Homework 3

Question 4

ANOVA Model:

$$y_{ij} = \mu + \alpha_i + e_{ij} \qquad i = 1,2 \quad j = 1,2 \dots 5$$

$$e_{ij} \overset{iid}{\sim} N(0, \sigma^2)$$

B Best
L Linear
U Unbiased
E Estimator

μ is the overall mean

α_i is the i^{th} element

e_{ij} are the error terms

$$E[y_{ij}] = \mu + \alpha_1, \quad j = 1 \dots 5$$

The first estimator for $\mu + \alpha_1$ is sample mean of group 1

$$\bar{y}_{1\cdot} = \frac{1}{5} \sum_{j=1}^{5} y_{1j}.$$

Since y_{1j} are independent and normally distributed. Compute mean and variance of $\bar{y}_{1\cdot}$.

$$E[\bar{y}_{1\cdot}] = \frac{1}{5} \sum_{j=1}^{5} E(y_{1j})$$

$$= \frac{1}{5} \times 5(\mu + \alpha_1) = \mu + \alpha_1.$$

Variance: $$Var(\bar{y}_{1\cdot}) = \frac{1}{5^2} \sum_{j=1}^{5} Var(y_{1j})$$

$$= \frac{1}{25} \times 5\sigma^2 = \sigma^2/5$$

$$\bar{y}_{1\cdot} \sim N\left(\mu + \alpha_1, \frac{\sigma^2}{5}\right)$$