (0 - 9) = 10
for 2nd digit is = 9
= 10 × 9 = 90 ways

=> for arrangement these are
four letters 4! = 24 way

Total = 26×25×9×10×24
= 650 × 90 × 24 = 1404000
Passwords.

=> with Repitation

for 1st & 2nd both have 26 options
1st & 2nd digit are 10 option

without arrangement

the sample space = 26 × 26 × 10 × 10
= 67600

Q5

Total books = 10
Select 5 book & arrange in order
condition 2 specific books are together.
so,
2 books as 1 group.
Remaing 3 book to select

$$= \binom{8}{3} = \frac{8!}{3!\,5!} = 56$$

for arrangment there are 4 units
because 2 book as 1 unit.

$$= 4! = 24 \text{ ways}.$$

and other 2 book also need arrangem

$$= 2! = 2 \text{ ways}.$$

total ways = $56 \times 24 \times 2 = 2688$

Without Restriction

$$P(10,5) = \frac{10!}{(10-5)!} = \frac{10!}{5!} =$$

$$= 10 \times 9 \times 8 \times 7 \times 6 = 30240$$

Q6 Total numbers (n) = 49
numbes of selection (r) = 6

$${}^nC_r = \frac{n!}{r!(n-r)!} = \binom{49}{6} = \frac{49!}{6!(49-6)!} = 13983816$$

=> And for "Atleast one even numbes"

total even numbers = 2, 4, 6 ... 48 = 24 even no.
odd numbers = 1, 3, 5, 7 = ... 49 = 25

choosing only odd no. = $\binom{25}{6}$ = 177,100

Atleast one even no =
13983816 − 177100 = 13806716

Q7 ① P(Both Red) = P(1st Red) = $\frac{5}{12}$
= P(2nd Red = $\frac{4}{11}$

$$P(\text{Both Red}) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33} = 0.1515$$

② P(2nd Red | 1st Red) = $\frac{4}{11}$ = 0.364 = P(B) = $\frac{5}{12}$ = 0.417

$$P(B|A) \neq P(B)$$

event are not independent.

Q10 a) P(red) $\Rightarrow$ using total probability

Formula

$$P(red) = P(A) \cdot P(red|A) + P(B) \cdot P(red|B) + P(C) \cdot P(red|C)$$

$P(red|A) = 3/5$

$P(red|B) = 1/5$ $\Rightarrow$ $P(A) = P(B) = P(C) = 1/3$

$P(red|C) = 2/5$

$$P(red) = \left(\frac{1}{3} \cdot \frac{3}{5}\right) + \left(\frac{1}{3} \cdot \frac{1}{5}\right) + \left(\frac{1}{3} \cdot \frac{2}{5}\right)$$

$$= \frac{3}{15} + \frac{1}{15} + \frac{2}{15} = \frac{6}{15} = \frac{2}{5} = 0.4$$

b) P(Box A | red)

$$P(A|red) = \frac{P(A) \cdot P(Red|A)}{P(Red)}$$

$$= \frac{\left(\frac{1}{3} \cdot \frac{3}{5}\right)}{0.4} = \frac{0.2}{0.4} = 0.5$$

Q11

Q8 ① $P(\text{coffee or tea}) = P(\text{coffee}) + P(\text{tea}) - P(\text{coffee} \cap \text{tea})$

$= 0.40 + 0.30 - 0.10$

$= 0.60 = 60\%$

② $P(\text{Tea}|\text{coffee}) = \dfrac{P(\text{Tea} \cap \text{coffee})}{P(\text{coffee})} = \dfrac{0.10}{0.40} = \dfrac{1}{4} = 0.25$

$= 25\%$

Q9 $P(\text{Disease}) = P(D) = (0.01)$

$P(\text{no Disease}) = P(D') = (0.99)$

$P(\text{Positive} | \text{Disease}) = P(T^+|D) = 0.95$

$P(\text{negative} | \text{no Disease}) = P(T^-|D') = 0.90$

$= P(T^+|D') = 0.10$

① $P(\text{Positive}) = P(T^+) = P(T^+|D) \cdot P(D) + P(T^+|D') \cdot P(D')$

$= (0.95 \times 0.01) + (0.10 \times 0.99)$

$= 0.0095 + 0.099$

$P(\text{Positive} = 0.1085 = 10.85\%$

② $P(\text{Disease} | \text{Positive})$ using Bayes' theorem

$$P(D|T^+) = \frac{P(T^+|D) \cdot P(D)}{P(T^+)} =$$

$$= \frac{0.95 \times 0.01}{0.1085} = \frac{0.0095}{0.1085} = 0.087$$

$= 8.76\%$

Q12 a) $P(X=x) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

$N=7$, $K=2$, $n=3$

$X=0 \quad = \dfrac{\binom{2}{0}\binom{5}{3}}{\binom{7}{3}} = \dfrac{1\times10}{35} = \dfrac{10}{35} = 0.285$

$x=1$

$= \dfrac{\binom{2}{1}\binom{5}{2}}{\binom{7}{3}} = \dfrac{2\times10}{35} = \dfrac{20}{35} = 0.571$

$x=2$

$= \dfrac{\binom{2}{2}\cdot\binom{5}{1}}{\binom{7}{3}} = \dfrac{5}{35} = 0.142$

Q13

a) Density function (verification)

$$\int_1^\infty 3x^{-4}\,dx = 3\left[\frac{x^{-3}}{3}\right]_1^\infty = -1[0-1] = 1$$

verified

b) $F(x)$ (CDF)

$$F(x) = \int_1^x 3t^{-4}\,dt = 3\left[\frac{t^{-3}}{3}\right] = 1-x^{-3}$$

c) $P(X>4)$

$$P(X>4) = 1-F(4) = 1-(1-4^{-3}) = 4^{-3} = \frac{1}{64}$$
$$= 0.0156$$

Q4 a) P(sum is 7 or 11)

total outcomes (S) $= 6 \times 6 = 36$

Sum of 7 outcomes = (1,6) (2,5) (3,4) (4,3) (5,2) (6,1)
total 6 outcomes

sum of 11 outcomes (S) = (5,6) (6,5)
2 outcomes.

$$P(A \cup B) = P(A) + P(B)$$

$$P(\text{sum } 7 \text{ or } 11) = \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$$

b) P(7 | sum is odd) => conditional probability.

odd sums (3, 5, 7, 9, 11)   18 => sums total odd

$$P(\text{sum}7 \mid \text{odd}) = \frac{\text{outcomes of } 7}{\text{total odd outcomes}}$$

$$= \frac{6}{18} = \frac{1}{3}$$

c) sum of 7 & first die even => check independence

$$P(\text{sum } 7) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{first die even}) = \frac{3}{6} = \frac{1}{2}$$

$$P(\text{sum } 7 \cap \text{Even}) = (2,5)\ (4,3)\ (6,1) = \frac{3}{36} = \frac{1}{12}$$

Checking $P(A \cap B) = P(A) \cdot P(B)$

$\frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$ independent