SGupta_HW03Q13

Question 13

Plot the PDF of NDVI for forested and deforested pixels, and the marginal distribution of NDVI averaging over categories.

The forested NDVI density : ff(x) = dbeta(x,25,10) The deforested NDVI density : fd(x) = dbeta(x,10,15) The marginal density f(x) = 0.9 * ff(x) + 0.1 * fd(x)

```r
# Create a grid for NDVI values between 0 and 1.
ndvi <- seq(0, 1, length.out = 1000)

# Find the PDFs for forested and deforested pixels.
pdf_forest <- dbeta(ndvi, shape1 = 25, shape2 = 10)
pdf_defor  <- dbeta(ndvi, shape1 = 10, shape2 = 15)

# Marginal PDF
pdf_marginal <- 0.9 * pdf_forest + 0.1 * pdf_defor

# Plot the PDFs.
plot(ndvi, pdf_forest, type = "l", lwd = 2, col = "green",
     ylim = c(0, max(pdf_forest, pdf_defor, pdf_marginal)),
     xlab = "NDVI", ylab = "Density", main = "NDVI PDFs")
lines(ndvi, pdf_defor, lwd = 2, col = "red")
lines(ndvi, pdf_marginal, lwd = 2, col = "blue")
legend("topright", legend = c("Forested: Beta(25,10)", "Deforested: Beta(10,15)", "Marginal")
       col = c("green", "red", "blue"), lwd = 2)
```

NDVI PDFs
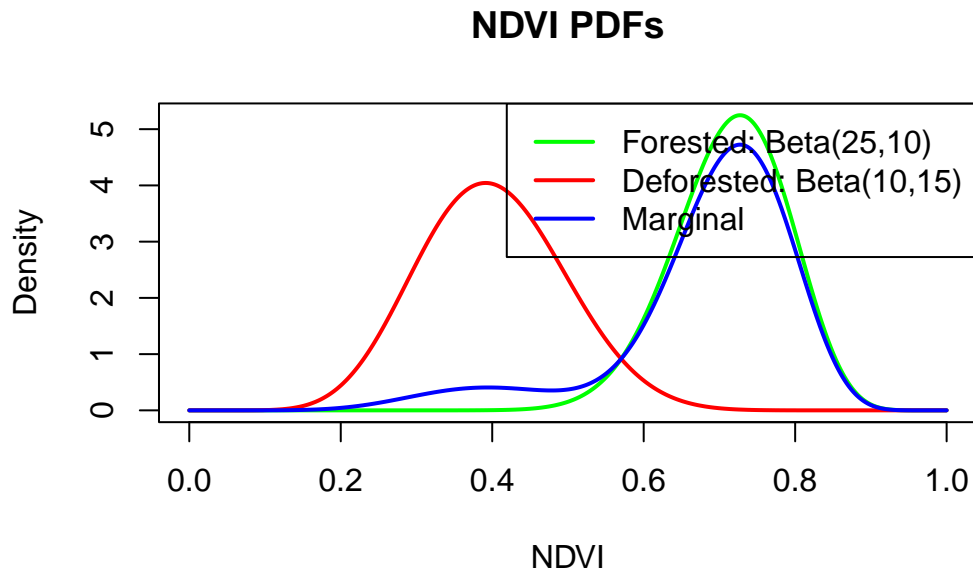
Give an expression for the probability that a pixel is deforested given its NDVI value, and plot this probability by NDVI.

Using Bayes' theorem, the probability that a pixel is deforested given its NDVI value is
p(deforested | x) = 0.1 dbeta(x;10,15) / (0.1 dbeta(x; 10,15) + 0.9 dbeta(x;25,10))

```
# Compute the weighted likelihoods.
p_deforest <- 0.1 * dbeta(ndvi, 10, 15)
p_forest <- 0.9 * dbeta(ndvi, 25, 10)

# Compute the posterior probability for deforestation.
posterior_deforest <- p_deforest / (p_deforest + p_forest)

# Plot the probability as a function of NDVI.
plot(ndvi, posterior_deforest, type = "l", lwd = 2, col = "red",
     xlab = "NDVI", ylab = "P(deforested | NDVI)",
     main = "Posterior Probability of Deforestation")
grid()
```

Posterior Probability of Deforestation

P(deforested | NDVI)
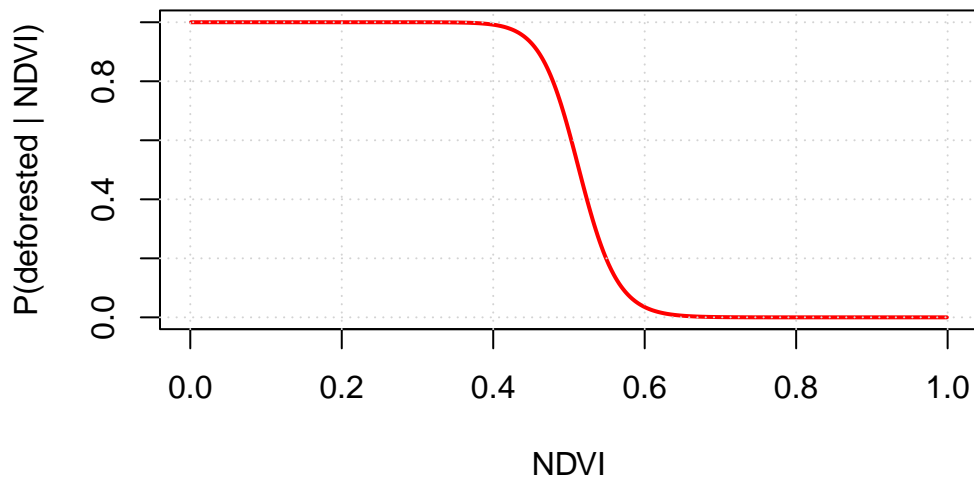
NDVI

You will classify a pixel as deforested if you are at least 90% sure it is deforested. Following this rule, give the range of NDVI that will lead to a pixel being classified as deforested.

classify a pixel as deforested if p(deforested | x) >= 0.9

```
# Identify NDVI values for which the posterior probability >= 0.9.

plot(ndvi, posterior_deforest, type = "l",
     xlab = "NDVI", ylab = "Posterior Probability",
     main = "Posterior Probability of Deforestation")
grid()

classified_deforest <- ndvi[posterior_deforest >= 0.9]

# Report the range.
ndvi_range <- range(classified_deforest)
print(ndvi_range)   # Prints the min and max NDVI values meeting the criterion.
```

```
[1] NA NA
```

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```
# Mark these on the plot:
abline(h = 0.9, lty = 2, col = "gray")
points(ndvi_range, c(0.9, 0.9), pch = 19, col = "blue")
text(ndvi_range, c(0.91, 0.91), labels = round(ndvi_range, 3), pos = 3, col = "blue")
```

Posterior Probability of Deforestation