TEORIA SISTEMELOR an II CB

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TEMA de CASĂ 1

Se introduc următoarele notații: n -numărul de ordine al studentului (din apelul grupei); a = n mod 7 + 1; b = n mod 6 + 1; c = n mod 8 + 1; e = n mod 4 + 1; d = ultima cifră a grupei.

I. Se consideră sistemul liniar neted (SL N) având realizarea de stare

$$\mathbf{A}=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a^2bc & -\left[a^2(b+c)+2abc\right] & -\left[a^2+2a(b+c)+bc\right] & -(2a+b+c) \end{bmatrix},\ \mathbf{B}=\begin{bmatrix}0\\0\\0\\1\end{bmatrix},\ \mathbf{x}_0=\begin{bmatrix}0\\0\\0\\(-1)^d\cdot e\end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} bd & b+d & 1 & 0 \\ ec & e & 0 & 0 \end{bmatrix}$$

$$\mathbf{ss}^T=\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & y_1 & y_2 \end{bmatrix}$$

Să se determine (numai) ss(b), dacă $\mathbf{x}_0$ este cel menționat, iar

$$u(t)=\left[b\cdot\sin 3(t-c)+d\cdot\cos 3(t-c)\right]\cdot 1(t-c)$$

II. Fie sistemul liniar discret (SL D) exprimat, intrare -ieșire, prin ecuația cu diferențe

$$y(t+4)+(b-2a)\cdot y(t+3)+\left(a^2-2ab\right)\cdot y(t+2)+a^2b\cdot y(t+1)=c\cdot u(t+1)+d\cdot u(t) \qquad (1)$$

a) Să se determine funcția de transfer a sistemului (condiții inițiale nule);
b) Să se determine, utilizând transformata Z, răspunsul sistemului (reprezentat ca în (1)) dacă

$$u(t)=1(t-1)\ ;\ y(0)=c\ ,\ y(1)=-e\ ,\ y(2)=-c\ ,\ y(3)=d \qquad (2)$$

c) Să se determine realizările (standard) de stare (inclusiv $\mathbf{x}_0$) pentru sistemul (1).

III. Să se analizeze stabilitatea internă şi externă a SL N (utilizând criteriul Hurwitz)

$$\mathbf{A}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & ce\left(a^2+b^2\right)^2 \\ 1 & 0 & 0 & 0 & 0 & (c-e)\left(a^2+b^2\right)^2 \\ 0 & 1 & 0 & 0 & 0 & 2ce\left(b^2-a^2\right)-\left(a^2+b^2\right)^2 \\ 0 & 0 & 1 & 0 & 0 & 2(c-e)\left(b^2-a^2\right) \\ 0 & 0 & 0 & 1 & 0 & ce-2\left(b^2-a^2\right) \\ 0 & 0 & 0 & 0 & 1 & c-e \end{bmatrix},\ \mathbf{B}=\begin{bmatrix} -c\left(a^2+b^2\right) & c\left(a^2+b^2\right) \\ a^2+b^2+2ac-c\left(a^2+b^2\right) & -a^2-b^2-2ac-c\left(a^2+b^2\right) \\ a^2+b^2+2ac-2a-c & a^2+b^2+2ac+2a+c \\ 1-2a-c & -1-2a-c \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

I. Se considera SL N avand realizarea de stare:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -75 & -130 & -68 & -14 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 5 & 6 & 1 & 0 \\ 9 & 3 & 0 & 0 \end{bmatrix} \qquad \mathbf{x_0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \end{bmatrix}$$

Sa se determine (numai) **ss(1)** = $\mathbf{x_1}$, stiind:

u(t) = [sin(3(t-3)) + 5cos(3(t-3))] · 1(t)

II. Fie SL D exprimat, intrare-iesire, prin ecuatia cu diferente:

y(t+4) – 9·y(t+3) + 15·y(t+2) + 25·y(t+1) = 3·u(t+1) + 5·u(t)

a) Sa se determine functia de transfer a sistemului (conditii initiale nule)

b) Sa se determine, utilizand transformata Z, raspunsul sistemului daca:

u(t) = 1(t-1)   y(0) = 3   y(1) = -3   y(2) = -3   y(3) = 5

c) Sa se determine realizarile (standard) de stare (inclusiv $\mathbf{x_0}$).

III. Sa se analizeze stabilitatea interna si externa a SL N (criteriul Hurwitz)

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 6084 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1108 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 57 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -78 & 78 \\ -22 & -134 \\ 43 & 69 \\ -12 & -14 \\ 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad]$$

Rezolvari:

I. $x(s) = \frac{1}{\chi(s)} (s\boldsymbol{I} - \boldsymbol{A})^* \cdot [\boldsymbol{x_0} + \boldsymbol{Bu}(s)]$

1) Calculez **u**(s) folosind transformarea Laplace si teorema intarzierii:

$=> \boldsymbol{u}(s) = e^{-3s} \cdot \mathcal{L}[(\sin(3t) + 5\cos(3t)) \cdot 1(\mathrm{t})]$

$=> \boldsymbol{u}(s) = e^{-3s} \cdot \frac{3}{s^2+9} + e^{-3s} \cdot \frac{5s}{s^2+9} = e^{-3s} \cdot \frac{5s+3}{s^2+9}$

2) Determin:

$$\boldsymbol{x_0} + \boldsymbol{Bu}(s) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \cdot e^{-3s} \cdot \frac{5s+3}{s^2+9} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-3s} \cdot \dfrac{5s+3}{s^2+9} - 3 \end{bmatrix}$$

3) Calculez:

$$s\boldsymbol{I} - \boldsymbol{A} = \begin{bmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -75 & -130 & -68 & -14 \end{bmatrix}$$
$$= \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ 0 & 0 & s & -1 \\ 75 & 130 & 68 & s+14 \end{bmatrix}$$

4) Aflu polinomul caracteristic χ(s)

$$\chi(s) = \det(s\boldsymbol{I} - \boldsymbol{A}) = s \cdot \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 130 & 68 & s+14 \end{vmatrix} + (-1)(-1) \cdot \begin{vmatrix} 0 & -1 & 0 \\ 0 & s & -1 \\ 75 & 68 & s+14 \end{vmatrix}$$
$$= s(s^2(s+14) + 130 + 68s) + 75$$
$$= s^4 + 14s^3 + 68s^2 + 130s + 75$$

5) Calculez (s**I** - **A**)* atat cat este necesar; se cere $\mathbf{x_1}$ deci este suficient sa aflu doar elementul aflat in coltul din dreapta-sus.

$$(s\boldsymbol{I}-\boldsymbol{A})^T = \begin{bmatrix} s & 0 & 0 & 75 \\ -1 & s & 0 & 130 \\ 0 & -1 & s & 68 \\ 0 & 0 & -1 & s+14 \end{bmatrix}$$

$$(s\boldsymbol{I}-\boldsymbol{A})^* = \begin{bmatrix} ? & ? & ? & 1 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

6) Folosind adjuncta calculata anterior, determin $x_1(s)$:

$$\boldsymbol{x}(s) = \frac{1}{\chi(s)} \cdot \begin{bmatrix} ? & ? & ? & 1 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-3s} \cdot \dfrac{5s+3}{s^2+9} - 3 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-3s} \cdot \dfrac{5s+3}{s^2+9} - 3 \\ ? \\ ? \\ ? \end{bmatrix} \cdot \frac{1}{s^4+14s^3+68s^2+130s+75}$$

$$=> x_1(s) = \frac{e^{-3s} \cdot \frac{5s+3}{s^2+9} - 3}{s^4+14s^3+68s^2+130s+75} = \frac{e^{-3s} \cdot (5s+3) - 3(s^2+9)}{(s^2+9)(s^4+14s^3+68s^2+130s+75)}$$

Caut radacini pentru

$$s^4+14s^3+68s^2+130s+75 = 0$$

Obs: coeficientii sunt numere intregi => sanse ca radacinile polinomului sa fie divizori ai termenului liberi (75).

Obs: s = -5 e radacina (dubla)

	1	14	68	130	75
s = -5	1	9	23	15	0
s = -5	1	4	3	0	

din schema lui Horner => $(s+5)^2(s^2+4s+3)=0$

$$=> (s+5)^2(s+1)(s+3)=0$$

$$=> \chi(s) = \frac{e^{-3s}\cdot(5s+3)-3(s^2+9)}{(s^2+9)(s+5)^2(s+1)(s+3)} = e^{-3s}\cdot\underbrace{\frac{5s+3}{(s^2+9)(s+5)^2(s+1)(s+3)}}_{(1)} - \underbrace{\frac{3}{(s+5)^2(s+1)(s+3)}}_{(2)}$$

Rescriu (1) sub forma de fractii simple:

$$\frac{5s+3}{(s^2+9)(s+5)^2(s+1)(s+3)} = \frac{As+B}{s^2+9}+\frac{C}{s+5}+\frac{D}{(s+5)^2}+\frac{E}{s+1}+\frac{F}{s+3}$$

$$\begin{aligned}(As+B)(s+5)^2(s+1)(s+3)&+C(s^2+9)(s+5)(s+1)(s+3)\\&+D(s^2+9)(s+1)(s+3)+E(s^2+9)(s+5)^2(s+3)\\&+F(s^2+9)(s+5)^2(s+1)=5s+3\end{aligned}$$

s^5: $A+C+E+F=0$

s^4: $14A+B+9C+D+13E+11F=0$

s^3: $68A+14B+32C+4D+64E+44F=0$

s^2: $130A+68B+96C+12D+192E+124F=0$

s^1: $75A + 130B + 207C + 36D + 495E + 315F = 5$

s^0: $75B + 135C + 27D + 675E + 225F = 3$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 14 & 1 & 9 & 1 & 13 & 11 & 0 \\ 68 & 14 & 32 & 4 & 64 & 44 & 0 \\ 130 & 68 & 96 & 12 & 192 & 124 & 0 \\ 75 & 130 & 207 & 36 & 495 & 315 & 5 \\ 0 & 75 & 135 & 27 & 675 & 225 & 3 \end{bmatrix} => \textit{transformari pe linii} =>$$

$=> A = \frac{-14}{1271}; B = \frac{-33}{5780}; C = \frac{-611}{9248}; D = \frac{-11}{136}; E = \frac{-1}{160}; F = \frac{1}{12}$

Rescriu (2) sub forma de fractii simple:

$$\frac{3}{(s+5)^2(s+1)(s+3)} = \frac{A}{s+5} + \frac{B}{(s+5)^2} + \frac{C}{s+1} + \frac{D}{s+3}$$

$$A(s+5)(s+1)(s+3) + B(s+1)(s+3) + C(s+5)^2(s+3) + D(s+5)^2(s+1) = 3$$

s^3: $A + C + D = 0$

s^2: $9A + 13C + B + 11D = 0$

s^1: $23A + 4B + 55C + 35D = 0$

s^0: $15A + 3B + 75C + 25D = 3$

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 9 & 1 & 13 & 11 & 0 \\ 23 & 4 & 55 & 35 & 0 \\ 15 & 3 & 75 & 25 & 3 \end{bmatrix} => \textit{operatii pe linii} => A = \frac{9}{32}; B = \frac{3}{8}; C = \frac{2}{32}; D = \frac{-3}{8}$$

$$x_1(s) = e^{-3s} \cdot \left(\frac{A_1 s + B_1}{s^2+9} + \frac{C_1}{s+5} + \frac{D_1}{(s+5)^2} + \frac{E_1}{s+1} + \frac{F_1}{s+3}\right) - \left(\frac{A_2}{s+5} + \frac{B_2}{(s+5)^2} + \frac{C_2}{s+1} + \frac{D_2}{s+3}\right)$$

Aplicand $\mathcal{L}^{-1}$ si utilizand teorema intarzierii:

$$x_1(t) = 1(t-3) \cdot \left(A_1 \cos\big(3(t-3)\big) + \frac{B_1}{3}\sin\big(3(t-3)\big) + C_1 e^{-5(t-3)} + D_1(t-3)e^{-5(t-3)} + E_1 e^{-(t-3)} + F_1 e^{-3(t-3)}\right) - 1(t) \cdot (A_2 e^{-5t} + B_2 t e^{-5t} + C_2 e^{-t} + D_2 e^{-3t})$$

$$x_1(t) = 1(t-3) \cdot \left(\frac{-14}{1271}\cos\big(3(t-3)\big) + \frac{-11}{5780}\sin\big(3(t-3)\big) + \frac{-611}{9248}e^{-5(t-3)} + \frac{-11}{136}(t-3)e^{-5t} + \frac{-1}{160}e^{-(t-3)} + \frac{1}{12}e^{-3(t-3)}\right) - 1(t) \cdot \left(\frac{9}{32}e^{-5t} + \frac{3}{8}te^{-5t} + \frac{3}{32}e^{-t} + \frac{-3}{8}e^{-3t}\right)$$

II. $y(z) = H(z) \cdot u(z) => H(z) = \frac{y(z)}{u(z)}$

fie $u(z) = \mathcal{Z}[u(t)]$

$y(z) = \mathcal{Z}[y(z)]$

$u(t+1) = zu(z) - zu(0)$

$y(t+1) = zy(z) - zy(0)$

$y(t+2) = z^2y(z) - z^2y(0) - \ z^1y(1)$

$y(t+3) = z^3y(z) - z^3y(0) - \ z^2y(1) - z^1y(2)$

$y(t+4) = z^4y(z) - z^4y(0) - \ z^3y(1) - z^2y(2) - z^1y(3)$

a) conditii initiale nule => u(0) = y(0) = y(1) = y(2) = y(3) = 0

$u(t+1) = zu(z)$

$y(t+1) = zy(z)$

$y(t+2) = z^2y(z)$

$y(t+3) = z^3y(z)$

$y(t+4) = z^4y(z)$

$=> y(z)(z^4 - 9z^3 + 15z^2 + 25z) = u(z)(3z+1)$

$=> \frac{y(z)}{u(z)} = \frac{3z+1}{z(z^3-9z^2+15z+25)}$

caut radacini pt: $z^3 - 9z^2 + 15z + 25 = 0$

<u>Obs:</u> coeficientii polinomului sunt reali => sansa ca radacinile sa fie divizori ai termenului liber (25)

<u>Obs:</u> z=-1 solutie

	1	-9	15	25
z = -1	1	-10	25	0

din schema lui Horner:

$=> (z+1)(z^2-10z+25)=0 => (z+1)(z-5)^2=0$

prin urmare $H(z) = \frac{3z+1}{z(z+1)(z-5)^2}$

b) $u(t+1) = zu(z)$ si $u(t+1) = \frac{z}{z-1} => u(z) = \frac{1}{z-1}$

$y(t+1) = zy(z) - 3z$

$y(t+2) = z^2y(z) - 3z^2 + 3z$

$y(t+3) = z^3y(z) - 3z^3 + 3z^2 + 3z$

$y(t+4) = z^4y(z) - 3z^4 + 3z^3 + 3z^2 - 5z$

$=> z^4y(z) - 3z^4 + 3z^3 + 3z^2 - 5z - 9z^3y(z) + 27z^3 - 27z^2 - 27z +$
$15z^2y(z) - 45z^2 + 45z + 25zy(z) - 75z = \frac{3z+5}{z-1}$

$=> y(z)(z^4 - 9z^3 + 15z^2 + 25z) = \frac{3z+5}{z-1} + 3z^4 - 3z^3 - 3z^2 + 5z - 27z^3 +$
$27z^2 + 27z + 45z^2 - 45z + 75z$

$$y(z) = \frac{3z^5 - 33z^4 + 99z^3 - 7z^2 - 59z + 5}{(z-1)(z^4 - 9z^3 + 15z^2 + 25z)}$$

$$\frac{y(z)}{z} = \frac{3z^5 - 33z^4 + 99z^3 - 7z^2 - 59z + 5}{(z-1)(z+1)(z-5)^2z^2}$$

Scriu $\frac{y(z)}{z}$ sub forma de fractii simple:

$$\frac{y(z)}{z} = \frac{A}{z-1} + \frac{B}{z+1} + \frac{C}{z-5} + \frac{D}{(z-5)^2} + \frac{E}{z} + \frac{F}{z^2}$$

$A(z+1)(z-5)^2z^2 + B(z-1)(z-5)^2z^2 + C(z-1)(z+1)(z-5)z^2 +$
$D(z-1)(z+1)z^2 + E(z-1)(z+1)(z-5)^2z + F(z-1)(z+1)(z-5)^2 =$
$3z^5 - 33z^4 + 99z^3 - 7z^2 - 59z + 5$

z^5: $A + B + C + E = 3$

z^4: $-9A - 11B - 5C + D - 10E + F = -33$

z^3: $15A + 35B - 4C + 24E - 10F = 99$

z^2: $25A - 25B + 5C - D + 10E + 24F = -7$

z^1: $-25E + 10F = -59$

z^0: $-25F = 5$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 3 \\ -9 & -11 & -5 & 1 & -10 & 1 & -33 \\ 15 & 35 & -4 & 0 & 24 & -10 & 99 \\ 25 & -25 & 5 & -1 & 10 & 24 & -7 \\ 0 & 0 & 0 & 0 & -25 & 10 & -59 \\ 0 & 0 & 0 & 0 & 0 & -25 & 5 \end{bmatrix} => \textit{operatii pe linii}$$

$=> A = \frac{745}{3737}; B = \frac{3366}{3173}; C = -\frac{531}{983}; D = \frac{831}{1090}; E = \frac{57}{25}; F = -\frac{1}{5}$

$$y(z) = \frac{zA}{z-1} + \frac{zB}{z+1} + \frac{zC}{z-5} + \frac{zD}{(z-5)^2} + E + \frac{F}{z}$$

$=> y(t) = A \cdot 1(t) + B(-1)^t \cdot 1(t) + C \cdot 5^t \cdot 1(t) + D \cdot 5^{t-1} C_t^1 \cdot 1(t) + E \cdot u_0(t) + F \cdot u_1(t)$

$=> y(t) = \frac{745}{3737} \cdot 1(t) + \frac{3366}{3173(-1)^t} \cdot 1(t) - \frac{531}{983} \cdot 5^t \cdot 1(t) + \frac{831}{1090} \cdot 5^{t-1} \cdot t \cdot 1(t) + \frac{57}{25} \cdot u_0(t) - \frac{1}{5} \cdot u_1(t)$

c) folosind functia de transfer de la punctul a)

$$H(z) = \frac{3z+1}{z(z+1)(z-5)^2}$$

determin polinomul caracteristic χ(s):

$$\chi(z) = \frac{3z+1}{z^4 - 9z^3 + 15z^2 + 25z + 0} = \frac{R(z)}{p(z)}$$

blocurile sunt de ordin 1 => dimensiunea realizarilor este $1 \cdot 4 = 4$

<u>Realizarea standard controlabila:</u>

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -25 & -15 & 9 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{C} = [\,1 \quad 3 \quad 0 \quad 0\,]$$

Realizarea standard observabila:

$$\overline{\boldsymbol{A}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -25 \\ 0 & 1 & 0 & -15 \\ 0 & 0 & 1 & 9 \end{bmatrix} \quad \overline{\boldsymbol{B}} = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\overline{\boldsymbol{C}} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$$

III. stabilitatea interna

realizare standard observabila => scriu direct polinomul caracteristic

$$\chi(s) = -6048 + 0s + 1108s^2 + 0s^3 - 57s^4 + 0s^5 + s^6$$

construiesc tabloul lui Hurwitz:

$$\boldsymbol{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -57 & 1108 & -6084 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -57 & 1108 & -6084 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -57 & 1108 & -6084 \end{bmatrix}$$

obs: $\det(\mathbf{H}_1)=0$ => SL N instabil intern.

stabilitatea externa

$$T(s) = \frac{R(s)}{\chi(s)} = \frac{[-78 \quad 78] + [-22 \quad -134]s + [43 \quad 69]s^2 + [-12 \quad -14]s^3}{s^6 - 57s^4 + 1108s^2 - 6084}$$

Caut c.m.m.d.c. pentru polinoamele:

$s^4 - 12s^3 + 43s^2 - 22s - 78$

$s^4 - 14s^3 + 69s^2 - 134s + 78$

$s6 - 57s^4 + 1108s^2 - 6084$

$$
\begin{array}{rrrrrrr|l}
\hline
s^6 & & -57s^4 & & +1108s^2 & & -6084 & s^4 - 12s^3 + 43s^2 - 22s - 78 \\
s^6 & -12s^5 & +43s^4 & -22s^3 & -78s^2 & & & \\
\cline{8-8}
\cline{1-7}
 & 12s^5 & -100s^4 & +22s^3 & +1186s^2 & & -6084 & s^2 + 12s + 44 \\
 & 12s^5 & -144s^4 & +516s^3 & -264s^2 & -936s & & \\
\cline{1-7}
 & & 44s^4 & -494s^3 & +1450s^2 & +936s & -6084 & \\
 & & 44s^4 & -528s^3 & +1892s^2 & -968s & -3432 & \\
\cline{1-7}
 & & & 34s^3 & -442s^2 & +1904s & -2652 &
\end{array}
$$

$$
\begin{array}{rrrrr|l}
\hline
s^4 & -12s^3 & +43s^2 & -22s & -78 & s^4 - 14s^3 + 69s^2 - 134s + 78 \\
\cline{6-6}
s^4 & -14s^3 & +69s^2 & -134s & +78 & 1 \\
\cline{1-5}
 & 2s^3 & -26s^2 & +112s & -156 &
\end{array}
$$

$s^4 - 12s^3 + 43s^2 - 22s - 78$

$2s^3 - 26s^2 + 112s - 156$

$34s^3 - 442s^2 + 1904s - 2652$

$$
\begin{array}{rrrrr|l}
s^4 & -\ 12\,s^3 & +\ 43\,s^2 & -\ 22\,s & -\ 78 & 2\,s^3 - 26\,s^2 + 112\,s - 156 \\
\cline{6-6}
s^4 & -\ 13\,s^3 & +\ 56\,s^2 & -\ 78\,s & & \frac{s}{2} + \frac{1}{2} \\
\hline
 & s^3 & -\ 13\,s^2 & +\ 56\,s & -\ 78 & \\
 & s^3 & -\ 13\,s^2 & +\ 56\,s & -\ 78 & \\
\hline
\end{array}
$$

$$
\begin{array}{rrrr|l}
34\,s^3 & -\ 442\,s^2 & +\ 1904\,s & -\ 2652 & 2\,s^3 - 26\,s^2 + 112\,s - 156 \\
\cline{5-5}
34\,s^3 & -\ 442\,s^2 & +\ 1904\,s & -\ 2652 & 17 \\
\hline
\end{array}
$$

=> c.m.m.d.c. al celor 3 polinoame: $2s^3 - 26s^2 + 112s - 156$

$$
\begin{array}{rrrrrrr|l}
s^6 & & -\ 57\,s^4 & & +\ 1108\,s^2 & & -\ 6084 & 2\,s^3 - 26\,s^2 + 112\,s - 156 \\
s^6 & -\ 13\,s^5 & +\ 56\,s^4 & -\ 78\,s^3 & & & & \\
\hline
 & 13\,s^5 & -\ 113\,s^4 & +\ 78\,s^3 & +\ 1108\,s^2 & & -\ 6084 & \frac{s^3}{2} + \frac{13\,s^2}{2} + 28\,s + 39 \\
 & 13\,s^5 & -\ 169\,s^4 & +\ 728\,s^3 & -\ 1014\,s^2 & & & \\
\hline
 & & 56\,s^4 & -\ 650\,s^3 & +\ 2122\,s^2 & & -\ 6084 & \\
 & & 56\,s^4 & -\ 728\,s^3 & +\ 3136\,s^2 & -\ 4368\,s & & \\
\hline
 & & & 78\,s^3 & -\ 1014\,s^2 & +\ 4368\,s & -\ 6084 & \\
 & & & 78\,s^3 & -\ 1014\,s^2 & +\ 4368\,s & -\ 6084 & \\
\hline
\end{array}
$$

$$=> T(s) = \frac{?}{\frac{s^3}{2}+\frac{13s^2}{2}+28s+39}$$

$$\boldsymbol{H} = \begin{bmatrix} 13/2 & 39 & 0 \\ 1/2 & 28 & 0 \\ 0 & 13/2 & 39 \end{bmatrix}$$

$\det(\mathbf{H}_1)=\frac{13}{2}>0$

$\det(\mathbf{H}_2)=\frac{325}{2}>0$

$\det(\mathbf{H}_3)=7098-\frac{1521}{2}>0$

=> SL N strict stabil extern.